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Robust Tracking Control for Fuzzy Markovian Jump Systems With Time-Varying Delay and Disturbances

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ABSTRACT In this paper, the robust trajectory tracking controller is proposed for a class of Takagi–Sugeno fuzzy Markovian jump time-varying delay systems with unknown uncertainties and disturbances. Precisely, uncertainty and disturbance estimation-based control design is developed for the addressed systems under uncertain environment. By exploiting the Lyapunov technique and Wirtinger-based integral inequalities, a sufficient condition is derived for ensuring the robust trajectory tracking performance. Precisely, an explicit form of the gain matrix to enhance the robustness of the controller is determined by solving the developed linear matrix inequalities. A single-link robot arm model is eventually provided to validate the potential of the designed control scheme.

INDEX TERMS Takagi-Sugeno fuzzy Markovian jump systems, tracking control design, uncertainty and disturbance estimator.

I. INTRODUCTION

Over the past few decades, stabilization of dynamical control systems has received considerable research attention due to its applications in various practical models such as robotic systems, manufacturing systems, electric power systems, aircraft control systems, mechanical systems, economic systems and so on [1], [2]. On the other hand, various dynamical control systems encounter abrupt variations in their structures and parameters, which exhibit random switching between the subsystems and such random process can be mathematically modeled as Markovian jump systems (MJSs) in a probability space. These abrupt variations may mislead the performance of the dynamical systems which prompt many researchers to study the stability of the MJSs via various type of robust controllers (see [3]–[8] and the references therein). Usually MJSs depend on two major aspects of the state trajectories, one is the usual state vector and another one depicts the random process within a finite set whose components are discrete in nature and usually denoted as system mode. In addition to that, the nonlinearities urge the systems to be more complicated and sometimes it may lead to instability of the MJSs directly due to their complicated dynamical behaviors. Shen *et al.* [5] derived a delay-dependent stability criterion in the framework of LMIs for the H_∞ performance analysis of MJSs in the presence of time-varying delays. For a continuous-time positive Markovian jump linear system, a new linear co-positive stochastic Lyapunov functional has been proposed in [6] for obtaining conditions for stochastic stability. Li and Yang [7] developed a mode-dependent intermediate estimator to estimate faults for a class of nonlinear MJSs with uncertain and unknown transition rates.

On another research front, fuzzy model based control design has been expeditiously developing among the researchers due to its effectiveness. Specifically the Takagi-Sugeno fuzzy model is an effective tool to tackle complicated nonlinear systems and this approach facilitates any nonlinear systems by its linear rule [9], [10]. Therefore, in recent years, various kinds of controllers have been developed to investigate the stability and stabilization for the various classes of continuous and discrete-time TSFSs [11]–[13]. Cheng *et al.* [14] developed a novel set of conditions to improve the efficiency of the transition rates for the fuzzy Markovian jump systems (TSFMJSs) via event-triggered approach. For a TSFMJSs, Shen *et al.* [15] designed an event-triggered H_∞ control scheme for obtaining finite-time boundedness. Further tracking control problems has been widely applied in several areas such as flight tracking control, robotic tracking control and missile tracking control. Therefore in the control field, it is of great significance to track a desired trajectory with optimal performance. Recently many important results on tracking control design based on the TSFSs have been discussed in [16]–[19]. Fan et al. [16] developed an adaptive fault-tolerant controller to achieve tracking performance between fault and reference signals for MJSs. An adaptive fuzzy tracking control scheme was studied in [17] for a class of uncertain nonlinear multiple-input- multiple-output systems. Yu *et al.* [18] investigated the output feedback tracking problem for TSFSs with input saturation.

Specifically disturbances and uncertainties such as external environmental disturbances, unmodeled dynamics and parameter perturbations can inevitably deteriorate the performance of practical control systems. In the past decades, in order to handle uncertainties and disturbances, a number of control techniques has been developed such as adaptive control, backstepping sliding mode control, disturbance observer based control, active disturbance rejection control and uncertainty and disturbance estimator (UDE) based control and so on (see [20]–[22] and the references therein). Among them, the UDE-based control technique is an effective one for uncertainty and disturbance rejection which is proposed by Zhong and Rees [23]. It should be noted that UDE-based control scheme is associated with the lumped disturbance estimation and reference tracking. In addition, it only requires bandwidth information to design the control and tackle singularity problem of the controller design. Thus, it is very easy to obtain the desired performance with less number of tuning parameters and hence it is applicable to a wider class of systems. It can be seen in literature [23]–[27] that the UDE-based control strategy has simple structure, robust nature and effective ability for handling unknown parametric variations and disturbances to ensure better tracking performance. In [24], a robust UDE-based Controller is designed for a class of non-affine nonlinear systems, which avoids inversion operation and ensures asymptotic stability. However, to the best of authors knowledge, there is no work reported regarding the UDE-based tracking control design for MJSs with time-varying delay by a fuzzy model approach which motivates this present study

In the light of the aforementioned studies, in this paper, we aim to present a novel UDE-based tracking control design for MJSs with time-varying delay by a fuzzy model approach. More precisely, the proposed approach is focused on to estimate and compensate the disturbances, time-varying delay and uncertainties. The major contributions of this manuscript are highlighted as follows:

- This is the first attempt to consider the UDE-based tracking control design with time-varying delay for the TSFMJSs with unknown uncertainties and external disturbances.
- The proposed UDE-based controller relaxes some general assumptions and the disturbance is extracted from the system dynamics and then estimated via a low-pass filter.
- The employed approach allows us to estimate and compensate the uncertainties and disturbances quickly. It is easy to implement and get robust tracking performance with less number of tuning parameters.
- Based on the Lyapunov functional technique and the Takagi-Sugeno fuzzy approach, a new delay dependent sufficient criterion is presented in terms of linear matrix inequalities such that the error dynamics is asymptotically stable. Moreover the developed algorithm is exploited to design the controller via optimization techniques.

II. SYSTEM AND CONTROL DESCRIPTIONS

Given probability space consists of sample space $\mathcal{S}$ and the probability measure $\mathcal{P}$ on the algebra of event $\mathcal{F}$, which can be represented as $(\mathcal{S}, \mathcal{F}, \mathcal{P})$. Consider a class of continuous MJSs with time-varying uncertainties and time-varying delay over the probability space which can be described as TSFMJSs in the following form

Plant rule p :

IF $\{\nu_1(t)$ is $M_{p1}\}$ and $\{\nu_2(t)$ is $M_{p2}\}$ $\cdots$ $\{\nu_k(t)$ is $M_{pk}\}$, ***THEN***

$$\dot{\chi}(t) = \bar{A}_p(\varrho(t))\chi(t) + \bar{A}_{dp}(\varrho(t))\chi(t-d(t)) + B_p(\varrho(t))u(t) + C_p(\varrho(t))\varpi(t), \quad (1)$$

where $p \in L = \{1, 2, \ldots, r\}$ and r represent number of *IF-THEN* rules; M_{pq} and $\nu_q(t)$, $(p \in L$ and $q = \{1, 2, \ldots, k\})$ represent fuzzy sets and premise variables; $\chi(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the state and control input vectors; $\varpi(t) \in \mathbb{R}^s$ is the exotic disturbance; $\bar{A}_p(\varrho(t))$ and $\bar{A}_{dp}(\varrho(t))$ denote $[A_p(\varrho(t)) + \Delta A_p(\varrho(t))]$ and $[A_{dp}(\varrho(t)) + \Delta A_{dp}(\varrho(t))]$ respectively, where $A_p(\varrho(t))$, $A_{dp}(\varrho(t))$, $B_p(\varrho(t))$ and $C_p(\varrho(t))$ are appropriate dimensioned constant matrices and also the unknown time-varying uncertain matrices are denoted as $\Delta A_p(\varrho(t))$ and $\Delta A_{dp}(\varrho(t))$; For known scalars $d > 0$ and $\tau > 0$, there exists a time-varying delay function $d(t)$ with the constraints $0 \leq d(t) \leq d$ and $\dot{d}(t) \leq \tau$. The transition probability rate matrix $\Lambda = \{\lambda_{ii'}\}$, $i, i' \in S$, represents the probabilities for the random process $\varrho(t)$ which jumps from the mode i to i' within a finite set $S = \{1, 2, \ldots, N\}$ is defined by

$$\mathcal{P}(\varrho(t+\delta t) = i' | \varrho(t) = i) = \begin{cases} \lambda_{ii'}\delta t + o(\delta t), & i \neq i', \\ 1 + \lambda_{ii'}\delta t + o(\delta t), & i = i', \end{cases} \quad (2)$$

where $\delta t > 0$ and $\lim_{\delta t \to 0}(o(\delta t)/\delta t) = 0$ with transition probability rates $\lambda_{ii'} \geq 0$ and $\lambda_{ii} = -\sum_{i'=1, i \neq i'}^{N} \lambda_{ii'} \leq 0$ for $i, i' \in S$; when $\varrho(t) = i$, $i \in S$. For brevity, ith mode associate matrices are denoted as: $\bar{A}_p(\varrho(t)) = \bar{A}_{pi}$; $\bar{A}_{dp}(\varrho(t)) = \bar{A}_{dpi}$; $B_p(\varrho(t)) = B_{pi}$ and $C_p(\varrho(t)) = C_{pi}$, for any $p \in L$ and $i \in S$.

According to the mode assumed by the Markov process, the overall TSFMJSs is inferred by a fuzzy blending of (1) as follows

$$\dot{\chi}(t) = \sum_{p=1}^{r} \zeta_p(\nu(t))[\bar{A}_{pi}\chi(t) + \bar{A}_{dpi}\chi(t-d(t)) + B_{pi}u(t) + C_{pi}\varpi(t)], \quad (3)$$

where $\zeta_p(\nu(t)) = \frac{\psi_p(\nu(t))}{\sum_{p=1}^{r}\psi_p(\nu(t))}$, $\psi_p(\nu(t)) = \prod_{q=1}^{k} M_{pq}(\nu_q(t))$. $M_{pq}(\nu_q(t))$ is the degree of the membership of $\nu_q(t)$ in M_{pq} . Further it is assumed that $\psi_p(\nu(t)) \geq 0$ and $\sum_{p=1}^{r}\psi_p(\nu(t)) = 1$, $\forall t > 0$. Then we obtain $\zeta_p(\nu(t)) \geq 0$ and $\sum_{p=1}^{r}\zeta_p(\nu(t)) = 1$, $\forall t > 0$. For notation simplification, let $\bar{\tilde{A}}_{pi} = \tilde{A}_{pi} + \Delta\tilde{A}_{pi}$, $\bar{\tilde{A}}_{dpi} = \tilde{A}_{dpi} + \Delta\tilde{A}_{dpi}$, $\tilde{A}_{pi} = \sum_{p=1}^{r}\zeta_p(\nu(t))A_{pi}$, $\Delta\tilde{A}_{pi} = \sum_{p=1}^{r}\zeta_p(\nu(t))\Delta A_{pi}$, $\tilde{A}_{dpi} = \sum_{p=1}^{r}\zeta_p(\nu(t))A_{dpi}$, $\Delta\tilde{A}_{dpi} = \sum_{p=1}^{r}\zeta_p(\nu(t))\Delta A_{dpi}$, $\tilde{B}_{pi} = \sum_{p=1}^{r}\zeta_p(\nu(t))B_{pi}$ and $\tilde{C}_{pi} = \sum_{p=1}^{r}\zeta_p(\nu(t))C_{pi}$. We obtain

$$\dot{\chi}(t) = \bar{\tilde{A}}_{pi}\chi(t) + \bar{\tilde{A}}_{dpi}\chi(t-d(t)) + \tilde{B}_{pi}u(t) + \tilde{C}_{pi}\varpi(t), \quad (4)$$

Further, a stable reference model is required to adopt UDE-based controller for the closed-loop system to meet the required specifications. The reference input is taken as

$$\dot{\chi}_n(t) = A_n\chi_n(t) + A_{dn}\chi_n(t-d(t)) + B_nc(t), \quad (5)$$

where $\chi_n(t) \in \mathbb{R}^n$ is the reference state vector, $c(t) \in \mathbb{R}^s$ is a piecewise continuous and uniformly bounded reference input, A_n, A_{dn} and B_n are appropriate dimensioned constant matrices.

The main objective of this work is to propose a feedback controller $u(t)$ for the system (1) such that $\chi(t)$ asymptotically tracks the reference trajectory $\chi_n(t)$, that is the tracking error $e(t) = \chi_n(t) - \chi(t)$ between the system and reference model converges to zero. To acquire this objective, the desired error dynamics is considered as

$$\dot{e}(t) = (A_n + K)e(t) + A_{dn}e(t-d(t)), \quad (6)$$

where K is the error feedback control gain matrix to be determined.

From equations (4), (5) and (6), we obtain

$$\tilde{B}_{pi}u(t) = A_n\chi(t) + A_{dn}\chi(t-d(t)) + B_nc(t) - \tilde{A}_{pi}\chi(t) - \tilde{A}_{dpi}\chi(t-d(t)) - u_d - Ke(t), \quad (7)$$

where the unknown lumped disturbance

$$u_d = \Delta\tilde{A}_{pi}\chi(t) + \Delta\tilde{A}_{dpi}\chi(t-d(t)) + \tilde{C}_{pi}\varpi(t). \quad (8)$$

Now the unknown dynamics are observed with the aid of known system dynamics in (4), which can be represented as

$$u_d = \dot{\chi}(t) - \tilde{A}_{pi}\chi(t) - \tilde{A}_{dpi}\chi(t-d(t)) - \tilde{B}_{pi}u(t). \quad (9)$$

However the control law cannot be accessible directly because it depends on the state derivative. Thus the UDE-based control strategy is implemented to conquer this issue in which, the unknown lumped disturbance is estimated by using a filter approach. Finally the lumped disturbance can be approximated by

$$\begin{aligned} u_{de} &= u_d * u_f(t), \\ &= [\dot{\chi}(t) - \tilde{A}_{pi}\chi(t) - \tilde{A}_{dpi}\chi(t-d(t)) \\ &\quad - \tilde{B}_{pi}u(t)] * u_f(t), \end{aligned} \quad (10)$$

where "$*$" denotes the convolution operator, $u_f(t)$ is the impulse response of the filter $U_f(t)$. Now, replacing the unknown lumped disturbance u_d in (7) by u_{de} in (10), the UDE-based control law is formulated as

$$\begin{aligned} \tilde{B}_{pi}u(t) &= A_n\chi(t) + A_{dn}\chi(t-d(t)) + B_nc(t) - Ke(t) \\ &\quad - \tilde{A}_{pi}\chi(t) - \tilde{A}_{dpi}\chi(t-d(t)) - [\dot{\chi}(t) - \tilde{A}_{pi}\chi(t) \\ &\quad - \tilde{A}_{dpi}\chi(t-d(t)) - \tilde{B}_{pi}u(t)] * u_f(t). \end{aligned}$$

From the above equation, we have

$$\begin{aligned} u(t) &= \tilde{B}_{pi}^{+}\Big[\mathcal{L}^{-1}\Big\{\frac{1}{1-U_f(s)}\Big\} * \big[A_n\chi(t) + B_nc(t) \\ &\quad + A_{dn}\chi(t-d(t)) - Ke(t)\big] - \tilde{A}_{pi}\chi(t) \\ &\quad - \tilde{A}_{dpi}\chi(t-d(t)) - \mathcal{L}^{-1}\Big\{\frac{sU_f(s)}{1-U_f(s)}\Big\} * \chi(t)\Big], \end{aligned} \quad (11)$$

where $\mathcal{L}^{-1}\{.\}$ is the inverse Laplace transform and $\tilde{B}_{pi}^{+} = (\tilde{B}_{pi}^T\tilde{B}_{pi})^{-1}\tilde{B}_{pi}^T$ is the pseudo inverse of $\tilde{B}_{pi}$.

The main objective of the proposed UDE-based control scheme is to make tracking error convergence rate increase with the increase of bandwidth γ. The selection of bandwidth of the low-pass filter plays a vital role to get the strong disturbance restrain capacity. We choose the low-pass filter in the following form

$$U_f(s) = \frac{\gamma}{s+\gamma}. \quad (12)$$

By imposing low-pass filter (12) in the proposed control law (11), we get

$$\begin{aligned} u(t) &= \tilde{B}_{pi}^{+}\Big\{ - \tilde{A}_{pi}\chi(t) - \tilde{A}_{dpi}\chi(t-d(t)) + A_n\chi_n(t) \\ &\quad + A_{dn}\chi_n(t-d(t)) + B_nc(t) + \gamma\Big[\Big(I - \frac{A_n+K}{\gamma}\Big) \\ &\quad \times e(t) - \frac{A_{dn}}{\gamma}e(t-d(t)) - (A_n+K)\int_0^t e(s)ds \\ &\quad - A_{dn}\int_0^t e(s-d(s))ds\Big]\Big\}. \end{aligned} \quad (13)$$

Besides to obtain the stability criterion for the system (6), the following Assumption and Lemma are needed:

Assumption 1: The bandwidth of the lumped disturbance is a known real constant.

Lemma 1 [22]: For a given positive definite matrix Υ, the inequality $\int_u^v \theta^T(s)\Upsilon\theta(s)ds \geq \frac{1}{v-u}\varsigma^T R\varsigma$ holds for all continuously differentiable function θ in $[u,v] \to \mathbb{R}^n$, where $R = \begin{bmatrix} -4\Upsilon & \frac{6}{v-u}\Upsilon \\ * & -\frac{12}{(v-u)^2}\Upsilon \end{bmatrix}$ and $\varsigma^T = [\int_u^v \theta^T(s)ds \ \int_u^v\int_s^v \theta^T(r)drds]$.

III. ASYMPTOTIC STABILITY CRITERION FOR TAKAGI-SUGENO FUZZY MARKOVIAN JUMP SYSTEMS

In this section, we pay attention to designing a novel uncertainity and disturbance estimator based feedback tracking controller for the considered TSFMJSs (1). To do the analysis, it is enough to attain the asymptotic stability for the error system (6) via proposed UDE-based tracking control law with the unknown gain matrix, which is presented in the sequel.

Theorem 1: For the given positive scalars d, τ, if there exist symmetric matrices $E_i > 0, i = (1, 2, 3, 4)$ and an appropriate dimensioned matrix Y such that the following LMI

$$\Phi = \begin{bmatrix} \Phi_1 & 0 & E_1A_{dn} & 0 & 0 \\ * & -E_2 & 0 & 0 & 0 \\ * & * & -(1-\tau)E_3 & 0 & 0 \\ * & * & * & -4E_4 & \frac{6}{d}E_4 \\ * & * & * & * & -\frac{12}{d^2}E_4 \end{bmatrix} < 0, \tag{14}$$

where $\Phi_1 = E_1A_n + A_n^TE_1^T + Y^T + Y + E_2 + E_3 + d^2E_4$ holds, then the error system (6) is asymptotically stable. Moreover the feedback controller gain matrix can be designed by $K = E_1^{-1}Y$.

Proof: For the stability analysis of the closed-loop system (6), we choose the Lyapunov-Krasovskii functional candidate as

$$\vartheta(t) = \sum_{j=1}^{3} \vartheta_j(t), \tag{15}$$

where

$$\vartheta_1(t) = e^T(t)E_1e(t),$$
$$\vartheta_2(t) = \int_{t-d}^{t} e^T(s)E_2e(s)ds + \int_{t-d(t)}^{t} e^T(s)E_3e(s)ds,$$
$$\vartheta_3(t) = d\int_{t-d}^{t}\int_{s}^{t} e^T(v)E_4e(v)dvds.$$

By computing the derivative of (15) with respect to time t, we obtain

$$\dot{\vartheta}_1(t) = 2e^T(t)E_1\dot{e}(t),$$
$$= 2e^T(t)E_1[(A_n+K)e(t) + A_{dn}e(t-d(t))], \tag{16}$$
$$\dot{\vartheta}_2(t) = e^T(t)(E_2+E_3)e(t) - e^T(t-d)E_2e(t-d) - (1-\tau)e^T(t-d(t))E_3e(t-d(t)), \tag{17}$$
$$\dot{\vartheta}_3(t) = e^T(t)d^2E_4e(t) - d\int_{t-d}^{t} e^T(s)E_4e(s)ds. \tag{18}$$

By applying Lemma 1, one can obtain the integral term in the aforementioned equation as

$$-d\int_{t-d}^{t} e^T(s)E_4e(s)ds \le \xi^T \begin{bmatrix} -4E_4 & \frac{6}{d}E_4 \\ * & -\frac{12}{d^2}E_4 \end{bmatrix} \xi, \tag{19}$$

where $\xi^T = \left[\int_{t-d}^{t} e^T(s)ds \ \int_{t-d}^{t}\int_{s}^{t} e(v)dvds\right]$. From the aforesaid equations (16)-(18), we get

$$\dot{\vartheta}(t) \le \psi^T(t)\Phi\psi(t), \tag{20}$$

where $\psi^T(t) = \left[e(t) \ e(t-d) \ e(t-d(t)) \ \int_{t-d}^{t} e(s)ds \ \int_{t-d}^{t}\int_{s}^{t} e(v)dvds\right]$ and the elements of Φ are same as in Theorem statement by letting $Y = E_1K$. If the matrix inequality in (14) holds, it follows that $\dot{\vartheta}(t) < 0$. Hence it can be concluded that the closed-loop error system (6) is asymptotically stable which completes the proof. □

When we ignore uncertainties and time-varying delay in the considered TSFMJSs and also in their reference system in (1) and (5), then consider the corresponding error system and UDE-based tracking controller as

$$\dot{e}(t) = (A_n + K)e(t), \tag{21}$$
$$u(t) = \tilde{B}_{pi}^{+}\Big\{-\tilde{A}_{pi}\chi(t) + A_n\chi_n(t) + B_nc(t) + \gamma\Big[\Big(I - \frac{A_n+K}{\gamma}\Big)e(t) - (A_n+K)\int_0^t e(s)ds\Big]\Big\}. \tag{22}$$

Corollary 1: Consider the error system (21) and UDE-based tracking controller (22). If there exist a symmetric matrix $E_1 > 0$ and a constant matrix Y with appropriate dimension such that the following LMI holds:

$$E_1A_n + A_n^TE_1^T + Y^T + Y < 0. \tag{23}$$

Then the error system (6) without time-varying delay term is asymptotically stable. Furthermore, if the aforesaid matrix inequalities have feasible solutions, by the relation $K = E_1^{-1}Y$ controller gain matrix is to be determined.

Proof: The derivation part is similar to that of Theorem 1 and we can easily obtain the inequality which is stated in the above corollary. This completes the proof. □

Note 1: It should be mentioned that the sufficient condition (14) in Theorem (1) ensures the asymptotic stability of the error system (6). From the proposed stability criterion in Theorem (1), we can easily obtain the value of gain matrix K.

Remark 1: The proposed UDE-based control scheme consists of set-point reference and the estimation of unknown dynamics and the disturbances. Here the unknown lumped disturbances are estimated by the filter approach and it can be seen that the error dynamics is also determined by the choice of filter so that the error dynamics decays as quickly as possible. Moreover the low-pass filter is employed to attenuate the uncertainty and disturbance signals with high-frequency.

IV. VALIDATION

This section validates the competence and prominence features of the proposed UDE-based tracking control design for TSFMJSs. In the sequel, the efficiency of the designed tracking controller against uncertainties and unknown disturbances is showed in first example whereas in the second

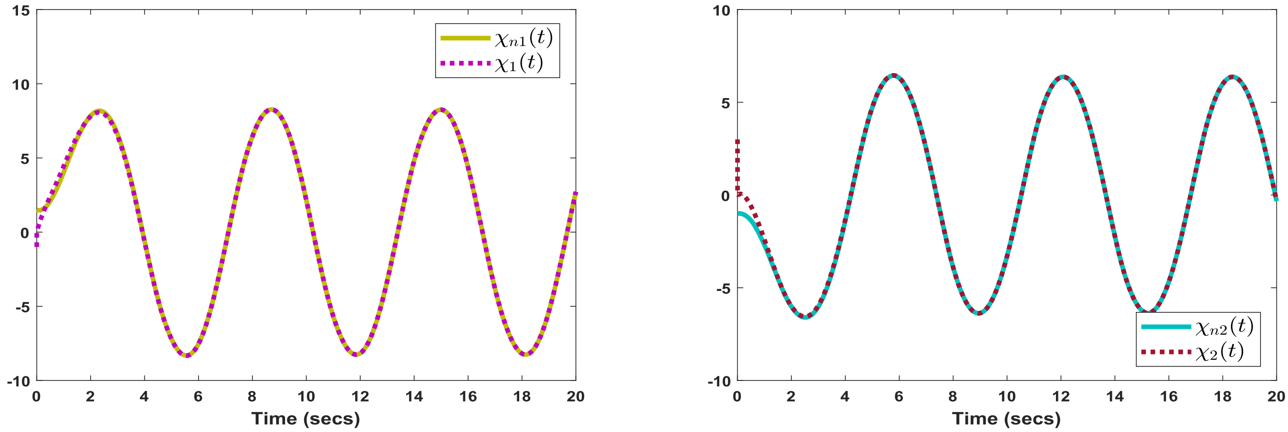

FIGURE 1. **State response of the closed-loop system.**

example, the single-link robot arm model [14] is used to verify our proposed control scheme.

Example 1: Consider an TSFMJSs (6) with two fuzzy rules and two modes:

$$A_{11} = \begin{bmatrix} -1 & -0.5 \\ 0 & -0.5 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} -1 & 2 \\ -3 & -1 \end{bmatrix},$$

$$_{12} = \begin{bmatrix} -2 & -0.3 \\ 0.7 & -1 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -0.45 & 1.1 \\ -0.3 & -4 \end{bmatrix},$$

$$\Delta A_{11} = \begin{bmatrix} 0.2\cos(t) & -0.5\sin(t) \\ 0.7\cos(0.1t) & -0.3\cos(t) \end{bmatrix},$$

$$\Delta A_{21} = \begin{bmatrix} -0.4\sin(0.2t) & 0.5\sin(t) \\ -0.7\cos(t) & -0.03\cos(0.3t) \end{bmatrix},$$

$$\Delta A_{12} = \begin{bmatrix} -0.9\sin^2(0.5t) & 0.4\sin(t) \\ -0.7\cos(2t)\sin(0.5t) & 0.49\cos(0.8t) \end{bmatrix},$$

$$\Delta A_{22} = \begin{bmatrix} -\sin(0.5t)\cos(0.5t) & 0.3\sin(t) \\ -0.5\cos(0.3t)\cos(0.7t) & 0.4\cos(0.9t) \end{bmatrix},$$

$$A_{d11} = \begin{bmatrix} 0.1 & 1.4 \\ -0.2 & 0 \end{bmatrix}, \quad A_{d21} = \begin{bmatrix} 0.5 & 0 \\ -0.8 & 0 \end{bmatrix},$$

$$A_{d12} = \begin{bmatrix} 0.2 & 0.3 \\ 0 & -0.4 \end{bmatrix}, \quad A_{d22} = \begin{bmatrix} 1 & 0.6 \\ 0 & 0.2 \end{bmatrix},$$

$$\Delta A_{d11} = \begin{bmatrix} -\sin(0.5t)\sin(1.5t) & 1.4\cos(t)\sin(0.3t) \\ -0.2\cos(2t)\sin(0.5t) & 0.9\cos(0.8t) \end{bmatrix},$$

$$\Delta A_{d21} = \begin{bmatrix} -1.8\sin(0.5t)\sin(0.5t) & 0.6\cos(3t) \\ -0.7\cos(2t)\sin(5t) & 2\cos(0.6t) \end{bmatrix},$$

$$\Delta A_{d12} = \begin{bmatrix} -0.9\sin^2(0.5t) & 1.4\sin(t) \\ -1.7\cos(2t) & 0.49\cos(0.8t) \end{bmatrix},$$

$$\Delta A_{d22} = \begin{bmatrix} -\sin(t)\cos(0.3t) & 0.3\sin(t)\sin(0.1t) \\ -2\sin(0.3t)\cos(0.7t) & 0.7\cos(0.9t) \end{bmatrix},$$

$$B_{pi} = \begin{bmatrix} 1 & 5 \\ 1 & 2 \end{bmatrix}, (p, i = 1, 2), \quad C_{11} = \begin{bmatrix} 0 \\ -1 \end{bmatrix},$$

$$C_{21} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$C_{12} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad C_{22} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Further the associated matrices for the reference model (5) are chosen as

$$A_n = \begin{bmatrix} 0.2 & 1 \\ -1.4 & -2.3 \end{bmatrix}, \quad A_{dn} = \begin{bmatrix} 0.1 & 0.3 \\ -0.2 & 0 \end{bmatrix}, \; B_n = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

Moreover, the corresponding membership functions are considered as: $\zeta_1(\chi_1(t)) = 1 - \frac{\sin^2(\chi_1(t))}{25}$ and $\zeta_2(\chi_1(t)) = \frac{\sin^2(\chi_1(t))}{25}$. Also, consider the external disturbances

$$\varpi(t) = \begin{cases} 12\cos(t), & t \geq 5, \\ 3, & 5 < t \leq 12, \\ 10\sin(\pi t), & 12 < t \leq 17, \\ 0, & \text{otherwise}, \end{cases}$$

and time-varying delay as $d(t) = |0.7\cos(t)|$ along with the upper bound $d = 0.7$ and its derivative bound $\tau = 0.5$. Based on the aforementioned system parameter values and resorting to MATLAB LMI toolbox, the established LMI condition (14) has feasible solution with the following control gain matrix: $K = \begin{bmatrix} -2.0903 & 0.3547 \\ 0.2974 & 0.5207 \end{bmatrix}$.

Based on the obtained control gain matrix K, the corresponding simulation results are presented in FIGURES 1-6 with the initial states $\chi(0) = \begin{bmatrix} -1 & 3 \end{bmatrix}^T$ and $\chi_n(0) = \begin{bmatrix} 1.5 & -1 \end{bmatrix}^T$ by selecting the bandwidth of the lumped disturbance as $\gamma = 1000$. FIGURE 1 exhibits the trajectories of the state and reference signals along with their estimation, under the proposed UDE-based control law. Thus the FIGURE 1 clearly shows that the proposed UDE-based tracking controller ensures good trajectory tracking performance in the presence of time-varying delay, uncertainty and disturbances in the system model. Moreover FIGURE 2 presents the state trajectories of the system (6) which fails to track the reference signals in the absence of the UDE-based controller. FIGURE 3 exploits the tracking accuracy between the actual state vector and its linear reference vector in the absence and presence of the UDE-based tracking controller. From these Figures, it is noted that the unknown external disturbances

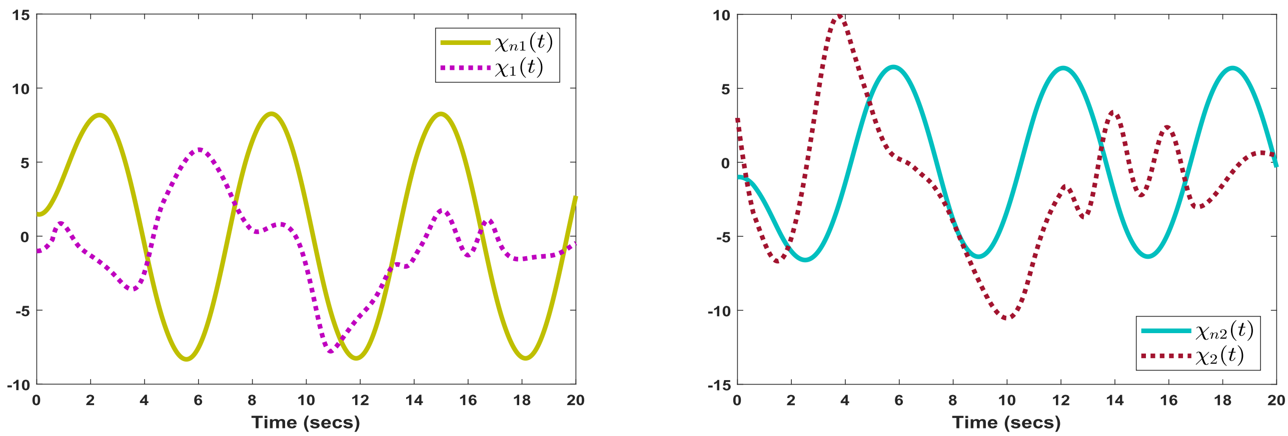

FIGURE 2. State response of the open-loop system.

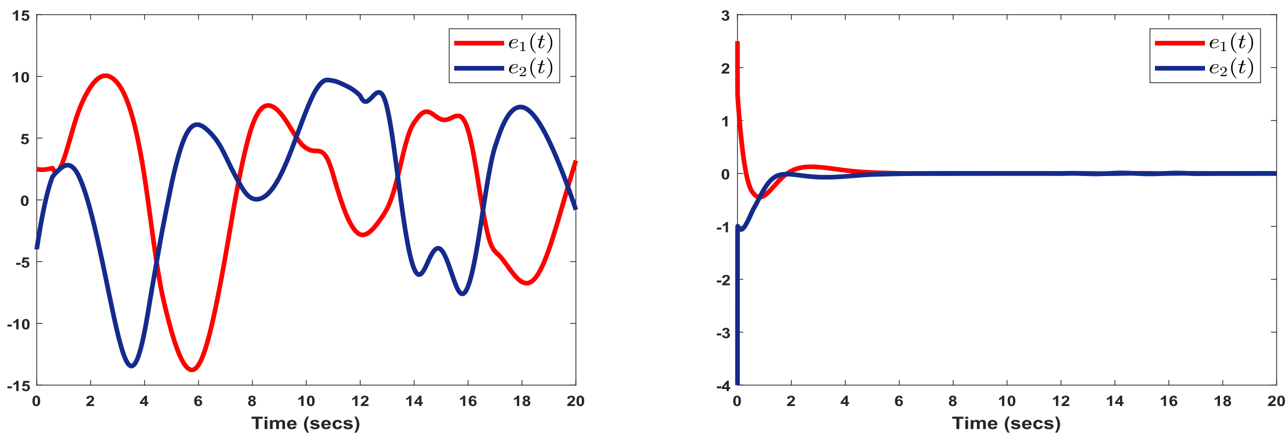

FIGURE 3. Trajectories of the error response .

are compensated within a short period. In addition, the control performance and lumped disturbances estimation are shown in FIGURES 4 and 5 respectively. The corresponding jump mode is given in FIGURE 6. It is evident from these figures that the proposed control strategy rejects the disturbance and also guarantees the asymptotic convergence of the error system (6). It is concluded from the simulation result that the proposed UDE-based tracking controller is more robust to tackle unknown disturbances and better reference tracking.

Example 2: In this example, to validate our proposed method, we consider the well-known single-link robot arm model as in [14]. Then the considered model can be expressed as TSFMJSs (4) without time-varying delay along with two fuzzy rules and three modes, the coefficient matrices associated with this model are given by

$$A_{11} = \begin{bmatrix} 0 & 1 \\ -gL & -D \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0 & 1 \\ -\beta gL & -D \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 0 & 1 \\ -gL & -0.8D \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0 & 1 \\ -\beta gL & -0.8D \end{bmatrix},$$

$$A_{13} = \begin{bmatrix} 0 & 1 \\ -gL & -0.5D \end{bmatrix}, \quad A_{23} = \begin{bmatrix} 0 & 1 \\ -\beta gL & -0.5D \end{bmatrix},$$

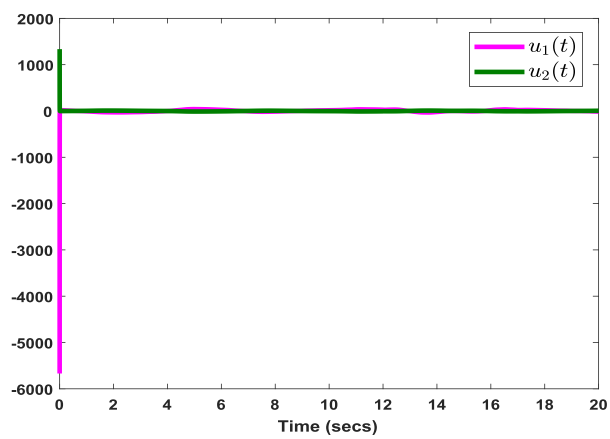

FIGURE 4. Control response.

$$B_{11} = B_{21} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_{12} = B_{22} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix},$$

$$B_{13} = B_{23} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \quad C_{11} = C_{21} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix},$$

$$C_{12} = C_{22} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \quad C_{13} = C_{23} = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}.$$

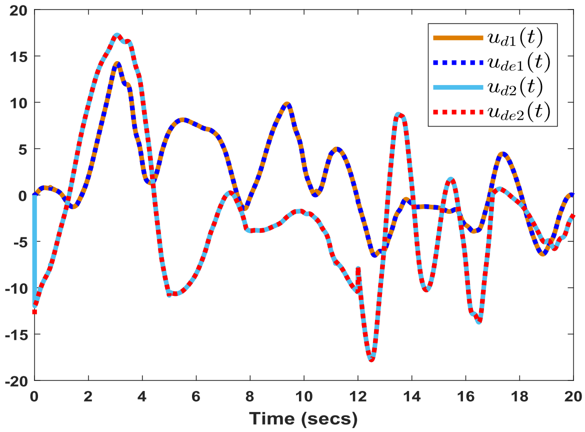

FIGURE 5. Estimation of lumped disturbance.

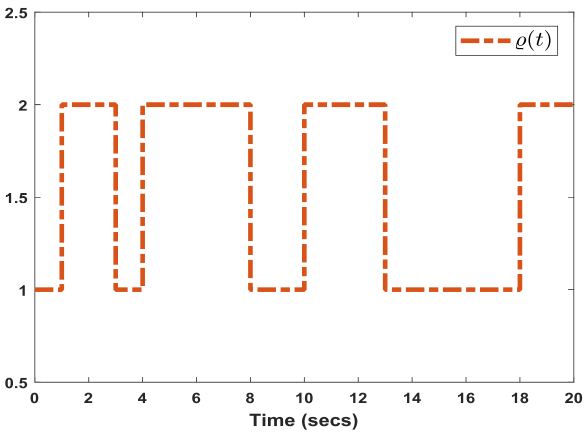

FIGURE 6. Jump mode.

Moreover the membership functions are

$$\zeta_1(\chi_1(t)) = \begin{cases} \dfrac{\sin(\chi_1(t)) - \beta\chi_1(t)}{\chi_1(t)(1-\beta)}, & \chi_1(t) \neq 0 \\ 1, & \chi_1(t) = 0 \end{cases}$$

and

$$\zeta_2(\chi_1(t)) = \begin{cases} \dfrac{\chi_1(t) - \sin(\chi_1(t))}{\chi_1(t)(1-\beta)}, & \chi_1(t) \neq 0 \\ 0, & \chi_1(t) = 0. \end{cases}$$

When $\beta = 0.01\pi^{-1}$, the membership functions of the addressed system are plotted in FIGURE 7. Moreover the corresponding parameter values of the model are borrowed from [14] with the acceleration due to gravity $g = 9.81$, length of the arm $L = 0.5$ and the viscous friction $D(t) = D = 0.2$. Since the mass M and the moment of inertia J have three different jumps, we can take $M_1 = J_1 = 1$, $M_2 = J_2 = 1.25$ and $M_3 = J_3 = 2$.

The linear reference model is given by

$$\dot{\chi}_n(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \chi_n(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} c(t), \qquad (24)$$

where $\chi_n(t) = \begin{bmatrix} \chi_{n1}(t) \ \chi_{n2}(t) \end{bmatrix}^T$ and $c(t) = (1/(1 + 0.05t))\sin(t)$. Based on the parameter values stated above,

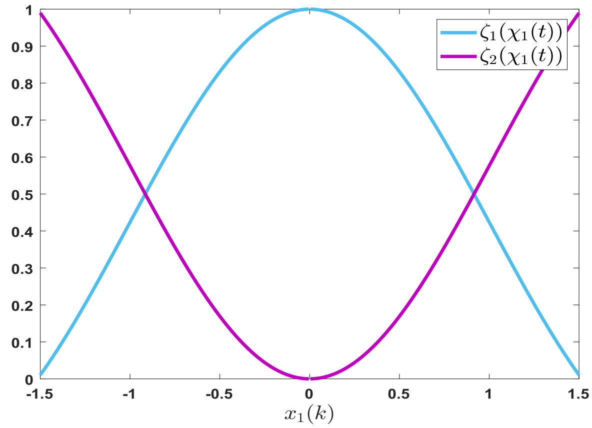

FIGURE 7. Membership function.

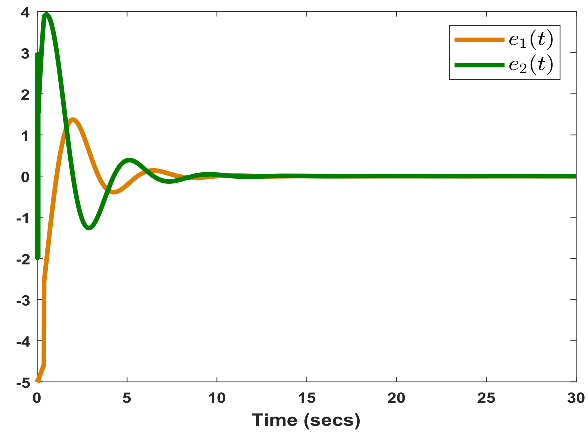

FIGURE 8. Trajectories of closed-loop error system.

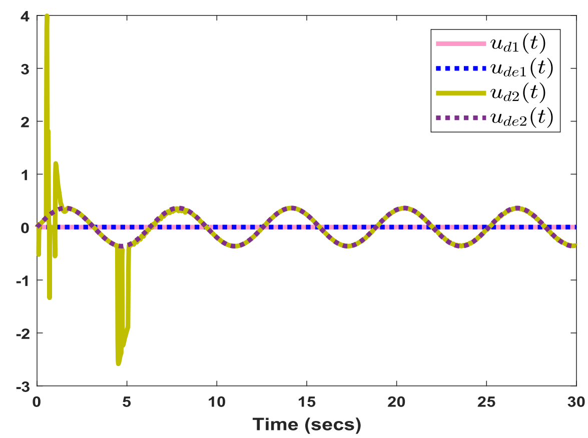

FIGURE 9. Estimation of disturbance.

it can be found that the linear matrix inequality (23) in corollary 1 has a feasible solution. According to this feasible solution, the state feedback control gain matrices are found to be $K = \begin{bmatrix} -0.5 & 0.5 \\ 0.5 & 2.5 \end{bmatrix}$. Now, for simulation purposes, we choose the initial states as $\chi(0) = \begin{bmatrix} 0.5\pi & -2 \end{bmatrix}^T$ and $\chi_n(0) = \begin{bmatrix} -2 & 1 \end{bmatrix}^T$ and the external disturbance as

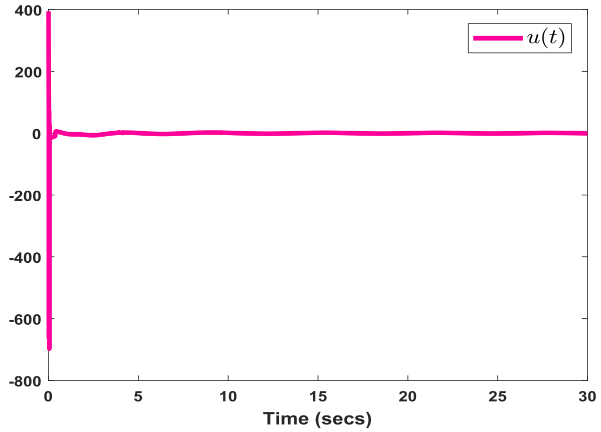

FIGURE 10. Control response.

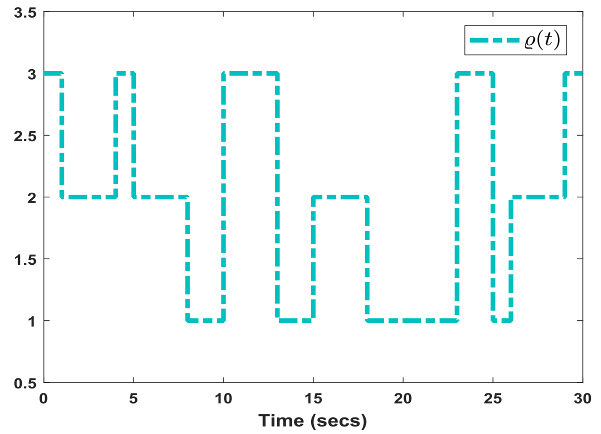

FIGURE 11. Jump mode.

$\varpi(t) = 1.2\sin(t)$. In addition, bandwidth value of the lumped disturbances can be taken as $\gamma = 100$.

Precisely FIGURE 8 depicts the error response in the presence of the control scheme (22) which reveals that the state signals track the reference signals under the proposed controller. Thus the UDE-based control law provides better tracking performance against the unknown lumped disturbances which are plotted in the FIGURE 9. In addition, the corresponding jump mode and proposed controller are given in FIGURES 10 and 11. From the simulation results, it can be seen that the designed novel UDE-based tracking controller for the TSFMJSs(1) guarantees better asymptotic reference tracking and disturbance rejection.

V. CONCLUSION

In this paper, we have designed an UDE-based tracking control to achieve better reference tracking and disturbance rejection for MJSs with time-varying delay by a fuzzy model approach. By developing an appropriate Lyapunov-Krasovskii functional and using Wirtinger-based single integral inequality, delay dependent sufficient criterion is presented in terms of linear matrix inequalities that admit the existence of a feasible solution to the error system. Consequently, by solving the developed linear matrix inequalities, the desired robust control gain matrix has been obtained. From the proposed UDE-based tracking control approach, the unknown uncertainties and disturbances can be estimated effectively. Finally simulation results are provided which reveal the excellent tracking and disturbance rejection capabilities of the proposed control design technique. The problem of designing UDE-based robust tracking control for fractional-order Takagi-Sugeno fuzzy singular Markovian jump systems subject to unknown time-varying delays is an untreated area which will be the topic of our future work.

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