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A Multivariate Control Chart for Monitoring Several Exponential Quality Characteristics Using EWMA

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ABSTRACT In this paper, we propose a new multivariate control chart for monitoring several exponential distributed characteristics. We transform each exponential variable into the one following an approximately normal distribution. The control statistic of the proposed chart is constructed using the exponentially weighted moving average. The performance of the proposed control chart is studied using the average run length according to the process shift. Some tables are presented for bivariate and trivariate cases. The application of the proposed control chart is discussed with the help of simulated data.

INDEX TERMS Multivariate control chart, average run length, exponential distribution.

I. INTRODUCTION

A control chart is one of the important tools in the industry for the quality enhancement of a product. These tools are important and applied at each stage of the manufacturing processes. The use of control charts in the industry minimizes the non-conforming product and alternatively increases the profit of the companies. The control charts are helpful in detecting the cause of variation in the manufacturing process. These are useful in indicating when the process has been shifted due to some controllable or uncontrollable factors.

Mostly, control charts are designed by assuming that the quality of interest follows the normal distribution. This assumption is not always true in practice because the quality of interest may follow some other non-normal distribution. The control chart constructed under the normal assumption may increase the type-I and/or type-II errors when the quality of interest does not follow a normal distribution. Therefore, the designing of the control chart for non-normal distribution is important. As mentioned by Santiago and Smith [29], the data which is not collected in a subgroup may follow a skewed distribution. The use of a normal distribution for this type of data is erroneous. More details can be seen in Schilling and Nelson [30] and Stoumbos and Reynolds [31]. As mentioned by Santiago and Smith [29] and Aslam *et al.* [3], an exponential distribution is better to consider for skewed data usually where the underlying characteristic is a time-related event. The control chart for an exponentially distributed quality characteristic is sometimes called a t-chart. Recently, Santiago and Smith [29], Aslam *et al.* [3] proposed t-charts for the exponential distribution using the transformation given by Nelson [25]. Some other works about the t-chart can be seen in Nelson [24], Kaminsky *et al.* [14], Gan [10], [11], Acosta-Mejia [1], Kittlitz [17], Chan *et al.* [6], Woodall and Ncube [35], Jones and Champ [13], Borror *et al.* [4], Mohammed [21], Xie *et al.* [36], Liu *et al.* [20], Woodall [33], Mohammed and Laney [22], Klevens *et al.* [18], Montgomery [23], Cheng and Chen [7], Szarka and Woodall [32], Zhang *et al.* [37], Ahmad *et al.* [2] and Joekes and Barbosa [12], FazalZarandi *et al.* [9], Borror *et al.* [5], and Fallahnezhad and Niaki [8].

In practice, we may have more than one variable of interest following the exponential distribution. For the monitoring of several variables simultaneously multivariate control charts have been widely used in the industry. The multivariate control charts have attracted researchers due to several reasons. According to Woodall [34] and Li *et al.* [19] " firstly monitoring several variables independently requires multiple charts to be handled in parallel, where each separate chart has a statistic to be updated and plotted from sample to sample and secondly, multivariate control charts appropriately describe and exploit the correlations among multiple variables and therefore provide general tools for monitoring multivariate processes''. So, the use of the univariate control chart for monitoring the several variables needs much time and efforts. The use of multivariate control chart is more economical in this situation [28]. Recently, Saghir and Lin [28] proposed the multivariate control chart for COM-Poisson distribution. Li *et al.* [19] proposed the multivariate control chart for multinomial distribution. Khoo *et al.* [16] proposed multivariate synthetic chart for Weibull distribution. More details about the applications of the multivariate control chart, the reader may refer to, Niaki and Abbasi [26], Zhang *et al.* [38], Khoo *et al.* [15], Niaki and Jahani [27] and Woodall [34].

By exploring the literature of t-charts, we note that no work has been done on the multivariate t-chart for the exponential distribution. In this manuscript, we will focus on the development of multivariate t-chart for exponential distribution using the transformation suggested by Nelson [25] as well as the exponentially weighted moving average statistic. The rest of the paper is set as follows: the designing of the proposed chart is given in section 2, a simulation study is given in the next section, some remarks are given in the last section.

II. DESIGNING OF THE PROPOSED CONTROL CHART

Suppose that the quality characteristic of interest(such as the time between events,) denoted by T, follows the exponential distribution with scale (mean) parameter θ. The transformed variable $T^* = T^{1/\beta}$ follows the Weibull distribution having shape parameter β and the scale parameter $\theta^{1/\beta}$, see Johnson *et al.* [40]. Later on, Nelson [25] suggested that $T^* = T^{1/3.6}$ changes the Weibull distributed variable to an approximate normal variable. The proposed control chart is for the monitoring of more than one exponential distributed variable at the same time.

Suppose that there are p independent exponentially distributed quality characteristics, denoted by $T_1, T_2, \ldots, T_p$. Let us assume that T_i follows an independent exponential distribution with the pdf of

$$f_{T_i}(x) = \frac{1}{\theta_i} e^{-\frac{x}{\theta_i}} \tag{1}$$

where θ_i is the mean of T_i. If we use the transform of $T_i^* = T_i^{1/3.6}$, then T_i^* follows an independent normal distribution with mean $\mu_i = E(T_i^*) = \theta_i^* . \Gamma\left(1 + \frac{1}{3.6}\right)$ and $\sigma_i^2 = Var(T_i^*) = \theta_i^{*2} . \left[\Gamma\left(1 + \frac{2}{3.6}\right) - \left(\Gamma\left(1 + \frac{1}{3.6}\right)\right)^2\right]$. where $\theta_i^* = \theta_i^{1/3.6}$.

We may use T_i^* in the control statistic but we propose the use of the EWMA statistic. Let T_{ij}^* be the i-th transformed variable at j-th subgroup. Then the EWMA statistic at j-th subgroup will be

$$M_{ij} = \lambda T_{ij}^* + (1-\lambda) M_{i,j-1} \tag{2}$$

where λ is a smoothing constant ranging between 0 and 1. The distribution of M_{ij} for a large j follows a normal distribution with mean and variance of

$$\mu_{M_i} = \theta_i^* . \Gamma\left(1 + \frac{2}{3.6}\right) \tag{3}$$

$$\sigma_{M_i}^2 = \frac{\lambda}{2-\lambda} [\theta_i^{*2} . \left[\Gamma\left(1+\frac{2}{3.6}\right) - \left(\Gamma\left(1+\frac{1}{3.6}\right)\right)^2\right] \tag{4}$$

We propose the following multivariate t-control chart, which will be operated at each subgroup until the process is declared as out-of-control.

Step-1: Draw a random sample of $(T_1, T_2, \ldots, T_p)$. Transform each variable to $T_i^* = T_i^{1/3.6}$, for $i = 1, \ldots, p$. Then, calculate the control statistic at the j-th subgroup as

$$Y_j = \sum_{i=1}^{p} \frac{M_{ij}^2}{\sigma_{M_i}^2}$$

where

$$M_{ij} = \lambda T_{ij}^* + (1-\lambda) M_{i,j-1}$$

Step-2: Declare the process as in-control if $Y_j \leq h$ and declare the process as out-of-control if $Y_j > h$.

Then the statistic

$$Y_j = \sum_{i=1}^{p} \frac{M_{ij}^2}{\sigma_{M_i}^2} \tag{5}$$

follows the non-central chi-square distribution with the degree of freedom p and the non-centrality parameter

$$\gamma = \sum_{i=1}^{p} \left(\frac{\mu_{M_i}^2}{\sigma_{M_i}^2}\right)$$

$$= \sum_{i=1}^{p} \left(\frac{\theta_i^{*2} . \Gamma\left(1+\frac{2}{3.6}\right)^2}{\frac{\lambda}{2-\lambda}[\theta_i^{*2} . \left[\Gamma\left(1+\frac{2}{3.6}\right) - \left(\Gamma\left(1+\frac{1}{3.6}\right)\right)^2\right]}\right)$$

$$= p . \frac{\Gamma\left(1+\frac{2}{3.6}\right)^2}{\frac{\lambda}{2-\lambda}\left[\Gamma\left(1+\frac{2}{3.6}\right) - \left(\Gamma\left(1+\frac{1}{3.6}\right)\right)^2\right]} \tag{6}$$

The mean and the variance of the random variable Y_j when the process is in control are as follows.

$$E(Y_j) = p + \gamma \quad \text{and } Var(Y_j) = 2(p + 2\gamma)$$

The probability of declaring the process as out of control when the process is actually in control is given as follows

$$P_{out}^0 = P(Y_j > h | \theta_i, i = 1, \ldots, p) \tag{7}$$

TABLE 1. **(Bivariate case): The values of ARL when r0 = 370 and λ = 0.30.**

c_2	UCL=186.384										
	c_1										
	1	1.05	1.1	1.2	1.3	1.4	1.5	1.75	2	2.5	3
1	370.00	294.57	237.17	158.48	109.77	78.46	57.66	29.55	17.04	7.35	4.05
1.05	294.57	236.08	191.29	129.36	90.60	65.42	48.54	25.41	14.93	6.63	3.75
1.1	237.17	191.29	155.94	106.66	75.50	55.06	41.22	22.03	13.17	6.02	3.48
1.2	158.48	129.36	106.66	74.55	53.83	39.99	30.46	16.92	10.45	5.05	3.04
1.3	109.77	90.60	75.50	53.83	39.59	29.91	23.15	13.32	8.49	4.31	2.70
1.4	78.46	65.42	55.06	39.99	29.91	22.97	18.04	10.73	7.03	3.74	2.43
1.5	57.66	48.54	41.22	30.46	23.15	18.04	14.37	8.81	5.93	3.29	2.21
1.75	29.55	25.41	22.03	16.92	13.32	10.73	8.81	5.79	4.13	2.52	1.82
2	17.04	14.93	13.17	10.45	8.49	7.03	5.93	4.13	3.10	2.05	1.57
2.5	7.35	6.63	6.02	5.05	4.31	3.74	3.29	2.52	2.05	1.54	1.29
3	4.05	3.75	3.48	3.04	2.70	2.43	2.21	1.82	1.57	1.29	1.15

The cumulative distribution function (cdf) of the non-central Chi-square distribution is as follows

$$F(x; p, \gamma) = e^{-\frac{\gamma}{2}}.\sum_{j=0}^{\infty} \frac{\left(\frac{\gamma}{2}\right)^j}{j!} Q(x; k + 2j) \tag{8}$$

where $Q(x; k + 2j)$ is the cdf of central chi-square distribution with the degree of freedom $k + 2j$,. The P^0_{out} using Eq. (7) can be rewritten as follows

$$P^0_{out} = 1 - F(h; p, \gamma)$$

or

$$P^0_{out} = 1 - e^{-\frac{\gamma}{2}} \sum_{j=0}^{\infty} \frac{\left(\frac{\gamma}{2}\right)^j}{j!} Q(h; k + 2j) \tag{9}$$

We will measure the performance of the proposed control chart using the average run length which is on the average sample needed for the indication when the process is to be going out of control. The in-control ARL is given by

$$ARL_0 = \frac{1}{P^0_{out}} \tag{10}$$

Now, we will derive the necessary measures for the proposed control chart when the process has been shifted to a new mean. When the process parameter θ_i is shifted to $c_i * \theta_i$, T^*_i approximately follows a normal distribution with new mean and variance of

$$\begin{aligned}\mu_{1,M_i} &= E\left(M_{ij}|c_i\theta_i, i = 1, \ldots, p\right)\\ &= c_i^{1/3.6}\theta_i^{*}.\Gamma\left(1 + \frac{1}{3.6}\right)\end{aligned} \tag{11}$$

$$\begin{aligned}\sigma^2_{1,M_i} &= Var\left(M_{ij}|c_i\theta_i, i = 1, \ldots, p\right)\\ &= c_i^{\frac{2}{3.6}} \frac{\lambda}{2-\lambda}\theta_i^{*2}.\left[\Gamma\left(1+\frac{2}{3.6}\right) - \left(\Gamma\left(1 + \frac{1}{3.6}\right)\right)^2\right]\end{aligned} \tag{12}$$

So, the control statistic $Y_j = \sum_{i=1}^{p} \frac{M^2_{ij}}{\sigma^2_{M_i}}$ for the shifted process follows the non-central Chi-square distribution with degree of freedom p and non-centrality parameter

$$\begin{aligned}\gamma_1 &= \sum_{i=1}^{p} \frac{\mu^2_{1,M_i}}{\sigma^2_{1,M_i}}\\ &= \sum_{i=1}^{p} \frac{c_i^{\frac{2}{3.6}}\Gamma^2\left(1 + \frac{1}{3.6}\right)}{\frac{\lambda}{2-\lambda}\left[\Gamma\left(1 + \frac{2}{3.6}\right) - \left(\Gamma\left(1 + \frac{1}{3.6}\right)\right)^2\right]}\\ &= \frac{\Gamma^2\left(1 + \frac{1}{3.6}\right)}{\frac{\lambda}{2-\lambda}\left[\Gamma\left(1 + \frac{2}{3.6}\right) - \left(\Gamma\left(1 + \frac{1}{3.6}\right)\right)^2\right]} \sum_{i=1}^{p} c_i^{\frac{2}{3.6}}\end{aligned} \tag{13}$$

The probability of being declared as out of control for the shifted process is given as follows

$$P^1_{out} = 1 - F(h; p, \gamma_1)$$

or

$$P^1_{out} = 1 - e^{-\frac{\gamma_1}{2}}.\sum_{j=0}^{\infty} \frac{\left(\frac{\gamma_1}{2}\right)^j}{j!} Q(h; k + 2j) \tag{14}$$

The out-of-control ARL for the shifted process is given as follows

$$ARL_1 = \frac{1}{P^1_{out}} \tag{15}$$

The ARLs are obtained using the following algorithm

1) Fix the value of r_0, the specified in-control ARL.
2) Determine h for which $ARL_0 \geq r_0$
3) Use h in Eqs. (14) and (15) to obtain ARL_1

We will present the ARL tables for bivariate ($p = 2$) and trivariate ($p = 3$) cases. Tables 1-6 are presented for bivariate case by considering when $r_0 = 200$, 300 and 370. The out-of-control ARLs are obtained according to various combinations of (c_1, c_2).

TABLE 2. (Bivariate case): The values of ARL whe r0 = 300 and λ = 0.30.

c_2	UCL=184.508										
	c_1										
	1	1.05	1.1	1.2	1.3	1.4	1.5	1.75	2	2.5	3
1	300.00	239.98	194.10	130.84	91.38	65.84	48.74	25.42	14.89	6.59	3.72
1.05	239.98	193.23	157.28	107.28	75.76	55.14	41.21	21.95	13.09	5.97	3.45
1.1	194.10	157.28	128.79	88.86	63.41	46.60	35.15	19.10	11.59	5.44	3.21
1.2	130.84	107.28	88.86	62.63	45.58	34.12	26.18	14.78	9.27	4.59	2.82
1.3	91.38	75.76	63.41	45.58	33.79	25.72	20.05	11.73	7.58	3.94	2.52
1.4	65.84	55.14	46.60	34.12	25.72	19.89	15.73	9.51	6.32	3.44	2.28
1.5	48.74	41.21	35.15	26.18	20.05	15.73	12.62	7.86	5.36	3.05	2.08
1.75	25.42	21.95	19.10	14.78	11.73	9.51	7.86	5.24	3.79	2.36	1.73
2	14.89	13.09	11.59	9.27	7.58	6.32	5.36	3.79	2.88	1.94	1.51
2.5	6.59	5.97	5.44	4.59	3.94	3.44	3.05	2.36	1.94	1.48	1.26
3	3.72	3.45	3.21	2.82	2.52	2.28	2.08	1.73	1.51	1.26	1.13

TABLE 3. (Bivariate case): The values of ARL when r0 = 200 and λ = 0.30.

c_2	UCL=180.79										
	c_1										
	1	1.05	1.1	1.2	1.3	1.4	1.5	1.75	2	2.5	3
1	200.00	161.49	131.81	90.42	64.20	46.98	35.30	19.05	11.50	5.37	3.16
1.05	161.49	131.25	107.80	74.81	53.70	39.69	30.10	16.58	10.20	4.90	2.95
1.1	131.81	107.80	89.06	62.51	45.33	33.83	25.89	14.55	9.10	4.50	2.77
1.2	90.42	74.81	62.51	44.80	33.12	25.17	19.59	11.43	7.38	3.84	2.46
1.3	64.20	53.70	45.33	33.12	24.94	19.26	15.23	9.19	6.12	3.34	2.22
1.4	46.98	39.69	33.83	25.17	19.26	15.11	12.12	7.56	5.17	2.95	2.03
1.5	35.30	30.10	25.89	19.59	15.23	12.12	9.85	6.33	4.44	2.64	1.87
1.75	19.05	16.58	14.55	11.43	9.19	7.56	6.33	4.34	3.22	2.09	1.59
2	11.50	10.20	9.10	7.38	6.12	5.17	4.44	3.22	2.50	1.76	1.41
2.5	5.37	4.90	4.50	3.84	3.34	2.95	2.64	2.09	1.76	1.38	1.20
3	3.16	2.95	2.77	2.46	2.22	2.03	1.87	1.59	1.41	1.20	1.10

TABLE 4. (Bivariate case): The values of ARL when r0 = 370 and λ = 0.50.

c_2	UCL113.78										
	c_1										
	1	1.05	1.1	1.2	1.3	1.4	1.5	1.75	2	2.5	3
1	370.00	313.21	267.06	197.99	150.23	116.34	91.72	54.11	34.50	16.68	9.53
1.05	313.21	266.16	227.77	170.04	129.85	101.16	80.20	47.91	30.88	15.20	8.82
1.1	267.06	227.77	195.61	147.00	112.95	88.49	70.54	42.65	27.77	13.91	8.18
1.2	197.99	170.04	147.00	111.86	86.95	68.86	55.45	34.29	22.77	11.78	7.12
1.3	150.23	129.85	112.95	86.95	68.32	54.65	44.42	28.05	18.98	10.12	6.26
1.4	116.34	101.16	88.49	68.86	54.65	44.13	36.18	23.30	16.03	8.80	5.57
1.5	91.72	80.20	70.54	55.45	44.42	36.18	29.90	19.62	13.72	7.73	5.00
1.75	54.11	47.91	42.65	34.29	28.05	23.30	19.62	13.41	9.72	5.82	3.95
2	34.50	30.88	27.77	22.77	18.98	16.03	13.72	9.72	7.27	4.58	3.24
2.5	16.68	15.20	13.91	11.78	10.12	8.80	7.73	5.82	4.58	3.15	2.39
3	9.53	8.82	8.18	7.12	6.26	5.57	5.00	3.95	3.24	2.39	1.91

Tables 1-3 show the out-of-control ARLs according to various combinations of (c_1, c_2) of the proposed control chart for p = 2 and λ = 0.30.

From Tables 1-3, we note following trends

1) We note the decreasing trend in ARL values when the shift parameters in one and/or both variables are increased.
2) The UCL decreases as r_0 decreases.

Tables 4-6 are presented for the case of $p = 2$ and λ = 0.50.

From these tables, we observe a slower decrease in ARLs as compared with the case of λ = 0.30.

Tables 7-10 are for the trivariate case, which shows the ARLs according to the shift parameter combination of two variables while the third variable remains in control for the case of p = 3 and λ = 0.50.

TABLE 5. **(Bivariate case): The values of ARL when r0 = 300 and λ = 0.50.**

c_2	UCL=112.313										
	c_1										
	1	1.05	1.1	1.2	1.3	1.4	1.5	1.75	2	2.5	3
1	300.00	254.83	218.01	162.66	124.18	96.72	76.68	45.82	29.55	14.58	8.48
1.05	254.83	217.29	186.57	140.16	107.69	84.37	67.26	40.69	26.53	13.32	7.86
1.1	218.01	186.57	160.75	121.57	93.97	74.04	59.34	36.33	23.93	12.22	7.31
1.2	162.66	140.16	121.57	93.08	72.78	57.96	46.92	29.38	19.73	10.41	6.39
1.3	124.18	107.69	93.97	72.78	57.51	46.26	37.80	24.16	16.52	8.98	5.65
1.4	96.72	84.37	74.04	57.96	46.26	37.55	30.95	20.17	14.03	7.85	5.05
1.5	76.68	67.26	59.34	46.92	37.80	30.95	25.71	17.07	12.06	6.92	4.55
1.75	45.82	40.69	36.33	29.38	24.16	20.17	17.07	11.80	8.64	5.26	3.62
2	29.55	26.53	23.93	19.73	16.52	14.03	12.06	8.64	6.52	4.18	3.00
2.5	14.58	13.32	12.22	10.41	8.98	7.85	6.92	5.26	4.18	2.92	2.24
3	8.48	7.86	7.31	6.39	5.65	5.05	4.55	3.62	3.00	2.24	1.82

TABLE 6. **(Bivariate case): The values of ARL when r0 = 200 and λ = 0.50.**

c_2	UCL=109.416										
	c_1										
	1	1.05	1.1	1.2	1.3	1.4	1.5	1.75	2	2.5	3
1	200.00	171.06	147.32	111.33	86.02	67.77	54.32	33.29	21.97	11.28	6.79
1.05	171.06	146.85	126.92	96.56	75.08	59.50	47.95	29.74	19.83	10.36	6.33
1.1	147.32	126.92	110.08	84.29	65.93	52.54	42.56	26.71	17.99	9.56	5.91
1.2	111.33	96.56	84.29	65.33	51.68	41.62	34.05	21.84	15.00	8.22	5.22
1.3	86.02	75.08	65.93	51.68	41.31	33.59	27.73	18.16	12.69	7.17	4.65
1.4	67.77	59.50	52.54	41.62	33.59	27.56	22.95	15.31	10.88	6.31	4.19
1.5	54.32	47.95	42.56	34.05	27.73	22.95	19.26	13.08	9.44	5.62	3.81
1.75	33.29	29.74	26.71	21.84	18.16	15.31	13.08	9.25	6.91	4.36	3.09
2	21.97	19.83	17.99	15.00	12.69	10.88	9.44	6.91	5.32	3.53	2.60
2.5	11.28	10.36	9.56	8.22	7.17	6.31	5.62	4.36	3.53	2.54	2.00
3	6.79	6.33	5.91	5.22	4.65	4.19	3.81	3.09	2.60	2.00	1.65

TABLE 7. **(Trivariate case): The values of ARL whe r0 = 370; p = 3 n and λ = 0.50.**

c_2	UCL=155.249, c_3=1										
	c_1										
	1	1.05	1.1	1.2	1.3	1.4	1.5	1.75	2	2.5	3
1	370.00	322.86	283.21	221.02	175.45	141.38	115.46	73.15	49.15	25.39	14.98
1.05	322.86	282.42	248.31	194.66	155.17	125.53	102.89	65.74	44.51	23.29	13.89
1.1	283.21	248.31	218.81	172.27	137.88	111.97	92.11	59.33	40.46	21.43	12.91
1.2	221.02	194.66	172.27	136.74	110.29	90.21	74.72	48.88	33.80	18.33	11.27
1.3	175.45	155.17	137.88	110.29	89.59	73.78	61.50	40.83	28.60	15.86	9.93
1.4	141.38	125.53	111.97	90.21	73.78	61.14	51.28	34.51	24.47	13.86	8.83
1.5	115.46	102.89	92.11	74.72	61.50	51.28	43.25	29.49	21.16	12.22	7.91
1.75	73.15	65.74	59.33	48.88	40.83	34.51	29.49	20.72	15.26	9.22	6.20
2	49.15	44.51	40.46	33.80	28.60	24.47	21.16	15.26	11.50	7.24	5.03
2.5	25.39	23.29	21.43	18.33	15.86	13.86	12.22	9.22	7.24	4.87	3.58
3	14.98	13.89	12.91	11.27	9.93	8.83	7.91	6.20	5.03	3.58	2.76

1) The values of ARL for $p = 3$ are larger than $p = 2$
2) The UCL for $p = 2$ is smaller than $p = 3$

III. SIMULATION STUDY

In this section, we will present the application of the proposed control chart by using the simulated data. For the purpose of simulation we generated the data by using the multivariate exponential distribution package in ''R- language'' name ''lcmix'', we consider the special case of bivariate exponential distribution by assuming that $p = 2$, further for the purpose of simulation for two independent variables we had taken the $\rho = 0.001$ and the variance as 0.50 First,

TABLE 8. **(Trivariate case):The values of ARL when r0 = 300; p = 3 and λ = 0.50.**

c_2	UCL=153.537, c_3=1										
	c_1										
	1	1.05	1.1	1.2	1.3	1.4	1.5	1.75	2	2.5	3
1	300.00	262.52	230.91	181.16	144.53	117.02	96.01	61.48	41.72	21.93	13.13
1.05	262.52	230.28	203.02	159.99	128.17	104.18	85.79	55.40	37.87	20.16	12.20
1.1	230.91	203.02	179.39	141.97	114.19	93.17	76.99	50.12	34.52	18.60	11.38
1.2	181.16	159.99	141.97	113.27	91.80	75.44	62.77	41.50	28.97	15.98	9.97
1.3	144.53	128.17	114.19	91.80	74.94	62.00	51.91	34.82	24.62	13.88	8.82
1.4	117.02	104.18	93.17	75.44	62.00	51.62	43.47	29.56	21.16	12.18	7.87
1.5	96.01	85.79	76.99	62.77	51.91	43.47	36.83	25.37	18.36	10.78	7.08
1.75	61.48	55.40	50.12	41.50	34.82	29.56	25.37	17.99	13.37	8.21	5.59
2	41.72	37.87	34.52	28.97	24.62	21.16	18.36	13.37	10.17	6.50	4.57
2.5	21.93	20.16	18.60	15.98	13.88	12.18	10.78	8.21	6.50	4.44	3.31
3	13.13	12.20	11.38	9.97	8.82	7.87	7.08	5.59	4.57	3.31	2.58

TABLE 9. **(Trivariate case): The values of ARL when r0 = 200; p = 3 and λ = 0.50.**

c_2	UCL=150.149, c_3=1										
	c_1										
	1	1.05	1.1	1.2	1.3	1.4	1.5	1.75	2	2.5	3
1	200.00	176.00	155.65	123.40	99.43	81.28	67.29	44.02	30.46	16.57	10.23
1.05	176.00	155.24	137.61	109.57	88.65	72.75	60.45	39.87	27.79	15.31	9.55
1.1	155.65	137.61	122.24	97.75	79.40	65.40	54.53	36.25	25.45	14.19	8.94
1.2	123.40	109.57	97.75	78.79	64.48	53.48	44.89	30.30	21.56	12.30	7.90
1.3	99.43	88.65	79.40	64.48	53.14	44.37	37.48	25.66	18.49	10.77	7.04
1.4	81.28	72.75	65.40	53.48	44.37	37.28	31.67	21.97	16.02	9.53	6.33
1.5	67.29	60.45	54.53	44.89	37.48	31.67	27.06	19.01	14.02	8.50	5.74
1.75	44.02	39.87	36.25	30.30	25.66	21.97	19.01	13.75	10.40	6.59	4.61
2	30.46	27.79	25.45	21.56	18.49	16.02	14.02	10.40	8.05	5.30	3.83
2.5	16.57	15.31	14.19	12.30	10.77	9.53	8.50	6.59	5.30	3.72	2.84
3	10.23	9.55	8.94	7.90	7.04	6.33	5.74	4.61	3.83	2.84	2.27

TABLE 10. **(Trivariate case): The values of ARL when r0 = 370; p = 3 and λ = 0.50.**

c_2	UCL=52.9877, c_3=1												
	c_1												
	1	1.01	1.02	1.03	1.04	1.05	1.1	1.5	2	2.5	3	5	8
1	370.01	362.79	355.77	348.93	342.27	335.79	305.81	160.38	85.67	52.02	34.47	11.08	4.29
1.01	362.79	355.74	348.87	342.18	335.68	329.34	300.03	157.68	84.39	51.32	34.05	10.98	4.27
1.02	355.77	348.87	342.15	335.62	329.26	323.06	294.40	155.04	83.14	50.64	33.64	10.88	4.24
1.03	348.93	342.18	335.62	329.23	323.01	316.95	288.92	152.47	81.92	49.97	33.24	10.79	4.22
1.04	342.27	335.68	329.26	323.01	316.93	311.00	283.58	149.96	80.73	49.31	32.84	10.70	4.20
1.05	335.79	329.34	323.06	316.95	311.00	305.21	278.38	147.50	79.56	48.67	32.45	10.61	4.17
1.1	305.81	300.03	294.40	288.92	283.58	278.38	254.28	136.07	74.08	45.65	30.62	10.18	4.06
1.5	160.38	157.68	155.04	152.47	149.96	147.50	136.07	77.86	45.19	29.29	20.47	7.63	3.36
2	85.67	84.39	83.14	81.92	80.73	79.56	74.08	45.19	27.90	18.98	13.82	5.76	2.80
2.5	52.02	51.32	50.64	49.97	49.31	48.67	45.65	29.29	18.98	13.43	10.09	4.60	2.42
3	34.47	34.05	33.64	33.24	32.84	32.45	30.62	20.47	13.82	10.09	7.79	3.83	2.15
5	11.08	10.98	10.88	10.79	10.70	10.61	10.18	7.63	5.76	4.60	3.83	2.34	1.58
8	4.29	4.27	4.24	4.22	4.20	4.17	4.06	3.36	2.80	2.42	2.15	1.58	1.26

20 observations are generated for from bivariate exponential distribution with $\theta_i = 0.21$, which is assumed to be in control. Next, we generate 30 more data from observations #21-50 for the shifted process ($c = 1.3$) transformed variables using $T_i^* = T_i^{1/3.6}$. We used $r_0 = 370$. The UCL is h = 186.38. We plotted the UCL and the data on the control chart in Figure 1.

From this Fig. 1, we note that the plotted values of Y for in control process for first 20 observations depicts our process is in control state We also plotted Y of the shifted process

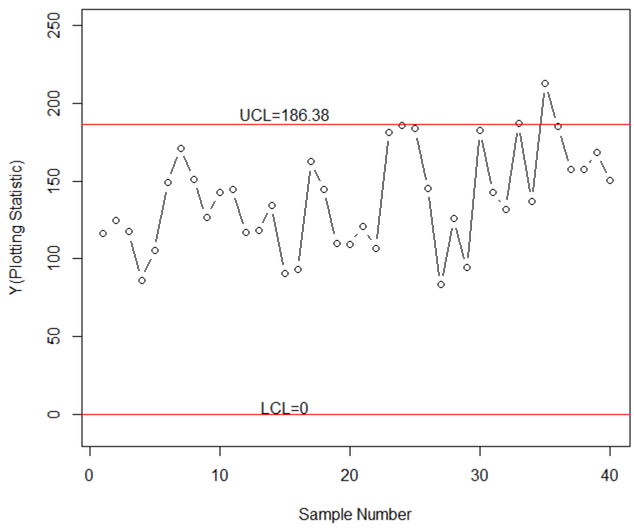

FIGURE 1. **The proposed chart for the simulated data.**

from observations #21-50 on the same chart with a shift of c = 1.50 in both the variables. From the chart, we note that the proposed control chart immediately detect the out-of-control signals before the 34^{th} observations. So, the proposed control chart is efficient in detecting the shifts in the process. The chart proposed by Santiago and Smith [29] shows the process is an in-control state in Figure 2.

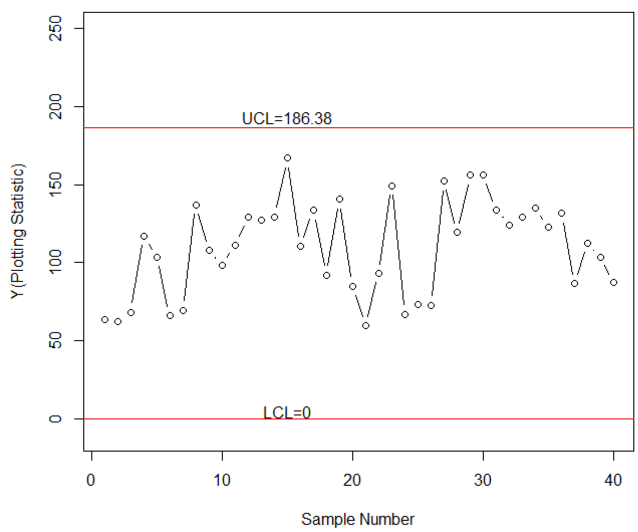

FIGURE 2. **The proposed chart by Santiago and Smith [29].**

IV. ADVANTAGES OF THE PROPOSED CHART

In order to access the efficiency of the proposed control chart, we have to compare the proposed new control chart scheme with the existing one. For the purpose of comparison, the average run length criteria of comparison of different control chart methodologies are considered as thumb rule by the researchers. In order to prove the validity of our proposed new control chart, we compared our charting procedure with the traditional Shewhart chart as in the case where we take $\lambda = 1.0$ our scheme reduces to Shewhart chart. Furthermore, we also considered the case where we have only one variable in order to monitor the process. Furthermore, we cannot assess the effectiveness of the proposed multivariate t-chart in the absence of a comparative study which should include competitor control charts. In fact, the ARL criterion is used to compare the performance of different control charts under given shifts in the process.In this section, we discuss the advantages of the proposed chart with two existing control chart. To compare the efficiency of charts, we selected the same values of all parameters. The control chart which provides the smaller values of ARL is said to be a more efficient control chart.

A. PROPOSED CHART VS TRADITIONAL CHART

The proposed chart reduces to traditional Shewhart type multivariate control chart when $\lambda = 1$. As indicated earlier comparison on the basis of ARL is traditional way for comparison of different control charts. For the purpose of comparison with the traditional Shewhart type multivariate t-charts, we constructed a separate Table 11 for the $r_0 = 370$ and p = 2. By comparing Table 11 with Table 1 of the proposed chart, it can be noted that for every combination of c_1 and c_2, the proposed chart provides the smaller values of ARLs. For example, when $c_1 = 1.05$ and $c_2 = 1.05$, the value of ARL from the proposed chart is 236 while it is 305 from the existing traditional control chart. So, the proposed chart is more efficient to indicate when the process is going out-of-control.

B. PROPOSED CHART VS SANTIAGO AND SMITH [29] CHART

The proposed chart is an extension of Santiago and Smith [29] chart. The proposed chart reduces to Santiago and Smith [29] chart when $p = 1$ and $\lambda = 1$. Now, we compare the efficiency of the proposed chart with Santiago and Smith [29] chart in terms of ARL. The values of ARLs for Santiago and Smith [29] chart are shown in Table 12.

By comparing Table 12 with Table 1 of the proposed chart, it can be noted that for every combination of c_1 and c_2, the proposed chart provides the smaller values of ARLs. For example, when $c_1 = 1.05$ and $c_2 = 1.05$, the value of ARL from the proposed chart is 236 while it is 332 from Santiago and Smith [29] control chart. So, the proposed chart is more efficient to indicate when the process is going out-of-control.

V. APPLICATION OF THE PROPOSED CHART

In this section, we discuss the application of proposed chart using temperature data and speed data. The same data is used by Zhang *et al.* [39]. According to Zhang *et al.* [39] "temperature (F) and wind speed (m.p.h.) measurements in Washington, DC., May-September, 1977; 133 observations

TABLE 11. (Bivariate case): The values of ARL when r0 = 370; p = 2 and λ = 1.

c_2	UCL=52.9876										
	c_1										
	1	1.05	1.1	1.2	1.3	1.4	1.5	1.75	2	2.5	3
1	370.00	335.78	305.81	256.12	216.99	185.71	160.38	114.94	85.67	52.01	34.47
1.05	335.78	305.20	278.38	233.81	198.61	170.40	147.50	106.25	79.56	48.67	32.45
1.1	305.81	278.38	254.28	214.15	182.38	156.85	136.06	98.51	74.08	45.65	30.62
1.2	256.12	233.81	214.15	181.29	155.13	134.01	116.74	85.32	64.69	40.42	27.42
1.3	216.99	198.61	182.38	155.13	133.34	115.66	101.15	74.58	56.98	36.07	24.73
1.4	185.71	170.40	156.85	134.01	115.66	100.72	88.41	65.72	50.57	32.40	22.44
1.5	160.38	147.50	136.06	116.74	101.15	88.41	77.86	58.33	45.19	29.29	20.47
1.75	114.94	106.25	98.51	85.32	74.58	65.72	58.33	44.46	34.98	23.27	16.62
2	85.67	79.56	74.08	64.69	56.98	50.57	45.19	34.98	27.90	18.98	13.82
2.5	52.01	48.67	45.65	40.42	36.07	32.40	29.29	23.27	18.98	13.43	10.09
3	34.47	32.45	30.62	27.42	24.73	22.44	20.47	16.62	13.82	10.09	7.79

TABLE 12. The values of ARL of Santiago and Smith [29] when r0 = 370; p = 1 and λ = 1.

	mean=22.0101		
UCL	32.5043	34.1062	34.9205
c	ARLs		
1	200.1067	300.201	370.002
1.05	175.8733	262.3691	322.4646
1.1	155.5284	230.766	282.8538
1.2	123.6452	181.582	221.4228
1.3	100.2023	145.7417	176.8605
1.4	82.5543	118.9866	143.7354
1.5	68.9962	98.5935	118.5875
1.75	46.4049	65.0182	77.4337
2	33.1068	45.5776	53.8032
2.5	19.1387	25.5579	29.7118
3	12.4693	16.2275	18.6212

TABLE 13. Temperature data (Zhang *et al.* [39]).

1025.81	829.85	534.79	497.67	497.67	461.82	461.82
461.82	427.25	427.25	393.96	393.96	393.96	361.96
331.25	331.25	331.25	301.83	301.83	301.83	301.83
273.72	273.72	246.92	246.92	246.92	246.92	246.92
246.92	221.43	221.43	221.43	221.43	221.43	221.43
197.27	197.27	197.27	197.27	197.27	197.27	197.27
197.27	174.43	174.43	174.43	174.43	174.43	174.43
152.94	152.94	152.94	152.94	152.94	152.94	152.94
152.94	152.94	132.79	132.79	132.79	132.79	132.79
132.79	132.79	132.79	114.00	114.00	114.00	114.00
114.00	114.00	96.58	96.58	96.58	96.58	96.58
80.54	80.54	80.54	80.54	80.54	80.54	80.54
65.88	65.88	65.88	65.88	65.88	65.88	65.88
65.88	65.88	65.88	52.64	52.64	52.64	52.64
52.64	52.64	40.81	40.81	40.81	30.42	30.42
30.42	30.42	30.42	30.42	30.42	30.42	30.42
21.49	21.49	21.49	21.49	21.49	21.49	14.05
14.05	14.05	14.05	14.05	14.05	14.05	14.05
14.05	8.12	8.12	8.12	3.75	3.75	0.00

made daily at noon. The dataset was obtained from Professor R. H. Shumway, Division of Statistics, University of California, Davis, and we are grateful to him for the use of his data." The data for both variables [39] showed that

TABLE 14. Wind speed data (Zhang *et al.* [39]).

0.00	0.33	0.70	0.84	0.84	0.84	0.84
0.84	1.00	1.00	1.00	1.34	1.34	1.52
1.91	1.91	2.12	2.12	2.12	2.12	2.56
2.56	2.56	2.56	2.56	2.79	2.79	3.28
3.28	3.28	3.28	3.53	3.53	3.53	3.53
3.53	3.53	3.79	3.79	4.33	4.33	4.61
4.61	4.61	5.19	5.19	5.19	5.19	5.49
5.49	5.49	5.49	6.11	6.11	6.11	6.11
6.11	6.11	6.43	6.43	6.75	6.75	6.75
7.42	7.42	7.76	7.76	7.76	8.47	8.47
8.82	8.82	8.82	8.82	8.82	9.56	9.56
9.56	9.94	9.94	9.94	10.70	10.70	10.70
10.70	10.70	10.70	11.10	11.10	11.10	12.30
12.30	12.71	13.56	13.56	13.56	13.56	13.56
13.98	14.85	14.85	14.85	15.30	15.30	15.30
15.74	15.74	19.49	19.98	19.98	19.98	20.98
20.98	20.98	21.99	21.99	23.02	23.54	23.54
24.59	25.12	25.12	28.98	30.12	30.70	34.23
34.23	36.06	38.54	43.04	49.10	49.10	66.65

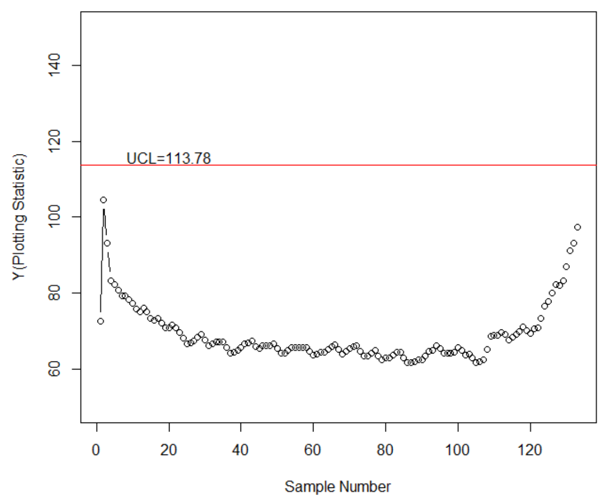

FIGURE 3. The proposed chart for the real data.

transformed data is well fitted to bivariate exponential distribution. The data of both variables reported in Tables 13-15. Let $r_0 = 370$, $p = 2$, and $\lambda = 0.5$.

TABLE 15. The values of Yj.

72.64	104.46	93.10	83.35	82.33	80.84	79.36
79.36	78.32	77.30	75.81	75.22	76.13	75.08
73.43	72.85	73.27	72.23	70.77	70.77	71.61
71.00	69.55	68.08	66.63	67.05	67.47	68.30
69.14	67.67	66.23	66.64	67.06	67.06	67.06
65.60	64.16	64.57	64.99	65.80	66.62	67.03
67.44	65.98	65.35	66.16	66.16	66.16	66.56
65.51	64.09	64.09	64.88	65.69	65.69	65.69
65.69	65.69	64.64	63.62	64.01	64.41	64.41
65.19	65.98	66.37	65.32	63.91	64.69	65.47
65.86	66.25	64.81	63.41	63.41	64.18	64.96
63.53	62.52	62.91	62.91	63.67	64.44	64.44
63.02	61.63	61.63	62.02	62.40	62.40	63.53
64.68	65.06	66.19	65.53	64.16	64.16	64.16
64.53	65.66	65.00	63.64	64.01	62.99	61.64
62.02	62.39	65.30	68.67	69.04	69.04	69.76
69.11	67.78	68.51	69.24	69.96	71.05	70.04
69.45	70.54	70.90	73.39	76.63	77.70	80.17
82.31	81.99	83.17	87.00	91.18	93.09	97.46

The UCL = 113.78 for this real data. The values of statistic Y_j are plotted on the control chart in Figure 3.

From Figure 3, we note that all values of statistic Y_j are within UCL which indicate that the process is in control.

VI. CONCLUDING REMARKS

In this paper, we proposed the multivariate t-chart for the exponential distribution where the quality characteristics of interest are independently distributed. The structure of the proposed is presented. The multivariate and bivariate cases are studies and tables are developed. The application of the proposed control chart using the simulation data from multivariate exponential is also presented in order to illustrate the application of proposed multivariate t-chart. The proposed control chart can be used to monitor more than one variable under study jointly by using the single proposed Multivariate t-chart. By comparing, bivariate case and trivariate case, we conclude that the UCL from the bivariate case is smaller than the trivariate case. Further, the ARLs from the bivariate case are smaller than the trivariate case and the ARLs decreases quickly in the bivariate case as shift increases. The proposed chart is more efficient than the existing control charts in terms of ARLs. The proposed control chart can be used in monitoring the health issues where we want to monitor the independent time events jointly. The proposed control chart can be extended for gamma distribution as a future research.

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Authors' photographs and biographies not available at the time of publication.