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# Output Feedback Thruster Fault-Tolerant Control for Dynamic Positioning of Vessels Under Input Saturation

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**ABSTRACT** In this paper, an output feedback fault-tolerant control (FTC) scheme is presented for the dynamic positioning of vessels subjected to external disturbances, dynamic uncertainties, and thruster saturation constraints under thruster faults. A high-gain observer is constructed to acquire the estimates of vessel velocities. An auxiliary dynamic system is introduced to compensate the effect of the thruster faults and saturation constraints. Neural network with iterative updating laws is employed to approximate the lumped uncertainties caused by unknown environmental disturbances and dynamic uncertainties of vessels. Furthermore, the control allocation is accomplished to distribute the FTC forces and moment among individual thrusters. It is theoretically proved that the derived FTC scheme has the capability to maintain the vessel position at the desired values and guarantees that all signals in the closed-loop control system are uniformly ultimately bounded. Finally, the simulation results demonstrate the effectiveness of the proposed control scheme.

**INDEX TERMS** Dynamic positioning, thruster faults, input saturation, high-gain observer, fault-tolerant control.

#### I. INTRODUCTION

The dynamic positioning (DP) technique has attracted an increasing interest due to the advantages of high maneuverability, high positioning accuracy and no limitation of water depth. This leads to its wide applications to marine operations, such as cable-laying, dredging, pipe-laying and platform supplying [1]. A number of results on the DP control have been reported adopting neural network (NN) / fuzzy control [2]–[5], sliding mode control [6], model-based observer [7], hybrid control [8], [9] and backstepping [10], [11]. The occurrence of faults were not considered in above control schemes. In practice, due to the harsh marine environment, seawater corrosion and erosion, the ship thrusters, sensors and other devices are prone to aging and damage. According to the International Marine Contractors Association (IMCA), thruster faults caused 21% of the incidents related to DP vessel [12]. To enhance the reliability and safety of DP control system, the DP control scheme should be capable of tolerating potential faults and maintaining desirable performance in the presence of thruster faults. One effective way to handle this problem is by means of fault-tolerant control (FTC) strategies [13]-[15]. Apart from the problem of thruster faults, the saturation characteristic of the thrusters is another challenging issue. Due to the physical constraint, the thruster can not execute arbitrary command. Therefore, if the thruster saturation characteristic is not considered in control design, it may result in degraded performance. At practice, speed logs are widely utilized to measure velocities. Unfortunately, measurements are usually different from the actual values, which are corrupted due to wind, waves and ocean currents as well as sensor noise. Hence in most cases, measurements of the vessel velocities are not available [16]. Therefore, the problems of the thruster fault, input saturation and unmeasurable velocities are obstacles for the control of the dynamic positioning of vessels.

Considering the thruster faults of vessels, a number of FTC strategies with fault detection and diagnosis (FDD) mechanism were addressed in [17]–[20]. To diagnose the thruster

faults, pre-set fault isolation logic [17], observer-based fault detection filter [18], unknown input observer-based detect and isolate scheme [19], observer-based time-varying detection criterion [20] and the fault state observer estimating the unknown effectiveness coefficients [21] were employed. It is clear that the main advantage of FDD mechanism used in [17]–[21] was to identify the nature and location of the fault, which is useful for reconfiguring the controller to ensure reliable operation of systems under thruster faults, but it also increased the system complexity. Beyond that, the unexpected disturbances and the saturation characteristic of the thrusters inevitably deteriorate the efficacy of FDD, which produces the probability of missed detections and degrades the performance of the FTC systems. Therefore, a more effective approach should be pursued for the security reliability of vessel control system. Recently, based on timevarying sliding mode, [22] proposed a finite-time trajectory tracking FTC for surface vessel without the need of FDD scheme.

Considering thruster saturation characteristic, [23]–[25] constrained the desired generalized forces computed from the controller such that the constraints on thruster control command were satisfied. Reference [23] proposed a proportional-integral controller for DP of marine vehicles, in which an anti-windup method was developed by mapping the thruster saturation constraint into a constraint set for the generalized forces. Further, [24] extended the work of [23] by deriving analytical tuning rules which simplified the control system. Reference [25] designed a novel nonlinear set-point-regulation controller for DP of marine craft using a port-Hamiltonian frame work, where an anti-windup compensator was designed to handle the thruster saturation. In [26], the thruster saturation was considered in control allocation, which made each thruster never exceed the amplitude by the inequality constraint. However, the strategy in [26] could not guarantee that the control forces and moment are fully allocated, especially under harsh sea conditions. It should be pointed out that the above results did not consider the possibility of the thruster faults. Therefore, the DP control algorithm in the presence of thruster saturation and thruster faults should be further investigated.

In practice, considering the fact that vessel velocities were unmeasurable, the state-observer, such as nonlinear passive observer [27]–[29], Luenberger-type observer [30] and adaptive NN observer [31], [32], were employed to provide the signal of vessel velocities. Apart from these, the high-gain observer has also been proved effective for estimating the system states [33], [34], which does not need the priori knowledge of the mathematical model of systems. In [35], the high-gain observer was constructed to estimate the velocities of dynamically positioned vessels in the presence of external disturbances and parametric uncertainties. Nevertheless, the problems of thruster faults and saturation constraints were not taken into consideration.

Inspired by the above observations, for the DP control of vessels subject to external disturbances, dynamic uncertainties, thruster saturation constraints and thruster faults, an output feedback FTC scheme with control allocation for DP of vessels is proposed in this paper. The high-gain observer is used to estimate the unmeasurable of vessel velocities, the auxiliary dynamic system is constructed to handle the effect of thruster faults and saturation constraints and NNs are employed to compensate for the unknown environmental disturbances and dynamics parameters. Furthermore, the FTC forces and moment computed from the FTC law are allocated to individual thrusters. In summary, the main contributions of this paper are as follows.

(1) To enhance the system reliability and safety of the DP control system, an auxiliary dynamic system is constructed for compensating the adverse effect caused by thruster faults and input saturation, such that the FDD mechanism is abandoned in our control design.

(2) NNs with online iterative updating mechanism are applied to approximate the lumped unknown nonlinearities derived from environmental disturbances and model dynamic uncertainties, which are easier to implement in digital processor.

(3) An output feedback FTC for DP system is developed based on the high gain observer in the absence of the priori knowledge of the mathematical model of dynamically positioned vessels.

The rest parts of this paper are organized as follows. Section 2 presents the nonlinear motion mathematical model of dynamically positioned vessels. Section 3 proposes the output feedback FTC scheme for DP of vessels subject to external disturbances, dynamic uncertainties, thruster saturation constraints and thruster faults. Section 4 provides simulation studies on a scale model vessel. Section 5 concludes this paper.

#### II. MOTION MATHEMATICAL MODEL OF DYNAMICALLY POSITIONED VESSELS

The kinematic equation (1a) and the dynamic equation (1b) of dynamically positioned vessels can be expressed as

$$\dot{\eta} = R\left(\psi\right)\upsilon\tag{1a}$$

$$M\dot{\upsilon} = -D\upsilon + \tau + d(t) \tag{1b}$$

where  $\eta = [x, y, \psi]^T$  represents the position vector of vessels in the earth-fixed frame and  $\upsilon = [u, v, r]^T$  denotes the velocity vector of vessels in the body-fixed frame.  $\tau = [\tau_1, \tau_2, \tau_3]^T$ is the control input vector produced jointly by the thrusters, consisting of the control forces  $\tau_1$  in surge and  $\tau_2$  in sway, and the moment  $\tau_3$  in yaw.  $d(t) \in R^3$  is the unknown time-varying disturbance acting on the vessel. The rotation matrix  $R(\psi)$  is

$$R(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1 \end{bmatrix}$$
(2)

with the property  $||R(\psi)|| = 1$ .  $||\cdot||$  denotes the 2-norm. *M* is the invertible, symmetric and positive definite inertia matrix. *D* is a linear damp matrix featured by strictly positive.

Assumption 1: (1) The inertia matrix M and the damp matrix D are unknown.

(2) The ship velocity vector  $v = [u, v, r]^T$  is the unmeasurable state.

(3) The environmental disturbance vector  $d(t) = [d_1(t), d_2(t), d_3(t)]^T$  is unknown, bounded and time-varying, i.e.  $|d_i(t)| \le \bar{d}_i$  with  $\bar{d}_i(i = 1, 2, 3)$  being a positive constant.

*Remark 1:* The inertia matrix M and the damp matrix D are concerned with the vessel operating conditions and characteristics, which are very hard to identify accurately. In addition, it is not easy to measure the vessel velocities due to the limitations of physical sensors. Furthermore, since the external environmental disturbances are constantly changing and have finite energy, the environmental disturbances are unknown, bounded and time-varying. It is obvious that Assumption 1 is reasonable [2], [35].

For a dynamically positioned vessel equipped with m azimuth thrusters, the control input vector  $\tau$  produced jointly by the thrusters is expressed by

$$\tau = A\left(\alpha\right)u_c\tag{3}$$

where  $u_c = [u_{c_1}, \dots, u_{c_m}]^T \in R^m$  with  $u_{c_i}$  being the designed control command of the *i*th thruster.  $A(\alpha) \in R^{3 \times m}$  is the configuration matrix whose *i*th column has the general form.

$$A(\alpha_i) = \begin{bmatrix} \cos(\alpha_i) \\ \sin(\alpha_i) \\ l_{x_i} \sin(\alpha_i) - l_{y_i} \cos(\alpha_i) \end{bmatrix}$$
(4)

with  $(l_{x_i}, l_{y_i})$  being the location of the *i*th thruster in the body-fixed frame.  $\alpha_i$  denotes the orientation angle of the *i*th thruster.  $\alpha_i \in [0, 180^\circ]$  denotes that the orientation angle is clockwise from the forward direction of the vessel, and  $\alpha_i \in [-180^\circ, 0]$  denotes that the orientation angle is anticlockwise from the forward direction of the vessel.

Due to the thruster saturation characteristic, the actual control command  $u_{p_i}$  of the *i*th thruster, can be described as follows

$$u_{p_{i}} = \operatorname{sat}(u_{c_{i}}) = \begin{cases} u_{p_{i}\max} & \text{if } u_{c_{i}} > u_{p_{i}\max} \\ u_{c_{i}} & \text{if } u_{p_{i}\min} \le u_{c_{i}} \le u_{p_{i}\max} \\ u_{p_{i}\min} & \text{if } u_{c_{i}} < u_{p_{i}\min} \end{cases} \quad i = 1, 2, 3 \quad (5)$$

where  $u_{p_i \max} > 0$  and  $u_{p_i \min} < 0$  are the known lower bound and upper bound of the *i*th thruster saturation constraint, respectively.

On the other hand, the thrusters inevitably undergo faults due to the long time operation in the complex ocean environment, which may deteriorate the positioning performance and even threaten the stability of the DP control system. In this paper, we consider the situation that there exist the thruster lose-of-effectiveness faults. The thruster effectiveness factors are introduced to express the thruster fault states. Suppose the effectiveness factor matrix  $F(t) = \text{diag}[\sigma_1(t), \dots, \sigma_m(t)]$ with  $\sigma_i(t) \in (0, 1]$ , which is unknown due to the unpredictability of the thruster faults.  $0 < \sigma_i(t) < 1$  indicates that the *i*th thruster is partial loss-of-effectiveness.  $\sigma_i(t) = 1$  indicates that the *i*th thruster is in the fault-free case [21].

The actual control input vector  $\tau^F$  produced by the thrusters is represented as

$$\tau^F = A(\alpha) F(t) u_p \tag{6}$$

Based on (6), the dynamic equation (1b) of dynamically positioned vessels under thruster faults is expressed by

$$\dot{\upsilon} = -M^{-1}D\upsilon + M^{-1}A(\alpha)F(t)u_p + M^{-1}d(t)$$
 (7)

Here, let  $J(t) = I_{m \times m} - F(t)$ ,  $J(t) = \text{diag}(j_1(t), \dots, j_m(t))$ and  $j_i(t) = 1 - \sigma_i(t)$ . Then, we obtain

$$\dot{\upsilon} = -M^{-1}D\upsilon + M^{-1}A(\alpha) u_p - M^{-1}A(\alpha) J(t)u_p + M^{-1}d(t)$$
(8)

The control objective of this paper is to develop an FTC scheme for DP of vessels in the presence of the unknown thruster faults and saturation constraints under Assumption 1, such that the vessel can arrive and be kept at the desired value  $\eta_d = [x_d, y_d, \psi_d]^T$  and all signals in the closed-loop control system are uniformly ultimately bounded (UUB).

#### **III. OUTPUT FEEDBACK FTC FOR DP OF VESSELS**

In this section, an output feedback FTC scheme with control allocation is proposed for dynamically positioned vessels in the presence of the unknown thruster faults and saturation constraints under Assumption 1.

#### A. HIGH-GAIN OBSERVER DESIGN FOR DP OF VESSELS

Because it is not easy to measure the velocities of vessels, in this subsection the high-gain observer is constructed to estimate the vessel velocities according to Lemma 1.

Lemma1 [36]: Consider the following linear system

$$\begin{cases} \varepsilon \dot{\chi}_i = \chi_{i+1} \ i = 1, \cdots, n-1 \\ \varepsilon \dot{\chi}_n = -\gamma_1 \chi_n - \cdots - \gamma_{n-1} \chi_2 - \chi_1 + y \end{cases}$$
(9)

where  $\varepsilon$  is a small positive constant, the parameters  $\gamma_1$  to  $\gamma_{n-1}$  are chosen to satisfy the condition that the polynomial  $s^n + \gamma_1 s^{n-1} + \cdots + \gamma_{n-1} s + 1$  is Hurwitz. If the system output y and its first n - 1 derivatives are bounded, i.e.,  $||y^{(k)}|| < Y_k$   $(k = 1, \cdots, n-1)$  with positive constants  $Y_k (k = 1, \cdots, n-1)$ , then the following properties holds:

n-1), then the following properties holds: (1)  $\frac{\chi_{k+1}}{\varepsilon^k} - y^{(k)} = -\varepsilon \phi^{(k+1)}$   $(k = 1, \dots, n-1)$  where  $\phi = \chi_n + \gamma_1 \chi_{n-1} + \dots + \gamma_{n-1} \chi_1$  with  $\phi^{(k)}$  denoting the *k*th derivative of  $\phi$ .

(2) There exist positive constants  $t^*$  and  $h_{k+1}$  only depending on  $Y_k(k = 1, 2, \dots, n-1)$ ,  $\varepsilon$  and  $\gamma_i(i = 1, \dots, n-1)$ , such that we have  $|\phi^{(k+1)}| \le h_{k+1}$ , for all  $t > t^*$ .

According to the boundedness of the position vector  $\eta$  and its first derivative  $\dot{\eta}$ , we can construct the high-gain observer for the DP system of vessels as follows

$$\hat{\hat{\eta}} = \frac{\chi_2}{\varepsilon} \tag{10}$$

In the light of (1b) and (10), the velocity vector estimate  $\hat{\upsilon}$  of the vessel can be denoted as

$$\hat{\upsilon} = R^{-1}(\psi) \frac{\chi_2}{\varepsilon} \tag{11}$$

where  $\chi_2 \in R^3$  is the state vector from the following linear system

$$\begin{cases} \varepsilon \dot{\chi}_1 = \chi_2 \\ \varepsilon \dot{\chi}_2 = -\gamma_1 \chi_2 - \chi_1 + \eta \end{cases}$$
(12)

According to (1a), (11) and the property (1) of Lemma 1, the velocity estimation error vector  $\tilde{v} = \hat{v} - v$  is obtained

$$\tilde{\upsilon} = R^{-1}(\psi)(\frac{1}{\varepsilon}\chi_2 - \dot{\eta}) = -\varepsilon R^{-1}(\psi)\ddot{\phi}$$
(13)

In the light of (13), ||R|| = 1 and the property (2) of Lemma 1, we have

$$\|\tilde{\upsilon}\| = \varepsilon \|\ddot{\phi}\| \le \varepsilon h_2 \tag{14}$$

where  $h_2$  is a positive constant.

It is obviously known from (14) and the property (2) of Lemma 1 that the velocity estimation error vector  $\tilde{v}$  is bounded.

#### **B. FTC DESIGN FOR DP OF VESSELS**

In this subsection, the FTC law is designed by combining the high-gain observer, the auxiliary dynamic system and NNs with backstepping technique. The control design procedures consist of the following two steps.

Step 1: The position error vector  $Z_1 \in \mathbb{R}^3$  is defined as

$$Z_1 = \eta - \eta_d \tag{15}$$

From (1a), the time derivative of (15) yields

$$\dot{Z}_1 = R(\psi)\upsilon \tag{16}$$

Design a virtual control law  $\varphi \in R^3$  as follows

$$\varphi = -R^{-1}(\psi)K_1Z_1 \tag{17}$$

where  $K_1 = K_1^T \in \mathbb{R}^{3 \times 3}$  is a positive definite design matrix. Step 2: Define the following vector  $Z_2 \in \mathbb{R}^3$ 

$$Z_2 = \hat{\upsilon} - \varphi \tag{18}$$

The time derivative of (18) is given as

$$\dot{Z}_2 = \dot{\hat{\upsilon}} - \dot{\varphi} \tag{19}$$

To handle the thruster saturation constraints and thruster faults, the auxiliary dynamic system is constructed as follows

$$\dot{\Upsilon}_{a} = -K_{a}\Upsilon_{a} - \frac{\left|Z_{2}^{T}A(\alpha)\Delta u\right| + \left|Z_{2}^{T}A(\alpha)u_{p}\right|}{\left\|\Upsilon_{a}\right\|^{2}}\Upsilon_{a}h\left(\left\|\Upsilon_{a}\right\|\right) \\ - \frac{0.5|A(\alpha)\Delta u|^{2} + 0.5|A(\alpha)u_{p}|^{2}}{\left\|\Upsilon_{a}\right\|^{2}}\Upsilon_{a}h\left(\left\|\Upsilon_{a}\right\|\right) \\ + 0.5A(\alpha)\Delta u + 0.5A(\alpha)u_{p}$$
(20)

where  $\Upsilon_a \in \mathbb{R}^3$  is the state vector of the auxiliary dynamic system,  $\Delta u = u_p - u_c$ ,  $K_a = K_a^T \in \mathbb{R}^{3 \times 3} > 0$  is a positive definite design matrix. The function  $h(||\Upsilon_a||)$  is given by

$$h\left(\|\Upsilon_{a}\|\right) = \begin{cases} 0, & \|\Upsilon_{a}\| \leq \Upsilon_{1} \\ 1 - \cos\left(\frac{\pi}{2}\sin\left(\frac{\pi}{2}\frac{\|\Upsilon_{a}\|^{2} - \Upsilon_{1}^{2}}{\Upsilon_{2}^{2} - \Upsilon_{1}^{2}}\right)\right), & otherwise \\ 1, & \|\Upsilon_{a}\| \geq \Upsilon_{2} \end{cases}$$

$$(21)$$

where  $\Upsilon_1$  and  $\Upsilon_2$  are arbitrarily small constants and  $0<\Upsilon_1<\Upsilon_2.$ 

The design FTC law  $\tau_c$  for dynamically positioned vessels is chosen as

$$\tau_c = A(\alpha)u_c$$
  
=  $-K_2Z_2 - R(\psi)Z_1 + \Upsilon_a + \hat{W}^T(t)\Phi(\hat{X})$  (22)

where  $K_2^T = K_2 \in \mathbb{R}^{3 \times 3}$  is a positive definite design matrix, and  $\hat{X} = [\eta^T, \hat{v}^T]^T \in \mathbb{R}^6$ .  $\hat{W}(t)$  is designed as

$$\hat{W}(t) = L\hat{W}(t-T) - \Gamma\Phi(\hat{X})Z_2^T$$
(23)

Here, T > 0 is the updating interval,  $L \in R^{3q \times 3q}$  is a positive definite diagonal matrix satisfying  $0 < L^T L < lI_{3q \times 3q}$  with 0 < l < 1 and  $I_{3q \times 3q}$  being a 3q-dimensional identity matrix, and  $\Gamma > 0$  is a design constant.  $\Phi(\hat{X}) = \left[\Phi_1^T(\hat{X}), \Phi_2^T(\hat{X}), \Phi_3^T(\hat{X})\right]^T \in R^{3q}$  with  $\Phi_i(\hat{X}) = \left[\Phi_{i_1}(\hat{X}), \cdots, \Phi_{i_q}(\hat{X})\right]^T \in R^q$  (i = 1, 2, 3) is the basis function vector. For all  $\hat{X} \in R^6$ ,  $\Phi_i(\hat{X})$  is bounded by a constant  $\Phi_M > 0$ , satisfying  $\left\|\Phi_i(\hat{X})\right\| \leq \Phi_M$ .

Consider the Lyapunov function candidate  $V_1$ 

$$V_1 = \frac{1}{2} Z_1^T Z_1 + \frac{1}{2} Z_2^T M Z_2 + \frac{1}{2} \Upsilon_a^T \Upsilon_a$$
(24)

The time derivative of  $V_1$  is

$$\dot{V}_1 = Z_1^T \dot{Z}_1 + Z_2^T M \dot{Z}_2 + \Upsilon_a^T \dot{\Upsilon}_a$$
(25)

Considering (16), (17), (18) and  $\tilde{\upsilon} = \hat{\upsilon} - \upsilon$ , we obtain

$$Z_{1}^{T}\dot{Z}_{1} = Z_{1}^{T}R(\psi)(Z_{2} + \varphi - \tilde{\upsilon})$$
  
=  $-Z_{1}^{T}K_{1}Z_{1} + Z_{1}^{T}R(\psi)Z_{2} - Z_{1}^{T}R(\psi)\tilde{\upsilon}$  (26)

In the light of (19) and (8), we have

$$Z_{2}^{T}M\dot{Z}_{2}$$

$$= Z_{2}^{T}\left(M\dot{\tilde{\upsilon}} + M\dot{\upsilon} - M\dot{\varphi}\right)$$

$$= Z_{2}^{T}\left[-D\upsilon + A(\alpha)u_{p} - A(\alpha)G(t)u_{p} + d(t) + M\dot{\tilde{\upsilon}} - M\dot{\varphi}\right]$$

$$\leq Z_{2}^{T}\left[A(\alpha)u_{c} - f(X)\right] + \left|Z_{2}^{T}A(\alpha)\Delta u\right| + \left|Z_{2}^{T}A(\alpha)u_{p}\right| \quad (27)$$

where  $f(X) = D\upsilon + M(-\dot{\tilde{\upsilon}} + \dot{\varphi}) - d(t)$  with  $X = [\eta^T, \upsilon^T]^T$ .

According to Assumption 1, M, D and d(t) are not used in control design. Here, the NNs are introduced to approximate the unknown function vector f(X)

$$f(X) = WT(t)\Phi(X) + E(X)$$
(28)

where  $W(t) \in R^{3q \times 3}$  is an unknown time-varying weight matrix satisfying  $||W(t)|| \leq W_M$  with  $W_M$  being a positive constant [32], q is the node number, E(X) is the approximation error vector satisfying  $||E(X)|| \leq E_M$  with  $E_M > 0$ being a constant. From Assumption 1, v is unknown. Thus, we employ  $\hat{X} = [\eta^T, \hat{v}^T]^T$  to replace X in control design, and then the estimate of f(X) is

$$\hat{f}(\hat{X}) = \hat{W}^T(t)\Phi(\hat{X})$$
(29)

Let  $\Omega = W(t) - LW(t - T)$  with  $||\Omega|| \le \Omega^*$  and  $\Omega^* = W_M(1 + ||L||)$ , and it follows that

$$\hat{\Omega} = \hat{W}(t) - L\hat{W}(t - T) = -\Gamma\Phi(\hat{X})Z_2^T$$
(30)

Define the estimation error of weight matrix W(t) as  $\tilde{W}(t) = \hat{W}(t) - W(t)$ , we have

$$\tilde{W}(t) = L\tilde{W}(t-T) + \hat{\Omega} - \Omega$$
(31)

Substituting (22), (28) and (29) into (27), we have

$$Z_{2}^{T}M\dot{Z}_{2} \leq Z_{2}^{T}[-W^{T}(t)\Phi(X) - E(X) - K_{2}Z_{2} - R(\psi)Z_{1} + \Upsilon_{a} + \hat{W}^{T}(t)\Phi(\hat{X})] + |Z_{2}^{T}A(\alpha)\Delta u| + |Z_{2}^{T}A(\alpha)u_{p}| = -Z_{2}^{T}K_{2}Z_{2} - Z_{2}^{T}R(\psi)Z_{1} + Z_{2}^{T}[\bar{E}(X) - E(X) + \Upsilon_{a} + \tilde{W}^{T}(t)\Phi(\hat{X})] + |Z_{2}^{T}A(\alpha)\Delta u| + |Z_{2}^{T}A(\alpha)u_{p}|$$
(32)

where  $\bar{E}(X) = -W^T(t)\Phi(X) + W^T(t)\Phi(\hat{X})$ . Since  $\Phi(X)$  and  $\Phi(\hat{X})$  are bounded, and  $||W^T(t)|| \le W_M$ , that is, there exists a constant  $\bar{E}_M > 0$  satisfying  $\|\bar{E}(X)\| \le \bar{E}_M$ .

In view of (20), we have

$$\begin{split} \Upsilon_{a}^{T} \dot{\Upsilon}_{a} &= -\Upsilon_{a}^{T} K_{a} \Upsilon_{a} \\ &- \left[ \left( \left| Z_{2}^{T} A(\alpha) \Delta u \right| + \left| Z_{2}^{T} A(\alpha) u_{p} \right| \right) h\left( \left\| \Upsilon_{a} \right\| \right) \right] \\ &- \left[ 0.5 \left( \left| A(\alpha) \Delta u \right|^{2} + \left| A(\alpha) u_{p} \right|^{2} \right) h\left( \left\| \Upsilon_{a} \right\| \right) \right] \\ &+ 0.5 \Upsilon_{a}^{T} \left[ A(\alpha) \Delta u + A(\alpha) u_{p} \right] \end{split}$$
(33)

Substituting (26), (32) and (33) into (25) yields

$$\begin{split} \dot{V}_{1} &\leq -Z_{1}^{T}K_{1}Z_{1} + Z_{1}^{T}R(\psi)Z_{2} - Z_{1}^{T}R(\psi)\tilde{\upsilon} - Z_{2}^{T}K_{2}Z_{2} \\ &- Z_{2}^{T}R(\psi)Z_{1} + Z_{2}^{T}[\bar{E}(X) - E(X) + \Upsilon_{a} + \tilde{W}^{T}(t)\Phi(\hat{X})] \\ &- \Upsilon_{a}^{T}K_{a}\Upsilon_{a} + \left|Z_{2}^{T}A(\alpha)\Delta u\right| (1 - h\left(\|\Upsilon_{a}\|\right)) \\ &+ \left|Z_{2}^{T}A(\alpha)u_{p}\right| (1 - h\left(\|\Upsilon_{a}\|\right)) \\ &- \left[0.5\left(|A(\alpha)\Delta u|^{2} + |A(\alpha)u_{p}|^{2}\right)h\left(\|\Upsilon_{a}\|\right)\right] \\ &+ 0.5\Upsilon_{a}^{T}\left[A(\alpha)\Delta u + A(\alpha)u_{p}\right] \\ &\leq -Z_{1}^{T}K_{1}Z_{1} - Z_{2}^{T}K_{2}Z_{2} - \Upsilon_{a}^{T}K_{a}\Upsilon_{a} - Z_{1}^{T}R(\psi)\tilde{\upsilon} \\ &+ Z_{2}^{T}[\bar{E}(X) - E(X) + \Upsilon_{a} + \tilde{W}^{T}(t)\Phi(\hat{X})] \\ &+ \left(\left|Z_{2}^{T}A(\alpha)\Delta u\right| + \left|Z_{2}^{T}A(\alpha)u_{p}\right|\right) [1 - h\left(\|\Upsilon_{a}\|\right)] \\ &+ 0.5\left(|A(\alpha)\Delta u|^{2} + |A(\alpha)u_{p}|^{2}\right) [1 - h\left(\|\Upsilon_{a}\|\right)] \\ &+ \frac{1}{8}\|\Upsilon_{a}\|^{2} \end{split}$$
(34)

Construct the following Lyapunov function

$$V_2 = V_1 + \frac{\xi}{2} \int_{t-T}^t tr \left[ \tilde{W}^T(s) \ \tilde{W}(s) \right] ds \tag{35}$$

where *tr* denotes the trace operation of a matrix. In the light of (34) and (31), the time derivative of  $V_2$  is

$$\begin{split} \dot{V}_{2} &= \dot{V}_{1} + \frac{\xi}{2} tr \left[ \tilde{W}^{T}(t) \tilde{W}(t) - \tilde{W}^{T}(t-T) \tilde{W}(t-T) \right] \\ &\leq -Z_{1}^{T} K_{1} Z_{1} - Z_{2}^{T} K_{2} Z_{2} - \Upsilon_{a}^{T} K_{a} \Upsilon_{a} - Z_{1}^{T} R(\psi) \tilde{v} \\ &+ Z_{2}^{T} [\bar{E}(X) - E(X) + \Upsilon_{a} + \tilde{W}^{T}(t) \Phi(\hat{X})] + \frac{1}{8} \|\Upsilon_{a}\|^{2} \\ &+ \left( \left| Z_{2}^{T} A(\alpha) \Delta u \right| + \left| Z_{2}^{T} A(\alpha) u_{p} \right| \right) [1 - h(\|\Upsilon_{a}\|)] \\ &+ 0.5 \left( |A(\alpha) \Delta u|^{2} + |A(\alpha) u_{p}|^{2} \right) [1 - h(\|\Upsilon_{a}\|)] \\ &+ \frac{\xi}{2} tr [\beta \tilde{W}^{T}(t) \tilde{W}(t) - (\beta - 1) \tilde{W}^{T}(t) \tilde{W}(t) \\ &- \tilde{W}^{T}(t-T) \tilde{W}(t-T)] \\ &\leq -Z_{1}^{T} K_{1} Z_{1} - Z_{2}^{T} K_{2} Z_{2} - \Upsilon_{a}^{T} K_{a} \Upsilon_{a} - Z_{1}^{T} R(\psi) \tilde{v} \\ &+ Z_{2}^{T} [\bar{E}(X) - E(X) + \Upsilon_{a}] + \frac{1}{8} \|\Upsilon_{a}\|^{2} \\ &+ \left( \left| Z_{2}^{T} A(\alpha) \Delta u \right| + \left| Z_{2}^{T} A(\alpha) u_{p} \right| \right) [1 - h(\|\Upsilon_{a}\|)] \\ &+ 0.5 \left( |A(\alpha) \Delta u|^{2} + |A(\alpha) u_{p}|^{2} \right) [1 - h(\|\Upsilon_{a}\|)] \\ &+ Z_{2}^{T} [L \tilde{W}(t-T) + \hat{\Omega} - \Omega]^{T} \Phi(\hat{X}) \\ &- \frac{\xi(\beta - 1)}{2} tr \left[ \tilde{W}^{T}(t) \tilde{W}(t) \right] + \frac{\xi}{2} tr \left[ \beta \tilde{W}^{T}(t) \tilde{W}(t) \right] \\ &- \frac{\xi}{2} tr \left[ \tilde{W}^{T}(t-T) \tilde{W}(t-T) \right] \end{split}$$

where  $\beta$  is a positive constant.

Let  $\xi\beta = \Gamma^{-1}$ , according to (31), we have

$$Z_{2}^{T} \left[ L\tilde{W} \left( t - T \right) + \hat{\Omega} - \Omega \right]^{T} \Phi \left( \hat{X} \right) + \frac{\xi}{2} tr \left[ \beta \tilde{W}^{T} \left( t \right) \tilde{W} \left( t \right) \right]$$

$$= \frac{\xi}{2} tr \left[ \beta \tilde{W}^{T} \left( t - T \right) L^{T} L\tilde{W} \left( t - T \right) + \beta \hat{\Omega}^{T} \hat{\Omega} + \beta \Omega^{T} \Omega + 2\beta \hat{\Omega}^{T} L\tilde{W} \left( t - T \right) - 2\beta \Omega^{T} L\tilde{W} \left( t - T \right) - 2\beta \hat{\Omega}^{T} \Omega \right]$$

$$+ Z_{2}^{T} \left[ L\tilde{W} \left( t - T \right) + \hat{\Omega} - \Omega \right]^{T} \Phi \left( \hat{X} \right)$$

$$= \frac{\xi \beta}{2} tr \left[ \tilde{W}^{T} \left( t - T \right) L^{T} L\tilde{W} \left( t - T \right) \right]$$

$$- \frac{\Gamma}{2} Z_{2}^{T} \left[ \Phi \left( \hat{X} \right) Z_{2}^{T} \right]^{T} \Phi \left( \hat{X} \right)$$

$$+ \frac{\xi}{2} tr \left[ \beta \Omega^{T} \Omega - 2\beta \Omega L\tilde{W} \left( t - T \right) \right]$$
(37)

Using Young's inequality, the following inequation holds

$$tr\left[-2\beta\Omega L\tilde{W}\left(t-T\right)\right] \leq \frac{\beta^{2}}{\kappa}\Omega^{*2} + tr\left[\kappa\tilde{W}^{T}\left(t-T\right)L^{T}L\tilde{W}\left(t-T\right)\right]$$
(38)

where  $\kappa$  is a positive constant.

Synthesizing (38) and  $\|\Omega\| \le \Omega^*$ , (37) can be written as

$$Z_{2}^{T}\left[L\tilde{W}\left(t-T\right)+\hat{\Omega}-\Omega\right]^{T}\Phi\left(\hat{X}\right)+\frac{\xi}{2}tr\left[\beta\tilde{W}^{T}\left(t\right)\tilde{W}\left(t\right)\right]$$

$$\leq \frac{\xi\beta}{2}tr\left[\tilde{W}^{T}\left(t-T\right)L^{T}L\tilde{W}\left(t-T\right)\right] + \frac{\xi\beta}{2}\Omega^{*2} + \frac{\xi\beta^{2}}{2\kappa}\Omega^{*2} + \frac{\xi\kappa}{2\kappa}tr\left[\tilde{W}^{T}\left(t-T\right)L^{T}L\tilde{W}\left(t-T\right)\right]$$
$$= \frac{\xi\left(\beta+\kappa\right)}{2}tr\left[\tilde{W}^{T}\left(t-T\right)L^{T}L\tilde{W}\left(t-T\right)\right] + \frac{\xi}{2}\left(\beta + \frac{\beta^{2}}{\kappa}\right)\Omega^{*2}$$
(39)

Substituting (39) into (36) yields

$$\begin{split} \dot{V}_{2} &\leq -Z_{1}^{T}K_{1}Z_{1} - Z_{2}^{T}K_{2}Z_{2} - Z_{1}^{T}R(\psi)\tilde{\upsilon} - \Upsilon_{a}^{T}K_{a}\Upsilon_{a} \\ &+ \frac{\|\Upsilon_{a}\|^{2}}{8} + Z_{2}^{T}[\bar{E}(X) - E(X) + \Upsilon_{a}] + \left(|Z_{2}^{T}A(\alpha)\Delta u| + |Z_{2}^{T}A(\alpha)u_{p}|\right)\left[1 - h\left(\|\Upsilon_{a}\|\right)\right] + 0.5\left(|A(\alpha)\Delta u|^{2} \\ &+ |A(\alpha)u_{p}|^{2}\right)\left[1 - h\left(\|\Upsilon_{a}\|\right)\right] \\ &- \frac{\xi(\beta - 1)}{2}tr\left[\tilde{W}^{T}(t)\tilde{W}(t)\right] \\ &- \frac{\xi(\beta - 1)}{2}tr\left[\tilde{W}^{T}(t - T)\tilde{W}(t - T)\right] \\ &+ \frac{\xi(\beta + \kappa)}{2}tr\left[\tilde{W}^{T}(t - T)L^{T}L\tilde{W}(t - T)\right] \\ &+ \xi\left(\beta + \frac{\beta^{2}}{\kappa}\right)\Omega^{*2} \end{split}$$
(40)

Using Young's inequality, we obtain

$$\begin{cases} -Z_{1}^{T}R(\psi)\tilde{\upsilon} \leq \frac{a_{1}}{2} \|Z_{1}\|^{2} + \frac{\|\tilde{\upsilon}\|^{2}}{2a_{1}} \\ Z_{2}^{T}\bar{E}(X) \leq \frac{a_{2}}{2} \|Z_{2}\|^{2} + \frac{\|\bar{E}(X)\|^{2}}{2a_{2}} \\ Z_{2}^{T}E(X) \leq \frac{a_{3}}{2} \|Z_{2}\|^{2} + \frac{\|E(X)\|^{2}}{2a_{3}} \\ Z_{2}^{T}\Upsilon_{a} \leq \frac{a_{4}}{2} \|Z_{2}\|^{2} + \frac{\|\Upsilon_{a}\|^{2}}{2a_{4}} \end{cases}$$
(41)

where  $a_1, a_2, a_3$  and  $a_4$  are positive constants.

If  $\|\Upsilon_a\| > \Upsilon_2$ ,  $h(\|\Upsilon_a\|) = 1$ . According to (41), then (40) can subsequently rewritten as

$$\dot{V}_{2} \leq -\left[\lambda_{\min}(K_{1}) - \frac{a_{1}}{2}\right] \|Z_{1}\|^{2} - \left[\lambda_{\min}(K_{2}) - \frac{a_{2} + a_{3}}{2} - \frac{a_{4}}{2}\right] \|Z_{2}\|^{2} - \left[\lambda_{\min}(K_{a}) - \frac{a_{4} + 4}{8a_{4}}\right] \|\Upsilon_{a}\|^{2} - \frac{\xi(\beta - 1)}{2} \left\|\tilde{W}(t)\right\|^{2} - \frac{\xi}{2}[1 - (\beta + \kappa)l] \left\|\tilde{W}(t - T)\right\|^{2} + \frac{\|\tilde{v}\|^{2}}{2a_{1}} + \frac{\bar{E}_{M}^{2}}{2a_{2}} + \frac{E_{M}^{2}}{2a_{3}} + \frac{\xi}{2} \left(\beta + \frac{\beta^{2}}{\kappa}\right) \Omega^{*2} \leq -J_{1} \|Z_{1}\|^{2} - J_{2} \|Z_{2}\|^{2} - J_{3} \|\Upsilon_{a}\|^{2} - J_{4} \left\|\tilde{W}(t)\right\|^{2} - J_{5} \left\|\tilde{W}(t - T)\right\|^{2} + \Theta_{1}$$

$$(42)$$

where  $\Theta_1 = \frac{(\epsilon h_2)^2}{2a_1} + \frac{\tilde{E}_M^2}{2a_2} + \frac{E_M^2}{2a_3} + \frac{\xi}{2} \left(\beta + \frac{\beta^2}{\kappa}\right) \Omega^{*2}$ , the constants  $J_1, J_2, J_3, J_4$  and  $J_5$  are given and should

satisfy as

$$\begin{cases} J_1 = \lambda_{\min}(K_1) - \frac{a_1}{2} > 0\\ J_2 = \lambda_{\min}(K_2) - \frac{a_2 + a_3 + a_4}{2} > 0\\ J_3 = \lambda_{\min}(K_a) - \frac{a_4 + 4}{8a_4} > 0\\ J_4 = \frac{\xi(\beta - 1)}{2} > 0\\ J_5 = 1 - (\beta + \kappa)l > 0 \end{cases}$$
(43)

Using Young's inequality, we have

$$\begin{cases} \left| Z_2^T A(\alpha) u_p \right| \le \frac{a_5}{2} \| Z_2 \|^2 + \frac{\left\| A(\alpha) u_p \right\|^2}{2a_5} \\ \left| Z_2^T A(\alpha) \Delta u \right| \le \frac{a_6}{2} \| Z_2 \|^2 + \frac{\| A(\alpha) \Delta u \|^2}{2a_6} \end{cases}$$
(44)

where  $a_5$  and  $a_6$  are positive constants.

If  $||\Upsilon_a|| \leq \Upsilon_2$ ,  $h(||\Upsilon_a||) < 1$ . Then, in view of (41) and (44), (40) can be rewritten as

$$\begin{split} \dot{V}_{2} &\leq -\left[\lambda_{\min}(K_{1}) - \frac{a_{1}}{2}\right] \|Z_{1}\|^{2} - \left[\lambda_{\min}(K_{2}) - \frac{a_{2} + a_{3}}{2}\right] \\ &\quad - \frac{a_{4} + a_{5} + a_{6}}{2} \|Z_{2}\|^{2} - \left[\lambda_{\min}(K_{a}) - \frac{a_{4} + 4}{8a_{4}}\right] \|\Upsilon_{a}\|^{2} \\ &\quad - \frac{\xi(\beta - 1)}{2} \left\|\tilde{W}(t)\right\|^{2} - \frac{\xi}{2} [1 - (\beta + \kappa)l] \left\|\tilde{W}(t - T)\right\|^{2} \\ &\quad + \frac{\|\tilde{\upsilon}\|^{2}}{2a_{1}} + \frac{\xi}{2} \left(\beta + \frac{\beta^{2}}{\kappa}\right) \Pi^{*2} + \frac{\bar{E}_{M}^{2}}{2a_{2}} + \frac{E_{M}^{2}}{2a_{3}} \\ &\quad + \frac{1 + a_{5}}{2a_{5}} \|A(\alpha)u_{p}\|^{2} + \frac{1 + a_{6}}{2a_{6}} \|A(\alpha)\Delta u\|^{2} \\ &\leq -D_{1}\|Z_{1}\|^{2} - D_{2}\|Z_{2}\|^{2} - D_{3}\|\Upsilon_{a}\|^{2} - D_{4}\left\|\tilde{W}(t)\right\|^{2} \\ &\quad - D_{5}\left\|\tilde{W}(t - T)\right\|^{2} + \Theta_{2} \end{split}$$
(45)

where  $\Theta_2 = \frac{(\epsilon h_2)^2}{2a_1} + \frac{\bar{E}_M^2}{2a_2} + \frac{E_M^2}{2a_3} + \frac{\xi}{2} \left(\beta + \frac{\beta^2}{\kappa}\right) \Omega^{*2} + \left(\frac{1}{2a_5} + \frac{1}{2}\right) \|A(\alpha) u_p\|^2 + \left(\frac{1}{2a_6} + \frac{1}{2}\right) \|A(\alpha) \Delta u\|^2$ , the constants  $D_1$ ,  $D_2$ ,  $D_3$ ,  $D_4$  and  $D_5$  are given and should satisfy as

$$\begin{cases} D_1 = \lambda_{\min}(K_1) - \frac{a_1}{2} > 0\\ D_2 = \lambda_{\min}(K_2) - \frac{a_2 + a_3 + a_4 + a_5 + a_6}{2} > 0\\ D_3 = \lambda_{\min}(K_a) - \frac{a_4 + 4}{8a_4} > 0\\ D_4 = \frac{\xi(\beta - 1)}{2} > 0\\ D_5 = 1 - (\beta + \kappa)l > 0 \end{cases}$$
(46)

Synthesizing (42) and (45) yileds

$$\dot{V}_{2} \leq -C_{1} \|Z_{1}\|^{2} - C_{2} \|Z_{2}\|^{2} - C_{3} \|\Upsilon_{a}\|^{2} - C_{4} \|\tilde{W}(t)\|^{2} - C_{5} \|\tilde{W}(t-T)\|^{2} + \Theta$$

$$(47)$$

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where  $C_1$ ,  $C_2$ ,  $C_3$ ,  $C_4$ ,  $C_5$  and  $\Theta$  are constants satisfying

$$\begin{cases} C_i = \min \{J_i, D_i\} \\ \Theta = \max \{\Theta_1, \Theta_2\}, \quad i = 1, \cdots, 5 \end{cases}$$
(48)

Therefore, we have the following theorem.

Theorem 1: Consider the closed-loop system consisting of dynamically positioned vessels (1) in the presence of the unknown thruster faults and saturation constraints under Assumption 1, the high-gain observer (11-12), the auxiliary dynamic system (20-21), the FTC law (22) and the adaptive law (23). The position error surface  $Z_1$  is guaranteed to be bounded and satisfies  $||Z_1|| \leq \sqrt{\frac{\Theta}{C_1} + \lambda_{\max}(M) \frac{\Theta}{C_2} + \frac{\Theta}{C_3} + \xi T \frac{\Theta}{C_4}}$  through appropriately choosing the design parameters  $\varepsilon$ ,  $\gamma_1$ ,  $K_1$ ,  $K_2$ ,  $K_a$ , L and  $\Gamma$ satisfying (43) and (46), while all signals in the closed-loop DP control system are guaranteed to be UUB.

Proof: It can be seen from (47) that either  $||Z_1|| > \sqrt{\frac{\Theta}{C_1}}$ , or  $||Z_2|| > \sqrt{\frac{\Theta}{C_2}}$ , or  $||\Upsilon_a|| > \sqrt{\frac{\Theta}{C_3}}$ , or  $\left\|\tilde{W}(t)\right\| > \sqrt{\frac{\Theta}{C_4}}$ , or  $\left\|\tilde{W}(t-T)\right\| > \sqrt{\frac{\Theta}{C_5}}$  renders  $\dot{V}_2(t) < 0$ . Thus, we have  $V_2 \le \frac{1}{2} \left( ||Z_1||^2 + \lambda_{\max}(M) ||Z_2||^2 + ||\Upsilon_a||^2 + \xi \int_{t-T}^t \left\|\tilde{W}(s)\right\|^2 ds \right)$  $\le \frac{1}{2} \left( \frac{\Theta}{C_1} + \lambda_{\max}(M) \frac{\Theta}{C_2} + \frac{\Theta}{C_3} + \xi \int_{t-T}^t \frac{\Theta}{C_4} dt \right)$ (49)

It is obviously observed from (49) that  $V_2$  is UUB. Further, according to (24) and (35),  $Z_1, Z_2, \Upsilon_a, \tilde{W}$  (*t*) and  $\tilde{W}$  (*t* - *T*) are UUB. Then,  $\eta, \varphi$  and  $\hat{v}$  are also UUB from (15), (17) and (18). Furthermore, v is UUB. It follows that all the signals in the closed-loop DP control system are UUB. From (35) and (49), we can obtain  $||Z_1|| \leq \sqrt{\frac{\Theta}{C_1} + \lambda_{\max}(M) \frac{\Theta}{C_2} + \frac{\Theta}{C_3} + \xi T \frac{\Theta}{C_4}}$ . Theorem 1 is thus proved.

#### C. CONTROL ALLOCATION FOR DP OF VESSELS

In order to effectively complete various tasks, the dynamically positioned vessels usually have a high degree of redundancy of thrusters. Thus, the DP control system is a typical overactuated control system, which needs to distribute the forces and moment among the thrusters. In this subsection, the FTC forces and moment computed from the FTC law are allocated to individual thrusters, which is solved by using the Lagrange multiplier method. The control allocation problem can be described as

$$\min_{u_c} u_c^T u_c \tag{50a}$$

$$\tau_c = A(\alpha) u_c \tag{50b}$$

It is seen from (50b) that  $\tau_c$  is a nonlinear function of  $u_c$  and the time-varying orientation angle  $\alpha$  of the azimuth thruster, which increases the complexity of control allocation. Therefore, to reduce the complexity of control allocation,

we decompose the control command  $u_{c_i}$  of the *i*th thruster into the surge force and the sway force as follows

$$\ell_{x_i} = u_{c_i} \cos(\alpha_i) \tag{51a}$$

$$\ell_{y_i} = u_{c_i} \sin(\alpha_i) \tag{51b}$$

Then the thruster control command  $u_{c_i}$  can be expressed as

$$u_{c_i} = \sqrt{\ell_{x_i}^2 + \ell_{y_i}^2}$$
(52)

Substituting (50) and (51) into (52), we have

$$\min_{\ell} \ell^T \ell \tag{53a}$$

$$\tau_c = A_\ell \ell \tag{53b}$$

where  $A_{\ell} = \begin{bmatrix} 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & \cdots & 0 & 1 \\ -l_{y_1} & l_{x_1} & \cdots & -l_{y_m} & l_{x_m} \end{bmatrix}, \ \ell = \begin{bmatrix} \ell_{x_1}, \ell_{y_1}, \cdots, \ell_{y_m} & \ell_{x_m} \end{bmatrix}$ 

 $\ell_{x_m}, \ell_{y_m}]^{T}$ . It is seen that the nonlinear constrained minimization problem (50) is transformed into a linear constrained minimization problem (52). As such, by means of the Lagrange multiplier method, the optimal solution  $\ell^*$  of the minimization problem can be obtained as follows

$$\ell^* = A_\ell^\dagger \tau_c \tag{54}$$

where  $\ell^* = \left[\ell_{x_1}^*, \ell_{y_1}^*, \cdots, \ell_{x_m}^*, \ell_{y_m}^*\right]^T$  and  $A_{\ell}^{\dagger} = A_{\ell}^{\mathrm{T}} \left(A_{\ell}^{\mathrm{T}} A_{\ell}\right)^{-1}$  is the pseudo inverse of  $A_{\ell}$ .

According to (54), the solution  $\ell^*$  forms the following vector

$$u_c^* = \left[\sqrt{\ell_{x_1}^{*2} + \ell_{y_1}^{*2}}, \cdots, \sqrt{\ell_{x_m}^{*2} + \ell_{y_m}^{*2}}\right]^T$$
(55)

which is the thruster control command vector.

#### **IV. SIMULATIONS AND COMPARISONS**

In this section, to assess performance of the proposed FTC scheme, simulations on a supply vessel model CyberShip I [37] are carried out in two cases.

Let the desired position and heading of the vessel be  $\eta_d = [0m, 0m, 20^\circ]^T$ . The initial conditions are set as  $\eta(0) = [-3m, -4.5m, 8^{\circ}]^{T}, \ \upsilon(0) = [0 \ m/s, 0m/s, 0^{\circ}/s]^{T},$  $\Upsilon_a(0) = [2, 2, 2]^T, \chi_1(0) = [-4.5, -4.5, 0.2]^T, \chi_2(0) =$  $[0.01, 0.01, 0.01]^{\tilde{T}}$  and  $\hat{W}(0) = 0_{24\times 3}$ . In the simulation, the M, D and the configuration of thrusters are captured in [37] in detail. The sigmoid basic function  $\frac{1}{1+\exp(-x)}$  is chosen as the activation function of NNs and the node number is chosen as q = 8. The environmental disturbances are  $d(t) = R^T b$  with  $\dot{b} = -H^{-1}b + \Lambda \bar{\omega}$ , which is the first-order Markov process.  $b \in R^3$  is a vector of bias forces and moment,  $H^{-1} = diag(0.02, 0.025, 0.025)$  is a bias time constant diagonal matrix,  $\bar{\omega}$  is a zero-mean Gaussian white noise vector and  $\Lambda = diag(0.25, 0.25, 0.15)$  is a diagonal magnitude matrix of  $\bar{\omega}$ . In addition, the design parameters are chosen as  $\varepsilon = 0.03$ ,  $\gamma_1 = 2$ ,  $K_a = diag (11.2, 11.5, 12.5)$ ,  $K_1 = diag(2, 2, 5), K_2 = diag(8.5, 8.5, 8), L = 0.28I_{24 \times 24},$  $\Gamma = 5.$ 



FIGURE 1. The simulation results in case 1. (a) Curves of vessel actual position and yaw angle. (b) Curves of the vessel velocities and their estimates. (c) Curves of control forces and moment. (d) Curves of thruster control commands. (e) Curves of learning behavior of NNs.

Case 1: The parameters M and D of the mathematical model are identified accurately in the ideal condition. The effectiveness factors are described

by

$\sigma_1(t) = 1$	(56a)
$\sigma_2(t) = 1$	(56b)



FIGURE 2. The simulation results in case 2. (a) Curves of vessel actual position and yaw angle. (b) Curves of the vessel velocities and their estimates. (c) Curves of control forces and moment. (d) Curves of thruster control commands. (e) Curves of learning behavior of NNs.

$$\sigma_3(t) = \begin{cases} 1 & 0 \le t \le 60\\ 0.7 + 0.2\cos(0.03t) & t > 60 \end{cases}$$
(56c)  
$$\sigma_4(t) = 1$$
(56d)

The simulation results of Case 1 are depicted in Fig. 1(a)-1(e). Fig. 1(a) shows the curves of vessel position

and heading, from which it is obvious that the proposed output feedback FTC scheme can force the vessel to be maintained at the desired position and heading in the simultanious presence of external disturbances, dynamic uncertainties, thruster saturation constraints and thruster faults. Fig. 1(b) illustrates the actual vessel velocities and their estimates. It is clear that the vessel velocities can be accurately estimated by the constructed high-gain observer. Fig. 1(c) shows the corresponding FTC forces and moment in this case, respectively. The thruster control commands are shown in Fig. 1(d), from which it is seen that results are also reasonable. Fig. 1(e) plots the uncertainties of vessels and the approximation results of NNs, which demonstrates that the uncertainties are efficiently approximated by the NNs. These results reveal that the output feedback FTC scheme can maintain the vessel position and heading at the desired values with satisfactory performance, while guaranteeing the uniform ultimate boundedness of all signals in the closed-loop DP control system, as proved in Theorem 1.

*Case 2:* The parameters M and D of the mathematical mode are changed to (1+10%)M and (1+10%)D. The effectiveness factors are presented by

$$\sigma_1(t) = \begin{cases} 1 & 0 \le t \le 120 \\ 0.7 & t > 120 \end{cases}$$
(57a)

$$\sigma_2(t) = 1 \tag{57b}$$

$$\sigma_3(t) = \begin{cases} 1 & 0 \le t \le 60\\ 0.6 + 0.3\sin(0.05t) & t > 60 \end{cases}$$
(57c)

$$\sigma_4(t) = 1 \tag{57d}$$

We adopt the same desired position and heading, initial conditions and control design parameters as the counterparts in Case 1. The simulation results are depicted in Fig. 2(a)-(e). It is observed from Fig. 1(a)-(e) and Fig. 2(a)-(e) that the proposed FTC scheme compensate for the effects of the thruster faults and uncertainties in the system dynamics.

#### **V. CONCLUSIONS**

This paper proposes the output feedback FTC scheme with control allocation for DP of vessels subject to external disturbances, dynamic uncertainties and thruster saturation constraints under thruster faults incorporating the high-gain observer, the auxiliary dynamic system, NNs into backstepping technique. Numerical simulations are performed to verify the capability of the proposed FTC laws in the presence of two cases of thruster faults, external disturbances and dynamic uncertainties. The simulation results verify the control performance of the proposed output feedback FTC scheme. The proposed scheme in this paper can be extended to other nonlinear MIMO systems, such as underwater vehicles and manipulators.

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