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Adaptive NN Tracking Control for Nonlinear Fractional Order Systems With Uncertainty and Input Saturation

CHANGHUI WANG AND MEI LIANG
School of Electromechanical and Automotive Engineering, Yantai University, Yantai 264005, China
Corresponding authors: Changhui Wang (wang_chaghui@ytu.edu.cn) and Mei Liang (mmglm@163.com)

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ABSTRACT In this paper, an adaptive neural network (NN) backstepping control method is designed for a class of uncertain fractional order nonlinear systems with external disturbance and input saturation, in which the fractional Lyapunov stability theory is used to construct the controller. The complicated unknown fractional order nonlinear function is approximated by a radial basis function (RBF) NN in each step, and the virtual control law and parameters update law are presented based on the backstepping algorithm procedures. At the final step, an adaptive RBF NN controller is constructed, in which no knowledge of system uncertainty and the upper bound of the disturbance is required. Then, a theorem is presented to address that the asymptotical convergence of the tracking error can be guaranteed. The effectiveness of the proposed scheme is illustrated by two simulation examples.

INDEX TERMS Adaptive control, fractional order, neural networks (NNs), input saturation, nonlinear system.

I. INTRODUCTION

Fractional order calculus is mentioned by Leibniz in 1695 for the first time [1], which has many interesting properties and some potential applications receiving lots of attention from engineers [2]–[6]. Recently, many results of the fractional order nonlinear systems regarding theory and applications are growing continuously [7]–[12]. The reason is that the model of the physical phenomena including viscoelastic structures or heat conduction can be established concisely and precisely using the fractional order differential equations [13]. In addition, the control performance of fractional order controllers is better than the classical controllers due to their general forms, hereditary and nonlocality [14]. Due to the existence of parameter uncertainties and noises, the uncertain fractional order system control is a very attractive and challenging research field, and several research results have been reported, such as the sliding mode control [15]–[17], the adaptive control [18]–[21], the neural network control [22], [23] and the fuzzy control [24], [25].

Adaptive backstepping control method is an important and widely used technique to control uncertain integer order nonlinear systems with triangular structure [26], [27], which establishes a systematic framework using intermediate variables recursively and constructs a Lyapunov function for the overall closed loop system. With the advantages (such as superior tracking, transient performance and global stability), the adaptive backstepping technique has been extended to control fractional order nonlinear systems by some scholars. There are mainly two classes of Lyapunov functions used to analyze the stability of the closed loop system. The first one is the direct Lyapunov method [4], [28], [29], in which an inequality lemma [30] is used to find fractional order Lyapunov candidate functions. The second one is the indirect Lyapunov method [20], [21], [31]–[33], in which the frequency distributed model of the fractional order system [34] is introduced so that the indirect Lyapunov method can be applied to design the controller.

Adaptive neural network (NN) control schemes have been found to be very useful for the control of integer order nonlinear systems, primarily by its inherent capability to model and control highly uncertain, nonlinear and complex systems [35], [36]. The radial basis function (RBF) NN is considered as a two-layer network [37], [38], which is one of the very effective tools in functional approximations and widely applied to handle uncertain information [39], [40]. So far,there are many research results on adaptive NN control of fractional order nonlinear systems. In [22], a robust adaptive NN control is presented for a fractional order rotational mechanical system with uncertainties and disturbances. In [41], an adaptive fractional sliding mode controller based on dual RBF NNs is given to enhance the performance of a three-phase shunt active power filter. In [42], an adaptive NN backstepping control method is proposed for a class of fractional order chaotic systems subjected to backlash-like hysteresis nonlinearities. Although lots of efforts have been put into fractional order algorithm, there are still short of results on how to apply backstepping control algorithm for uncertain fractional order system with input saturation.

Input saturation is a usually occurred nonlinearity in control devices, which is the most dangerous nonlinearity in the closed-loop control systems. Input saturation can gravely restrict the performance of system, or result in oscillations in the output response, which may lead to instability and divergence of the system. Therefore, the effect of input saturation should be considered when designing and implementing nonlinear fractional control systems. Motivated by the above analyses and discussions, an adaptive NN backstepping control method is given for a class of uncertain fractional order nonlinear systems with external disturbance and input saturation in this paper. Compared with the extant results, the contributions of this paper are summarized as:

1) It is the first time handling tracking control problem for a class of uncertain fractional order nonlinear system subjected to input saturation based on RBF NN and backstepping recursive algorithm. Comparing [25], the structure of the uncertain integer order nonlinear systems considered in this paper is a triangular structure.

2) Based on backstepping control scheme, the RBF NN is used to approximate the fractional order nonlinear functions in each step of the backstepping, and the fractional order adaptation laws are given to estimate the parameters of the RBF NN. Then, an adaptive NN fractional order backstepping controller is constructed based on the fractional Lyapunov stability theory.

3) A fractional order compensator is designed to approximate the unknown upper bound from the external disturbance and input saturation, which can ensure system stability and the tracking error asymptotical convergence.

The remainder of this article is organized as follows. In Section 2, the basic definitions on the fractional integrals and derivatives and the preliminary results on fractional order systems are presented. The detailed controller design and the stability analysis of the fractional order control system is given in Section 3. In Section 4 simulation results are presented to illustrate the validity of the proposed approach. Conclusions are given in Section 5.

II. PRELIMINARIES

The fractional order integrodifferential operator is an extended concept of the integer order integrodifferential operator [43]. There are several definitions regarding the fractional derivative, and the Caputo definition is one of the most used in engineering applications. The fractional order integral of continuous function $f(t)$ with respect to t and the lower terminal t_0 is defined as follow

$$ {}_{t_0}I_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau \tag{1}$$

where $\Gamma(\alpha) = \int_0^{\infty} e^{-t} t^{\alpha-1} dt$ is the well-known Eulers Gamma function. The $\alpha - th$ Caputo fractional derivative is defined by

$$ {}_{t_0}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} \frac{f^{(n)}(t)}{(t-\tau)^{\alpha+1-n}} d\tau \tag{2}$$

where $n-1 < \alpha < n, n \in \mathbb{Z}^+$ and ${}_{t_0}D_t^{\alpha}$ is the classical $\alpha - th$ order derivative operator. To simplify the notation, ${}_{t_0}D_t^{\alpha}$ is abbreviated as D^{α} when $t_0 = 0$.

The Laplace transform on (2) can be given as

$$ \int_0^{\infty} e^{-st} D^{\alpha} f(t)\, d\tau = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0) \tag{3}$$

where $F(s)$ is the Laplace transform of $f(t)$.

Definition 1 [2]: The Mittag-Leffler function is defined as

$$ E_{\alpha,\gamma}(\zeta) = \sum_{k=0}^{\infty} \frac{\zeta^k}{\Gamma(\alpha k - \gamma)} \tag{4}$$

where ζ is a complex number, and α, γ are positive constants.

The Laplace transform of (4) is

$$ L\left(t^{\gamma-1} E_{\alpha,\gamma}\left(-at^{\alpha}\right)\right) = \frac{s^{\alpha-\gamma}}{s^{\alpha}+a} \tag{5}$$

Lemma 1 [2]: For a complex number β and two real numbers α, ν satisfying $0 < \alpha < 1$ and

$$ \frac{\pi\alpha}{2} < \nu < \min\{\pi, \pi\alpha\} \tag{6}$$

the following equation holds for $\forall n \geq 1$:

$$ E_{\alpha,\beta}(\zeta) = -\sum_{j=1}^{n} \frac{1}{\Gamma(\beta - \alpha j)\,\zeta^j} + o\left(\frac{1}{|\zeta|^{n+1}}\right) \tag{7}$$

and $\nu \leq |arg(\zeta)| \leq \pi$ when $|\zeta| \to \infty$.

Lemma 2 [2]: Let α satisfy $0 < \alpha < 2$ and β be an arbitrary real number. If there exists a positive constant μ such that $\left(\pi\alpha/2\right) < \mu \leq \min\{\pi, \pi\alpha\}$, then one has

$$ \left|E_{\alpha,\beta}(\zeta)\right| \leq \frac{C}{1+|\zeta|} \tag{8}$$

where $C > 0$, $\mu \leq |arg(\zeta)| \leq \pi$, and $|\zeta| \geq 0$.

Definition 2 [7]: $g : [0, b) \to I$ is a continuous and strictly increases function satisfying $g(0) = 0$, then g belong to class-K.

Lemma 3 [7]: Assume that the origin is an equilibrium point of a nonautonomous fractional order nonlinear system

$$ D^{\alpha} x(t) = f(t, x(t)) \tag{9}$$

where $f : I \times \Omega \to \mathbb{R}$ is Lipschitz continuous. If there is a Lyapunov function $V(t, x(t))$ and class-K functions g_i $(i = 1, 2, 3)$ to satisfy

$$\begin{aligned} g_1(\|x(t)\|) &\le V(t, x(t)) \le g_2(\|x(t)\|) \\ D^\alpha V(t, x(t)) &\le -g_3(\|x(t)\|) \end{aligned} \tag{10}$$

then (9) is asymptotically stable.

Lemma 4 [30]: If $x(t)$ is a smooth function, then

$$\frac{1}{2} D^\alpha \left(x(t)^T x(t) \right) \le x(t)^T D^\alpha (x(t)), \quad \forall t \in I \tag{11}$$

The RBF NN is a feed-forward neural network with three layers: input layer, hidden layer and output layer. Any smooth nonlinear functions can be approximated by the RBF NN with any precision.

Lemma 5 [44]: For any smooth function $f(X)$ on a compact set Ω and an expected accuracy ε, there exists a RBF NN $\theta^{*T} \vartheta(X)$ so that

$$f(X) = \theta^{*T} \vartheta(X) + \delta(X), |\delta(X)| \le \varepsilon \tag{12}$$

i.e.

$$\theta^* = \arg\min_\theta \left[\sup_X \left| f(X) - \theta^T \vartheta(X) \right| \right] \tag{13}$$

where $\theta = \begin{bmatrix} \theta_1 & \theta_2 & \dots & \theta_l \end{bmatrix} \in \mathbb{R}^l$ is the weight vector, $X \in \Omega$ is the input vector, $l > 1$ is the neural network node number; $\vartheta(X) = [\vartheta_1(X), \vartheta_2(X), \dots, \vartheta_l(X)]^T$, and $\vartheta_i(X)$ being chosen as the common Gaussian functions with the form

$$\vartheta_i(X) = \exp\left(\frac{-(X - \mu_i)^T (X - \mu_i)}{\eta^2} \right), \quad i = 1, 2, \dots, l \tag{14}$$

where $\mu_i = (\mu_{i1}, \mu_{i2}, ..., \mu_{in})^T$ is the center of the respective field and η is the width of the Gaussian function.

III. ADAPTIVE NEURAL NETWORK BACKSTEPPING CONTROLLER DESIGN

Consider the fractional order nonlinear systems described as follows

$$\begin{cases} D^\alpha x_1 = x_2 + f_1(\bar{x}_1) \\ D^\alpha x_2 = x_3 + f_2(\bar{x}_2) \\ \vdots \\ D^\alpha x_{n-1} = x_n + f_{n-1}(\bar{x}_{n-1}) \\ D^\alpha x_n = bu(\upsilon(t)) + f_n(\bar{x}_n) + d(t) \\ y(t) = x_1 \end{cases} \tag{15}$$

where $0 < \alpha < 1$ is the system commensurate fractional order, $x = \begin{pmatrix} x_1 & x_2 & \dots & x_n \end{pmatrix}^T \in \mathbb{R}^n$, $\bar{x}_i = \begin{pmatrix} x_1 & x_2 & \dots & x_i \end{pmatrix}^T \in \mathbb{R}^i$ are the state vectors, and $y \in \mathbb{R}$ is the output, $f_i(\bar{x}_i)$, $i = 1, \dots, n$ are the unknown smooth nonlinear function, $d(t) \in \mathbb{R}$ is an unknown external disturbance, $u(\upsilon(t))$ is an input saturation defined as [45]

$$u = sat(\upsilon(t)) = \begin{cases} u_{\max}, & \upsilon \ge u_{\max} \\ \upsilon, & u_{\min} < \upsilon < u_{\max} \\ u_{\min}, & \upsilon \le u_{\min} \end{cases} \tag{16}$$

where υ is the input signal of the input saturation nonlinearity, $u_{\max} > 0$ and $u_{\min} < 0$ are the unknown constants.

For (16), the input saturation function can be approximated by a smooth piecewise function defined as

$$\begin{aligned} g(\upsilon) &= \begin{cases} u_{\max} \cdot \tanh\left(\frac{\upsilon}{u_{\max}}\right), & \upsilon \ge 0 \\ u_{\min} \cdot \tanh\left(\frac{\upsilon}{u_{\min}}\right), & \upsilon < 0 \end{cases} \\ &= \begin{cases} u_{\max} \cdot \dfrac{e^{\frac{\upsilon}{u_{\max}}} - e^{-\frac{\upsilon}{u_{\max}}}}{e^{\frac{\upsilon}{u_{\max}}} + e^{-\frac{\upsilon}{u_{\max}}}}, & \upsilon \ge 0 \\ u_{\min} \cdot \dfrac{e^{\frac{\upsilon}{u_{\min}}} - e^{-\frac{\upsilon}{u_{\min}}}}{e^{\frac{\upsilon}{u_{\min}}} + e^{-\frac{\upsilon}{u_{\min}}}}, & \upsilon < 0 \end{cases} \end{aligned} \tag{17}$$

and $sat(\upsilon(t))$ in (16) can be rewritten as

$$u = sat(\upsilon) = g(\upsilon) + \Delta(\upsilon) \tag{18}$$

where $\Delta(\upsilon) = sat(\upsilon) - g(\upsilon)$ is bounded, satisfying

$$\begin{aligned} |\Delta(\upsilon)| &= |sat(\upsilon) - g(\upsilon)| \\ &\le \max\{u_{\max}(1 - \tanh(1)), u_{\min}(\tanh(1) - 1)\} \\ &= D \end{aligned} \tag{19}$$

With the theorem of the mean, there exists a constant μ, $0 < \mu < 1$ to make

$$g(\upsilon) = g(\upsilon_0) + g_{\upsilon_\mu}(\upsilon - \upsilon_0) \tag{20}$$

where

$$\begin{aligned} g_{\upsilon_\mu} &= \left.\frac{\partial g(\upsilon)}{\partial \upsilon}\right|_{\upsilon = \upsilon_\mu} \\ \upsilon_\mu &= \mu\upsilon + (1 - \mu)\upsilon_0 \end{aligned} \tag{21}$$

Remark 1: Mean value theorem: For a function $p(x)$, if it is continuous in the interval $[c, d]$ and derivable in the interval (c, d), then there exists at least one point ξ, $c < \xi < d$, such that: $p(c) - p(d) = \dot{p}(\xi)(c - d)$ holds.

By selecting $\upsilon_0 = 0$, (20) can be represented as

$$g(\upsilon) = g_{\upsilon_\mu} \upsilon \tag{22}$$

Substituting (18) and (22) into (15), a new expression of the system (15) can be obtained as

$$\begin{cases} D^\alpha x_1 = x_2 + f_1(\bar{x}_1) \\ D^\alpha x_2 = x_3 + f_2(\bar{x}_2) \\ \vdots \\ D^\alpha x_{n-1} = x_n + f_{n-1}(\bar{x}_{n-1}) \\ D^\alpha x_n = b g_{\upsilon_\mu} \upsilon + f_n(\bar{x}_n) + b\Delta(\upsilon) + d(t) \\ y(t) = x_1 \end{cases} \tag{23}$$

Let y_d be a known smooth reference signal. Our aim is to establish a proper controller υ such that the tracking error $e_1 = y - y_d$ converges to an arbitrary small region of zero. Then, a recursive backstepping algorithm is presented, which can be separated as the following steps.

Step 1: Based on Lemma 5, the unknown function $f_1(\bar{x}_1)$ from (23) can be approximated by a RBF NN as follow:

$$\hat{f}_1(\bar{x}_1, \theta_1) = \theta_1^T \vartheta_1(\bar{x}_1) \tag{24}$$

where parameter estimation $\theta_1 \in \mathbb{R}^{m_1}$. The ideal parameter θ_1^* is given by

$$\theta_1^* = \arg\min_{\theta_1}\left[\sup_{x_1}\left|f_1(\bar{x}_1) - \hat{f}_1(\bar{x}_1,\theta_1)\right|\right] \tag{25}$$

Let

$$\begin{aligned}\tilde{\theta}_1 &= \theta_1^* - \theta_1\\ \varepsilon_1(\bar{x}_1) &= \hat{f}_1\left(\bar{x}_1,\theta_1^*\right) - f_1(\bar{x}_1)\end{aligned} \tag{26}$$

be the parameter estimation error and the optimal approximation error, respectively. Assume that the optimal approximation error remain bounded, one can obtain

$$|\varepsilon_1(\bar{x}_1)| \leq \bar{\varepsilon}_1 \tag{27}$$

where $\bar{\varepsilon}_1 > 0$. One can obtain

$$\begin{aligned}&\hat{f}_1(\bar{x}_1,\theta_1) - f_1(\bar{x}_1)\\ &\quad= \hat{f}_1(\bar{x}_1,\theta_1) - \hat{f}_1\left(\bar{x}_1,\theta_1^*\right) + \hat{f}_1\left(\bar{x}_1,\theta_1^*\right) - f_1(\bar{x}_1)\\ &\quad= \theta_1^T\vartheta_1(\bar{x}_1) - \theta_1^{*T}\vartheta_1(\bar{x}_1) + \varepsilon_1(\bar{x}_1)\\ &\quad= -\tilde{\theta}_1^T\vartheta_1(\bar{x}_1) + \varepsilon_1(\bar{x}_1)\end{aligned} \tag{28}$$

It follows from (15), (26) and (28) that

$$\begin{aligned}D^\alpha e_1 &= D^\alpha \bar{x}_1 - D^\alpha y_d\\ &= \bar{x}_2 + f_1(\bar{x}_1) - D^\alpha y_d\\ &= \bar{x}_2 + f_1(\bar{x}_1) - \hat{f}_1(\bar{x}_1,\theta_1) + \hat{f}_1(\bar{x}_1,\theta_1) - D^\alpha y_d\\ &= \bar{x}_2 + \tilde{\theta}_1^T\vartheta_1(\bar{x}_1) - \varepsilon_1(\bar{x}_1) + {\theta_1}^T\vartheta_1(\bar{x}_1) - D^\alpha y_d\end{aligned} \tag{29}$$

Let a virtual control input $\upsilon_1(e,\bar{x}_1,y_d)$ be

$$\upsilon_1(e,\bar{x}_1,y_d) = -{\theta_1}^T\vartheta_1(\bar{x}_1) - k_{11}e_1 - k_{21}sign(\bar{x}_1) + D^\alpha y_d \tag{30}$$

where k_{11} and k_{21} are design parameters. Let

$$e_2 = x_2 - \upsilon_1 \tag{31}$$

Substituting (30) and (31) into (29) gives

$$D^\alpha e_1 = e_2 - k_{11}e_1 - k_{21}sign(e_1) + \tilde{\theta}_1^T\vartheta_1(\bar{x}_1) - \varepsilon_1(\bar{x}_1) \tag{32}$$

Selecting the Lyapunov function candidate V_1 as

$$V_1 = \frac{1}{2}e_1^2 + \frac{1}{2\sigma_1}\tilde{\theta}_1^T\tilde{\theta}_1 \tag{33}$$

and design a fractional order adaptation law as follow

$$D^\alpha\theta_1 = \sigma_1\vartheta_1(\bar{x}_1)e_1 \tag{34}$$

where $\sigma_1 > 0$ is the design parameter. Due to the fractional order derivative of a constant is zero, based on (34) one can obtain

$$D^\alpha\tilde{\theta}_1 = D^\alpha\theta_1^* - D^\alpha\theta_1 = -D^\alpha\theta_1 \tag{35}$$

Then, according to (32), (34), (35) and Lemma 4, one has

$$\begin{aligned}D^\alpha V_1 &= \frac{1}{2}D^\alpha\left(e_1^2\right) + \frac{1}{2\sigma_1}D^\alpha\left(\tilde{\theta}_1^T\tilde{\theta}_1\right)\\ &\leq e_1\left(e_2 - k_{11}e_1 - k_{21}sign(e_1) + \tilde{\theta}_1^T\vartheta_1(\bar{x}_1) - \varepsilon_1(\bar{x}_1)\right)\\ &\quad - \frac{1}{\sigma_1}\tilde{\theta}_1^T(\sigma_1\vartheta_1(\bar{x}_1)e_1)\\ &= e_1e_2 - k_{11}e_1e_1 - k_{21}sign(e_1)e_1 - \varepsilon_1(\bar{x}_1)e_1\\ &\leq e_1e_2 - k_{11}e_1^2 - (k_{21} - \bar{\varepsilon}_1)|e_1|\end{aligned} \tag{36}$$

Step 2: From (23) and (31), one has

$$\begin{aligned}D^\alpha e_2 &= D^\alpha x_2 - D^\alpha\upsilon_1\\ &= x_3 + f_2(\bar{x}_2) - D^\alpha\upsilon_1\\ &= x_3 + F_2(\bar{x}_2)\end{aligned} \tag{37}$$

where $F_2(\bar{x}_2) = f_2(\bar{x}_2) - D^\alpha\upsilon_1$ is an unknown function. Just like step 1, $F_2(\bar{x}_2)$ can be approximated by a RBF NN as follow

$$\hat{F}_2(\bar{x}_2,\theta_2) = \theta_2^T\vartheta_2(\bar{x}_2) \tag{38}$$

where parameter estimation $\theta_2 \in \mathbb{R}^{m_2}$. Then, (37) can be presented as follow

$$\begin{aligned}D^\alpha e_2 &= x_3 + F_2(\bar{x}_2)\\ &= x_3 + \tilde{\theta}_2^T\vartheta_2(\bar{x}_2) - \varepsilon_2(\bar{x}_2) + \theta_2^T\vartheta_2(\bar{x}_2)\end{aligned} \tag{39}$$

where $\varepsilon_2(\bar{x}_2) = \hat{F}_2\left(\bar{x}_2,\theta_2^*\right) - F_2(\bar{x}_2)$, satisfying $|\varepsilon_2(\bar{x}_2)| \leq \bar{\varepsilon}_2$, $\bar{\varepsilon}_2 > 0$.

Define the estimated error $\tilde{\theta}_2 = \theta_2^* - \theta_2$, and the following equation can be obtained using Caputo's definition

$$D^\alpha\tilde{\theta}_2 = D^\alpha\theta_2^* - D^\alpha\theta_2 = -D^\alpha\theta_2 \tag{40}$$

Let a virtual control input be

$$\upsilon_2 = -{\theta_2}^T\vartheta_2(\bar{x}_2) - k_{12}e_2 - k_{22}sign(e_2) - e_1 \tag{41}$$

where k_{12} and k_{22} are design parameters. Let

$$e_3 = x_3 - \upsilon_2 \tag{42}$$

Substituting (41) and (42) into (39) gives

$$D^\alpha e_2 = e_3 - k_{12}e_2 - k_{22}sign(e_2) + \tilde{\theta}_2^T\vartheta_2(\bar{x}_2) - \varepsilon_2(\bar{x}_2) - e_1 \tag{43}$$

Selecting the Lyapunov function V_2 as

$$V_2 = V_1 + \frac{1}{2}e_2^2 + \frac{1}{2\sigma_2}\tilde{\theta}_2^T\tilde{\theta}_2 \tag{44}$$

and design a fractional order adaptation law as follow

$$D^\alpha\theta_2 = \sigma_2\vartheta_2(\bar{x}_2)e_2 \tag{45}$$

where $\sigma_2 > 0$ is the design parameter. Then, according to (40), (43), (45) and Lemma 4, one has

$$\begin{aligned}D^\alpha V_2 &= D^\alpha V_1 + e_2D^\alpha(e_2) - \frac{1}{\sigma_2}\tilde{\theta}_2^T D^\alpha\theta_2\\ &\leq e_1e_2 - k_{11}e_1^2 - (k_{21} - \bar{\varepsilon}_1)|e_1|\\ &\quad + e_2e_3 - k_{12}e_2^2 - k_{22}sign(e_2)e_2 + \tilde{\theta}_2^T\vartheta_2(\bar{x}_2)e_2\\ &\quad - \varepsilon_2(\bar{x}_2)e_2 - e_1e_2 - \tilde{\theta}_2^T\vartheta_2(\bar{x}_2)e_2\\ &\leq e_2e_3 - k_{11}e_1^2 - k_{12}e_2^2 - (k_{21} - \bar{\varepsilon}_1)|e_1| - (k_{22} - \bar{\varepsilon}_2)|e_2|\end{aligned} \tag{46}$$

Step i, $3 \le i \le n-1$: Let

$$e_i = x_i - \upsilon_{i-1} \tag{47}$$

Just like the procedures in step 1 and 2, one has

$$\begin{aligned} D^{\alpha} e_i &= x_{i+1} + f_i(\bar{x}_i) - D^{\alpha} \upsilon_{i-1} \\ &= x_{i+1} + F_i(\bar{x}_i) \end{aligned} \tag{48}$$

where $F_i(\bar{x}_i) = f_i(\bar{x}_i) - D^{\alpha}\upsilon_{i-1}$ is an unknown function, and υ_{i-1} is a virtual control input. Let

$$\hat{F}_i(\bar{x}_i, \theta_i) = \theta_i^T \vartheta_i(\bar{x}_i) \tag{49}$$

where parameter estimation $\theta_i \in \mathbb{R}^{m_i}$.

Define the estimated error $\tilde{\theta}_i = \theta_i^* - \theta_i$, and the following equation can be obtained

$$D^{\alpha}\tilde{\theta}_i = D^{\alpha}\theta_i^* - D^{\alpha}\theta_i = -D^{\alpha}\theta_i \tag{50}$$

From (49), (48) can be rewritten as follow

$$D^{\alpha} e_i = x_{i+1} + \tilde{\theta}_i^T \vartheta_i(\bar{x}_i) - \varepsilon_i(\bar{x}_i) + \theta_i^T \vartheta_i(\bar{x}_i) \tag{51}$$

where $\varepsilon_i(\bar{x}_i) = \hat{F}_i\left(\bar{x}_i, \theta_i^*\right) - F_i(\bar{x}_i)$, satisfying $|\varepsilon_i(\bar{x}_i)| \le \bar{\varepsilon}_i$, $\bar{\varepsilon}_i > 0$.

Let a virtual control input be

$$\upsilon_i = -\theta_i{}^T \vartheta_i(\bar{x}_i) - k_{1i} e_i - k_{2i} sign(e_i) - e_{i-1} \tag{52}$$

where k_{1i} and k_{2i} are design parameters.

Substituting (47) and (52) into (51) gives

$$D^{\alpha} e_i = e_{i+1} - k_{1i} e_i - k_{2i} sign(e_i) + \tilde{\theta}_i^T \vartheta_i(\bar{x}_i) - \varepsilon_i(\bar{x}_i) - e_{i-1} \tag{53}$$

Selecting the Lyapunov function V_i as

$$V_i = V_{i-1} + \frac{1}{2} e_i^2 + \frac{1}{2\sigma_i} \tilde{\theta}_i^T \tilde{\theta}_i \tag{54}$$

and a fractional order adaptation law is designed as

$$D^{\alpha}\theta_i = \sigma_i \vartheta_i(\bar{x}_i) e_i \tag{55}$$

where $\sigma_i > 0$, then its derivative based on (50), (53), (55) and Lemma 4 is expressed as

$$\begin{aligned} D^{\alpha} V_i &= D^{\alpha} V_{i-1} + e_i D^{\alpha}(e_i) - \frac{1}{\sigma_i} \tilde{\theta}_i^T D^{\alpha}\theta_i \\ &\le e_{i-1} e_i - \sum_{k=1}^{i-1} \left(k_{1k} e_k^2 + (k_{2k} - \bar{\varepsilon}_k) |e_k| \right) \\ &\quad + e_i e_{i+1} - k_{1i} e_i^2 - k_{2i} sign(e_i) e_i + \tilde{\theta}_i^T \vartheta_i(\bar{x}_i) e_i \\ &\quad - \varepsilon_i(\bar{x}_i) e_i - e_{i-1} e_i - \frac{1}{\sigma_i} \tilde{\theta}_i^T (\sigma_i \vartheta_i(\bar{x}_i) e_i) \\ &\le e_i e_{i+1} - \sum_{k=1}^{i} \left(k_{1k} e_k^2 + (k_{2k} - \bar{\varepsilon}_k) |e_k| \right) \end{aligned} \tag{56}$$

Step n: Let

$$e_n = x_n - \upsilon_{n-1} \tag{57}$$

From (23) and (57), one has

$$\begin{aligned} D^{\alpha} e_n &= b g_{\upsilon_\mu} \upsilon + f_n(x) + b\Delta(\upsilon) + d(t) - D^{\alpha}\upsilon_{n-1} \\ &= b g_{\upsilon_\mu} \upsilon + b\Delta(\upsilon) + d(t) + F_n(x) \end{aligned} \tag{58}$$

where υ_{n-1} is a virtual control input, and $F_n(x) = f_n(x) - D^{\alpha}\upsilon_{n-1}$ is an unknown function. Let

$$\hat{F}_n(x, \theta_n) = \theta_n^T \vartheta_n(x) \tag{59}$$

where parameter estimation $\theta_n \in \mathbb{R}^{m_n}$. Define the estimated error $\tilde{\theta}_n = \theta_n^* - \theta_n$, and one can get the following equation

$$D^{\alpha}\tilde{\theta}_n = D^{\alpha}\theta_n^* - D^{\alpha}\theta_n = -D^{\alpha}\theta_n \tag{60}$$

From (59), (58) can be rewritten as

$$D^{\alpha} e_n = b g_{\upsilon_\mu} \upsilon + b\Delta(\upsilon) + d(t) + \tilde{\theta}_n^T \vartheta_n(x) - \varepsilon_n(x) + \theta_n^T \vartheta_n(x) \tag{61}$$

where $\varepsilon_n(x) = \hat{F}_n\left(x, \theta_n^*\right) - F_n(x)$, satisfying $|\varepsilon_n(\bar{x}_n)| \le \bar{\varepsilon}_n$, $\bar{\varepsilon}_n > 0$.

The following assumption is presented to proceed.

Assumption 1: The external disturbance $d(t)$ is bounded, i.e., $|d(t)| \le \bar{d}$, for $\forall t \ge 0$ where constant $\bar{d} > 0$ is unknown.

According to (19) and Assumption 1, one can find that $b\Delta(\upsilon) + d(t)$ is bounded with an unknown upper bound h^*, i.e

$$|b\Delta(\upsilon) + d(t)| \le |b| D + \bar{d} = h^* \tag{62}$$

Let us establish the controller u as follow

$$\upsilon(t) = \frac{1}{b g_{\upsilon_\mu}} \left(-\theta_n^T \vartheta_n(x) - k_{1n} e_n - (k_{2n} + h) sign(e_n) - e_{n-1} \right) \tag{63}$$

where k_{1n} and k_{2n} are design parameter, and h is the estimation of the unknown constant h^*.

Let $\tilde{h} = h^* - h$, then the following equation can be obtained

$$D^{\alpha}\tilde{h} = D^{\alpha} h^* - D^{\alpha} h = -D^{\alpha} h \tag{64}$$

Substituting (57) and (63) into (61) gives

$$\begin{aligned} D^{\alpha} e_n &= -k_{1n} e_n - (k_{2n} + h) sign(e_n) - e_{n-1} \\ &\quad + b\Delta(\upsilon) + d(t) + \tilde{\theta}_n^T \vartheta_n(x) - \varepsilon_n(x) \end{aligned} \tag{65}$$

Selecting the Lyapunov function V_n as

$$V_n = V_{n-1} + \frac{1}{2} e_n^2 + \frac{1}{2\sigma_n} \tilde{\theta}_n^T \tilde{\theta}_n + \frac{1}{2\rho} \tilde{h}^2 \tag{66}$$

where $\sigma_n, \rho > 0$. To update θ_n and h, design the following fractional order adaptation laws

$$D^{\alpha}\theta_n = \sigma_n \vartheta_n(x) e_n \tag{67}$$

and

$$D^{\alpha} h = \rho |e_n| \tag{68}$$

Just like the procedures in step i, $3 \le i \le n-1$, the derivative of V_n is

$$\begin{aligned} D^{\alpha} V_n &= D^{\alpha} V_{n-1} + e_n D^{\alpha}(e_n) - \frac{1}{\sigma_n} \tilde{\theta}_n^T D^{\alpha}\theta_n - \frac{1}{\rho} \tilde{h} D^{\alpha}\tilde{h} \\ &\le e_{n-1} e_n - \sum_{k=1}^{n-1} \left(k_{1k} e_k^2 + (k_{2k} - \bar{\varepsilon}_k) |e_k| \right) \end{aligned}$$

$$
\begin{aligned}
& - k_{1n}e_n^2 - (k_{2n}+h)\left|e_n\right| - e_{n-1}e_n + (b\Delta(\upsilon)+d(t))\, e_n \\
& + \tilde{\theta}_n^T \vartheta_n(x)\, e_n - \varepsilon_n(x)\, e_n - \tilde{\theta}_n^T \vartheta_n(x)\, e_n - \tilde{h}\left|e_n\right| \\
\le & -\sum_{k=1}^{n-1}\Big(k_{1k}e_k^2 + (k_{2k}-\bar{\varepsilon}_k)\left|e_k\right|\Big) \\
& - k_{1n}e_n^2 - (k_{2n}+h-\bar{\varepsilon}_n)\left|e_n\right| + (b\Delta(\upsilon)+d(t))\, e_n \\
& - \tilde{h}\left|e_n\right| \\
\le & -\sum_{k=1}^{n-1}\Big(k_{1k}e_k^2 + (k_{2k}-\bar{\varepsilon}_k)\left|e_k\right|\Big) \\
& - k_{1n}e_n^2 - (k_{2n}+h-\bar{\varepsilon}_n)\left|e_n\right| + h\left|e_n\right| - \tilde{h}\left|e_n\right| \\
\le & -\sum_{k=1}^{n}\Big(k_{1k}e_k^2 + (k_{2k}-\bar{\varepsilon}_k)\left|e_k\right|\Big) \qquad (69)
\end{aligned}
$$

Based on above design procedures, the follow theorem gives the stability of the closed loop system.

Theorem 1: Consider the fractional order system (15) with input saturation, if the NN controller is designed as (63) with (30), (41) and (52), and the fractional adaptation laws are chosen as (34), (45), (55), (67) and (68), then under a proper choice of design parameters $k_{1i} > 0$, $k_{2i} > \bar{\varepsilon}_i$, $\sigma_i > 0$ and $\rho > 0$, $i = 1, 2 \ldots, n$, the tracking error $e_1 = y - y_d$ tend to zero asymptotically when $t \to \infty$.

Proof: According to step 1, 2, i, n, if the control input is designed as (63) with (30), (41) and (52), and the adaptation laws are chosen as (34), (45), (55), (67) and (68), then under a proper choice of design parameters $k_{1i} > 0$, $k_{2i} > 0$, $\sigma_i > 0$ and $\rho > 0$, $i = 1, 2 \ldots, n$, the derivative of Lyapunov function V_n satisfy

$$D^\alpha V_n \le -\sum_{k=1}^{n}\Big(k_{1k}e_k^2 + (k_{2k}-\bar{\varepsilon}_k)\left|e_k\right|\Big) \qquad (70)$$

Denote $s^2(t) = \frac{1}{2}\sum\limits_{l=2}^{n} e_l^2 + \sum\limits_{i=1}^{n} \frac{1}{2\sigma_i}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2\rho}\tilde{h}^2$, one gets $V_n = \frac{1}{2}e_1^2 + s^2(t)$. Then, $e_1^2 \le 2V_n$. From (70) one gets

$$D^\alpha V_n \le -k_{11}e_1^2 \qquad (71)$$

The fractional order integral of (71) is obtained as

$$V_n(t) - V_n(0) \le -k_{11} I_t^\alpha e_1^2(t) \qquad (72)$$

Based on (72), one gets $e_1^2 \le 2V_n(0) - 2k_{11} I_t^\alpha e_1^2(t)$. There exists $\lambda(t) \ge 0$ such that

$$e_1^2(t) = 2V_n(0) - 2k_{11} I_t^\alpha e_1^2(t) - \lambda(t) \qquad (73)$$

Denoting $E_1(s) = L\left(e_1^2(t)\right)$ and $\Gamma(s) = L(\lambda(t))$, the Laplace transform of (73) is described as

$$E_1(s) = \frac{2V_n(0)}{s} - 2k_{11}\frac{1}{s^\alpha}E_1(s) - \Gamma(s) \qquad (74)$$

That is

$$E_1(s) = 2V_n(0)\frac{s^{\alpha-1}}{s^\alpha + 2k_{11}} - \frac{s^\alpha}{s^\alpha + 2k_{11}}\Gamma(s) \qquad (75)$$

Based on (5), the solution of (75) can be obtained as

$$e_1^2(t) = 2V_n(0)\, E_\alpha\left(-2k_{11}t^\alpha\right) - \lambda(t)\left(t^{-1}E_{\alpha,0}\left(-2k_{11}t^\alpha\right)\right) \qquad (76)$$

Since that $E_{\alpha,0}(-2k_{11}t^\alpha) \ge 0$ and $t^{-1} \ge 0$, then one can get

$$e_1^2(t) \le 2V_n(0)\, E_\alpha\left(-2k_{11}t^\alpha\right) \qquad (77)$$

Based on Lemmas 1-3, the tracking error $e_1 = y - y_d$ tend to zero asymptotically.

Remark 2: Theorem 1 proposes the stability analysis of the proposed adaptive NN backstepping algorithm in this paper, in which the fractional order stability criterion is adopted and the complicated unknown function derived from differentiating a compound function with a fractional order is approximated by the RBF NN in each step.

Remark 3: The input saturation function is approximated by a smooth piecewise function and rewritten as a linear function by mean value theorem, which is convenient for controller design.

Remark 4: The unknown upper bound h^* is determined by the external disturbance and input saturation. To estimate the unknown upper bound h^*, a fractional order compensator is used, which can ensure system stability and the tracking error asymptotical convergence.

Remark 5: $D^{\alpha_i}\upsilon_{i-1}$ contains the error form the parameter estimation θ_{i-1} for the ideal parameter θ_{i-1}^* to approximate unknown function $f_i(\bar{x}_i)$, which is not precise signal. Then, $D^{\alpha_i}\upsilon_{i-1}$ is considered as the unknown function in this paper, which is conducive to implement the controller comparing $D^{\alpha_i}\upsilon_{i-1}$ considered as the known function.

Remark 6: The sign function employed in the controller (63) with (30), (41) and (52) may result the chattering phenomenon. To eliminate the chattering phenomenon, the sign function can be replaced by the continuous function, such as *arctan* (10·). Although the asymptotical stability of the tracking error may not be ensured, the tracking error may get into a smaller range of zero.

IV. SIMULATION

The following simulation two examples results are put forward to illustrate the effectiveness of the presented method.

There are several independent packages providing the necessary functionality for working with fractional order systems and controllers [46], [47], such as CRONE toolbox, Ninteger and FOMCON toolbox. The FOMCON toolbox for MATLAB is a fractional order calculus based toolbox for system modeling and control design, which is used in this paper.

A. EXAMPLE 1

Consider a commensurate fractional order nonlinear system as follows

$$\begin{cases} D^{\alpha}x_1 = x_2 - 0.08x_1^2 \\ D^{\alpha}x_2 = x_3 + \dfrac{x_2 - 0.3x_1^2}{1 + x_1^4} \\ D^{\alpha}x_3 = 3u(\upsilon) - e^{-x_1^2}x_2\sin(5x_3) + d(t) \end{cases} \tag{78}$$

where $\alpha = 0.5$, and the initial state $x(0) = \begin{bmatrix} 0.5 & 1 & 2 \end{bmatrix}^T$. $f_1(x_1) = -0.08x_1^2$, $f_2(\bar{x}_2) = \frac{x_2 - 0.3x_1^2}{1+x_1^4}$ and $f_3(\bar{x}_3) = e^{-x_1^2}x_2\sin(5x_3)$ are the unknown nonlinear functions. The input saturation $u(\upsilon)$ is described as

$$u(\upsilon) = sat(\upsilon(t)) = \begin{cases} 5, & \upsilon \geq 5 \\ \upsilon, & -5 < \upsilon < 5 \\ -5, & \upsilon \leq -5 \end{cases} \tag{79}$$

The reference signal y_d is chosen as $\sin(t+0.3)$ and will be tracked by the system output, the disturbance signal $d(t)$ is chosen as $0.1(\sin(t) + \cos(t))$. $sign(\cdot)$ is replaced by $arctan(10\cdot)$ in the presented controller to avoid the chattering phenomenon.

The membership functions are chosen to deal with the unknown nonlinear terms

$$\vartheta(x_1) = \exp\left(-\left(x_1 - x_{k_1}\right)^2/0.25\right)$$

$$\vartheta(\bar{x}_2) = \exp\left(-\frac{\left(x_1 - x_{k_1}\right)^2}{0.25} - \frac{\left(x_2 - x_{k_2}\right)^2}{0.25}\right)$$

$$\vartheta(\bar{x}_3) = \exp\left(-\frac{\left(x_1 - x_{k_1}\right)^2}{0.25} - \frac{\left(x_2 - x_{k_2}\right)^2}{0.25} - \frac{\left(x_3 - x_{k_3}\right)^2}{0.25}\right)$$

$$\begin{aligned} x_{k_1} &\in \{0.5k_1 - 2 | \, k_1 = 1, 2, \ldots, 6\} \\ x_{k_2} &\in \{k_2 - 2 | \, k_2 = 1, 2, 3\} \\ x_{k_3} &\in \{0.5k_3 - 1.5 | \, k_3 = 1, 2, \ldots, 5\} \end{aligned} \tag{80}$$

where the initial condition is $\theta_1(0) = 0_{6\times 1}$, $\theta_2(0) = 0_{18\times 1}$ and $\theta_3(0) = 0_{90\times 1}$.

The design parameters are chosen as $k_{11} = k_{12} = k_{13} = 13$, $k_{21} = k_{22} = k_{23} = 0.01$, $\sigma_1 = \sigma_2 = \sigma_3 = 3$, $g_{\upsilon_\mu} = 1$ and $\rho = 2$.

Fig. 1, 2, 3, 4, 5 and 6 show corresponding simulation results. Fig. 1 shows trajectories of system output y and reference signal y_d, demonstrating that the tracking error $e_1 = y - y_d$ can be guaranteed to be small enough and have a rapid convergence with the unknown system model and input saturation. However, the tracking error e_1 gets into a smaller range of zero and does not stop at zero. There are two main reasons for this result: 1) $arctan(10\cdot)$ is used to replace $sign(\cdot)$ to eliminate the chattering phenomenon, so that asymptotical convergence of the tracking error can not beassured; 2) There is approximation error from unknown nonlinear functions approximated by RBF NN.

Fig. 2 indicates the trajectories of the system states x_2 and x_3, which displays the boundedness of the systems states. Fig. 3 and Fig. 4 depict the norm of parameters estimation of the RBF NN and the estimation of the upper

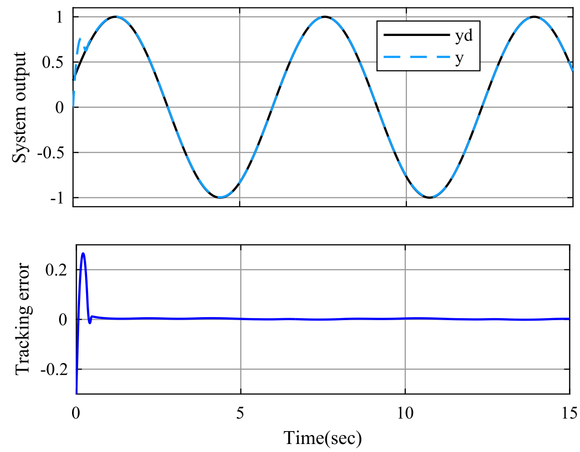

FIGURE 1. Trajectories of y, y_d and tracking error.

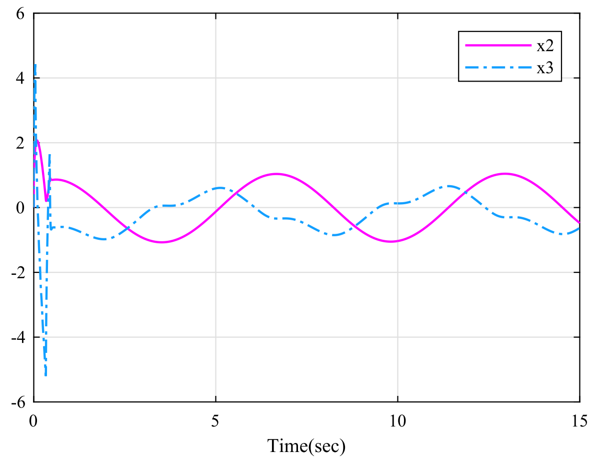

FIGURE 2. State trajectories x_2 and x_3.

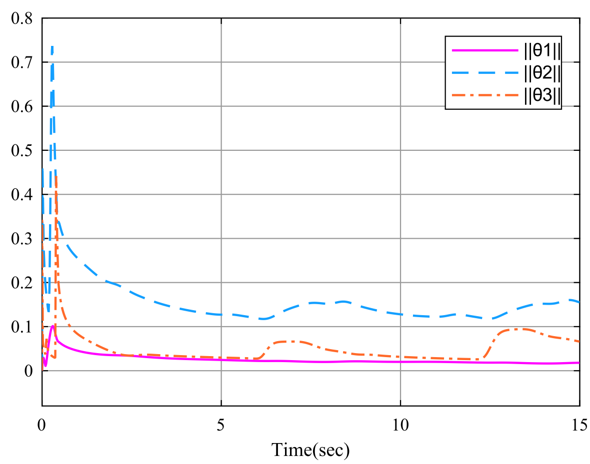

FIGURE 3. Evolution of the norm of the RBF NN parameters.

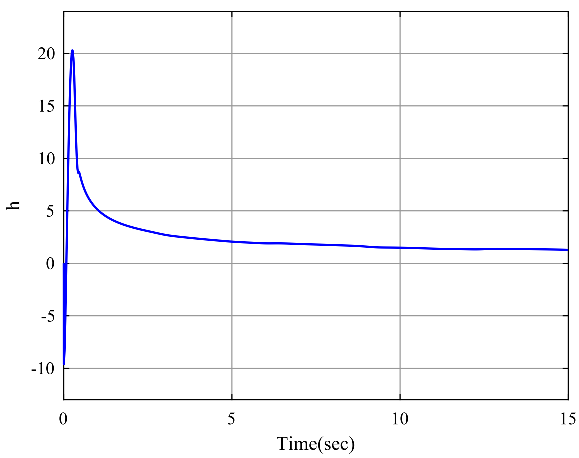

FIGURE 4. Evolution of the estimation of the upper bound of *h*.

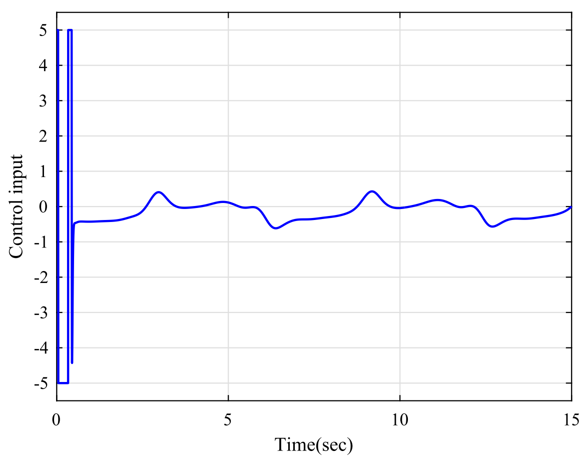

FIGURE 5. Evolution of the control input.

bound of h are bounded, and Fig. 5 shows the system input saturation $u(\upsilon)$ is bounded as well, which is demonstrating the boundedness of the signals in the closed loop adaptive system.

B. EXAMPLE 2

Consider the fractional order Chua-Hartley's system [48]

$$\begin{cases} D^{0.98}x_1 = x_2 + \frac{10}{7}\left(x_1 - x_1^3\right) \\ D^{0.98}x_2 = x_3 + 10x_1 - x_2 \\ D^{0.98}x_3 = u(\upsilon) - \frac{100}{7}x_2 + d(t) \end{cases} \tag{81}$$

where the initial state $x(0) = \begin{pmatrix} -2 & -1 & 1 \end{pmatrix}^T$. The input saturation $u(\upsilon)$ is described as

$$u(\upsilon) = sat(\upsilon(t)) = \begin{cases} 15, & \upsilon \geq 15 \\ \upsilon, & -15 < \upsilon < 15 \\ -15, & \upsilon \leq -15 \end{cases} \tag{82}$$

The reference signal y_d is chosen as $\sin(t+0.1)$ and will be tracked by the system output, the disturbance signal $d(t)$ is chosen as $0.3(\sin(t) + \cos(t))$. The membership functions are chosen as

$$\begin{aligned} \vartheta(x_1) &= \exp\left(-\left(x_1 - x_{k_1}\right)^2/0.25\right) \\ \vartheta(\bar{x}_2) &= \exp\left(-\frac{\left(x_1 - x_{k_1}\right)^2}{0.25} - \frac{\left(x_2 - x_{k_2}\right)^2}{0.25}\right) \\ \vartheta(\bar{x}_3) &= \exp\left(-\frac{\left(x_2 - x_{k_2}\right)^2}{0.25}\right) \\ x_{k_1} &\in \{0.5k_1 - 2 | \, k_1 = 1, 2, \ldots, 6\} \\ x_{k_2} &\in \{k_2 - 2 | \, k_2 = 1, 2, 3\} \end{aligned} \tag{83}$$

The design parameters are chosen as $k_{11} = k_{12} = k_{13} = 23$, $k_{21} = k_{22} = k_{23} = 0.1$, $\sigma_1 = \sigma_3 = 0.1$, $g_{\upsilon_\mu} = 1$ and $\sigma_2 = \rho = 0.01$.

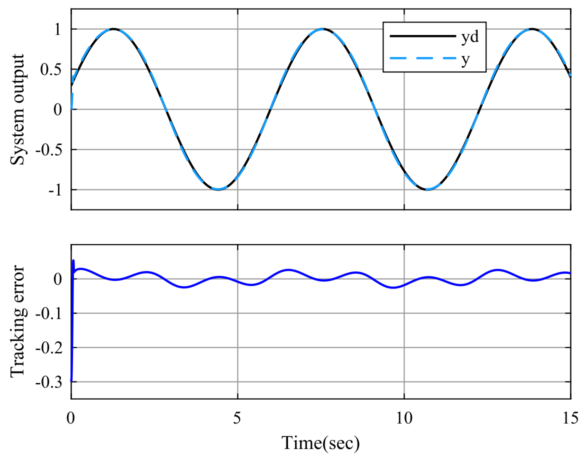

FIGURE 6. Trajectories of y, y_d and tracking error.

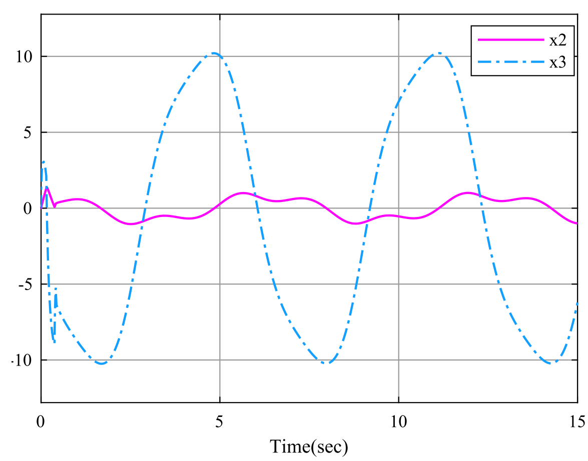

FIGURE 7. State trajectories x_2 and x_3.

Fig. 6 presents the tracking error e_1 can be guaranteed to be small enough. Fig. 7 indicates the trajectories of the system states x_2 and x_3, revealing that the systems states are bounded. Fig. 8 and Fig. 9 depict the norm of parameters estimation of the RBF NN and the estimation of the upper bound of h

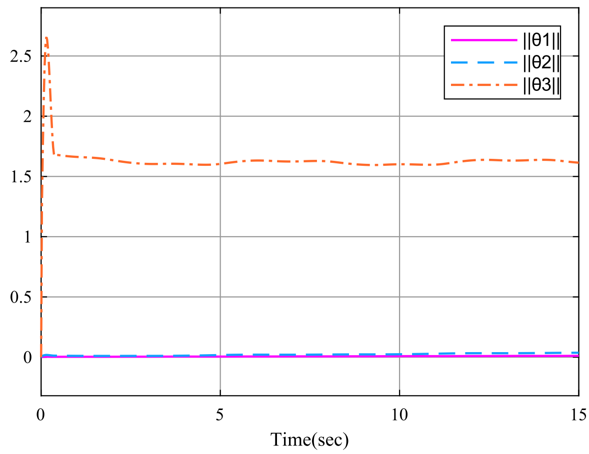

FIGURE 8. Evolution of the norm of the RBF NN parameters.

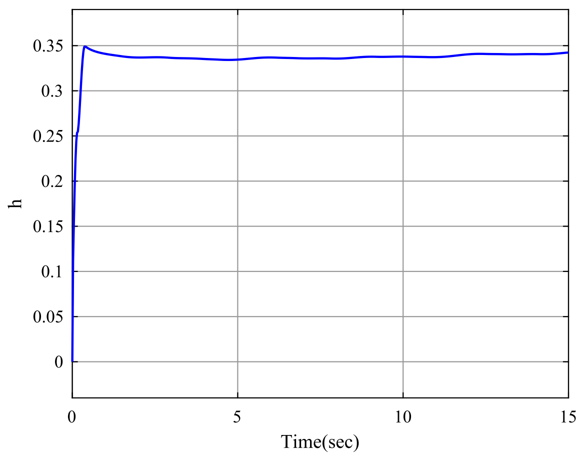

FIGURE 9. Evolution of the estimation of the upper bound of *h*.

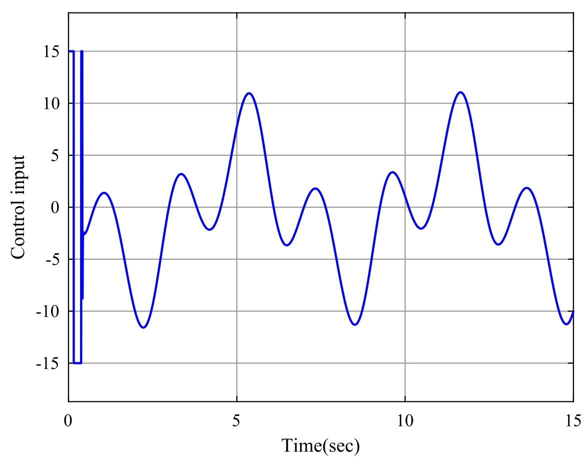

FIGURE 10. Evolution of the control input.

are bounded, and Fig. 10 shows the input saturation $u(\upsilon)$ is bounded as well.

All of the aforesaid simulation results show that the system output follows the desired reference signal in a small enough region, all the closed loop signals are bounded, and the effectiveness of the presented adaptive RBF NN backstepping control scheme is demonstrated.

V. CONCLUSION

In this paper, an adaptive NN backstepping tracking control scheme is presented for a class of uncertain fractional order nonlinear systems with external disturbance and input saturation. By using the RBF NN approximate approach and the adaptive backstepping scheme, the tracking control problem with the restricted condition of the unknown nonlinear terms and input saturation is studied, and a robust adaptive NN controller is constructed based on the fractional Lyapunov stability theory. Under the proposed control scheme, the tracking error of the closed-loop system can reach a small enough neighborhood of zero. The effectiveness of proposed method demonstrated by two simulation examples.

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CHANGHUI WANG received the Ph.D. degree in control science and engineering from the Harbin Institute of Technology, Harbin, China, in 2016.

He has been with the School of Electromechanical and Automotive Engineering, Yantai University, Yantai, China, since 2017. His current research interests include automotive engine, system identification, adaptive control, and intelligent systems and control.

MEI LIANG received the Ph.D. degree in instrument science and technology from the Harbin Institute of Technology, Harbin, China, in 2017.

She has been with the School of Electromechanical and Automotive Engineering, Yantai University, Yantai, China, since 2017. Her current research interests include radiation thermometry, medical instruments, system identification, and intelligent systems and control.