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Underdetermined Blind Source Separation of Synchronous Orthogonal Frequency-Hopping Signals Based on Tensor Decomposition Method

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ABSTRACT This paper considers the problems of synchronous orthogonal frequency-hopping (FH) signals separation and direction of arrival (DOA) estimation in the underdetermined situation. First of all, in order to estimate the mixing matrix of the FH signals, we stack the two-dimensional covariance matrices of the received FH signals into a third-order tensor, and then, an estimation approach based on the tensor decomposition method is developed to calculate this mixing matrix. Next, to separate these FH signals, a modified source recovery method is proposed. Finally, the DOAs of the FH signals are also obtained. Numerical results demonstrate that compared with the existing algorithms, the proposed algorithm not only presents superior performance but also requires less sparsity limitation and cost time.

INDEX TERMS Direction of arrival, frequency-hopping signals, mixing matrix, tensor decomposition.

I. INTRODUCTION

Frequency-hopping (FH) signal has been widely used in communication and radar systems due to the advance anti-jamming ability and low probability of interception [1], [2]. FH signal also presents a severe challenge to reconnaissance system in the realistic civilian and military applications. The FH signal reconnaissance usually contains two steps: (i) parameters estimation and (ii) signal sorting. Estimating parameters is the main task in the practical applications [3]–[5]. Therefore, estimating hop timing [10], frequency [11] and direction of arrival (DOAs) [15], [19], [25] are vital to sort multiple users without prior information. Time-frequency (TF) analysis is widely used to estimate the FH signal parameters [6]–[8]. The TF analysis algorithm proposed in [8] used the Wigner-Ville distribution (WVD), and it was shown that this algorithm can achieve better resolution, but incur severe cross-term interference. An algorithm based on Smoothed Pseudo Wigner-Ville distribution (SPWVD) to estimate the hopping period proposed in [7]. To estimate the hop rate of the FH signal, an algorithm based on Reassigned Smoothed Pseudo Wigner-Ville distribution (RSPWVD) was proposed in [6]. Both algorithms proposed in [6] and [7] can overcome the cross-term interference. However, these algorithms can only handle one single FH signal.

In order to deal with the parameters estimation of multiple FH signals, many methods have been proposed [9], [10]. An algorithm was proposed in [9] based on the principle of dynamic programming, which combines two-dimensional harmonic retrieval and low-rank decomposition to estimate FH signals parameters. A new approach based on sparse linear regression to estimate the hop timing was proposed in [10]. Although these algorithms can estimate the parameters of multiple FH signals, the subsequent signal sorting can be executed only when these FH signals are with different hop timing rules.

In order to sort synchronous orthogonal FH signals, the hop timings of which are with the same rule, Blind Source Separation (BSS) technology is thus introduced [11], [12], which has been widely used in audio signal processing, biomedical signal processing, wireless communication, and radar signals processing [13], [14], [16]–[18], [20]. In the most practical situations, the number of the received signals is less than that of source signals, which is called Underdetermined Blind Source Separation (UBSS) problem. The UBSS problem can be solved by two steps: (i) the mixing matrix estimation and (ii) the source recovery. TF analysis is also an effective way to solve the UBSS problem [21]–[24], [26]–[30]. The algorithms in [11] and [12] separated FH signals in the underdetermined situation via TF analysis. Specifically, the algorithm in [11] constructed the TF ratio matrix, and it assumed that there exists only one FH signal at each TF point. Particularly, it clusters the Euclidean distance between different columns of the TF ration matrix to calculate the mixing matrix. However, the clustering method in [11] not only requires much cost time, but also ignores the time-varying structure of the FH mixing matrix. The algorithm in [12] considered the time-varying structure of the FH mixing matrix, and estimated the hop timings by calculating the number of FH signals at each sample with the assumption that each TF point is a single source point. However, the methods in [11] and [12] are limited by the harsh sparse constraints, and thus the performance is not satisfactory.

In order to improve the performance of separating FH signals, this paper is inspired by using a multi-linear algebra tool named Canonical Polyadic (CP) decomposition which is also known as Parallel Factor (PARAFAC) decomposition [31], [32] to estimate the mixing matrix of FH signals. In this paper, the two-dimensional covariance matrices of FH signals are stacked into a third-order tensor, and then the CP decomposition can deal with the UBSS problem of FH signals. Next, an improved FH signals recovery method is proposed to separate FH signals, and the parameters of FH signals, including the frequencies and the DOAs, is also estimated. Because the DOAs of hopping fragments in an FH signal are the same, the FH signals can be sorted based on the estimated DOAs. This paper aims to separate the synchronous orthogonal FH signals and estimate the parameters of synchronous orthogonal FH signals. The proposed work includes two contributions. The proposed method can reduce the limitation of the signal sparsity. Also, the proposed method can reduce the cost time while improves the estimation performance.

The rest of the paper is organized as follows. The problem is formulated in Section II. In Section III, the proposed method is introduced. In Section IV, the simulation results are provided to demonstrate the validity of the proposed algorithm. Finally, the conclusions are given in Section V.

Notation: a^* denotes the conjugate of a. $\circ$ denotes the outer product. $\odot$ denotes the Khatri-Rao product. $E[\cdot]$ denotes expectation operation. $r(\mathbf{A}) = N$ means that the rank of the matrix $\mathbf{A}$ is equal to N. $\underline{\mathbf{A}}$ represents the submatrix of $\mathbf{A}$ without the first row. $\overline{\mathbf{A}}$ represents the submatrix of $\mathbf{A}$ without the last row. $\mathbf{A}(m:n,:)$ represents the submatrix of $\mathbf{A}$ consisting of the rows from m to n of $\mathbf{A}$. $\mathbf{A}^*$, $\mathbf{A}^{-1}$, $\mathbf{A}^{\mathrm{H}}$, $\mathbf{A}^{\mathrm{T}}$ and $\mathbf{A}^{\dagger}$ denote as conjugate, inverse, conjugate-transpose, transpose and Moore-Penrose pseudo-inverse of $\mathbf{A}$. $\mathrm{Im}(a)$ means the imaginary part of a. $\|\mathbf{A}\|$ denotes the l_2 norm of $\mathbf{A}$.

II. THE PROBLEM FORMULATION

Suppose that the FH signals $\mathbf{s}(t) = [s_1(t), \ldots, s_N(t)]^{\mathrm{T}}$ impinge instantaneously onto an M-element array. The nth FH signal is written as:

$$s_n(t) = b_n(t)e^{j(2\pi f_n(t)t+\varphi_n(t))}, \quad 0 < t \leq T, \tag{1}$$

where $b_n(t)$, $f_n(t)$ and $\varphi_n(t)$ are the amplitude, instantaneous frequency, and phase of the nth FH signal, respectively. T is the observation time. The propagation time-delay from the mth antenna to the first antenna of the nth source can be formulated as follow:

$$\tau_{m,n} = \frac{1}{c}(m-1)r\cos\theta_n, \quad m = 1, \ldots, M, \quad n = 1, \ldots, N, \tag{2}$$

where c denotes the speed of the light, r denotes the element spacing, and θ_n denotes the DOA of the nth FH signal. Assume that the FH signals are not hopping in the delay time, thus

$$\begin{aligned} b_n(t-\tau_{m,n}) &\approx b_n(t), \\ f_n(t-\tau_{m,n}) &\approx f_n(t), \\ \varphi_n(t-\tau_{m,n}) &\approx \varphi_n(t). \end{aligned} \tag{3}$$

The mth received signal can be formulated as:

$$x_m(t) = \sum_{n=1}^{N} s_n(t)e^{-j2\pi f_n(t)\tau_{m,n}} + v_m(t), m = 1, \ldots, M, \tag{4}$$

where $v_m(t)$ is the additive Gaussian noise signal with mean 0 and variance σ^2. Then, the vector formulation of (4) can be expressed as:

$$\mathbf{x}(t) = \mathbf{A}(t)\,\mathbf{s}(t) + \mathbf{v}(t), \quad 0 < t \leq T, \tag{5}$$

where $\mathbf{x}(t) = [x_1(t), \ldots, x_M(t)]^{\mathrm{T}}$, $\mathbf{v}(t) = [v_1(t), \ldots, v_M(t)]^{\mathrm{T}}$, and $\mathbf{s}(t) = [s_1(t), \ldots, s_N(t)]^{\mathrm{T}}$. Also, $\mathbf{A}(t)$ is called the mixing matrix, and formulated by:

$$\mathbf{A}(t) = \begin{bmatrix} e^{-j2\pi f_1(t)\tau_{1,1}} & \cdots & e^{-j2\pi f_N(t)\tau_{1,N}} \\ e^{-j2\pi f_1(t)\tau_{2,1}} & \cdots & e^{-j2\pi f_N(t)\tau_{2,N}} \\ \vdots & \ddots & \vdots \\ e^{-j2\pi f_1(t)\tau_{M,1}} & \cdots & e^{-j2\pi f_N(t)\tau_{M,N}} \end{bmatrix}. \tag{6}$$

When the number of the receiving antennas M is smaller than the number of the FH signals N, the mixing system is underdetermined. In this situation, sorting the synchronous orthogonal FH signals becomes a UBSS problem.

III. THE PROPOSED METHOD

To solve the UBSS problem, the proposed method consists of two steps: (i) estimating the mixing matrix and (ii) recovering the FH signals. In the following, we present the proposed method in detail.

A. THE MIXING MATRIX ESTIMATION METHOD

In this subsection, the method to estimate the mixing matrix of the synchronous orthogonal FH signals is introduced. According to (2) and (6), it is noticed that the mixing matrix $\mathbf{A}(t)$ is a Vandermonde factor. Because the carrier frequency of the synchronous orthogonal FH signal is varying in each hop duration, the mixing matrix also varies over the different hop durations. If the hop timings are known, the mixing matrix keeps constant in the hop duration. In this paper, the hop timings are estimated firstly by the method in [35].

Thus, in the pth hop duration, without considering the noise interference, the mixing matrix and the received FH signals can be respectively rewriter as follows:

$$\mathbf{A} = \begin{bmatrix} 1 & \cdots & 1 \\ a_{2,1} & \cdots & a_{2,N} \\ \vdots & \ddots & \vdots \\ a_{2,1}^{M-1} & \cdots & a_{2,N}^{M-1} \end{bmatrix}, \tag{7}$$

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) = \sum_{n=1}^{N} \mathbf{a}_n s_n(t), t_{p-1} < t \leq t_p, \tag{8}$$

where $a_{2,n} \triangleq e^{-j2\pi f_n r \cos\theta_n / c}$ is the second element in the nth column of the mixing matrix, f_n is the frequency of the nth FH signal in this hop duration, and $\mathbf{a}_n = \left[1, a_{2,n} \ldots, a_{2,n}^{M-1}\right]^{\mathrm{T}}$ is the nth column of the mixing matrix.

Suppose that N FH signals are independent of each other in the time domain. The kth spatial covariance matrix of the received FH signals is written as:

$$\mathbf{C}_k = E\left[\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t+\gamma_k)\right] = \mathbf{A}\mathbf{P}_k\mathbf{A}^{\mathrm{H}}, \tag{9}$$

where $\mathbf{P}_k = E\left[\mathbf{s}(t)\mathbf{s}^{\mathrm{H}}(t+\gamma_k)\right] \in \mathbb{C}^{N\times N}$ is a diagonal matrix and γ_k is the kth delay time. Let $\{\mathbf{C}_1, \mathbf{C}_2 \ldots, \mathbf{C}_K\}$ denote K spatial covariance matrices with different delay time and stack the matrixes $\{\mathbf{C}_1, \mathbf{C}_2 \ldots, \mathbf{C}_K\}$ into a tensor $\mathcal{X} \in \mathbb{C}^{M\times M\times K}$. Thus, the element in $\mathcal{X}$ is represented as:

$$\mathcal{X}_{i,j,k} = (\mathbf{C}_k)_{i,j}, \tag{10}$$

where $(\mathbf{C}_k)_{i,j}$ represents the ith element in the jth column of the matrix $\mathbf{C}_k$. Define a matrix $\mathbf{D} \in \mathbb{C}^{K\times N}$ and the kth element in the nth column $d_{k,n}$ is represented as:

$$d_{k,n} = (\mathbf{P}_k)_{n,n}, k = 1, \ldots, K, n = 1, \ldots, N, \tag{11}$$

where $(\mathbf{P}_k)_{n,n}$ means the nth element in the nth column of the matrix $\mathbf{P}_k$, and N is the number of the FH signals. Then, the tensor $\mathcal{X}$ can be written as [33]:

$$\mathcal{X} = \sum_{n=1}^{N} \mathbf{a}_n \circ \mathbf{a}_n^* \circ \mathbf{d}_n, \tag{12}$$

where $\mathbf{d}_n$, $\mathbf{a}_n = \left[1, a_{2,n}, \ldots, a_{2,n}^{M-1}\right]^{\mathrm{T}}$, and $\mathbf{a}_n^* = \left[1, a_{2,n}^*, \ldots, a_{2,n}^{*M-1}\right]^{\mathrm{T}}$ are the nth columns of $\mathbf{D}$, $\mathbf{A}$ and $\mathbf{A}^*$, respectively. If the rank of the tensor $\mathcal{X}$ is N, (12) is called CP decomposition [34]. The element in the tensor can also be written as:

$$\mathcal{X}_{i,j,k} = \sum_{n=1}^{N} a_{2,n}^{i-1} a_{2,n}^{*j-1} d_{k,n}. \tag{13}$$

The mixing matrix $\mathbf{A}$ is considered as the first factor matrix of the tensor $\mathcal{X}$. The matrix $\mathbf{A}^*$ and the matrix $\mathbf{D}$ are considered as the second and the third matrix factors, respectively. In order to obtain the unique result of the mixing matrix estimation, there exist some limitations. The unique result means that (12) is the only possible combination of the tensor with the exception of the elementary indeterminacies of scaling and permutation. The unique mixing matrix estimator can be obtained under the following conditions [31]:

$$r\left(\mathbf{A}(1:K_1,:) \odot \mathbf{A}^*(1:K_2,:)\right) = N, \tag{14}$$

$$r\left(\mathbf{A}(1:L_1,:) \odot \mathbf{A}^*(1:L_2,:) \odot \mathbf{D}\right) = N, \tag{15}$$

where $K_1 + L_1 = M + 1$ and $K_2 + L_2 = M + 1$. M is the number of antennas and N is the number of FH signals. The process of obtaining the unique solution is shown in Appendix. Construct a tensor $\mathcal{Y} \in \mathbb{C}^{K_1\times K_2\times L_1\times L_2\times K}$ as follows:

$$\mathcal{Y}_{k_1,k_2,l_1,l_2,k} = \mathcal{X}_{k_1+l_1-1,k_2+l_2-1,k}. \tag{16}$$

The matrix representation of the tensor $\mathcal{Y}$ can be written as follows:

$$\begin{aligned}\mathbf{Y} &= \left(\mathbf{A}(1:K_1,:) \odot \mathbf{A}^*(1:K_2,:)\right) \\ &\quad \times \left(\mathbf{A}(1:L_1,:) \odot \mathbf{A}^*(1:L_2,:) \odot \mathbf{D}\right) \\ &= \begin{bmatrix} \mathcal{Y}_{1,1,1,1,1} & \mathcal{Y}_{1,1,1,1,2} & \cdots & \mathcal{Y}_{1,1,L_1,L_2,K} \\ \mathcal{Y}_{1,2,1,1,1} & \mathcal{Y}_{1,2,1,1,2} & \cdots & \mathcal{Y}_{1,2,L_1,L_2,K} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{Y}_{K_1,K_2,1,1,1} & \mathcal{Y}_{K_1,K_2,1,1,2} & \cdots & \mathcal{Y}_{K_1,K_2,L_1,L_2,K} \end{bmatrix}.\end{aligned} \tag{17}$$

It means that when the matrix $\mathbf{D}$ is rank deficient, the unique mixing matrix estimation can still be obtained. In summary, the proposed algorithm is given in Algorithm 1.

Algorithm 1 The Mixing Matrix Estimation Algorithm

1: Input: The receiving FH signals $\mathbf{x(t)} = [x_1(t), \cdots, x_M(t)]^{\mathrm{T}}$
2: Step1: Calculate the spatial covariance matrices $\{\mathbf{C}_1, \mathbf{C}_2 \ldots, \mathbf{C}_K\}$ based on (9) and stack these matrices into the tensor $\mathcal{X} \in \mathbb{C}^{M\times M\times K}$.
3: Step2: Construct the matrix $\mathbf{Y}$ in (17) based on (16).
4: Step3: Calculate the Singular Value Decomposition of $\mathbf{Y} = \mathbf{U}\Sigma\mathbf{V}^{\mathrm{H}}$.
5: Step4: Set $\mathbf{U}_1 = \mathbf{U}(1:(K_1-1)K_2,:) \in \mathbb{C}^{(K_1-1)K_2\times N}$ and $\mathbf{U}_2 = \mathbf{U}(K_2+1:K_1K_2,:) \in \mathbb{C}^{(K_1-1)K_2\times N}$, and calculate the eigenvector $\mathbf{Z} = \left[a_{2,1}, \ldots, a_{2,N}\right]$ of $\mathbf{U}_1^{\dagger}\mathbf{U}_2$ according to (26) and (31).
6: Step5: Estimate the mixing matrix $\hat{\mathbf{A}}$ based on (32).
7: Output: The mixing matrix estimation $\hat{\mathbf{A}}$.

B. THE FH SIGNALS RECOVERY AND DOA ESTIMATION METHOD

In this subsection, the FH signals recovery method is introduced. Since the FH signals separation methods in [11] and [12] are limited by the strict sparsity condition, a novel method is exploited to reduce the sparse limitation.

The Short Time Fourier Transformation (STFT) is a linear TF analysis method, and the STFT representation of (8) is written as:

$$\mathbf{X}(t,f) = \mathbf{A}\mathbf{S}(t,f), \tag{18}$$

where $\mathbf{X}(t,f)$ and $\mathbf{S}(t,f)$ are the STFT of $\mathbf{x}(t)$ and $\mathbf{s}(t)$, respectively.

Suppose that there exists g $(g \leq M)$ FH signals at each TF point and g is a variable number. To improve the performance of the recovery method, the residual power is considered at each TF point. The orthogonal project matrix of the mixing matrix is utilized to calculate the residual power, and a threshold is set to approximate the residual power at each TF point. If the residual power of g FH signals $\Gamma(g)$ is less than the set threshold, the corresponding g FH signals at this TF point can be determined. If the residual power of a small number of FH signals is already less than the threshold, it is unnecessary to calculate the residual power of more FH signals, because we only need to detect signals that make major contribution to this TF point. The main steps of the proposed method are summarized in Algorithm 2. The Algorithm 2 aims to estimate the TF representation of g FH signals at the TF point (t,f). In Algorithm 2, $\mathbf{A}_g^{(i)} = [\mathbf{a}_{i_1}, \ldots, \mathbf{a}_{i_g}]$, $1 \leq i \leq C_N^g$, $C_N^g = N(N-1)\cdots(N-g+1)/N!$, and the columns are arranged in ascending order of subscript, and the subscript is a random integer in the set $\{1, 2, \ldots, N\}$. The threshold of the residual power is calculated by $\varepsilon \|\mathbf{X}(t,f)\|_2$, where ε is an experience value.

Algorithm 2 The FH Signals Recovery Algorithm

1: Input: the mixing matrix $\mathbf{A}$
2: **for** $g = 1 : M$ **do**
  $\Gamma(g) = \min_i \left\| \mathbf{Q}_g^{(i)} \mathbf{X}(t,f) \right\|_2$, where $\mathbf{Q}_g^{(i)} = \mathbf{I} - \mathbf{A}_g^{(i)} \left(\mathbf{A}_g^{(i)H} \mathbf{A}_g^{(i)} \right)^{-1} \mathbf{A}_g^{(i)H}$ is the orthogonal project matrix of the mixing matrix $\mathbf{A}_g^{(i)}$.
3: **if** $\Gamma(g) < \varepsilon \|\mathbf{X}(t,f)\|_2$ **then**
    $\hat{\mathbf{A}}_g = \arg\min \left\| \mathbf{Q}_g^{(i)} \mathbf{X}(t,f) \right\|_2$, where $\hat{\mathbf{A}}_g$ is the corresponding mixing matrix of the g FH signals.
    $\hat{\mathbf{S}}_g(t,f) = \left(\hat{\mathbf{A}}_g^H \hat{\mathbf{A}}_g \right)^{-1} \hat{\mathbf{A}}_g \mathbf{X}(t,f)$, where $\hat{\mathbf{S}}_g(t,f)$ is the TF representation of the g estimated FH signals at the TF point (t,f).
    break
4: **end if**
5: **end for**
6: Output: The TF representation $\hat{\mathbf{S}}_g(t,f)$.

Utilize inverse STFT to recover FH signals $\hat{\mathbf{s}}(t) = [\hat{s}_1(t), \ldots, \hat{s}_N(t)]^{\mathrm{T}}$. Then, the frequencies $\hat{f}_n$, $n \in \{1, 2, \ldots, N\}$ are estimated by the fast Fourier transformation (FFT) operator. According to (7), the DOAs of FH signals in the hop duration are estimated by

$$\hat{\theta}_n = \arccos \frac{-\operatorname{Im}\left(\ln \hat{a}_{2,n}\right) c}{2\pi \hat{f}_n r}, \tag{19}$$

where $\hat{a}_{2,n}$ is the second value in the nth column of the estimated mixing matrix $\hat{\mathbf{A}}$. DOAs cannot change in a short time. The DOAs of the segments belonging to the same FH signal are the same. Therefore, comparing the DOAs of FH signals in each hop duration is an effective way to sort FH signals.

TABLE 1. The parameters of FH signals.

Signals	Frequency(MHz)	DOA(degree)
Signal1	27, 33	100
Signal2	55, 61	80
Signal3	46, 52	30
Signal4	60, 66	40

IV. SIMULATIONS

In this section, numerical simulation results are given to demonstrate the advantages of the proposed algorithms. In all simulations, the number of antenna $M = 3$. Suppose the length of data L=256 and the number of FH signals N=4. The parameters of FH signals are shown in Table 1. The hop timings are estimated by the method in [35]. In the following simulations, signal-to-noise ratio (SNR) is written as follow:

$$\mathrm{SNR} = 10\log_{10}\left(\frac{\|\mathbf{s}(t)\|_2^2}{N\sigma^2}\right), \tag{20}$$

where σ^2 is the variance of the additive Gaussian noise.

The performance of the proposed methods is evaluated by using the following measures.

First, the Mean Square Error (MSE) of the estimated mixing matrix is shown as follow:

$$\mathrm{MSE} = 10\log_{10}\left(\frac{\left\|\mathbf{A} - \hat{\mathbf{A}}\right\|_2^2}{N}\right), \tag{21}$$

where $\mathbf{A}$ and $\hat{\mathbf{A}}$ denote the true mixing matrix and the estimated mixing matrix, respectively. A small MSE indicates superior quality.

Second, the correlation coefficient between the source signals and the separated signals is shown as follow:

$$\zeta = \frac{1}{N} \sum_{n=1}^{N} \frac{\left| \mathrm{E}\left(s_n(t)\, \hat{s}_n^*(t) \right) \right|}{\sqrt{\mathrm{E}\left(|s_n(t)|^2 \right)} \sqrt{\mathrm{E}\left(\left| \hat{s}_n(t) \right|^2 \right)}}, \tag{22}$$

where $s_n(t)$ and $\hat{s}_n(t)$ denote the nth source signal and the nth separated signal, respectively. A large ζ indicates superior quality.

Third, the Normalized Mean Square Error (NMSE) of the estimated frequencies is shown as follow:

$$\mathrm{NMSE} = \sqrt{\frac{1}{N} \sum_{n=1}^{N} \frac{\left(f_n - \hat{f}_n \right)^2}{f_n^2}}, \tag{23}$$

where f_n and $\hat{f}_n$ denote the true frequency and the estimated frequency of the nth FH signal, respectively. A small NMSE represents high estimation accuracy.

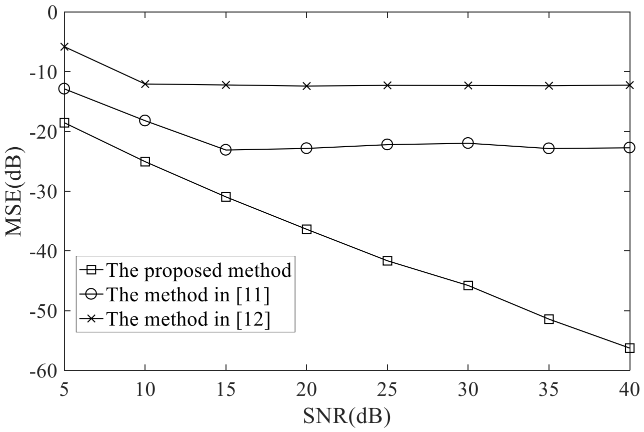

FIGURE 1. The MSE comparison of different methods.

Fourth, the Root Mean Square Error (RMSE) of the estimated DOA is shown as follow:

$$\text{RMSE}=\sqrt{\frac{\sum_{n=1}^{N}\left(\theta_n-\hat{\theta}_n\right)^2}{N}}, \tag{24}$$

where θ_n and $\hat{\theta}_n$ denote the original DOA and the estimated DOA of the nth signal, respectively. A small RMSE represents high estimation accuracy.

In the following simulations, the proposed method is mainly compared with the methods in [11] and [12].

A. THE MIXING MATRIX ESTIMATION

First, the noiseless case is considered. The original mixing matrix $\mathbf{A}$ and the estimated mixing matrix $\hat{\mathbf{A}}$ are shown at the bottom of this page. It can be seen that $\hat{\mathbf{A}}$ is the same with $\mathbf{A}$. It means that the proposed method can exactly estimate the mixing matrix in the noiseless case. It is because that the proposed method does not exploit clustering methods to approximate the result.

Second, the noise case is considered. The mean MSE values over 100 Monte Carlo trails for varying SNR are shown in Fig. 1. It can be seen that the proposed method can get smaller MSE values than other algorithms. Due to the limitation of the sparsity, the MSE performance of the methods in [11] and [12] is not satisfactory. The methods in [11] and [12] exploit clustering methods to approximate the mixing matrix, and thus the MSE performance of the two methods is smooth when SNR increases to a certain value. The proposed method does not utilize the clustering methods or iteration methods to approximate the mixing matrix, and it can exactly estimate the mixing matrix in the noiseless case. So the MSE of the proposed algorithm is decreasing with the increasing SNR.

TABLE 2. The CPU time (s) of the methods.

SNR(dB)	5	10	15	20	25	30
The proposed method	0.0188	0.0192	0.0193	0.0196	0.0191	0.0193
The method in [11]	6857	480.7	168.9	163.4	162.2	162.7
The method in [12]	1.774	1.992	1.981	2.001	1.988	2.109

Third, in order to analyze the complexity of these methods, CPU time is calculated. Our simulations are performed in MATLAB R2016b using Intel Xeon CPU E5-1620 3.50GHz processor. The operating system is Microsoft Windows 10 and the memory is 32GB. The average CPU time (s) against SNR level is shown in Table 2. The main cost step of the methods in [11] and [12] is determining satisfactory TF points. The method in [11] calculates the Euclidean distance of the columns of the TF matrix to determine the TF points belonging to the same FH signal, and it costs so much time at this step. The proposed method costs less CPU time because it is a simple algebraic operation, and it does not include the steps of selecting TF points.

Fourth, the effect of hop timing error on the MSE of the estimated mixing matrix is discussed. Fig. 2 shows the impact of different hop timing errors on the mixing matrix estimation with different data lengths. If the length of the data L=256, the MSE is smaller than -20 dB when the hop timing error is smaller than 10 samples. If the length of data L=512, the MSE is smaller than -20 dB when the hop timing error is smaller than 20 samples. Although the performance is affected by the hop timing error, it can be concluded that the proposed method can still obtain satisfactory performance when the hop timing error is not large. When the hop timing error is smaller than 10 samples with SNR from 10 dB to 40 dB, the proposed method is an effective way to separate FH signals and estimate the parameters of FH signals. The hop timing error which is calculated by the method in [35] is shown in Fig. 3. The hop timing error is smaller than 4 samples when SNR is changing from 10 dB to 40 dB, and it meets the requirement of the proposed method for the hop timing accuracy.

B. THE FH SIGNALS RECOVERY AND PARAMETERS ESTIMATION

First, the correlation coefficients of the separated signals in the time domain are shown in Fig. 4. The correlation coefficients of the proposed method are higher than that of

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.9700 + 0.2430i & 0.8775 - 0.4795i & -0.4926 - 0.8703i & -0.9989 - 0.0477i \\ 0.8819 + 0.4715i & 0.5402 - 0.8415i & -0.5147 + 0.8574i & 0.9954 + 0.0953i \end{bmatrix}$$

$$\hat{\mathbf{A}} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.9700 + 0.2430i & 0.8775 - 0.4795i & -0.4926 - 0.8703i & -0.9989 - 0.0477i \\ 0.8819 + 0.4715i & 0.5402 - 0.8415i & -0.5147 + 0.8574i & 0.9954 + 0.0953i \end{bmatrix}$$

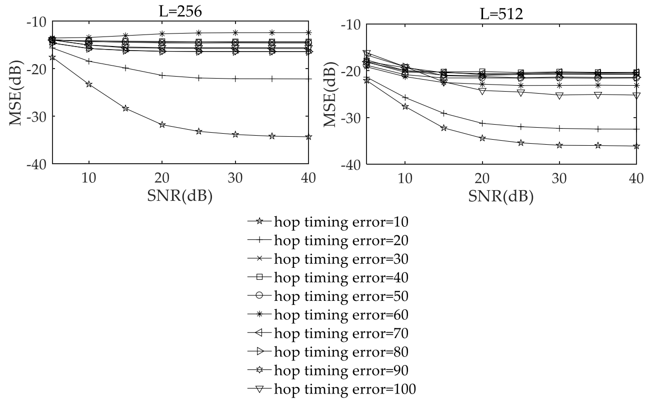

FIGURE 2. The effect of hop timing error on the mixing matrix estimation.

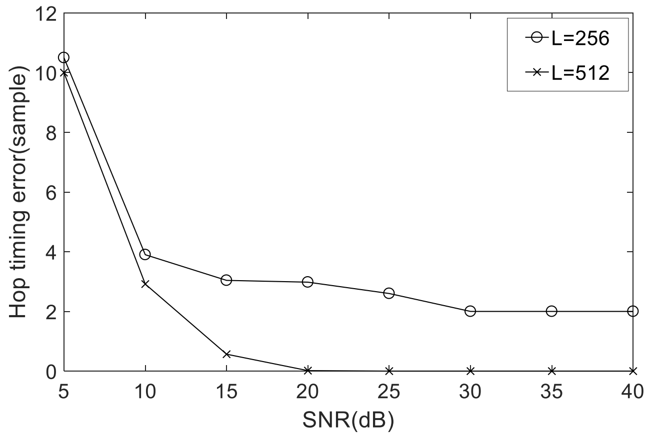

FIGURE 3. The hop timing error for different data lengths.

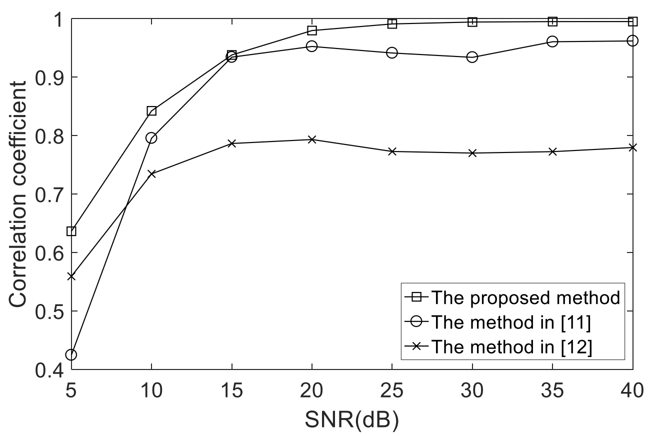

FIGURE 4. The correlation coefficient comparison of different methods.

other methods with SNR from 5 dB to 40 dB. The correlation coefficients are closed to 1 with SNR from 25 dB to 40 dB. It can be concluded that the signals recovered by the proposed method are more similar to the original signals.

Second, the NMSE of the estimated frequencies is shown in Fig. 5. It can be seen that the proposed method obtains smaller NMSE of the estimated frequencies, and the proposed method achieves better performance. The method in [11] has the large NMSE in low SNR because it is difficult to select

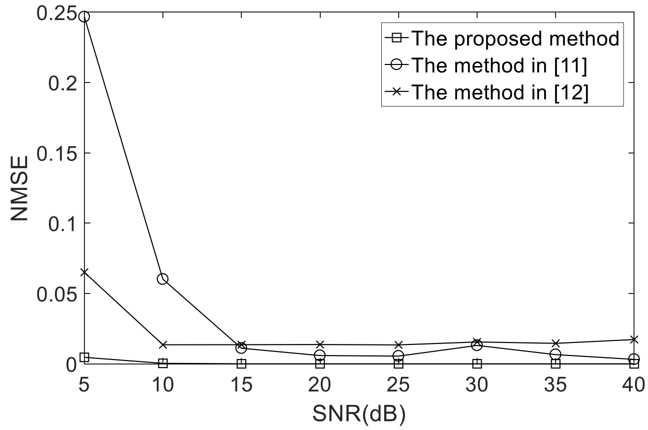

FIGURE 5. The NMSE of the methods.

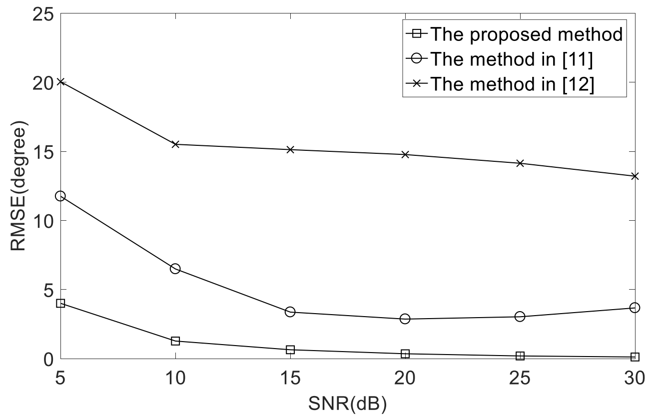

FIGURE 6. The RMSE comparison of different methods.

TABLE 3. The RMSE (degree) of the proposed method.

SNR		15dB	20dB	30dB
Hop duration1	DOA Estimation) (degree)	99.3558 79.8497 30.2542 39.4464	100.0536 80.0481 30.0902 39.7457	100.1464 79.9409 30.0846 39.9509
	RMSE	0.6444	0.3540	0.1210
Hop duration2	DOA Estimation) (degree)	100.0677 80.2330 29.2983 40.1982	100.1530 79.9648 29.7969 40.3015	99.8355 80.0248 30.0464 40.0910
	RMSE	0.7822	0.3941	0.1412

satisfactory TF points in low SNR, and this also leads to bad mixing matrix estimation performance. The method in [12] obtains large NMSE because this method clusters frequencies at each sample in the TF domain and the frequency error is large.

Third, the RMSE of DOA is shown in Fig. 6. Since the performance of the estimated frequencies and the estimated mixing matrix is not satisfactory, the methods in [11] and [12] achieve large RMES of the estimated DOAs. It can be seen that the proposed method obtains smaller RMSE than other methods. It means that the proposed method obtains more accurate estimation.

Fourth, to sort FH signals, DOAs of different hop durations are calculated. Table 3 lists the mean values of RMSE and the estimated DOAs of two hop durations. It can be seen that the RMSE is smaller than 1 degree when SNR is higher than 15dB. The error of the estimated DOA is mainly caused by the error of the estimated mixing matrix in low SNR. In the future work, it will be researched to estimate the mixing matrix in low SNR.

V. CONCLUSIONS

This paper proposes a novel UBSS and parameters estimation method of FH signals. The proposed method exploits the tensor decomposition and an improved recovery method to separate the FH signals and estimate the parameters of FH signals in the underdetermined situation. The proposed method obtains more accurate mixing matrix estimation and higher correlation coefficient of separated FH signals with less cost time. The estimated frequencies and DOAs are also more satisfactory than other methods. In the future, how to improve the performance of mixing matrix estimation in low SNR will be researched.

APPENDIX

This part is shown how to estimate the mixing matrix from the matrix $\mathbf{Y}$.

Let $\mathbf{Y} = \mathbf{U}\Sigma\mathbf{V}^{\mathrm{H}}$ denotesthe Singular Value Decomposition (SVD) of $\mathbf{Y}$. If the conditions are satisfied in (14) and (15), there exists a nonsingular matrix $\mathbf{G} \in \mathbb{C}^{N\times N}$ such that

$$\mathbf{UG} = \mathbf{A}\left(1 : K_1, :\right) \odot \mathbf{A}^*\left(1 : K_2, :\right). \tag{25}$$

According to the Vandermonde structure of the mixing matrix $\mathbf{A}$, it can be obtained as follow:

$$\left(\bar{\mathbf{A}}\left(1 : K_1, :\right) \odot \mathbf{A}^*\left(1 : K_2, :\right)\right)\mathbf{Z} = \bar{\mathbf{A}}\left(1 : K_1, :\right) \odot \mathbf{A}^* \tag{26}$$

where $\mathbf{Z} = \left[a_{2,1}, \ldots, a_{2,N}\right]$.

According to (25), it can be implied that

$$\mathbf{U}_1\mathbf{G} = \bar{\mathbf{A}}\left(1 : K_1, :\right) \odot \mathbf{A}^*\left(1 : K_2, :\right), \tag{27}$$

$$\mathbf{U}_2\mathbf{G} = \bar{\mathbf{A}}\left(1 : K_1, :\right) \odot \mathbf{A}^*\left(1 : K_2, :\right), \tag{28}$$

where $\mathbf{U}_1 = \mathbf{U}\left(1 : \left(K_1 - 1\right)K_2, :\right) \in \mathbb{C}^{(K_1-1)K_2\times N}$, $\mathbf{U}_2 = \mathbf{U}\left(K_2 + 1 : K_1K_2, :\right) \in \mathbb{C}^{(K_1-1)K_2\times N}$.

According to (26), (27) and (28), the following equalities can be obtained

$$\begin{aligned}\mathbf{U}_2\mathbf{G} &= \bar{\mathbf{A}}\left(1 : K_1, :\right) \odot \mathbf{A}^*\left(1 : K_2, :\right)\\ &= \left(\bar{\mathbf{A}}\left(1 : K_1, :\right) \odot \mathbf{A}^*\left(1 : K_2, :\right)\right)\mathbf{Z} = \mathbf{U}_1\mathbf{GZ}.\end{aligned} \tag{29}$$

Then, it can be obtained that

$$\mathbf{U}_2 = \mathbf{U}_1\hat{\mathbf{Z}}, \tag{30}$$

where $\hat{\mathbf{Z}} = \mathbf{GZG}^{-1}$ is the Eigen Value Decomposition (EVD) of $\hat{\mathbf{Z}}$.

Assume that $r\left(\bar{\mathbf{A}}\left(1 : K_1, :\right) \odot \mathbf{A}^*\left(1 : K_2, :\right)\right) = N$, the representation of $\hat{\mathbf{Z}}$ is written as follow:

$$\hat{\mathbf{Z}} = \mathbf{U}_1^{\dagger}\mathbf{U}_2. \tag{31}$$

From EVD of $\hat{\mathbf{Z}}$, the vector $\mathbf{Z} = \left[a_{2,1}, \ldots, a_{2,N}\right]$ can be obtained. According to the Vandermonder structure of the mixing matrix $\mathbf{A}$ in (8), it can be obtained as follow:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ a_{2,1} & a_{2,2} & \ldots & a_{2,N} \\ \vdots & \vdots & \vdots & \vdots \\ a_{2,1}^{M-1} & a_{2,2}^{M-1} & \ldots & a_{2,N}^{M-1} \end{bmatrix} \in \mathbb{C}^{M\times N}. \tag{32}$$

The proof is completed.

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