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Improvement in the Electromechanical Properties of a Partially Diced Piezoelectric Disc Transducer

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ABSTRACT Piezoelectric discs vibrating in the thickness mode are widely used in underwater electroacoustic and electromechanical applications. It is well known and established that the first fundamental mode along the thickness of the disc determines the transducer resonant frequency and the disc diameter determines the radiation surface. For certain applications, the frequency, radiation pattern, and power requirement determine certain thickness-to-diameter ($h/2a$) ratio, however, at these aspect ratios, the vibrational velocity of the piezoelectric disc is non-uniform across its surface due to the combination of coupled vibration in thickness and radial directions. These higher order radial modes decrease the electromechanical and electroacoustical transformation, resulting in a performance degradation of the transducer. Previous theoretical analysis on piezoelectric cylinders as a function of thickness-to-diameter ratio under three different polarizations, namely the radial, axial, and circumferential, was published by Sarangapani *et al.*, and the coupled vibration for piezoelectric disc of certain thickness-to-diameter ratio has been studied analytically by Aronov. This paper investigates the effect of the electromechanical performance of the transducer using piezoelectric disc of certain aspect ratios where coupled vibration in thickness and radial direction exists. We also propose a solution to overcome the limitations of a piezoelectric disc with non-ideal thickness-to-diameter ratio by a partially diced design. A finite-element model is used to calculate the dynamics of the diced disc and in comparison with a uniform disc to illustrate the reduction of coupled vibration in thickness and radial directions. The electromechanical coupling coefficient is shown to improve greatly over the uniform disc by optimizing certain dicing parameter. An equivalent circuit based on Mason's equivalent circuit model is derived for the diced disc under the assumption of a one-degree-of-freedom system. The electromechanical properties, the frequency spectrum, and the effective coupling coefficient are presented and show good agreement with the experimental results.

INDEX TERMS Electroacoustic transducers, disc piezoelectric transducers, coupled vibrations, coupling coefficient, effective electromechanical coupling coefficient.

SYMBOLS

e_{33} :	piezoelectric stress constant
$\varepsilon_{33}^S, \varepsilon_{33}^T$:	piezoelectric permittivity under constant strain and constant stress.
c_{33}^D :	elastic stiffness under constant electric displacement.
s_{33}^D, s_{33}^E:	elastic compliance under constant electric displacement and constant electric field.
d_{33} :	piezoelectric charge constant
h_{33} :	piezoelectric constant
k_t :	thickness mode coupling coefficient
k_{33}:	33 mode coupling coefficient
k_{33eff}:	effective electromechanical coupling coefficient
C_b, C_d:	clamped capacitance of piezoelectric bar and disc
V^D, κ:	sound speed and wave number in the direction of polarization
n_b:	electromechanical transformation coefficient of a bar
n_d:	electromechanical transformation coefficient of a disk
M_{eqv} :	effective mass of the piezoelectric transducer
C_m^E :	effective compliance of the piezoelectric transducer
r_{ml}:	mechanical loss in the piezoelement
A_{bar}, A_{disk} :	Surface area of piezoelectric bar and disc
h_{bar}, h_{disk} :	thickness of piezoelectric bar and disc
ρ :	piezoelectric material density

T_1, T_2, and $S_{1,2}$ are the stress and strain along axes (1,2) perpendicular to polarization direction (3).

I. INTRODUCTION

Piezoelectric cylinders polarized through the thickness (electrodes on their end-surfaces) and vibrating along the axial direction are widely used for broadband underwater acoustic sonar applications, such as in depth sounder, fish finder and Acoustic Doppler Current Profilers. It is well known that the resonant frequencies and effective coupling coefficients of finite sized piezoelectric cylinder are functions of its height-to-diameter ratio $h/2a$, where h is the height and a is the radius of the cylinder as shown in Figure 1. For the cases, where the aspect ratio is $\ll 1$, the cylinder is termed disc and when $h/2a \gg 1$, it is termed as rod or bar. In either of the above mentioned case, the vibration of the disc or rod can be considered as having a single degree of freedom in the axial direction under simple boundary conditions (i.e. predominantly thickness or 33 mode). When the aspect ratio is greater than 0.1 and lesser than 0.5 ($0.1 < h/2a < 0.5$) the vibration of the cylinder cannot be considered as a single degree of freedom system as the vibration is strongly coupled in both axial and radial directions [1], [2]. The coupled vibration reduces the electromechanical properties of the piezoelement and therefore the effective electromechanical coupling coefficient under that mode of vibration and results in generating spurious modes that corrupts the frequency spectrum and the acoustic radiation pattern and as a result, the piezoelement becomes a less efficient transducer [1]. Therefore broadband electroacoustic transducers using the piezoceramic discs have been designed, manufactured and used for underwater electroacoustic application where the aspect ratio is either much lesser than unity or much greater than unity.

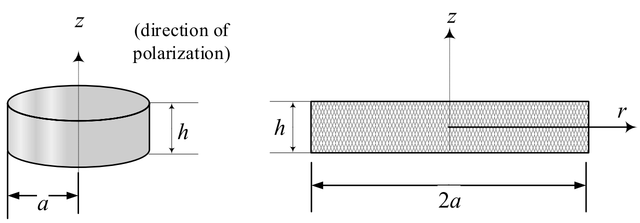

FIGURE 1. Illustration of geometry of the finite size cylinder where *h* is the height and *a* is the radius of the cylinder. The polarization axis is along the height/thickness of the disc (*z*) and the direction along the radius is represented by (*r*).

For example, Brumley *et al.* [3] uses a disc transducer with an aspect ratio much smaller than unity for Doppler Current Profiler. Such an approach employing disc transducers is common today whereby the sound is radiated in a direction normal to the face of the piezoelectric acoustic transducer to achieve a directed beam.

Aronov *et al.* [2] developed an analytical model of the electromechanical transduction in the piezoelectric discs and rods using the energy method approach and including the effects of coupled vibrations for an aspect ratio of $0.5 < h/2a < 1.5$. Expressions for the electromechanical properties and the effective electromechanical coupling coefficient were derived in this range of aspect ratio. However, for aspect ratios in the range of $0.1 < h/2a < 0.5$, due to higher radial modal coupling with fundamental axial mode, the coupled two-degree-of-freedom system is not applicable. Experimental data shows poor effective coupling coefficient for fundamental axial mode in this range of aspect ratio, therefore Aronov recommends to avoid a piezoelectric disc with such aspect ratios for designing an disc transducer vibrating using the thickness mode. As a result, such piezoelectric discs are not applicable for broadband high power applications due to the inherent non-uniform vibrations along the surface of the disc and the reduced directional factor.

However, in practice, beyond the frequency range of 100 kHz, piezoelectric ceramics in the shape of discs and bars provide the best possible option for electroacoustic transducers designs with desired operation frequency, range, power and directivity. We propose a diced disc design (hybrid design) that overcomes the disadvantages of the uniform disc having aspect ratio in the range of $0.1 < h/2a < 0.5$. Both analytical and experimental studies reveal that the new design can effectively reduce the coupled vibrations resulting in a single-degree-of-freedom vibration in the axial direction. As a result the electromechanical coupling coefficient can be greatly improved for the operation mode and the transducer can be used in high power applications.

The hybrid design of a piezoelectric disc considers a solid piezoceramic disc that is partially diced through its height (or thickness). The piezoceramic disc has electrodes applied to the end-surfaces and polarized in the axial direction before dicing. Dicing is along two perpendicular axes on one end surface. After dicing, plural rectangular bars remain in junction with a thin disc that has a thickness-to-diameter ratio $h/2a < 0.1$. Dicing pitch and blade width is flexible as long as the bars have an aspect ratio of the height to the lateral dimensions >2.5. The resulting piezoelement composes of an array of bars mechanically connected to a thin disc. Although, in this study, we start from a uniform disc with $0.1 < h/2a < 0.5$, this method can be applicable to piezoceramic disc of any aspect ratio if necessary. The technique is similar to using composite transducers where several single piezoceramic pieces are assembled together and bonded using a common base [4] but composite transducers are extremely difficult to manufacture especially at frequencies >100 kHz because of the small size of the individual piezoceramic elements used and high cost of manufacturing compared to the technique discussed in this paper.

A Finite Element Model was also used to study the vibrational behavior of the diced piezoelectric disc under different dicing conditions. The FEM used in this paper is fairly general and does not explain the inherent energy conversions. An analytical model is therefore developed based on the one-degree-of-freedom assumption of the diced disc to provide a better physical understanding of the electromechanical energy conversion and optimizing the electroacoustic transducer. Results from the FEM and experiment indicate very good comparison with analytical model.

The paper is organized in the following manner. Section I explains the electrical characterization of the single degree of freedom system of a mechanical system such as a bar or a disc where the aspect ratio is much greater or less than unity. The effective coupling coefficient of the disc with increasing aspect ratio is provided and compared with experiment results in Section II. The hybrid design modeled with an equivalent electromechanical circuit is illustrated in Section III. The FEM is briefly described in Section IV followed by conclusions based on the experimental and analytical results in Section V.

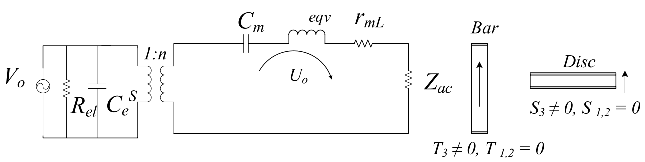

FIGURE 2. **Equivalent electrical circuit representation of a one-degree-freedom system represented by cylinder with two extremities of the height-to-diameter aspect ratio. T_3 and S_3 are the stress and strain along polarization direction (3) respectively. $T_{1,2}$, and $S_{1,2}$ are the stress and strain along axes perpendicular (1,2) to polarization direction (3).**

II. PIEZOELECTRIC BARS AND DISCS

A piezoelectric disc with aspect ratio $h/2a$ (either $h/2a \ll 1$ or $h/2a \gg 1$) polarized along the axial direction can be represented as a single-degree-of-freedom system around the fundamental resonance along the thickness [1], [5]. The disc or a bar can be represented in terms of its equivalent electromechanical circuit as shown in Figure 2 [6], [7].

For a disc with aspect ratio ($h/2a < 0.1$), the electromechanical properties and the equivalent parameters vibrating in the fundamental thickness mode are given here [2], [8]:

$$\begin{aligned} M_{eqv} &= \frac{1}{2}\rho A_{disk} h_{disk}, \quad C_m^E = \frac{2h_{disk}}{\pi^2 A_{disk}} s_{33}^D, \quad n = \frac{e_{33}A_{disk}}{h_{disk}} \\ r_{mL} &= \frac{\omega M_{eqv}}{Q_m}, \quad Q_m : \text{mechanical quality factor} \\ Z_{ac} &= \text{radiation acoustic impedance} \end{aligned} \tag{1.1}$$

where A_{disc} and h_{disc} are surface area and thickness of the disc. The expression for the effective electromechanical coupling coefficient of the disc given here as, $k_t^2 = \frac{e_{33}^2}{\varepsilon_{33}^S c_{33}^D}$.

For a bar with $h/2a > 2.5$, the equivalent electromechanical parameters vibrating in the fundamental height mode are given here [5], [9]:

$$\begin{aligned} M_{eqv} &= \frac{1}{2}\rho A_{bar} h_{bar} \\ C_m^E &= \frac{2s_{33}^E h_{bar}}{\pi^2 A_{bar}} \frac{1-k_{33}^2}{1-8/\pi^2 k_{33}^2} \quad n = \frac{2d_{33}A_{bar}}{S_{33}^E h_{bar}} \end{aligned} \tag{1.2}$$

where A_{bar} and h_{bar} are surface area and height of the bar. The expression for the effective electromechanical coupling coefficient of the bar is given here as, $k_{33}^2 = \frac{d_{33}^2}{\varepsilon_{33}^T s_{33}^E}$.

For intermediate aspect ratio ($0.1 < h/2a < 1.5$) that involves coupled vibration in both height and radial directions, the analysis can be referred to [2]. For this case, multiple radial modes are coupled with fundamental axial mode in the frequency spectrum, resulting in a reduction of effective coupling coefficient and electromechanical parameters in the piezoelement and the equivalent electromechanical circuit given in Figure 2 is not valid.

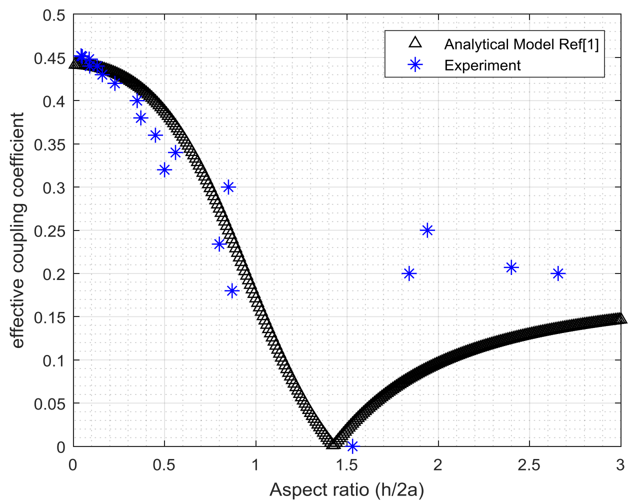

FIGURE 3. **Comparison of the effective electromechanical coupling coefficient between the experimental results and analytical model for piezoelectric discs for various aspect ratios.**

Figure 3 provides the comparison of the effective electromechanical coupling coefficient between the experimental results and analytical model for various aspect ratios. The analytical model was developed based on energy method using the approach described in [2] and an experimental investigation was conducted with various samples of different aspect ratios. This graph was plotted for piezoelectric discs made of PZT-4 Navy TYPE 1 material. As we can see from Figure 3, k_{eff} reduces as the aspect ratio increases and goes to zero around an aspect ratio of 1.5. At the aspect ratio of 1.5, the coupling between the radial modes and the axial modes is the greatest and results in no energy conversion along the axial direction. This is due to the increase in the stress T_1,T_2 as the aspect ratio increases and the resultant piezoelectric constants in the radial direction (d_{31}) is of opposite sign to the piezoelectric constant in the axial direction (d_{33}). Beyond the aspect ratio of 1.5, the k_{eff} increases as the effect of stress T_1, T_2 along the radial direction decreases. Although the experiment follows the nature of the analytical model [2], it is difficult to predict the effective coupling coefficient in the range of $0.1 \sim 0.5$ due to the existence of strong coupling between the thickness mode and the higher order radial modes.

III. HYBRID DICED-DISC TRANSDUCER

In order to reduce the effect of stress T_1, T_2, and to increase the electromechanical parameters, we propose a new hybrid design that employs dicing of the disc through its height direction (i.e. polarizing direction) resulting in a piezoelement composed of plural rectangular bars in junction with a thin base disc. The hybrid design is shown in Figure 4.

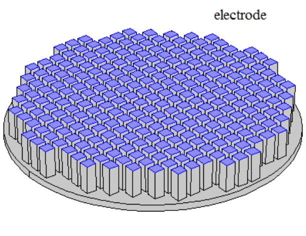

FIGURE 4. Illustration of the hybrid diced disc design with multiple pillars. The electrodes are applied to top/bottom surfaces of the hybrid disc.

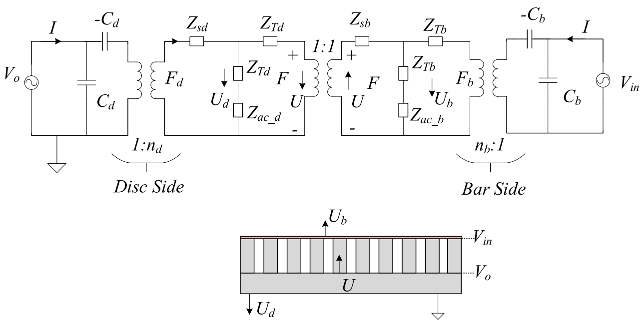

FIGURE 5. Equivalent electrical circuit for the diced piezo-disc. The disc and bar are represented by 3-port circuit, which are mechanically joined together at their interface. V_0 is the electrical potential at the interface. U_d and U_b are velocities at the disc surface and bar surface respectively. U is the velocity at the interface. F_d, F_b and F represent equivalent forces developed across the disk, bar and at their interface respectively.

Dicing results in suppressing the coupled vibration and effectively increases the vibration along the direction of the polarization. Dicing in 2D results in multiple bars (pillars) with lateral dimensions much smaller than height ($h/2a \gg 1$). These pillars vibrate in 33 mode and employ the longitudinal piezoelectric effect and result in having the maximum effective coupling coefficient of k_{33eff}. The remaining base, disc, has small thickness-to-diameter ratio ($h/2a \ll 1$) and vibrates in thickness mode with coupling coefficient of k_t. The diced piezoelectric element does not have the problem of coupled vibrations and behaves as a single degree of freedom system near its fundamental resonance frequency along the thickness. The equivalent electrical circuit of the hybrid diced piezoelement is shown in Figure 5. In the equivalent circuit, the motional impedances for a thin disc and a tall bar are represented as equivalent network impedance in terms of Z_{sd}, Z_{td} and Z_{sb}, Z_{tb}. The acoustic loading on the surface of disc and bar is represented as Z_{ac_d} and Z_{ac_b} respectively. C_d and C_b are clamped electrical capacitance of the disc and bar respectively. The electromechanical parameters for the hybrid diced disc are derived based on the parameters given here below,

$$
\begin{aligned}
C_d &= \frac{\varepsilon_{33}^S A_{disk}}{h_{disk}}, \quad \kappa = \frac{\omega}{V^D} = \omega\sqrt{\frac{\rho}{c_{33}^D}}, \quad n_d = C_d h_{33} \\
Z_0 &= \rho V^D A_{disk} \quad Z_{Td} = jZ_0 \tan\left(\frac{\kappa h_{disk}}{2}\right), \\
Z_{Sd} &= -jZ_0 \csc\left(\kappa h_{disk}\right) \\
Z_d^{ac} &= \rho c A_{disk} \quad \text{acoustic radiation impedance}
\end{aligned} \tag{1.3}
$$

$$
\begin{aligned}
C_b &= \frac{\varepsilon_{33}^S M A_{bar}}{h_{bar}}, \quad \kappa = \frac{\omega}{V^D} = \omega\sqrt{\frac{\rho}{c_{33}^D}}, \quad n_b = C_b h_{33} \\
Z_0 &= \rho V^D M A_{bar} \quad Z_{Tb} = jZ_0 \tan\left(\frac{\kappa h_{bar}}{2}\right), \\
Z_{Sb} &= -jZ_0 \csc\left(\kappa h_{bar}\right) \\
Z_b^{ac} &= M Z_{bar}
\end{aligned} \tag{1.4}
$$

where Z_{bar} is the acoustic impedance of a length-extension bar and M is the number of bars. The loss of the piezoelement is taken into consideration by assuming complex valued material parameters of ε_{33}^S, c_{33}^D, h_{33} [12].

In general, the radiation impedance of a thin piezoelectric disc Z_{ac_d} is real and proportional to the surface area of the disc. On the other side, for a length extension bar where lateral dimension is much smaller than the wavelength, the radiation impedance is reactive and is proportional to $j\omega\rho A_{bar}^{3/2}$. One of the advantages of using a hybrid design is that the acoustic loading of the design can be tailored to obtain an optimum value allowing better matching between input power and electromechanical coupling.

From equations (1.3) (1.4), the electrical impedance in air can be obtained as $Z = \frac{V_{in}}{I}$ (with $Z_{ac\_d} = Z_{ac\_b} = 0$). The set of equation that govern the operation of the piezoelement can be represented in the following form,

$$
\begin{aligned}
F_d &= (U_d + U)\, Z_{Sd} + U_d Z_{Td} \\
F &= U_d Z_{Td} - U Z_{Td} \\
F_b &= (U_b - U)\, Z_{Sb} + U_b Z_{Tb} \\
-F &= U_b Z_{Tb} + U Z_{Tb} \\
F_d &= n_d \left[V_0 - \frac{n_d}{j\omega\,(-C_d)}\,(U_d + U) \right] \\
F_b &= n_b \left[V_{in} - V_0 - \frac{n_b}{j\omega\,(-C_b)}\,(U_b - U) \right] \\
I &= j\omega C_d V_0 + n_d\,(U_d + U) \\
I &= j\omega C_b\,(V_{in} - V_0) + n_b\,(U_b - U)
\end{aligned} \tag{1.5}
$$

Here, I is the input current and the time dependent factor $e^{(jwt)}$ is omitted. The material parameters are: $\varepsilon_{33}^S = 560$, $c_{33}^D = 1.2 \times 10^{11}$Pa, $e_{33} = 15\text{C/m}^2$, $h_{33} = \frac{e_{33}}{\varepsilon_{33}^S}$V/m.

An experimental investigation of the dicing process was completed on a piezoelement in the shape of disc according to the geometry shown in Figure 1. The resonance and anti-resonance frequencies including the effective electromechanical coefficient were measured on the samples before and after the dicing process using the standard resonance/antiresonance method [10] with an HP4194A impedance analyzer. The discs were first placed on a sacrificial plate made out of polycarbonate. A small groove with a thickness of about 0.010'' was designed in the polycarbonate to seat the piezoelectric ceramic in the polycarbonate groove during dicing. The dicing process was performed using a CNC machine with a diamond blade (Buehler.com, IL). During the process of dicing, care has to be taken to lubricate the diamond blade continuously to obtain a smooth cut. A picture of the piezoelement during the dicing process is shown below in Figure 6 and a picture of the diced piezoelement is shown in Figure 7. The aspect ratio of the piezoelement shown in the figure is 0.12.

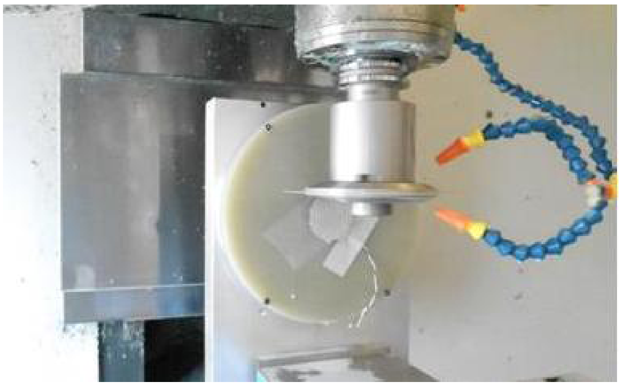

FIGURE 6. **Picture of the dicing process of the ceramic. The ceramic is attached to a sacrificial plate and diced using a diamond blade in the CNC.**

FIGURE 7. **An example picture of the diced piezoelement.**

From the equivalent electrical circuit shown in Figure 5, the modeled electrical admittance of the hybrid diced design is plotted against the frequency normalized to its fundamental axial resonant frequency as shown in Figure 8. The experimental data shown agrees very well with modeled results. The admittance of a uniform disc having multiple coupled resonances in the frequency spectrum is also shown for comparison. The aspect ratios of the disc and the hybrid design are 0.12. Navy Type I material was used for the model and the two designs. From the admittance plot, in the case of a piezoelectric disc, it is difficult to identify the major thickness vibration mode without applying additional boundary constraints due to the strong coupling between thickness and radial vibrations of the disc. On the other hand,

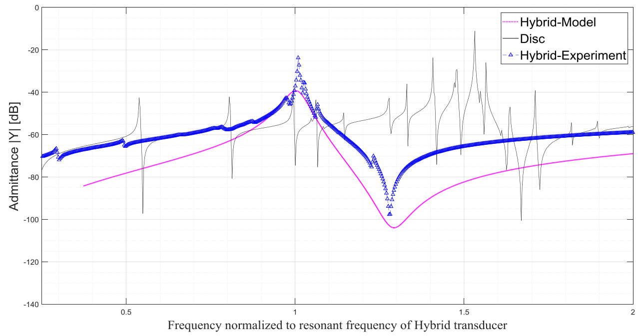

FIGURE 8. **Electrical Admittance plotted against frequency normalized to axial resonant frequency of a hybrid diced transducer (partially diced transducer). The model predicts well with the experiment results and suppression of the radial modes as observed in the diced disc. The effective coupling coefficient of the hybrid design is improved by 70% compared to that of a uniform disc.**

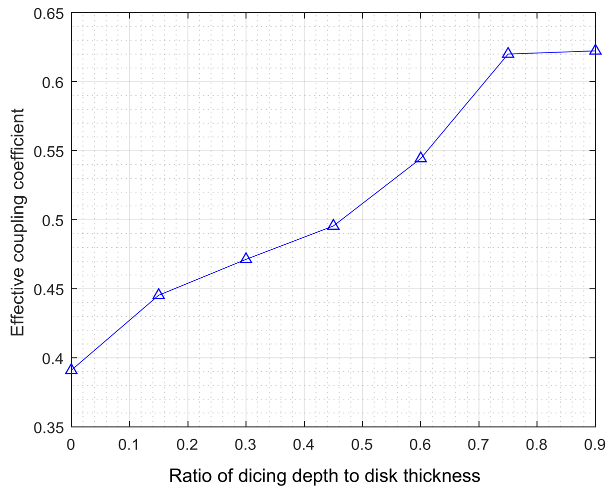

FIGURE 9. **Experimental data of the effective coupling coefficient (k_{eff}) plotted as a function of dicing depth for the disc of aspect ratio of 0.12. As seen from the graph, the coupling coefficient increases as the dicing of the disc is increased and tries to approach the maximum effective coupling coefficient of 0.64 of a single degree of freedom system.**

there is a significant reduction in the coupling modes in the hybrid design (partially diced disc) as shown in Figure 8. The effective coupling coefficient increases from a $k_t \sim 0.35$ to the maximum of $k_{33eff} \sim 0.64$. The dicing process increases the utilization of the longitudinal piezoelectric effect in (33 mode) along the thickness direction where the strain is maximum under that mode of vibration. Therefore, by properly dicing the disc through the thickness, a single resonant frequency can be obtained in the axial mode and the associated electromechanical properties can be improved.

Figure 9 shows the plot of the experimental results of the effective coupling coefficient as the disc is diced with aspect ratio of 0.12. As seen from Figure 9, the effective coupling coefficient of the disc increases as the dicing depth is increased and gradually approaches the maximum effective coupling coefficient of k_{33eff} of 0.64 for a length extension bar using the longitudinal piezoelectric effect.

The inherent non uniform vibrations on the surface of a uniform piezoelectric disc may prevent it from being used in high power applications. It is well known that increasing the mechanical quality factor Q_m under electrical limiting conditions, will result in increase in the acoustic power. Also, decreasing the Q_m, under mechanical limiting conditions will result in increase in the acoustic power [11]. A tradeoff can be sought using the hybrid diced design where its Q_m can be selected based on design requirement and loading condition. In practical cases, a fully diced disc is difficult for handling and assembly, therefore partial dicing is implemented where the mechanical strength of the disc as well as the improved electromechanical properties of the bar are preserved.

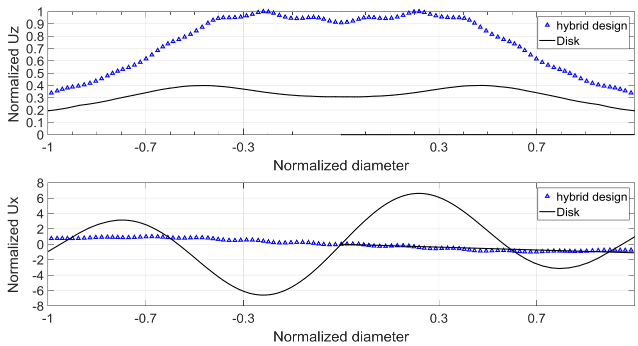

FIGURE 10. Comparison of mode shapes along the diameter of the planar surface at the resonant frequency of diced disc and uniform disc of the same aspect ratio of 0.12. Ux is the horizontal displacement and Uz is the vertical displacement.

IV. FEM MODELING

Finite Element Method (FEM) is one of the effective ways of studying the vibration behavior of geometries shown in Figure 4. In this FEM model we calculate the dynamic vibration of a diced piezoelectric disk shown in Figure 7 over a broad frequency range. All the surfaces vibrate under free-stress condition which simulates the vibration of the piezoceramic in air. The excitation voltage is applied across the top/bottom electrodes. The displacement and velocity of every node of the diced disk can be obtained from the FEM model. The resonant frequency is found by plotting the electrical admittance of the diced disk over the frequency range. The corresponding modeshape of the piezoelectric disc and the hybrid disc are calculated at resonance and are plotted as a function of normalized displacement across the surface of the disk in Figure 10. The results of single resonant peak and the modeshape proved the essential one-degree-of-freedom vibration of the diced geometry based on which the electromechanical properties can be obtained using the equivalent electrical circuit shown in Figure 5. The piezoelectric parameters used in the FEM are given below and the mode shapes are given in Figure 10.

Elastic matrix:

$$\left[C^E\right]=\begin{bmatrix} 13.9 & 7.78 & 7.43 & & & \\ & 13.9 & 7.43 & & & \\ & & 11.5 & & & \\ & & & 2.56 & & \\ & & & & 2.56 & \\ & & & & & 3.06 \end{bmatrix}\times 10^{10}\ N/m^2$$

Coupling matrix:

$$[e]=\begin{bmatrix} & & & & 12.7 & \\ & & & 12.7 & & \\ -5.2 & -5.2 & 15.1 & & & \end{bmatrix} C/m^2$$

Dielectric Matrix:

$$\left[\varepsilon^S\right]=\begin{bmatrix} 762.5 & & \\ & 762.5 & \\ & & 663.2 \end{bmatrix}\varepsilon_0$$

$\tan\delta_e = 0.004$ and $\tan\delta_m = 0.002$. (The unfilled values in the above matrices are zero)

The resonant mode shape of the hybrid diced disc of aspect ratio of 0.12 and a uniform disc of the same aspect ratio is plotted in Figure 10. It is noted that the mode shape is taken in the planar surfaces along the diameter with free boundary conditions. As seen from Figure 10, the radial displacement of the hybrid diced disc is virtually zero. The axial displacement is three times higher than the corresponding disc with the same excitation voltage. It is noted for both cases, the mode shape is not strictly planar. The amplitudes are approximately reduced by half near the edge of the discs for both cases. This is due to the fact that the disc has finite radius, where the axial stress must reduce to zero at the free edges. A distribution of axial stress must be established that allow the change from maximum at the center of the disc to zero at the edge of the disc. This uneven mode shape can only be studied with FEM with full elastic theory, where one-degree-freedom system would inherently assume a uniform, piston like amplitude distribution. Nevertheless, a one-degree-freedom system is fairly good approximation for the hybrid diced design, while not applicable for the uniform disc where coupled vibration results in a quite none uniform distribution of radial displacement.

V. CONCLUSION

The electromechanical properties of a diced piezoelectric disc are calculated and modeled using analytical method and compared favorably with experiment. The equivalent electromechanical circuit for the diced disc is derived as a single degree of freedom system. In addition, FEM was used to model the vibrations of the diced disc in comparison to a uniform disc of the same aspect ratio. The non uniform mode shape results from the coupled axial and radial modes are illustrated for uniform disc and suppression of such coupled vibration is shown for diced disc, which further validate the single degree of freedom system of the diced disc design. The electrical admittance and effective coupling coefficient can be readily obtained from the equivalent circuit, which further can be used to optimize dicing parameters for the electroacoustic transducer designs. In this paper, we demonstrated a potential 70% improvement in electromechanical coupling coefficient from a diced disc design due to greatly suppressed radial coupling from axial vibration. The effective electromechanical coupling coefficient of the hybrid transducer reaches the maximum value of ~0.64 due to the increase of the longitudinal piezoelectric effect (33-mode) compared to that of a disc.

We also provide experiment data to prove the non-uniform vibration and effect of the coupled vibration of the disc transducer as the aspect ratio is increased. Without dicing, the effective coupling coefficient decreases almost to zero as the aspect ratio is increased to 1.5. The hybrid disc design reduces the negative effect of the coupled vibration and the non-uniform vibrations happening in the disc and increases the electromechanical properties and the effective electromechanical coupling coefficient.

The effect of the dicing along the thickness or height of the piezoelectric hybrid disc is experimentally studied and plotted. Although dicing could reduce piezoceramic volume, the merit of dicing is to improve the electromechanical efficiency of the transducer system by removing significantly spurious radial modes. This can also result in an improved radiation directivity of the transducer by reducing the side lobes associated with these radial modes. Future work will be focused on directional factor and the high power applications of the assembled transducer.

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