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# Target Tracking Based on Incremental Center Differential Kalman Filter With Uncompensated Biases

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**ABSTRACT** To mitigate the negative effects of the sensor measurement biases for the maneuvering target, a novel incremental center differential Kalman filter (ICDKF) algorithm is proposed. Based on the principle of independent incremental random process, the incremental measurement equation is modeled to preprocess the sensor measurement biases. Then, a general ICDKF algorithm is proposed by augmenting the process and measurement noises into the state vector to mitigate the negative effects of the sensor biases. For the system with additive noises, an additive ICDKF algorithm is derived by introducing the incremental measurement equation to reduce the measurement biases. Numerical simulations for four types of sensor biases are designed to demonstrate that the proposed ICDKF can effectively mitigate the measurement biases compared to the CDKF.

**INDEX TERMS** Center differential Kalman filter, incremental measurement equation, target tracking, systematic bias, offset bias.

## I. INTRODUCTION

The classic Kalman filter (KF) is optimal when the disturbances of the process and measurement systems can be modeled as Gaussian white noises. Any colored noises or biases in the systems may degrade the filtering performance, especially the sensor measurement biases in the target tracking system [1], [2]. To improve the tracking performance, the sensor measurement biases in the tracking system should be mitigated or compensated.

Many methodologies are proposed to mitigate the negative effects of the measurement biases. The first idea is augmenting the biases into the state vector to estimate them and then compensating the measurements. But, implementing this augmented strategy may load infeasible computation or even diverge for ill-conditioned systems. Haessig and Friedland [3] proposed a paralleling reduced-order filtering to separate the bias estimates, and demonstrated that it is equivalent to the above augmenting strategy. Lin *et al.* [4] estimated the biases by using the local unassociated track estimates at a single time. Schmidt presented a consider Kalman filter to solve the systematic biases, which considers the cross-covariance between the biases and the states, but not estimates them [5]–[7]. Based on the sensitivity of the model uncertainties (including biases) and desensitized optimal control methodology, the desensitized Kalman filter is proposed to reduce these sensitivities [8], [9]. Kai *et al.* [10] devised a robust extended Kalman filter with stochastic uncertainties in the non-linear discrete-time system models. Xie *et al.* [11] designed a $H_\infty$ filtering for linear discrete-time systems with norm-bounded parameter uncertainties in both state and output matrices. Habibi [12] developed a smooth variable structure filter to guarantee stability given an upper bound for uncertainties and noise levels. Ben Hmida *et al.* [13] presented a robust three-stage Kalman filter for linear stochastic discrete-time varying systems with unknown inputs to give an unbiased minimum-variance estimation. Based on the principle of independent incremental random process [14], Fu *et al.* proposed an incremental filtering concept to mitigate the systematic biases in original measurement equation by reconstructing the measurement equation [15], [16]. In addition, multiple model adaptive estimation [17], set-valued estimation [18] and so on, are proposed to reduce the negative effects of the systematic biases.

Estimating unknown sensor measurement biases is an important problem for the target tracking system. Uncorrected biases can lead to large tracking errors and multiple unknown formations for the same target [1]. Zhang *et al.* [19] used the consider Kalman filter (also called Schmidt-Kalman filter) algorithm to solve the combined problem of "out-of-sequence" measurement with residual biases from multiple sensors. Lin *et al.* [2] provided an exact method for the multiple sensor bias estimation problem based on local tracks. Xiaoquan *et al.* [20] designed a new unbiased conversion approach to compensate the biases for target tracking by obtaining the covariance of the conversion. Bar-Shalom [21] used static-rotator targets of opportunity to estimate the position biases of the airborne ground in moving target indicator radars.

The center differential Kalman filter (CDKF) is a state estimation methodology to solve nonlinear system problems. The CDKF is obtained by using the multivariable extension of the Stirling interpolation principle to approximate the nonlinear function [9], [22].To deal with the biases in the ranging and azimuth measurements for the target tracking, an incremental nonlinear measurement equation is introduced into the CDKF, and a new incremental center differential Kalman filter (ICDKF) is proposed to mitigate the negative effects of the measurement biases for the nonlinear system.

The structure of this paper is as follows. Section II analyzes the biases of the target tacking measurement equation, and sets up the incremental measurement equation. In Section III derives the proposed ICDKF algorithm for general discrete dynamic system and measurement model. The ICDKF algorithm for system with additive noises is derived in Section IV. Four numerical simulations with different biases are run to demonstrate the performance of the proposed ICDKF comparing with the CDKF algorithm. The conclusion is summarized in Section V.

## II. TARGET TRACKING MODEL WITH BIASES

Consider a target (aircraft, missile, etc.) that moves in a two-dimensional plane $XOY$. The motion states of the target in the horizontal direction ($X$-axis direction) and the vertical direction ($Y$-axis direction), which includes position, velocity and acceleration, are represented by vector $x_k = [x_k, y_k, \dot{x}_k, \dot{y}_k, \ddot{x}_k, \ddot{y}_k]^T$ at time $t_k$.The discrete dynamic model of the target is defined by

$$x_k = f(x_{k-1}, u_{k-1}, w_{k-1}) \tag{1}$$

where $u_k$ is the control input vector, and $w_k$ is the zero mean Gaussian white noises with covariance $Q_k$.

The discrete-time target dynamic model may be constant velocity (CV) model, constant acceleration (CA) model, Singer model, "Current" model, or constant turn model (CT) with known turn rate and unknown turn rate [23]. In this work, the sensor, which measures the range and azimuth of the target, is a radar sensor or an imaging sensor. The sensor is fixed in a certain position or in a moving air vehicle. The measurement model with biases of the target in the polar coordinates at time $t_k$ is denoted by [1], [2], [24]

$$z_k = h(x_k, b_k, v_k) = \begin{bmatrix} (1+s_k^r)r_k + o_k^r + v_k^r \\ (1+s_k^\phi)\phi_k + o_k^\phi + v_k^\phi \end{bmatrix} \tag{2}$$

where $r_k$ and $\phi_k$ are the true range and azimuth; $b_k = \left[s_k^r, s_k^\phi, o_k^r, o_k^\phi\right]^T$ is the bias vector, in which $s_k^r$ and $s_k^\phi$ are the scale biases for the range and azimuth, and $o_k^r$ and $o_k^\phi$ are the offset biases (also called systematic errors) of the range and azimuth; the measure noises $v_k^r$ and $v_k^\phi$ of the range and azimuth are the zero mean Gaussian white noises with covariance $\sigma_r^2$ and $\sigma_\phi^2$, respectively, and are assumed mutually independent with $w_k$.

Here, the biases $b_k$ of the sensor can be modeled as a constant variable, a suddenly-change variable or a Gauss-Markov random variable [1], [24]. Then, Eq.(2) can be re-writed as

$$z_k = \begin{bmatrix} r_k \\ \phi_k \end{bmatrix} + \Gamma_k b_k + \begin{bmatrix} v_k^r \\ v_k^\phi \end{bmatrix} \tag{3}$$

where

$$\Gamma_k = \begin{bmatrix} r_k & 0 & 1 & 0 \\ 0 & \phi_k & 0 & 1 \end{bmatrix} \tag{4}$$

When the sampling time of the system is small, the difference of the bias value for two adjacent measurement vectors $z_k$ and $z_{k-1}$ is relatively small, that is to say, $\Delta b_k = \Gamma_k b_k - \Gamma_{k-1} b_{k-1}$ is small. This difference $\Delta b_k$ in the measurement vector $\Delta z_k = z_k - z_{k-1}$ can be modeled as zero mean Gaussian white noises or even neglected. Moreover, from the principle of independent incremental random process [14], it can be seen that $\Delta z_k$ and $\Delta z_{k-1}$ meet the independent requirement of measurement data in filter processing, respected to the direct measurement values $z_k$ and $z_{k-1}$. Then, based on the above discussion, the incremental measurement for the sensor is modeled as

$$\begin{aligned} \Delta z_k &= h\left(x_k, b_k, x_{k-1}, b_{k-1}, v_k^*\right) \\ &= h(x_k, b_k, v_k) - h(x_{k-1}, b_{k-1}, v_{k-1}) \\ &= \begin{bmatrix} r_k - r_{k-1} + v_k^{*r} \\ \phi_k - \phi_{k-1} + v_k^{*\phi} \end{bmatrix} \end{aligned} \tag{5}$$

where $v_k^{*r} = v_k^r - v_{k-1}^r$ and $v_k^{*\phi} = v_k^\phi - v_{k-1}^\phi$ are zero mean Gaussian white noises with covariance $2\sigma_r^2$ and $2\sigma_\phi^2$, respectively, and are also independent with $w_k$.

## III. INCREMENTAL CENTER DIFFERENCE KALMAN FILTER

In this work, two cases with different form of the noises for the process and measurement models. One is the above general discrete-time dynamic equation and incremental measurement equation, which is denoted as

$$x_k = f(x_{k-1}, u_{k-1}, w_{k-1}) \tag{6}$$

$$\Delta z_k = h\left(x_k, b_k, x_{k-1}, b_{k-1}, v_k^*\right) \tag{7}$$

Another one is the discrete dynamic equation and incremental measurement equation with additive noises, which is modeled by

$$x_k = f(x_{k-1}, u_{k-1}) + w_{k-1} \tag{8}$$
$$\Delta z_k = h(x_k, b_k) - h(x_{k-1}, b_{k-1}) + v_k^* \tag{9}$$

where the direct measurement equation with additive noises is given by

$$z_k = h(x_k, b_k) + v_k \tag{10}$$

### *A. GENERAL ICDKF ALGORITHMS*

For discrete model Eqs.(6) and (7), the process and measurement noises are augmented into the state vector to estimate together with the state. Based on the center differential transformation [22], the state distribution including the mean and covariance is represented by a Gaussian random variable with sigma-points. The sequential incremental measurement vectors with unnegligible biases are introduced into the CDKF, and the ICDKF is presented to enhance the estimation accuracies. The general ICDKF for incremental measurement equation is derived as follow.

Firstly, initializing the state and covariance

$$\hat{x}_0 = E[x_0] \tag{11}$$
$$P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \tag{12}$$

Secondly, based on the posteriori state $\hat{x}_{k-1}^+$ and covariance $P_{k-1}^+$ at time $t_{k-1}$, augmenting the process noise $w_{k-1}$ into the state vector and the process covariance $Q_{k-1}$ into the covariance matrix

$$\hat{x}_{k-1}^{+\alpha} = [\hat{x}_{k-1}^{+T}, w_{k-1}^T]^T \tag{13}$$
$$P_{k-1}^{+\alpha} = \begin{bmatrix} P_{k-1}^+ & 0 \\ 0 & Q_{k-1} \end{bmatrix} \tag{14}$$

and then evaluating time-updated sigma-points

$$\chi_{k-1}^{\alpha} = [\hat{x}_{k-1}^{+\alpha}, \hat{x}_{k-1}^{+\alpha} \pm \lambda\sqrt{P_{k-1}^{+\alpha}}] \tag{15}$$

Evaluating the propagated sigma-points

$$\chi_{k/k-1}^{x} = f(\chi_{k-1}^{\alpha x}, \chi_{k-1}^{\alpha w}, u_{k-1}) \tag{16}$$

Estimating the priori state and covariance

$$\hat{x}_k^- = \sum_{i=0}^{2n} w_i^{(m)} \chi_{i,k/k-1}^x \tag{17}$$

$$\begin{aligned} P_k^- = \sum_{i=1}^{n} [& w_i^{(c_1)}(\chi_{i,k/k-1}^x - \chi_{n+1,k/k-1}^x) \\ & \times (\chi_{i,k/k-1}^x - \chi_{n+1,k/k-1}^x)^T \\ & + w_i^{(c_2)}(\chi_{i,k/k-1}^x + \chi_{n+i,k/k-1}^x - 2\chi_{0,k/k-1}^x) \\ & \times (\chi_{i,k/k-1}^x + \chi_{n+i,k/k-1}^x - 2\chi_{0,k/k-1}^x)^T] \end{aligned} \tag{18}$$

Thirdly, augmenting the process noise $v_k^*$ into the state vector and the process covariance $2R_k$ into the covariance matrix, and resampling the sigma-points

$$\hat{x}_{k/k-1}^{\beta} = [\hat{x}_k^-, v_k^*] \tag{19}$$
$$P_{k/k-1}^{+\beta} = \begin{bmatrix} P_k^- & 0 \\ 0 & 2R_k \end{bmatrix} \tag{20}$$
$$\chi_{k/k-1}^{\beta} = [\hat{x}_{k/k-1}^{+\beta}, \hat{x}_{k/k-1}^{+\beta} \pm \lambda\sqrt{P_{k/k-1}^{+\beta}}] \tag{21}$$

Evaluating the propagated sigma-points for the measurement equation, and estimating the priori measurement

$$\Delta Z_{k/k-1} = h(\chi_{k/k-1}^{\beta x}, \hat{b}_k, \hat{x}_{k-1}^{+x}, \hat{b}_{k-1}, \chi_{k/k-1}^{\beta v}) \tag{22}$$
$$\Delta\hat{z}_k^- = \sum_{i=0}^{2n} w_i^{(m)} \Delta Z_{i,k/k-1} \tag{23}$$

where, $\hat{b}_k$ is the pre-estimate of the biases at time $t_k$. Estimating the innovation covariance matrix

$$\begin{aligned} P_{zz,k}7 = \sum_{i=1}^{n} [& w_i^{(c_1)}(\Delta Z_{i,k/k-1} - \Delta Z_{n+i,k/k-1}) \\ & \times (\Delta Z_{i,k/k-1} - \Delta Z_{n+i,k/k-1})^T \\ & + w_i^{(c_2)}(\Delta Z_{i,k/k-1} + \Delta Z_{n+i,k/k-1} - 2\Delta Z_{0,k/k-1}) \\ & \times (\Delta Z_{i,k/k-1} + \Delta Z_{n+i,k/k-1} - 2\Delta Z_{0,k/k-1})^T] \end{aligned} \tag{24}$$

Estimating the cross-covariance matrix

$$P_{xz,k} = \sqrt{w_1^{(c_1)} P_k^-}(\Delta Z_{1:n,k/k-1} - \Delta Z_{n+1:2n,k/k-1})^T \tag{25}$$

Fourthly, the gain matrix is obtained by

$$K_k = P_{xz,k} P_{zz,k}^{-1} \tag{26}$$

Lastly, estimating the posterior state and the posterior covariance matrix

$$\hat{x}_k^+ = \hat{x}_k^- + K_k(\Delta z_k - \Delta\hat{z}_k^-) \tag{27}$$
$$P_k^+ = P_k^- - K_k P_{zz,k} K_k^T \tag{28}$$

where the weights of the sigma-points are given by $(L = 2n + 1)$

$$w_0^{(m)} = \frac{\lambda^2 - L}{\lambda^2}, \quad w_i^{(m)} = \frac{1}{2\lambda^2} \tag{29}$$
$$w_i^{(c_1)} = \frac{1}{4\lambda^2} \tag{30}$$
$$w_i^{(c_2)} = \frac{\lambda^2 - 1}{4\lambda^2} \tag{31}$$

where $\lambda$ denotes the half-step length in the center differential principle, and it's optimal value is the Kurtosis of the distribution. For the Gaussian distribution, it's optimal value is $\sqrt{3}$ [9].

### B. ICDKF ALGORITHMS WITH ADDITIVE NOISES

To reduce the computation cost, the additive noises for Eqs.(8) and (9) are not incorporated into the state to generate the sigma-points, and its covariance are compensated into the priori covariance and the innovation covariance [25]. The formulations of the ICDKF algorithm with additive noises are given in the next.

Firstly, initializing the state $\hat{x}_0 = E[x_0]$ and covariance $P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]$.

Secondly, based on the posteriori state $\hat{x}_{k-1}^+$ and covariance $P_{k-1}^+$ at time $t_{k-1}$, evaluating time-update sigma-points

$$\chi_{k-1} = [\hat{x}_{k-1}^+, \hat{x}_{k-1}^+ \pm \lambda\sqrt{P_{k-1}^+}] \tag{32}$$

Evaluating the propagated sigma-points

$$\chi_{k/k-1}^* = f(\chi_{k-1}, u_{k-1}) \tag{33}$$

Estimating the priori state and covariance

$$\hat{x}_k^- = \sum_{i=0}^{2n} w_i^{(m)} \chi_{i,k/k-1}^* \tag{34}$$

$$\begin{aligned} P_k^- = \sum_{i=1}^{n} [& w_i^{(\check{c}c_1)\check{c}}(\chi_{i,k/k-1}^* - \chi_{n+i,k/k-1}^*) \\ & \times (\chi_{i,k/k-1}^* - \chi_{n+i,k/k-1}^*)^T \\ & + w_i^{(c_2)\check{c}}(\chi_{i,k/k-1}^* + \chi_{n+i,k/k-1}^* - 2\chi_{0,k/k-1}^*) \\ & \times (\chi_{i,k/k-1}^* + \chi_{n+i,k/k-1}^* - 2\chi_{0,k/k-1}^*)^T] + Q_{k-1} \end{aligned} \tag{35}$$

Thirdly, resampling the sigma-points

$$\chi_{k/k-1}^r = [\hat{x}_k^-, \hat{x}_k^- \pm \lambda\sqrt{P_k^-}] \tag{36}$$

Evaluating the propagated sigma-points for the measurement equation, and estimating the priori measurement

$$\Delta Z_{k/k-1} = h(\chi_{k/k-1}^r, \hat{b}_k) - h(\hat{x}_{k-1}^+, \hat{b}_{k-1}) \tag{37}$$

$$\Delta\hat{z}_k^- = \sum_{i=0}^{2n} w_i^{(m)} \Delta Z_{i,k/k-1} \tag{38}$$

Estimating the innovation covariance matrix

$$\begin{aligned} P_{zz,k} = \sum_{i=1}^{n} [& w_i^{(c_1)}(\Delta Z_{i,k/k-1} - \Delta Z_{n+i,k/k-1}) \\ & \times (\Delta Z_{i,k/k-1} - \Delta Z_{n+i,k/k-1})^T \\ & + w_i^{(c_2)}(\Delta Z_{i,k/k-1} + \Delta Z_{n+i,k/k-1} - 2\Delta Z_{0,k/k-1}) \\ & \times (\Delta Z_{i,k/k-1} + \Delta Z_{n+i,k/k-1} - 2\Delta Z_{0,k/k-1})^T] + 2R_k \end{aligned} \tag{39}$$

Estimating the cross-covariance matrix

$$P_{xz,k} = \sqrt{w_1^{(c_1)} P_k^-}(\Delta Z_{1:n,k/k-1} - \Delta Z_{n+1:2n,k/k-1})^T \tag{40}$$

Fourthly, obtaining the gain matrix

$$K_k = P_{xz,k} P_{zz,k}^{-1} \tag{41}$$

Lastly, estimating the posterior state and the posterior covariance matrix

$$\hat{x}_k^+ = \hat{x}_k^- + K_k(\Delta z_k - \Delta\hat{z}_k^-) \tag{42}$$

$$P_k^+ = P_k^- - K_k P_{zz,k} K_k^T \tag{43}$$

## IV. NUMERICAL SIMULATION

To evaluate the performance of the proposed ICDKF, consider a target moves with a nearly constant velocity in a two-dimensional plane. The dynamic model in Cartesian coordinate system is given by [26]

$$x_k = \Phi x_{k-1} + w_{k-1} \tag{44}$$

where

$$\Phi = \begin{bmatrix} 1 & 0 & T & 0 & \frac{T^2}{2} & 0 \\ 0 & 1 & 0 & T & 0 & \frac{T^2}{2} \\ 0 & 0 & 1 & 0 & T & 0 \\ 0 & 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{45}$$

An unmanned aerial vehicle (UAV), which carries an airborne radar, moves with a nearly constant velocity in the same plane to track the target. The radar on the UAV, which position is $(x_k^{UAV}, y_k^{UAV})$ at time $t_k$, provides the distance $r_k$ and the azimuth $\phi_k$ between the UAV and the target, and the measurement equation with offset biases is

$$\Delta z_k = \begin{bmatrix} r_k - r_{k-1} \\ \phi_k - \phi_{k-1} \end{bmatrix} + \begin{bmatrix} v_k^{*r} \\ v_k^{*\phi} \end{bmatrix} \tag{46}$$

where

$$r_k = \sqrt{(x_k - x_k^{UAV})^2 + (y_k - y_k^{UAV})^2} + o_k^r \tag{47}$$

$$\phi_k = \arctan\left(\frac{y_k - y_k^{UAV}}{x_k - x_k^{UAV}}\right) + o_k^{\phi} \tag{48}$$

In simulations, the initial state of the target is $x_0 = [1000, 5000, 10, 50, 2, -4]^T$, and the corresponding covariance is $P_0 = diag\,[100, 100, 1, 1, 0.1, 0.1]$; the process noise covariance is $Q_k = diag\left[1, 1, 0.1^2, 0.1^2, 0.01^2, 0.91^2\right]$, and the measurement noise covariance is $R_k = diag\left[10^2, 0.001^2\right]$; the initial state of the UAV is $x_0^{UAV} = [900, 4900, 1, 15, 0, 0]^T$, and the process noises of the CV model for the UAV is $Q_k^{UAV} = 0.1Q_k$. The simulation lasts 25s, and the sampling time is $T = 0.5s$. The trajectories of the UAV and the target are plotted in Fig.1. Five hundred Monte Carlo simulations are run to evaluate the performance of the ICDKF and the CDKF. The root mean squared errors (RMSE) of position and velocity for 500 times simulation are calculated, respectively, as

$$PRMSE_k = \sqrt{\frac{1}{500}\sum_{i=1}^{500}\left[(x_{k,1}^i - \hat{x}_{k,1}^i)^2 + (x_{k,2}^i - \hat{x}_{k,2}^i)^2\right]} \tag{49}$$

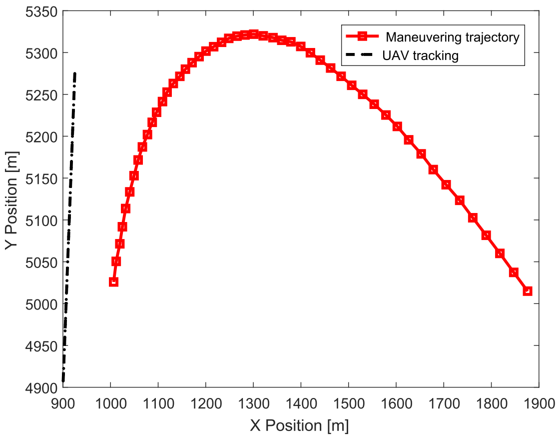


**FIGURE 1.** UAV target crossing scenario.

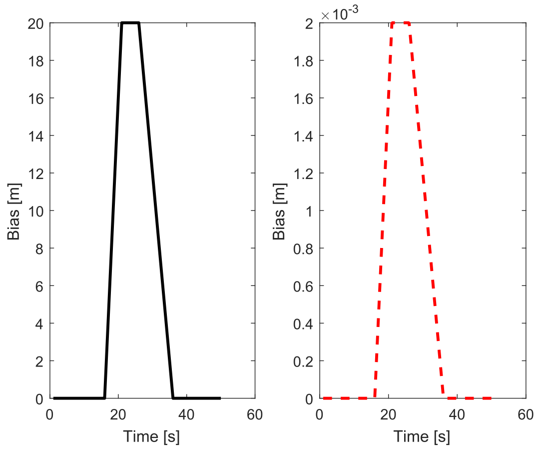


**FIGURE 2.** Values of the systematic biases.

$$VRMSE_k = \sqrt{\frac{1}{500}\sum_{i=1}^{500}\left[(x^i_{k,3}-\hat{x}^i_{k,3})^2+(x^i_{k,4}-\hat{x}^i_{k,4})^2\right]} \tag{50}$$

where $x^i_k$ is the true state, and $\hat{x}^i_k$ is the state estimate of the $i$th Monte Carlo run.

For the systematic errors $o^r_k$ and $o^\phi_k$ , four cases are considered in the simulations, which include zero mean, constant bias, suddenly-change bias and stationary first-order Markov process.

*Case 1:* Zero mean. The biases in this case are zero mean, that is to say, there is no biases in the measurement model.

*Case 2:* Random constant bias. The biases of the radar are the random constant values, which are distributed to the normal distribution with mean $\bar{b} = [20m, 0.002rad]^T$ and covariance $Q^b_k = diag\left[1^2, 0.0005^2\right]$.

*Case 3:* Suddenly-change bias. The biases of the radar suddenly increase, last sometimes, and then decrease slowly. The values of the biases are plotted in Fig. 2.

*Case 4:* Stationary first-order Markov process. The stationary first-order Markov process is driven by zero-mean Gaussian white noises, and its state transition equation is

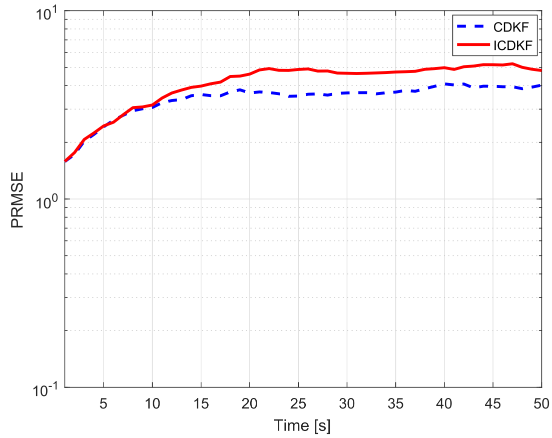


**FIGURE 3.** Position RMSEs of the CDKF and ICDKF for case 1.

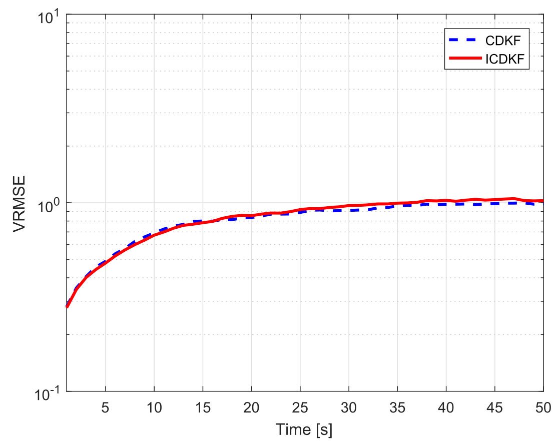


**FIGURE 4.** Velocity RMSEs of the CDKF and ICDKF for case 1.

given by

$$b_k = M_{k-1}b_{k-1} + w^b_{k-1} \tag{51}$$

where the state transition matrix is $M_{k-1} = diag\,[0.99, 0.99]$, the initial bias is $b_0 = [8m, 0.001rad]^T$, the process noises $w^b_k$ are zero-mean white noises with covariance $Q^b_k$.

The RMSEs for position and velocity of the CDKF and the ICDKF for four cases are shown in Figs.3-10, respectively. The means of the four RMSEs are listed in Table 1.

From Figs.3 and 4, it can be seen that the RMSEs of the position and velocity for the proposed ICDKF are all slightly less than those of the CDKF, because the CDKF is optimal when the measurements have no biases.

Figs.5 and 6 show the RMSEs of position and velocity for two filters when the measurements have random constant biases in case 2. The performance of the proposed ICDKF is much better than the CDKF, which is disturbed by the random constant biases.

Figs.7 and 8 clearly show the RMSEs for two filters in case 3. Because the biases suddenly change from zero to non-zero, slowly vary and steady at constant values, and then slowly decrease to zero, the CDKF is disturbed by

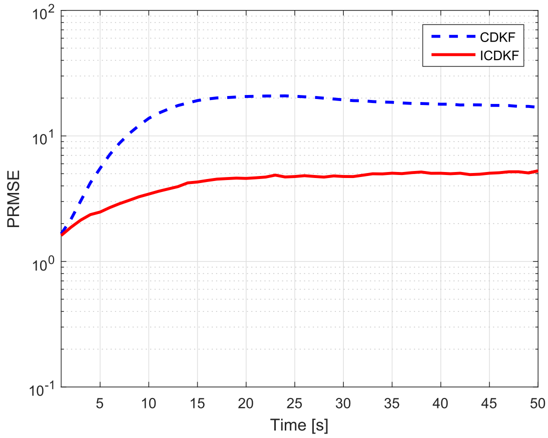


**FIGURE 5.** Position RMSEs of the CDKF and ICDKF for case 2.

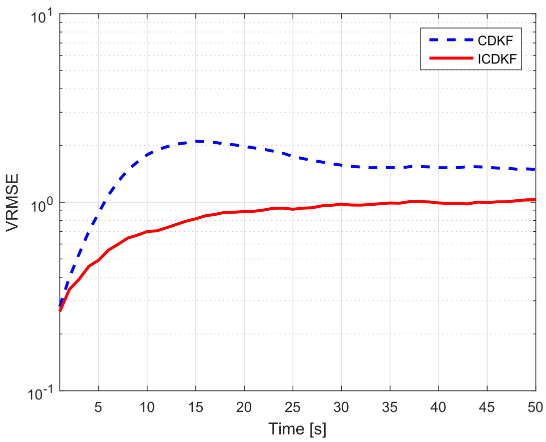


**FIGURE 6.** Velocity RMSEs of the CDKF and ICDKF for case 2.

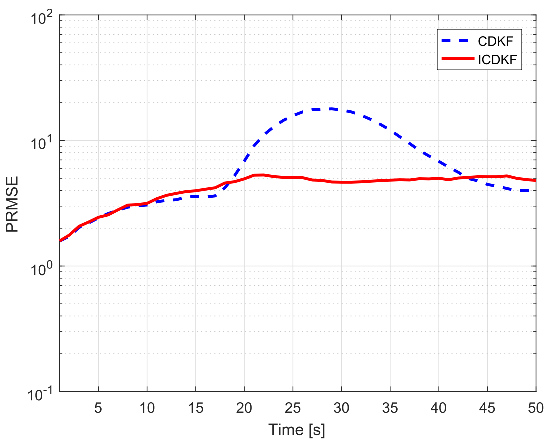


**FIGURE 7.** Position RMSEs of the CDKF and ICDKF for case 3.

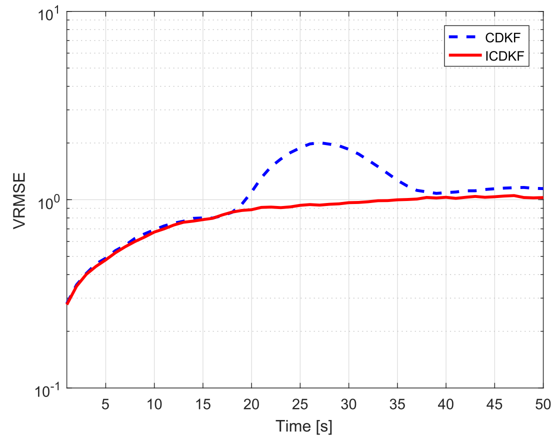


**FIGURE 8.** Velocity RMSEs of the CDKF and ICDKF for case 3.

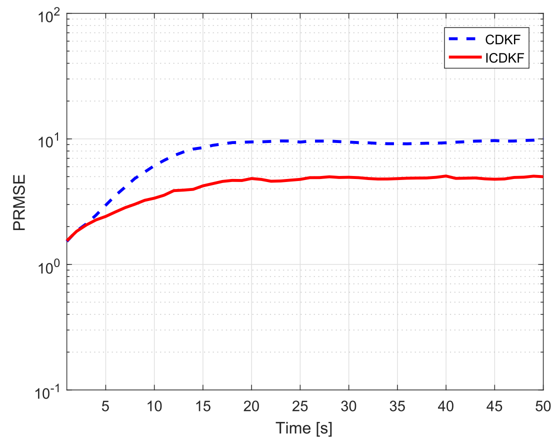


**FIGURE 9.** Position RMSEs of the CDKF and ICDKF for case 4.

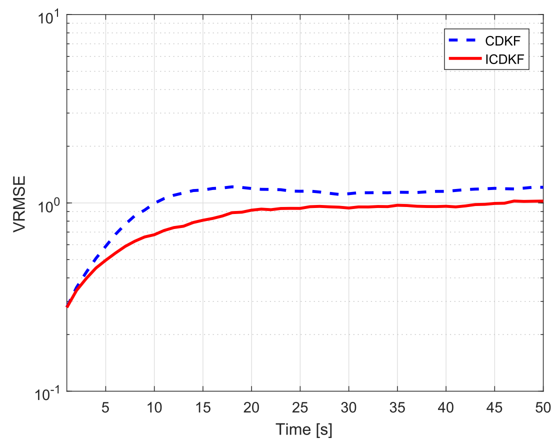


**FIGURE 10.** Velocity RMSEs of the CDKF and ICDKF for case 4.

the emergent biases, and has a worse performance than the ICDKF when the biases emerge.

For case 4, the RMSEs of position and velocity for the two filters are shown in Figs.9 and 10. The biases of the stationary first-order Markov process with initial bias $b_0$ are less than $\bar{b}$. Under this sort of case, the two RMSEs of the proposed ICDKF are also all smaller than those of the CDKF.

Finally, from Figs.5-10, it can be seen that the proposed ICDKF is superior to that of the CDKF in most cases, and has smaller state estimate errors for all three cases (random constant biases, suddenly-change biases and time-varying biases). For Table 1, it has the same results as the results from the Figs.5-10. That is to say, the proposed ICDKF can mitigate the negative effects of the biases in the measurement

**TABLE 1.** The RMSEs of the position and velocity.

| Scenario | Case 1 RMSE | | Case 2 RMSE | | Case 3 RMSE | | Case 4 RMSE | |
|---|---|---|---|---|---|---|---|---|
| | Position (m) | Velocity (m/s) | Position (m) | Velocity (m/s) | Position (m) | Velocity (m/s) | Position (m) | Velocity (m/s) |
| CDKF | 3.46 | 0.83 | 16.09 | 1.56 | 7.43 | 1.11 | 8.08 | 1.06 |
| ICDKF | 4.23 | 0.84 | 4.28 | 0.85 | 4.35 | 0.87 | 4.27 | 0.84 |

information and has a better estimate accuracy compared with the CDKF.

## V. CONCLUSION

In this paper, the ICDKF algorithm is proposed to mitigate the negative effects of the systematic biases in measurement information for target tracking by using mobile airborne radar. Firstly, the target tracking model with systematic biases is discussed and the incremental measurement equation is modeled based on the principle of independent incremental random process. Secondly, the general ICDKF algorithm is proposed by augmenting the process and measurement noises into the state vector. Thirdly, to reduce the computation cost and the negative biases, the additive ICDKF algorithm is derived by introducing the additive noises and incremental equation. Lastly, numerical simulations for four cases are run to demonstrate the effective of the proposed ICDKF algorithm to mitigate the measurement biases comparing with the CDKF algorithm.

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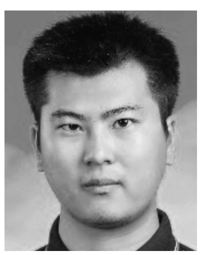

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