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Truss Topology Optimization Based on a Birth/Death Element Approach

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ABSTRACT In this paper, an alternative approach for topology optimization of truss-type structures is presented. The structural design is translated to a constrained discrete optimization problem based on weight reduction, with performance considerations included in its objective function. The constraints are related to the maximum global stress and to the individual strain energy density. Each member has a standard profile and its absence/presence in the structural representation is implemented with a binary coding; in order to avoid singularities while in the structural analysis, a scheme for simulating the effects of missing members by means of the birth/death element technique is applied. An energy-based approach is employed to detect those elements that not contribute to the overall stiffness. The optimization problem is solved by applying a Modified Binary Differential Evolution algorithm, and the graph theory is applied in parallel with the optimization process to discard unfeasible structures and reduce execution time. The structural performance is evaluated by an execution-time static analysis based on the finite-element method, considering the behavior in a 3-D environment and using commercial software to reduce the uncertainty in this step. The presented proposal is implemented in ANSYS APDL, using as case studies two different structures with a specific load case. The obtained results show a volume reduction of more than 40% off the initial structures, indicating that the proposed approach can be a high-quality tool for structural design in real engineering problems.

INDEX TERMS Truss design, FEM, topology optimization, birth/death element, metaheuristic.

I. INTRODUCTION

Trusses are 2D or 3D structures composed by lineal members connected to joints for supporting loads with compression or stress [1]. They are used in a wide variety of engineering applications and many structures in the real world can be modeled with them, such as roofs, bridges, electric lines towers, among others [2]. These structural systems employ trusses because of their cheapness and simple processes of manufacture, transport, and storage. Normally, they include several elements characterized by diverse parameters: length, cross-section area, shape, material, etc.

Structural optimization is the design and development of structures to get the maximum profitability of the available resources [3], considering the material limitations, environmental impact, and economic competition that generate requirements of weight, cost, and reliability [4]. A general difference in the optimization of truss design with respect to other types of structures is the application of the Finite Element Method (FEM) for the structural analysis. Generally, the number of state variables in truss problems is limited; so, it is not required an advanced FEM analysis. The structural optimization for trusses is important because it can be used to simplify the development of new concepts or preliminary models, for their further analysis and detailed design. From the optimization point of view, the difficulty of truss design problems increases when more complex structures are addressed, adding design variables or constraints. Usually, the weight of the structure is considered as the base for structural optimization [5], with constraints such as displacement, deflection, profitability, stress, critical buckling load, and natural frequency varying in dependency to the specific optimization problem.

The optimal design of structures can be determined as a combination of up to four optimization problems, corresponding to issues of material, dimension, topology, and geometry. The material optimization is focused on the properties of the manufacturing materials, while in dimensional optimization the cross-section areas are optimized as is also optimized the thickness of the continuous structures, to obtain the minimum total weight [6]. The topological optimization is related to the connectivity or to the problem of absence/presence of elements. Finally, the geometry optimization determines the optimum nodal coordinates, assuming a fixed topology [7]. The optimization of geometry and topology has received less attention, although them can considerably improve the design process [25].

The first work on topological optimization was presented by Michell *et al.* [8] more than a century ago, determining the solutions for lighter trusses and introducing the general theory of this field. A number of works were developed in the subsequent decades, mainly in the continuous-problem area. The works on topological truss optimization with discrete variables are less frequent since the application of mathematical methods to these problems is a complex task [9].

Classic methods have been applied for the structural optimization, such as sensibility analysis or approximation concepts, which provide efficient solutions to solve truss optimization problems; however, as stated before they present deficiencies when addressing big and complex problems [17], [18], and specially they are inefficient for representing the connectivity between elements. The non-gradient and metaheuristic methods such as evolutive algorithms have demonstrated their ability to search for the global optimum in these problems [3]. The best-known approaches for defining search spaces are based on the use of ground structures [12], which are formed by all possible connections between nodes and are controlled by the length of the element. In this method, the elimination/inclusion of members is developed sequentially until achieving the design objective [13], [14]. Truss Topology Optimization (TTO) faces two main drawbacks: the possibility of being trapped in a local optimum grows due to the high number of members in the structure; and it can lead to isolated or singular solutions since the stress constraints are not applied over the eliminated members [15].

Metaheuristics have been successfully implemented as an alternative to mathematical methods, for solving structural problems of optimum design with complex constraints and non-convex objective functions, with continuous, discrete and mixed variables [19], [20]. The element-placement problem in rigid-node trusses is a discrete case; the number of possible variants depends on the number of nodes and is quite considerable even in the case of small-scale structures. A natural strategy for TTO is the use of Genetic Algorithm (GA), adapting the solution to the constraints and objective function [16]. Algorithms such as Differential Evolution (DE) [22], Particle Swarm Optimization (PSO) [23], Ant Colony Optimization (ACO) [24], among others, have produced quality results in the solution of structural problems.

Mashayekhi *et al.* [21] used a two-step method for topology optimization based on two-layer mesh reliability, applying the Mobile Asintota Method (MAM) and ACO. The results of MAM are used to enhance ACO through four modifications based on the importance rate of the topology, finding the stress types in elements, changing the threshold of cross-sections and modifying the process to generate random structures. Deb and Gulati [10] proposed a method for TTO with a modified genetic algorithm to solve the weight minimization problem by determining the cross-section areas of the members subject to geometrical, stability, stress, displacement and size constraints. In [38], a Single-Loop Deterministic Method (SLDM) was developed by approximating deterministic constraints and using mix variables. In the Reliability-Based Design Optimization (RBDO), problems can be transformed into approximate deterministic optimization cases, and be solved with a combination of SLDM and an improved ED. In [39], the size, shape and topology of spatial and planar trusses are optimized by the Fully Stressed DesignâĂ"Grey WolfâĂ"Adaptive Differential Evolution (FSD-GWADE) algorithm, considering the stiffness matrix properties for stability checking. A penalized objective function is employed to handle the displacement and stress constraints.

Shakya *et al.* [40] used a ground-based representation to detect unwanted elements in trusses, with an element-removal algorithm to translate the representation code to its corresponding geometrical form and mapping it in kinematic stable solutions. A multi-population PSO is employed to solve the truss topology optimization problem. In [41], the strain energy of trusses is minimized with a limited amount of nodes by applying a simple heuristic based on the alternative direction method of multipliers. An overlapping-member treatment is implemented to supply a chain (a set of members) replacing the corresponding nodes by a simple longer member. Cui *et al.* [42] developed a two-level approximation method for truss topology optimization, with local member buckling constraints on member intersections and overlaps, considering this phenomena in the objective function. The mix variable optimization problem is solved by GA, and the structure is replaced by explicit approximation functions in the execution process.

Most of the work on optimum structural design is related to the optimization of cross-sections solving a standard problem, based on weight and strain energy and subject to response constraints. The topology optimization is obtained by discarding members whose cross sections are lower than a minimum. Elements cannot be eliminated from the structure if they are subject to buckling constraints [11]. The structural analysis is carried out by an approximate function or by assembling the global stiffness matrix given the element matrix. Other disadvantage is the necessity to apply special treatments for avoiding overlapped members and unstable element cases; in large-scale problems, the members are grouped to assign the same cross-sections reducing the number of design variables. Some methods improve the algorithm convergence by only considering the cross-section area, without taking into account the transversal shape (square, circular, etc.); in other approaches, the FEM results are approximated or require more complex considerations for overlapping elements or unstable cases.

This work presents a truss-design topological optimization based on an alternative approach, whose objective is to produce the lightest structure. It considers the present elements and evaluates the structural performance by energetic media, fulfilling the response requirements for a load case. Geometry criteria based on graph theory is applied for analyzing the load paths by means of the adjacency matrix, reducing the computing time by discarding non-feasible configurations. The Birth/Dead Element technique is employed for the representation of necessary/unnecessary elements and their conditions in the FEM model. Unnecessary elements are identified by their Strain Energy Density *SED* response, that is lower than a established threshold. The proposed methodology was programmed in FORTRAN, while the performance analysis was obtained by a static subroutine based on the Finite Element Method; both procedures were implemented in the Programming Design Language environment of ANSYS®. The design problem is translated to an optimization case that in turn is solved by the Modified Binary Differential Evolution algorithm. Two well-known case studies were used to verify the performance of the proposal, analyzing loaded structures and obtaining their minimum weight while still supporting the established loads. The results show a weight reduction of 42.78% and 91.81% off the original weight, indicating that this approach can be used in the design of 2D and 3D structures for real engineering problems, such as the manufacturing of large-dimension passenger-transport vehicles (buses, trains, ships, etc.)

This work is organized as follows: Section 2 describes the proposed approach, detailing both the problem conversion of the design problem to a topological optimization case and the applied solution method. The ANSYS® implementation of the proposed approach is presented in Section 3, while Section 4 shows the cases studies and the analysis of the corresponding results. Finally, Section 5 includes the conclusions and future work.

II. PROPOSED APPROACH FOR STRUCTURAL TOPOLOGICAL OPTIMIZATION

The main objective of this approach is the weight reduction of a reticular structure by eliminating redundant elements, but still satisfying the original operational constraints. The initial structure is defined as a discrete media, knowing the number of elements n and the connections between them.

There are different options to represent design variables in structural optimization: binary string coding, pre-shape generation, and graph translation [26]. The first two methods are commonly used in topological optimization, converting each element in a variable and then modifying its state in the structure. The binary string coding was implemented in this proposal, expressing as 1/0 if an element is active/inactive. Thus, the design variable vector corresponds to expressions (1) and (2).

$$\vec{x} = x_1, x_2, \ldots\ldots, x_n \tag{1}$$

$$x_i \in (0, 1) \tag{2}$$

It is possible to focus the problem on the elements, taking into account that their characteristics are similar and the total volume V_T of the truss depends on them, as is shown in Eq. (3). The gravity force and the material density are considered as constant, and the design problem is of a topological character (geometry). Therefore, the weight directly depends on the Number of Active Elements *NEA* in the structure, as expressed in Eq. 4:

$$V_T = \sum_{i=1}^{n} v_i \cdot x_i \tag{3}$$

$$NEA = N(\vec{x}) = \sum_{i=1}^{n} x_i; \quad \forall\, x_i = 1 \tag{4}$$

If the structures have similar characteristics and predefined lengths, it is difficult to differentiate them by a simple element count; for this reason, the Strain Energy Density *SED* is applied as a structural response measure [27], making it possible to quantify the strain energy (flexibility) per area/volume of material to determine which members are necessary. Two conditions must be met:

1) The structure has to be fully stressed with an equitable distribution, that is, the present elements must have a similar response.
2) The strain energy must be decreased for a stiffness increment.

For the first condition, it is proposed that the difference between the most stressed element (SED_{max}) and the less stressed member (SED_{min}) must be decreased to search a workload equilibrium between all present elements; the aforementioned condition is shown in expression (5). The second condition is valid when the modifications are applied over an initial structure; however, when the optimization starts from a set of possible structural configurations, the stiffness in the worst case tends to zero; for this proposal, the SED_{max} must be decreased to keep the stiffness according to expression (6). The objective function of the optimization problem is presented in expression (7); the parameters α_1, α_2, α_3 are normalization values, since the terms in the linear combination have different ranges of magnitude, depending on the specific load case.

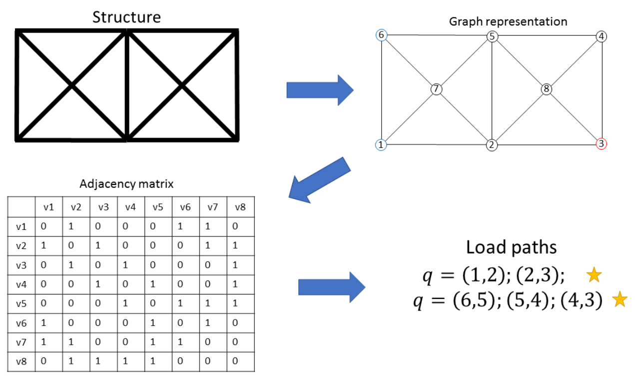
$$min\ U(\vec{x}) = SED_{max}(\vec{x}) - SED_{min}(\vec{x}) \tag{5}$$

$$min\ K(\vec{x}) = SED_{max}(\vec{x}) \tag{6}$$

$$min\, f(\vec{x}) = \alpha_1 \cdot N(\vec{x}) + \alpha_2 \cdot U(\vec{x}) + \alpha_3 \cdot K(\vec{x}) \tag{7}$$

A. DESIGN CONSIDERATIONS

Truss design is subject to a set of requirements to ensure that the designed structure supports the required load. The structures must meet basic criteria such as geometry, maximum effort, and displacement, as well as additional criteria of manufacturing, buckling and frequency response, derived from the design problem and the operation ranges. This proposal only considers the basic issues; the first group (geometric character) makes it possible to determine if the structure will be analyzed by FEM, reducing the computation time. By checking the topology of the structure it can be determined if there are connections between the base nodes (load and support nodes); in other words, it is verified that the structure is able to transmit the applied force through a set of elements to the support nodes. One way to corroborate this condition is by graph theory [28], as seen in Fig. 1. Variables a, b, and c store a predefined value that is used to indicate if the constraints are not fulfilled.



	v1	v2	v3	v4	v5	v6	v7	v8
v1	0	1	0	0	0	1	1	0
v2	1	0	1	0	0	0	1	1
v3	0	1	0	1	0	0	0	1
v4	0	0	1	0	1	0	0	1
v5	0	0	0	1	0	1	1	1
v6	1	0	0	0	1	0	1	0
v7	1	1	0	0	1	1	0	0
v8	0	1	1	1	1	0	0	0

FIGURE 1. Load path checking by graphs.

It is verified that there is at least one continuous connection between the loading points and the fixed support points, taking the nodes as vertices and the members as edges. The adjacency matrix is use to verify the load paths, establishing initial points (load nodes) and arrival points (support nodes). If there is at least one valid connection, then $q = 1$ and the structure is candidate to be analyzed; on the other hand, $q = 0$ and the structure is penalized with a high value in its variable a, as shown in expression (8), preventing the structural analysis to be performed on an inviable structure.

$$a = \begin{cases} 1\text{E}12 & if\ q = 0 \\ 0 & if\ q = 1 \end{cases} \tag{8}$$

Next, it is considered the maximum stress σ_{max}, commonly used to measure the structural performance, establishing that the maximum stress in a structure must not exceed the admissible stress σ_{ad} [29], as indicated in equation (9).

$$\sigma_{max} < \sigma_{ad} \tag{9}$$

The allowable stress is a relation between the selected material and a safety factor *fs* established by the designer, as in Eq. (10). For this case, the global maximum is considered, since the stress per element is less than or equal to this value. If the maximum stress is greater than the design stress (allowable), the variable *b* stores the difference between them, making it possible to verify how deficient a geometry is, as indicated in expression (11).

$$\sigma_{ad} = \sigma_u / fs \tag{10}$$

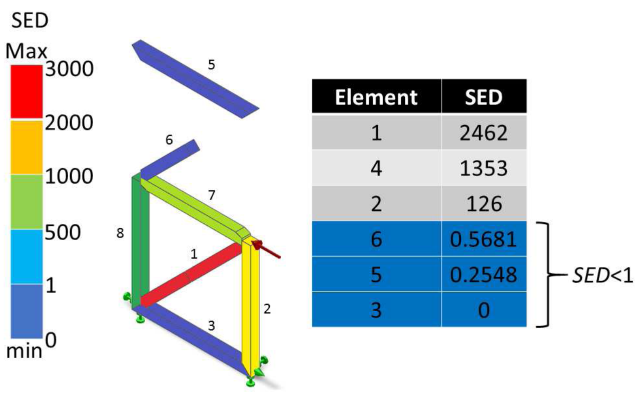
$$b = \begin{cases} \sigma_{max} - \sigma_{ad} & if\ \sigma_{max} > \sigma_{ad} \\ 0 & if\ \sigma_{max} \leq \sigma_{ad} \end{cases} \tag{11}$$

By means of the *SED* is possible to identify the active elements in the structure ($x_i = 1$) that do not contribute to the stiffness (discardable members); for those elements, their *SED* is in the range [0,1], since the element is a direct connection between support nodes. It also applies for elements isolated from the main geometry and those that are not connected to other(s), as shown in Fig. 2. This process is made by sorting the SED_i values in descending order. It is proposed that when this performance is detected, these elements are discarded from the total SED value SED_T of the structure, accordingly to expressions (12) to (14).

$$SED_T(\vec{x}) = \sum_{i=1}^{n} SED_i \cdot x_i; \quad i = 1, 2, \ldots, n \tag{12}$$

$$h = SED_T(\vec{x}) - SED_i \tag{13}$$

$$c = \begin{cases} 0 & if\ SED_i > 1; \\ h & if\ SED_i < 1; \end{cases} \tag{14}$$



Element	SED
1	2462
4	1353
2	126
6	0.5681
5	0.2548
3	0

FIGURE 2. Necessary/unnecessary element check.

Finally, the nonlinear optimization problem is established in expressions (15) to (19) for the design of a structural system, based on volume minimization and considering stress and energy constraints:

Find

$$\vec{x} = [x_1, x_2, \ldots\ldots, x_n]^T \tag{15}$$

to minimize

$$f(\vec{x}) = \alpha_1 \cdot N(\vec{x}) + \alpha_2 \cdot U(\vec{x}) + \alpha_3 \cdot K(\vec{x}) \tag{16}$$

subject to

$$g_1\left(\overrightarrow{x}\right) = \sigma_{max} - \sigma_{adm} \leq 0 \quad (17)$$

$$g_2\left(\overrightarrow{x}\right) = SED_i - 1 \leq 0 \quad (18)$$

$$h_1\left(\overrightarrow{x}\right) = q - 1 = 0 \quad (19)$$

B. DIFFERENTIAL EVOLUTION ALGORITHM

The algorithm of Differential Evolution (DE), proposed by Storn and Price [30], has proved to be one of the most efficient global search methods to solve continuous optimization problems [31]. DE has been successfully applied on diverse fields, including constrained optimization, multi-objective problems, and the solution of real engineering design cases [32]. The main idea behind DE is the creation of new test candidates as offspring of a base population, trying to generate better solutions; itl iterates through the population and creates the test candidates by vectorial differences, applying operations of mutation and crossover. DE has three control parameters: scaling factor F, population size NP, and crossover factor CR [33].

The algorithm generates an initial population randomly, and the mutation aims to generate variations for displacing the solution vectors toward the correct direction and magnitude; there are different versions of DE taking into account how the mutation is implemented. The simplest form is ED/ran/1/bin, represented in Eq. (20), where F is a value in the range [0,1] that controls the vector difference, with $r_1 \neq r_2 \neq r_3 \neq i$:

$$v_{i,G} = x_{r1,G} + F \cdot \left(x_{r3,G} - x_{r2,G}\right) \quad (20)$$

The crossover or recombination merges the information from the parent and mutant vectors into the descendant. Each element of the child vector is taken from the original or the mutated vectors, and j_{rand} is used to ensure that at least one parameter of such mutation is considered, where CR is a value in the range [0,1], as shown in Fig. 3.

FIGURE 3. Selection of variables between original and mutated vectors.

The selection is carried out by means of a tournament, where both the father and offspring are compared based on their aptitude and the best survives to the next generation, as shown in expression (21).

$$\overrightarrow{x}_{i,G+1} = \begin{cases} \overrightarrow{u_{i,G}} & \text{if } f\left(\overrightarrow{u_{i,G}}\right) < f\left(\overrightarrow{x_{i,G}}\right) \\ \overrightarrow{x_{i,G}} & \text{other case} \end{cases} \quad (21)$$

III. IMPLEMENTATION

A square profile under the UNE-EN-10210-1-2007 standard is assigned to every member in the structure; its characteristics are shown in Table 1.

TABLE 1. Structural profile characteristics.

Characteristics	Profile 1
Cross section area (m^2)	0.121E-2
Torsional inertial moment IXX (m^4)	0.176E-5
Inertial moment area IYY (m^4)	0.118E-5
Inertial moment area IZZ (m^4)	0.118E-5
Thickness TKZ (m)	0.08
Thickness TKY (m)	0.08

Although the topologies are represented in binary form indicating presence/absence, a different consideration for eliminating/adding elements is applied, since the elimination introduces inconsistencies in the static analysis. For this reason, a Birth/Dead Element strategy is applied [34]. Active elements are given the label *ealive*, while the inactive ones are labeled as *edead* and deactivated from the truss, so they are not considered for the structural analysis; however, they are only suspended (standby), as shown in Fig. 4, and can be activated if required. The stiffness of *edead* members is reduced by a factor, so the equations that govern the system can be calculated; for this work was considered the factor proposed by ANSYS (1E-6). Loads associated with *edead* still appear in the load vector although they are set to zero; similarly, mass, tension, damping, specific heat and other effects are set to zero. The mass and energy of the deactivated elements are not included in the quantification of the model, and neither are considered in the FEM analysis.

FIGURE 4. Alive/dead element.

A set of initial structures is generated, and each variable is randomly assigned a value of 1/0, corresponding to the label *ealive*/*edead* (see Fig. 5). Subsequently, each structure is evaluated by a strategy that allows to differentiate between feasible/unfeasible trusses; if the generated structure fails to fulfill the constraints, the value of its Constraint Violation *CV* calculated by expression (22) is greater than zero and consequently, the objective function *OF* is penalized with a high-value *P*, as indicated in expression (23). Otherwise,

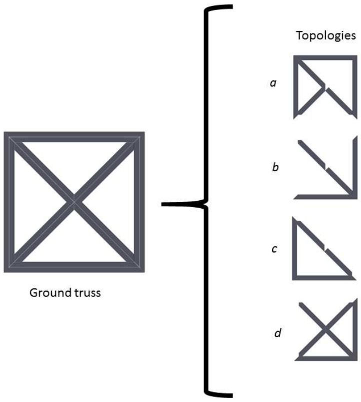


FIGURE 5. Initial population exemplifying for *Np*=4.

if CV=0, OF is calculated by expression (16).

$$CV = \begin{cases} 0 & \text{no constraint violation} \\ a & \text{connection constraint} \\ b+c & \text{stress and SED constraint} \end{cases} \tag{22}$$

$$OF = \begin{cases} f\left(\vec{x}\right) & \text{if } CV = 0 \\ P & \text{if } CV > 0 \end{cases} \tag{23}$$

A. MODIFIED BINARY DIFFERENTIAL EVOLUTION MBDE

MBDE [35] is a version of Differential Evolution used when the design variables are of discrete nature, such as in this implementation where the absence/presence of elements is represented with binary integer values. As mentioned before, the mutated vector is created by randomly choosing three different vectors r_1, r_2 and r_3, and then a modification for binary handling is applied. The algebraic operations are performed based on the behavior of logical gates: *XOR* gate ($\oplus$) is applied for the difference between vectors, while the multiplication is implemented with an *AND* gate ($\otimes$), and the sum uses an *OR* gate ($\odot$). The complete expression is in Eq. (24), where F is dynamically adjusted per generation as indicated in expression (25).

$$v_{i,G} = x_{r3,G} \odot (F \otimes (x_{r1,G} \oplus x_{r2,G})) \tag{24}$$

$$F = round(rand(0,1)) \tag{25}$$

The generation of the child vector is carried out if the conditions of CR and j_{rand} are fulfilled, combining the values of the original and the mutated vector (see Fig. 6). The selection is by a tournament, comparing father and child by means of the rules of Deb [36], that are as follows: 1) between two feasible individuals the one with better aptitude is selected; 2) between a feasible and an unfeasible individual, the feasible is chosen; and, 3) between two unfeasible individuals, the one with the lower constraint violation is selected. The pseudocode of MBED is shown in Algorithm 1.

IV. CASE STUDIES

In order to test the proposed approach, two case studies were considered. In both cases, the problem consists of minimizing the total weight starting from a defined initial structure, to obtain a truss that still fulfills the design constraints. The specific details of the case studies and their corresponding results are presented in the following subsections.

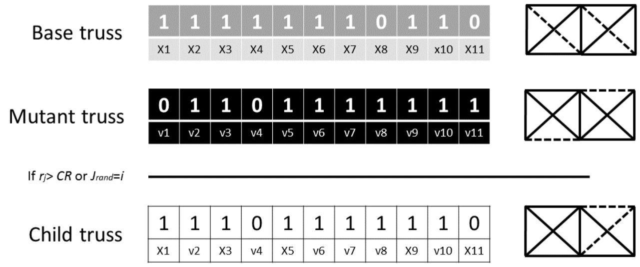


FIGURE 6. Crossover in BDE.

Algorithm 1 MBDE

begin
  Set parameters NP, CR, G_{max};
  Generate randomly a initial population x_{ij}^0; $i = 1, \ldots, NP$;
  Evaluate population individuals $\overrightarrow{x_{i,0}}$ computing its OF and CV;
  repeat
    Define F by Eq. 25;
    for $i = 1$ *a* NP **do**
      select randomly $r1 \neq r2 \neq r3 \neq i$;
      j_{rand} = round(rand(1,D)) (D, number of variables);
      for $j = 1$ *to* D **do**
        create mutated vector v_{ij}^g by Eq. 24;
      end
      for $j = 1$ *a* D **do**
        create child u_{ij}^g by binomial crossover;
        evaluate child computing its OF and CV;
      end
      if u_i^g *is bettre than* x_i^g*(Deb rules)* **then**
        $x_i^g = u_j^i$;
      else
        $x_i^g = x_i^{g-1}$;
      end
    end
    G=G+1;
  until $G = G_{max}$;
end

For this work, the structure was analyzed without considering superimposed elements; so, for each element intersection, a connection node was generated. The selected material is structural steel ASTM A36, with a elasticity modulus $E = 200\ GPa$, Poisson's ratio $v = 0.3$ and tensile ultimate strength $\sigma_u = 400\ MPa$; as a safety factor it was considered $fs = 1.5$. The parameters α_1, α_2 and α_3 are set to 100, 1/1000 and 1/10, respectively. The algorithm was implemented on a computing platform with an Intel Core(TM) i7@5930K 3.5 GHz processor and 32 GB of RAM DDR4, programmed in ANSYS® APDL Release 18.1. Thirty simulations were carried out for each case study, with the following configuration: population Np=100, crossover Cr=0.9, and F dynamically changing per iteration. The maximum number of iterations G_{max} was 100 for the first case and 1000 for the second.

A. CASE STUDY 1 (CS1)

The structure shown in Fig. 7 is formed by fifteen elements; this truss supports two point loads $P_1 = P_2 = 10,000\ N$ and is embedded in its leftmost ends (FS), with L=2 m. The weight of the initial structure is $W = 2,373.34\ kg$, with the profile and material previously mentioned.

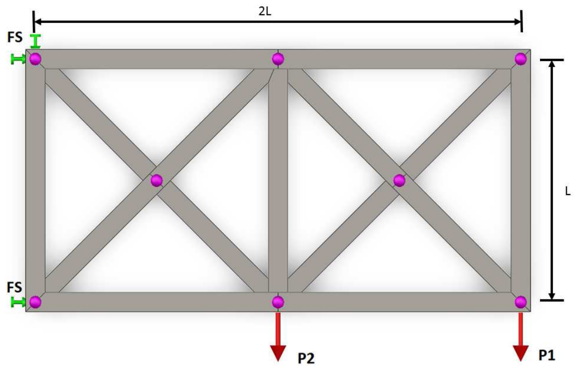


FIGURE 7. Structure for case 1.

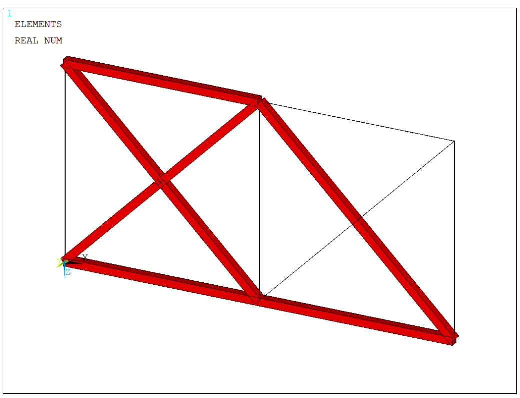


FIGURE 8. Optimum topology for CS1.

Fig. 8 shows the resulting truss, which has a weight of $W = 1.358.16\ kg$, representing a volume reduction of 42.78% off the original; the security factor is $fs = 7.03$, indicating that the structure can support higher loads without failing. Table 2 presents a comparison between the result of method A (see Fig. 9), applied in [37], and the proposed approach labeled as B; as can be seen, the final truss obtained in this work surpasses the reported results in the cited reference.

The corresponding stress of the obtained truss is $\sigma_{max} = 37.9\ MPa$, shown in Fig. 10; this value does not exceed the allowable stress, and its global SED level SED_{max} is 2706.31 J/m^3 with a difference between maximum and minimum SED_{diff} of 2030.3 J/m^3.

Figures 11 and 12 show the convergence of the proposed MBDE implementation for the best value obtained after the

TABLE 2. Results for both optimization processes.

Element	$A_i(mm^2)$	A	$A_i(mm^2)$	B
1	674.893	1	1,216	1
2	499.909	1	1,216	1
3	0.100	0	0	0
4	0.100	0	0	0
5	211.499	1	1,216	1
6	0.100	0	0	0
7	421.531	1	1,216	1
8	459.771	1	1,216	1
9	421.531	1	1,216	1
10	459.771	1	1,216	1
11	211.499	1	0	0
12	706.978	1	1,216	1
13	0.129	0	0	0
14	706.978	1	1,216	1
15	0.129	0	0	0
$V_T\ (mm^3)$	8.91591E6	0	1.76E6	0

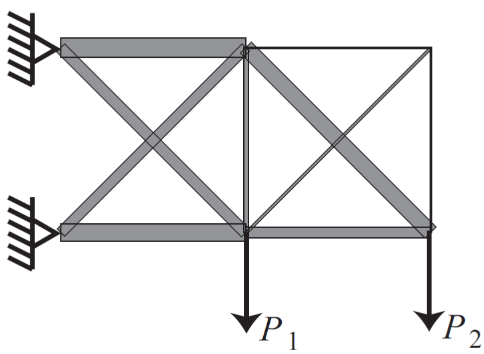


FIGURE 9. Reported optimum topology.

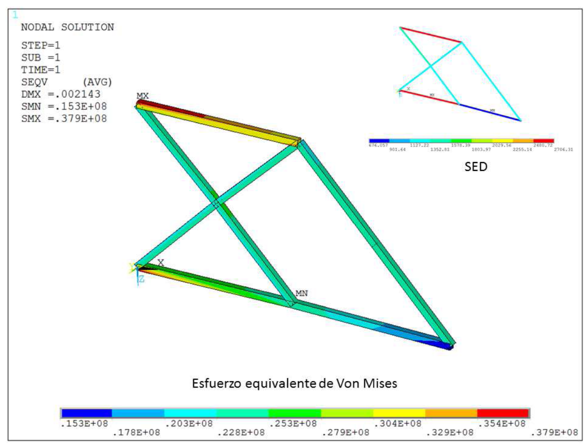


FIGURE 10. Truss stress distribution and SED layout.

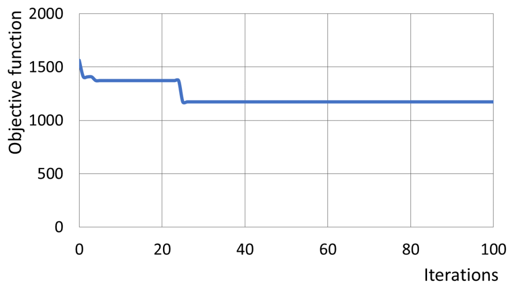


FIGURE 11. Objective-function convergence graph for CS1.

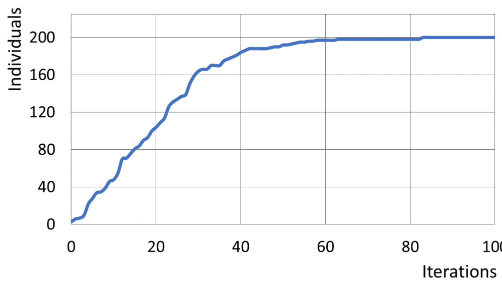


FIGURE 12. Feasible-individuals convergence graph for CS1.

thirty simulations, presenting the objective function and the feasible individuals, respectively. As can be seen in both graphs, the algorithm has a good performance since at the end of the iterations all its individuals are feasible.

B. CASE STUDY 2 (CS2)

The structure shown in Fig. 13 supports a point load $P = 10,000\ N$, and is supported on its left and right lower lateral ends, with $L = 1\ m$ and a weight $W = 3,239.03\ kg$, using the profile and material previously established.

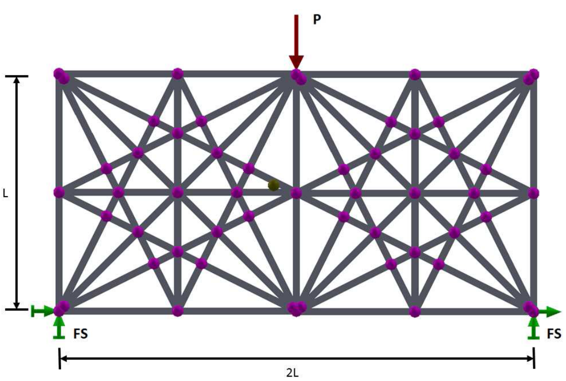


FIGURE 13. Structure for case study 2.

The obtained result is shown in Fig. 14. The total weight is $W = 265.2\ kg$, representing a 91.81% of volume removed from the original structure; in spite of being quite simple, this geometry has a safety factor of $fs = 39.45$, demonstrating that the resulting truss can support a higher load margin.

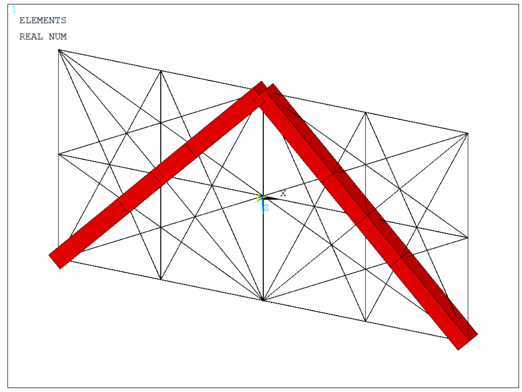


FIGURE 14. Optimal geometry for case study 2.

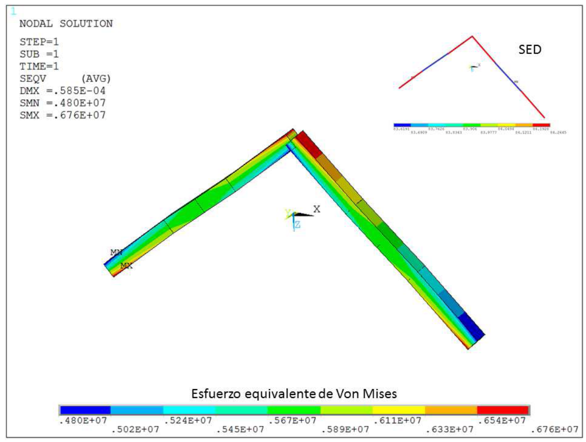


FIGURE 15. Stress distribution and SED layout of load case.

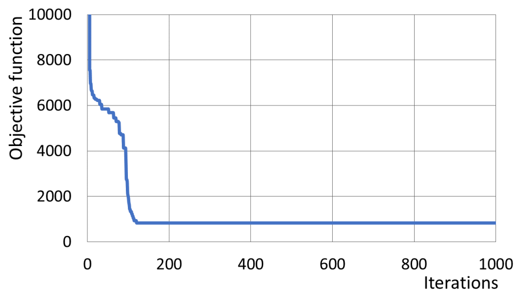


FIGURE 16. Objective-function convergence graph for CS2.

The stress of the resulting truss, shown in Fig. 15, is $\sigma_{max} = 6.76\ MPa$, which does not exceed the allowable stress. The global SED level SED_{max} is 84.27 J/m^3, with a difference between maximum and minimum of 0.65 J/m^3, this means that the elements have a good distribution of energy throughout the geometry, and they are acting synergistically.

Figures 16 and 17 show the convergence of the proposed MBDE for the mesh of the CS2, presenting the objective function and the feasible individuals, respectively. As can be seen in both graphs, the algorithm has a good performance, since at the end of the iterations all its individuals are feasible.

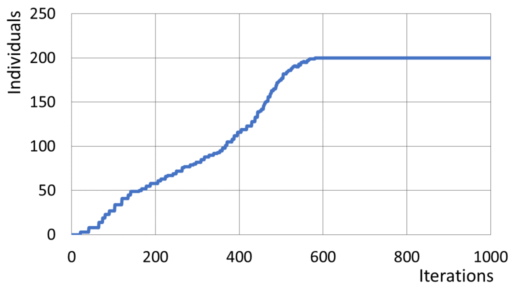


FIGURE 17. Feasible-individuals convergence graph for CS2.

It is remarkable that the behavior is quite similar to the corresponding to the implemented solution for CS1, in spite of CS2 is a much more complex problem because of its higher mesh density, and this complexity is reflected in the generation of feasible solutions.

V. CONCLUSIONS

This paper presents an alternative approach for topological design of truss-type structures, by addressing it as an optimization problem based on weight minimization, that is solved with a metaheuristic algorithm. For this objective, the management of necessary and unnecessary elements uses a scheme considering alive/dead members in order to carry out the structural analysis. The objective function is a linear combination composed by the design goal and a performance metric to evaluate the quality of the generated structures. Simultaneously to the optimization process, a FEM static analysis is applied to the best result already obtained at the end of each cycle of the metaheuristic algorithm; by means of a geometric constraint the computation time is reduced, since any unsuitable structure is not analyzed. The FEM analysis is programmed in a subroutine, so it can be executed whenever it is required.

The metaheuristic applied for solving the optimization problem is the differential evolution algorithm, with a series of modifications to handle the design variables by executing the mutation process using logical gates to create the bit differences in each chromosome of the individual. The obtained results show feasible geometries with weight reductions of more than 40% with respect to the original trusses, fulfilling favorably the structural requirements despite having a lower number of elements, verifying its viability by the FEM analysis.

The use of the Birth/Death Element strategy for identifying members in topological optimization allows to analyze structures that have no instabilities in the stiffness matrix, unlike traditional methods that use a predefined value in some of the element characteristics (rigidity, cross-sectional area). Additionally, the use of the adjacency matrix makes it possible to verify the trajectories along the structure for the transmission of the load. The obtained results indicate that the proposed approach can be used as a high-quality tool in engineering applications for structural optimization tasks.

As a future work, it is considered the analysis of different load cases applied to diverse structures, variating both the material and the type of mesh. It is also contemplated to implement the profile selection from a database, to determine its effect on the final result when the size and topology of the structure are simultaneously optimized.

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