

Received September 19, 2018, accepted October 25, 2018, date of publication October 31, 2018, date of current version November 30, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2878882

# Dynamical Analysis of Memristive Unified Chaotic System and Its Application in Secure Communication

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This work was supported in part by the National Natural Science Foundation of China under Grant 61471192 and Grant 61371169 and in part by the Priority Academic Program Development of Jiangsu Higher Education Institutions through the project.

**ABSTRACT** In this paper, a novel memristive unified chaotic system is proposed. The fundamental dynamical behaviors of such system are investigated by calculating phase portraits, Poincaré map, Lyapunov exponents, and bifurcation diagram. Also, the parameters and initial conditions that affect the properties of the memristive system are studied. In addition, a new scheme employing the adaptive parameter modulation synchronization of two memristive unified chaotic systems is proposed. In this scheme, the original signal is modulated into the parameter of the memristive unified systems, and the corresponding receiver system can succeed in recovering the original signal by utilizing the estimated parameter. Furthermore, the proposed memristive unified chaotic system can be applied into secure communication. The simulation results prove the effectiveness of the proposed scheme.

**INDEX TERMS** Memristive unified chaotic system, dynamical analysis, parameter modulation, secure communication.

## I. INTRODUCTION

In the past decades, different chaotic systems have been reported, and the unified chaotic system and its application have attract many researchers' attention, such as unified chaotic system and its application in secure communication [1], control for a unified chaotic system using output feedback controllers [2], Identical and non-identical synchronization of unified chaotic system [3], Adaptive synchronization and anti-synchronization of TSUCS and Lu unified chaotic systems [4], Adaptive control of fractional-order unified chaotic systems [5], Hybrid chaos control of continuous unified chaotic systems [6], dynamics of a fractional-order simplified unified system [7], Robust synchronization of uncertain fractional-order chaotic unified systems [8]. Furthermore, the memristor is a physically new, fourth basic circuit element with memory characteristics [9], [10]. Unlike the other three basic circuit elements of resistance, capacitance, and inductance which have both linear and nonlinear properties. However, Memristor is a pure non-linear circuit element. The dynamic modeling and analysis of such memristive chaotic circuits or memristive

chaotic systems have developed for several years, such that, initial value effect for a memristor-based chaotic system [11], Uncertain destination dynamics of a memristive system [12]. A hyperjerk memristive system [13], [14], The modeling methodology for memristive systems [15], a simplest fractional-order delayed memristive system [16], Multi-scroll hidden attractors in a 5D memristive system [17], Window functions of memristive systems [18], A memristive time-delay system [19], Memristor-based Lorenz hyper-chaotic system [20], Modeling of memristor-based chaotic systems using filter [21]. In order to realize the engineering application of chaotic systems, Many schemes have been reported, such as a chaotic communication scheme based on adaptive synchronization [22], Exponential synchronization of chaotic system [23], The design and its application in secure communication and image encryption [24]–[38]. Based on these approach, the parameter modulation synchronization can be applied to chaotic secure communication. The effective realization of signal transmission is a key part of chaotic secure communication technology At present, the memristive unified chaotic system and its adaptive parameter modulation

synchronization have not been reported, especially the parameters and initial value conditions affect the memristive unified chaotic system.

The purpose of this paper is to prompt secure security in the transmission of signal. and a simple memristive unified chaotic system and its adaptive parameter modulation synchronization scheme in secure communication are presented. Firstly, the model of memristive unified system is proposed. Also its functional dynamic behaviors including phase portraits, Poincaré map, time series, power spectra, Lyapunov exponents and bifurcation diagram are analyzed. Secondly, The proposed scheme for parameter modulation synchronization of identical memristive unified system is studied. Thirdly, the original signal is applied to secure communication and the simulation results verify the effectiveness of this scheme.

The rest of this paper is organized as follows. In Sect.2, The model of memristive unified chaotic system is proposed and its dynamics behaviors are studied in Sect.3, The adaptive synchronization scheme of identical memristive unified system is proposed in Sect.4 .The synchronization of this system via proposed scheme and its simulation results are given in section 5. Finally, conclusions are drawn in Sect 6.

## II. THE MEMRISTIVE UNIFIED CHAOTIC SYSTEM

The memductance  $W(\phi)$  is described by

$$q(\phi) = a\phi + b\phi^3 \tag{1a}$$

$$W(\phi) = a + 3b\phi^2 \tag{1b}$$

Where,  $a$  and  $b$  are positive parameters,  $\phi$  represents internal state variables of memristor.  $q$  represents the charge. And The memristive unified chaotic system is described in form as

$$\begin{cases} \dot{x}_1 = [25\alpha(t) + 10](x_2 - x_1) \\ \dot{x}_2 = [28 - 35\alpha(t)]x_1 - x_1x_3 + [29\alpha(t) - 1]x_2 \\ \quad - 0.1\alpha(t)W(x_4)x_1 \\ \dot{x}_3 = x_1x_2 - [\alpha(t) + 8]x_3/3 \\ \dot{x}_4 = x_1 \end{cases} \tag{2}$$

Eq.(2) is a 4-dimensional autonomous differential system equation which is called the memristive unified chaotic system. For numerical simulations, the parameters  $\alpha$ ,  $b$  and  $c$  take the values

$$\alpha = 1, \quad a = 1/7b = 2/7 \tag{3}$$

and the initial conditions of the system(2) are selected as

$$(x_1(0), x_2(0), x_3(0), x_4(0)) = (1, 0, 20, c). \tag{4}$$

When  $c = 0$ , Fig. 1 shows the projections of the memristive unified system (2). Fig. 1(a) shows a single scroll chaotic attractor, and Fig. 1 (b) shows a double scroll attractor, Fig1(c) is the projection of  $v - i$  which are across the memristor.

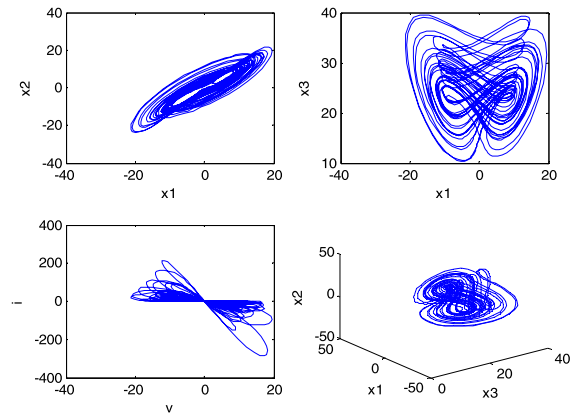


FIGURE 1. The projection phase of system (2) in plane (a)  $x_1, x_2$ , (b)  $x_1, x_3$ , (c)  $v, i$  (d).

## III. DYNAMICS ANALYSIS

### A. EQUILIBRIUM POINTS AND STABILITY

We set  $\dot{x}_1 = \dot{x}_2 = \dot{x}_3 = \dot{x}_4 = 0$ , the equilibrium points of the 4D system(2) are obtained as

$$O = \{(x_1, x_2, x_3, x_4)|x_1 = x_2 = x_3 = 0, x_4 = c\} \tag{5}$$

Where  $c$  is an arbitrary constant, and the Jacobian matrix of system(2) at equilibrium points  $O$  is (6) as shown at the bottom of this page.

Where  $W_1 = a + 3bc^2$  and  $W_{11} = 6bc$ .

The eigenvalues polynomial condition is

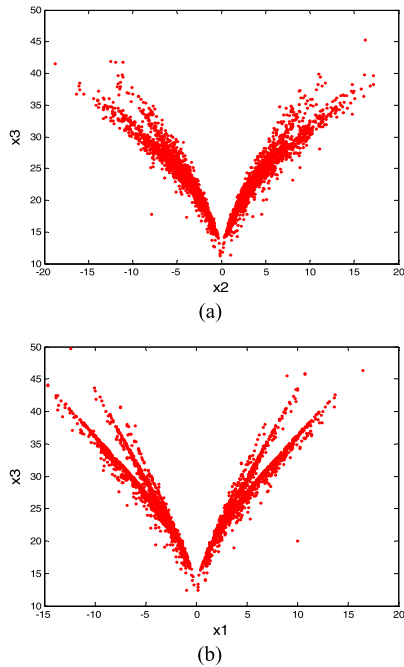
$$\begin{aligned} \lambda [\lambda + (\alpha + 8)/3] \cdot \{[\lambda + (25\alpha + 10)](\lambda - 29\alpha + 1) \\ - [(28 - 35\alpha) - 0.1\alpha W_1](25\alpha + 10)\} \\ + 0.1\alpha W_{11}(25\alpha + 10)[\lambda + (\alpha + 8)/3] = 0 \end{aligned} \tag{7}$$

According to Routh-Hurwitz stability condition, when we choose appropriate parameters  $\alpha$  and  $c$ , the stability of system(2) can be obtained.

$$\begin{aligned} J &= \begin{bmatrix} -(25\alpha + 10) & 25\alpha + 10 & 0 & 0 \\ (28 - 35\alpha) - x_3 - 0.1\alpha W_1 & 29\alpha - 1 & -x_1 & -0.1\alpha W_{11} \\ x_2 & x_1 & -(\alpha + 8)/3 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} -(25\alpha + 10) & 25\alpha + 10 & 0 & 0 \\ (28 - 35\alpha) - 0.1\alpha W_1 & 29\alpha - 1 & 0 & -0.1\alpha W_{11} \\ 0 & 0 & -(\alpha + 8)/3 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{6}$$

**B. POINCARÉ MAP, LYAPUNOV EXPONENTS AND BIFURCATION DIAGRAM**

Choosing the parameter values  $\alpha = 1$ ,  $a = 1/7$  and  $b = 2/7$ , The calculated Lyapunov exponents of the system (2) are  $LE_1 = 1.8043$ ,  $LE_2 = -0.0009$ ,  $LE_3 = -0.0427$ ,  $LE_4 = -11.7607$ , and Lyapunov fractional dimension  $D_L = 3.1497$ . In addition, the Poincaré map in Figs. 2 (a) and (b) present the folding proper-ties of chaos in different planes.



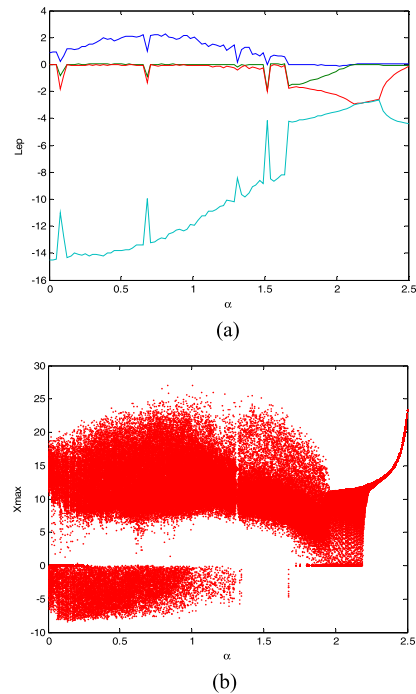
**FIGURE 2.** Poincaré maps of the system (2) in different plane (a)  $x_0=0$ , (b)  $y_0=0$ .

**C. THE PARAMETERS AFFECT THE SYSTEM**

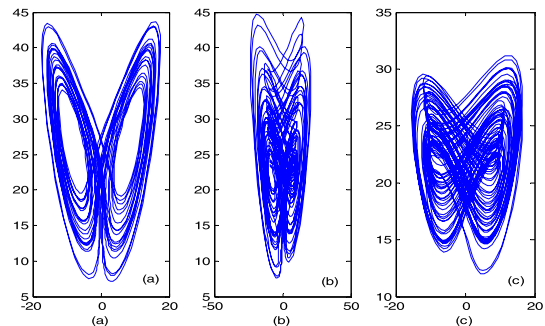
When changing the parameter  $\alpha \in [0, 2.5]$ , The Lyapunov exponents spectrum and bifurcation diagram are shown in Fig.3(a) and(b), respectively, From Fig. 3(a), it can be observed that the system has a positive Lyapunov exponent, two zero-valued Lyapunov exponents and a negative Lyapunov exponent. From Fig. 3(b), the path leading to chaos can be observed from the system. The boundary point turns into an unstable periodic orbit, enters chaotic orbit after bifurcation, and there are several periodic orbits and quasi-periodic orbits in the chaotic zone.

When the system (2) is in a chaotic state, a double scroll chaotic attractor can be formed when different parameter values  $\alpha(t)$  are taken. The Lorenz, Chen and Lü chaotic attractors are shown in Figs.4(a),(b) and (c), respectively

When the initial value  $x_4(0)$  of the system (2) changes in the interval  $[-18, 18]$ , the Lyapunov exponent spectrum and the bifurcation diagram of the system (2) changing with the initial value  $x_4(0)$  are shown in Figures 5(a) and 5(b) respectively. From the Lyapunov exponent spectrum, we can see that the dynamical behavior of system (2) has changed with the



**FIGURE 3.** Bifurcation diagram and Lyapunov exponents of the system (2) when changing the value of parameter  $\alpha \in [0, 2.5]$  for the parameters  $a = 1/7$ ,  $b = 2/7$ . (a) Bifurcation diagram (b) Lyapunov exponents.



**FIGURE 4.** The different parameter values  $\alpha$  of system(2) (a)  $\alpha = 0$ , (b)  $\alpha = 0.8$ , (c)  $\alpha = 1.2$ .

initial value, Combined with the bifurcation diagram, we can see that when the system (2) is in a chaotic state, the double-scroll chaotic attractor and the single-scroll chaotic attraction can be formed. Thus, there are two different chaotic states. Under a typical system initial value, the single scroll chaotic attractor generated by the system (2) is shown in Fig. 6(a). In the single scroll chaotic state, there is an upper attractor or a lower attractor, as shown in Fig. 6 (b).

**IV. THE PROPOSED COMMUNICATION SCHEME**

In this paper, we will consider using the parameter  $\alpha$  of memristive unified chaotic system (2) to transmit the useful information. As the transmitted signal is bounded, The original information signal  $f(t)$  satisfies

$$m \leq f(t) \leq 2M - m \tag{8}$$

where  $m$  and  $M$  are known scalar factors.

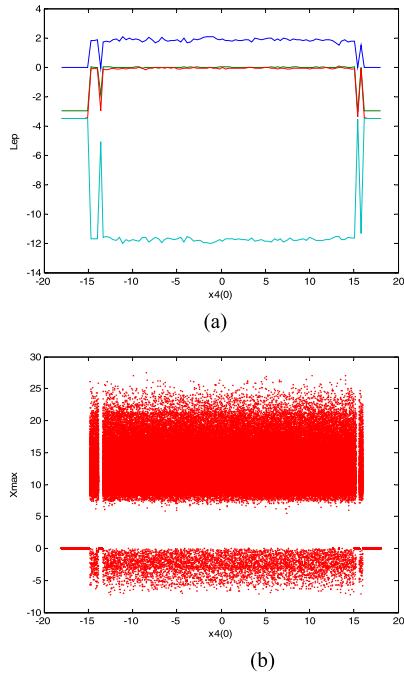


FIGURE 5. Bifurcation diagram and Lyapunov exponents of the system(2) when changing the initial value  $x_4(0) \in [-18, 18]$ . (a) Bifurcation diagram (b) Lyapunov exponents.

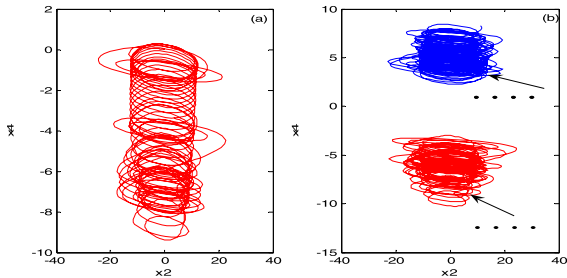


FIGURE 6. The chaotic attractor rely upon the initial conditions (a) single-scroll attractor ( $c = 0$ ), (b) upper attractor ( $c = 5$ ) and lower attractor ( $c = -5$ ).

And a parameter  $\alpha(t)$  is defined as the following:

$$\alpha(t) = \frac{f(t) - m}{M - m} \tag{9}$$

Obviously, the new parameter  $\alpha(t)$  satisfies  $\alpha(t) \in [0, 2]$ .

The unified chaotic system is described by

$$\dot{X} = \Phi(X) \tag{10}$$

where the state vector  $X = (x_1, x_2, x_3, x_4)$ . The communication scheme using adaptive synchronization of system [22], [23] is shown in Fig.7. The transmitter consists of a memristive unified chaotic system  $X$  and a function  $F$ . The receiver is composed of an identical memristive unified chaotic system  $Y$ , nonlinear controller  $U_i$  and the function  $F^{-1}$ . The original signal  $f(t)$  is modulated into the parameter  $\alpha$ , that is,  $\alpha(t) = F[f(t)]$ . and  $\hat{\alpha}(t) = \Gamma(X, Y)$  which is estimated parameter. Thus the original information  $\hat{f}(t)$  can be recovered by  $\hat{f}(t) = F^{-1}[\hat{\alpha}(t)]$ .

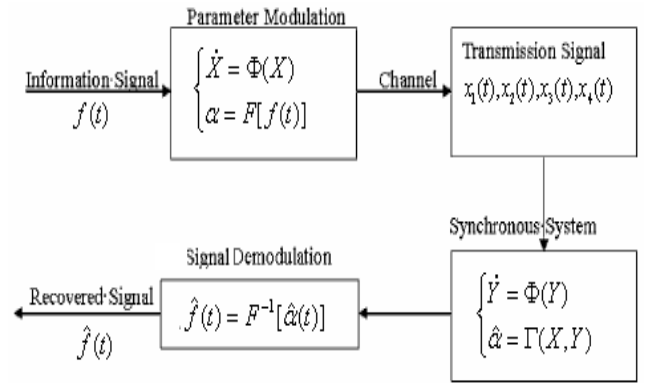


FIGURE 7. Communication scheme of the memristive unified chaotic systems.

### V. APPLICATION IN SECURE COMMUNICATION

The master system (11) and the slave system (12) are described, respectively, as follow

$$\begin{cases} \dot{x}_1 = [25\alpha(t) + 10](x_2 - x_1) \\ \dot{x}_2 = [28 - 35\alpha(t)]x_1 - x_1x_3 \\ \quad + [29\alpha(t) - 1]x_2 - 0.1\alpha(t)W(x_4)x_1 \\ \dot{x}_3 = x_1x_2 - [\alpha(t) + 8]x_3/3 \\ \dot{x}_4 = x_1 \end{cases} \tag{11}$$

and

$$\begin{cases} \dot{y}_1 = [25\hat{\alpha} + 10](y_2 - y_1) + u_1 \\ \dot{y}_2 = [28 - 35\hat{\alpha}]y_1 - y_1y_3 \\ \quad + [29\hat{\alpha} - 1]y_2 - 0.1\hat{\alpha}W(y_4)y_1 + u_2 \\ \dot{y}_3 = y_1y_2 - [\hat{\alpha} + 8]y_3/3 + u_3 \\ \dot{y}_4 = y_1 + u_4 \end{cases} \tag{12}$$

Where  $\hat{\alpha}(t)$  is the parameter of the slave system which needs to be estimated, and  $u_i (i = 1, 2, 3, 4)$  are the nonlinear controllers.

Subtracting Eq. (11) from Eq. (12) and the error dynamical system between Eqs. (11) and (12) can be obtained

$$\begin{cases} \dot{e}_1 = [25\hat{\alpha} + 10](e_2 - e_1) + 25(\hat{\alpha} - \alpha)(x_2 - x_1) + u_1 \\ \dot{e}_2 = (28 - 35\hat{\alpha})e_1 - e_1e_3 + (29\hat{\alpha} - 1)e_2 - x_3e_1 - x_1e_3 \\ \quad - (\hat{\alpha} - \alpha)(35x_1 - 29x_2) - 0.1\hat{\alpha}W(y_4)y_1 \\ \quad + 0.1\alpha W(x_4)x_1 + u_2 \\ \dot{e}_3 = e_1e_2 + x_1e_2 + x_2e_1 - (\hat{\alpha} + 8)e_3/3 \\ \quad - (\hat{\alpha} - \alpha)x_3/3 + u_3 \\ \dot{e}_4 = e_1 + u_4 \end{cases} \tag{13}$$

Where  $e_1 = y_1 - x_1, e_2 = y_2 - x_2, e_3 = y_3 - x_3, e_4 = y_4 - x_4$ .

Theorem 1: with the adaptive control law (14)

$$\begin{cases} u_1 = (25\hat{\alpha} + 9)e_1 + (10\hat{\alpha} - 38)e_2 \\ u_2 = -29\hat{\alpha}e_2 + x_3e_1 + 0.1\hat{\alpha}W(y_4)y_1 - 0.1\alpha W(x_4)x_1 \\ u_3 = -x_2e_1 + (\hat{\alpha} + 5)e_3/3 \\ u_4 = -e_1 - e_4 \end{cases} \quad (14)$$

and a parameter update rule (15)

$$\dot{\hat{\alpha}}(t) = -25(x_2 - x_1)e_1 + 35x_1e_2 - 29x_2e_2 + x_3e_3/3 + \dot{f}/(M - m) \quad (15)$$

Such that the states of the slave system (12) and the master system (11) are globally synchronized asymptotically

$$\lim_{t \rightarrow \infty} \|e\| = 0 \quad (16)$$

Where  $e = (e_1, e_2, e_3, e_4)^T$ .

Proof: Choose the following Lyapunov function

$$V(t) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_\alpha^2), \quad (17)$$

where  $e_\alpha = \hat{\alpha}(t) - \alpha(t)$ .

Taking the time derivative of the Lyapunov function along the trajectory of the error system (9), we obtain

$$\begin{aligned} \dot{V}(t) &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + e_\alpha\dot{e}_\alpha \\ &= e_1 \{ [25\hat{\alpha} + 10](e_2 - e_1) + 25[\hat{\alpha} - \alpha](x_2 - x_1) + u_1 \} \\ &\quad + e_2 \{ (28 - 35\hat{\alpha})e_1 + (29\hat{\alpha} - 1)e_2 - e_1e_3 - x_3e_1 - x_1e_3 \\ &\quad - e_\alpha(35x_1 - 29x_2) \\ &\quad - 0.1\hat{\alpha}W(y_4)y_1 + 0.1\alpha W(x_4)x_1 + u_2 \} \\ &\quad + e_3 \{ e_1e_2 + x_2e_1 + x_1e_2 - (\hat{\alpha} + 8)e_3/3 - e_\alpha x_3/3 + u_3 \} \\ &\quad + e_4(e_1 + u_4) + e_\alpha [\dot{\hat{\alpha}} - \dot{\alpha}] \end{aligned} \quad (18)$$

Substituting Eqs. (14) and (15) into Eq. (18), we obtain

$$\begin{aligned} \dot{V}(t) &= e_1 \{ (35\hat{\alpha} - 28)e_2 - e_1 + 25e_\alpha(x_2 - x_1) \} \\ &\quad + e_2 \{ (28 - 35\hat{\alpha})e_1 - e_2 - e_1e_3 - x_1e_3 \\ &\quad - e_\alpha(35x_1 - 29x_2) \} \\ &\quad + e_3 \{ e_1e_2 + x_1e_2 - e_3 - e_\alpha x_1/3 \} + e_4 \{ e_1 - e_1 - e_4 \} \\ &\quad + e_\alpha \{ -25(x_2 - x_1)e_1 + 35x_1e_2 - 29x_2e_2 + x_3e_3/3 \} \\ &= -(e_1^2 + e_2^2 + e_3^2 + e_4^2) \\ &= -e^T e. \end{aligned} \quad (19)$$

And  $\dot{V}(t)$  is a negative semidefinited function, According to Barbalat's lemma, we obtain

$$\int_0^t \|e\|^2 dt \leq \int_0^t e^T e dt \leq \int_0^t -\dot{V} dt = V(0) - V(t) \leq 0 \quad (20)$$

Thus, we get  $e_1 \rightarrow 0, e_2 \rightarrow 0, e_3 \rightarrow 0, e_4 \rightarrow 0,$

$\hat{\alpha} = \alpha$  as  $t \rightarrow \infty$ , i.e.,

$$\lim_{t \rightarrow \infty} \|e\| = 0. \quad (21)$$

Therefore, the states of the slave system (12) and the master system (11) are globally synchronized asymptotically, and the original information signal can be recovered as the follow:

$$\hat{f}(t) = (M - m)\hat{\alpha}(t) + m, \quad (22)$$

Where  $\hat{f}(t)$  denotes the recovered signal.

### VI. SIMULATION RESULTS

In this section, the effectiveness of the proposed modulated scheme is verified, The initial states of the master system (11) and the slave system (12) are chosen as  $[x_1(0), x_2(0), x_3(0), x_4(0)] = [1, 2, 3, 4]$  and  $[y_1(0), y_2(0), y_3(0), y_4(0)] = [-1, -2, -3, -4]$ , respectively. and the initial estimated parameter  $\hat{\alpha}(0) = 0$ .

The original information signal is chosen as  $f(t) = 5 + \sin(t) + \sin(2t) + \sin(3t) + \sin(4t)$ ,  $M = 8$  and  $m = 2$ . And the modulated parameter  $\alpha(t)$  can be obtained as

$$\alpha(t) = \frac{3 + \sin(t) + \sin(2t) + \sin(3t) + \sin(4t)}{6} \quad (23)$$

The synchronization errors of the memristive unified chaotic systems (11) and (12) via the adaptive control law (14) and the update rule (14) is shown in Fig. 8. it shows that the response

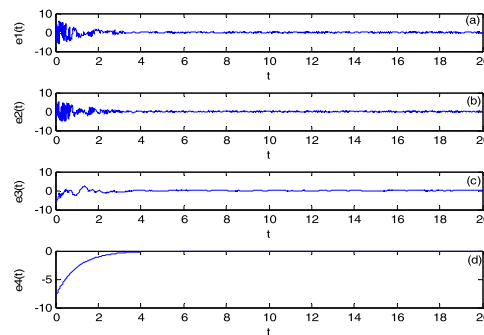


FIGURE 8. The synchronization response of  $e(t)$  between system (11) and (12).

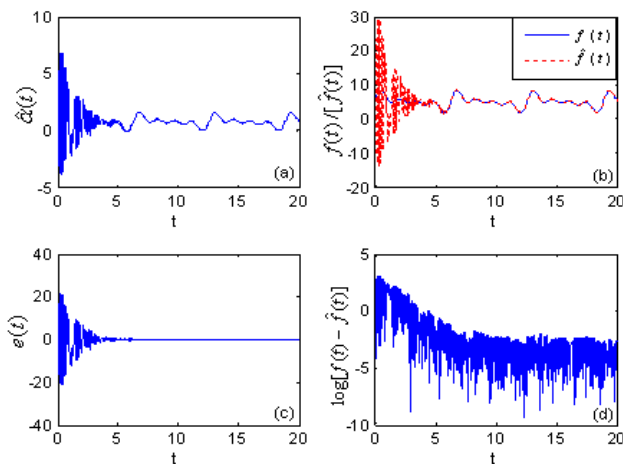


FIGURE 9. The responses of simulation results for the proposed scheme. (a) the estimated parameter  $\hat{\alpha}(t)$ , (b) the original signal  $f(t)$  and the recovered signal  $\hat{f}(t)$ , (c) the error  $e(t) = f(t) - \hat{f}(t)$  response, (d) the time domain response of  $\log[f(t) - \hat{f}(t)]$ .

of  $e(t)$  converges to zero after  $t > 4s$ . And the response of the estimated parameter  $\hat{\alpha}(t)$  is shown in Fig 9(a). Fig. 9(b) shows the original signal  $f(t)$  and the recovered signal  $\hat{f}(t)$ . The error  $e(t) = f(t) - \hat{f}(t)$  is displayed in Fig. 9(c). It is easy to find that the original signal  $f(t)$  can be recovered accurately after  $t \approx 4s$ . It is clearly shown that the small fluctuation for mismatch  $|f(t) - \hat{f}(t)| \geq 2^{-8.849}$  in Fig.9(d).

From Figs. 8 and 9, we can see that the original signal  $f(t)$  can be modulated in parameter  $\alpha$  and it can be recovered quickly without distortion, the simulation results prove the effectiveness of the proposed scheme. Particularly, it is significant for the memristive unified chaotic system can be applied into secure communication.

## VII. CONCLUSION

Magnetically controlled memristive unified chaotic systems are different from common chaotic systems in that their dynamic characteristics depend not only on the system parameters but also on the initial conditions of the system. Therefore, they have some distinctive nonlinear dynamic properties. The dynamic characteristics of the system parameters and initial conditions change are studied. Then a new adaptive synchronization scheme is proposed. Taking the memristive unified chaotic system introduced in this paper as an example, The useful information signal is modulated in one parameter of the system. independently of the state variables of the system and based on the Lyapunov stability theory, an effective nonlinear controller is designed to realize the self-adaptive synchronization of the homogeneous system. At the same time, it is applied to the secure communication and contains a useful information with different frequencies. And the original signal can be recovered from the receiver without distortion, it has potential application prospects in radar, secure communication, and compressed sensing.

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