IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received September 26, 2018, accepted October 18, 2018, date of publication October 31, 2018, date of current version November 30, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2877404

# Fault-Tolerant Topology Control Towards *K*-Channel-Connectivity in Cognitive Radio Networks

**XUAN LI[1], JUNHUI ZHAO[1,2], (Senior Member, IEEE), YU YAO[1], TIANQING ZHOU[1], YI GONG[3], (Senior Member, IEEE), AND LEI XIONG[4]**

[1]School of Information Engineering, East China Jiaotong University, Nanchang 330013, China
[2]School of Electronic and Information Engineering, Beijing Jiaotong University, Beijing 100044, China
[3]Department of Electrical and Electronic Engineering, Southern University of Science and Technology, Shenzhen 518055, China
[4]State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing 100044, China

Corresponding author: Junhui Zhao (junhuizhao@hotmail.com)

This work was supported in part by the Natural Science Foundation of China under Grant 61861018, Grant 61861017, Grant 61471031, Grant 61661021, and Grant 61271204, in part by the Key Technology Research and Development Program of Jiangxi Province under Grant 20171BBE50057, in part by the Natural Science Foundation of Jiangxi Province of China under Grant 20181BAB211013 and Grant 20181BAB211014, and in part by the State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, under Contract RCS2017K009.

**ABSTRACT** In a cognitive radio network (CRN), connectivity is essential for the information exchange between secondary users (SUs). However, the unpredictable activities of primary users (PUs) may result in an unconnected network. Most of the existing works could only guarantee the CRN's connectivity with one channel reclaimed by PU, without considering a more general case that PUs request multiple channels simultaneously, and thus, a network partition may occur more likely. In this paper, first, *k-channel-connectivity* is defined to derive a CRN that remains connected whenever any $k-1$ channels are occupied concurrently. Then, we propose both centralized and distributed topology control algorithms to ensure both the $k$-channel-connected and conflict-free properties. Particularly, it is accomplished by ensuring that any $k-1$ independent sets (i.e., groups of SUs transmitting on the same channel) are not any vertex-cut set of the CRN. Next, the correctness of both the algorithms is verified via theoretical analysis; meanwhile, the analysis demonstrates that the proposed algorithms can achieve the target with a reasonable computation complexity, and in particular, the distributed one can work with limited local information. Finally, simulation results reveal that the proposed algorithms enable the reduction of not only the required channels but also the power consumption of the CRN.

**INDEX TERMS** Cognitive radio networks, multiple channel, topology control, $k$-channel-connectivity.

## I. INTRODUCTION

Cognitive radio is deemed as a novel approach for the utilization improvement of spectrum resource [1]. Cognitive users could access the licensed spectrum only if they do not interfere with the owner users. In term of the priority of spectrum utilization, the cognitive users and owner users are named as secondary users (SUs) and primary users (PUs), respectively [2]. To implement the capability of perceiving current network conditions and exchanging the information, SUs form a cognitive radio network (CRN) so that end-to-end quality of experience can be enabled [3]. When a channel of a CRN is reclaimed by a PU, for the protection of PU's transmission, the SUs have to vacate the channel. In particular, when PUs reclaim multiple channels simultaneously, the SUs operating on the occupied channels have to seek for other unoccupied ones for their transmissions; if there exist no available channels for SUs, their transmissions have to be suspended. It follows that, the communication between SUs may be interrupted due to activities of PUs [4]. Therefore, it is desired that the information remains exchanged in a CRN whenever multiple channels are occupied by PUs.

In ad hoc networks, users may be unavailable due to the battery depletion or uncertain weather factors. To protect end-to-end transmissions from being interrupted by invalid users, controlling the network's topology is an efficient approach to address the problem [5]–[12]. In particular, most of existing works treat the topology of a network as a directed graph firstly. Then, after removing the invalid users out of the graph, efficient algorithms are designed to maintain the network connectivity through guaranteeing that the users in the remaining graph still connect with each other. However, compared to ad hoc networks, the topology control in CRNs poses more challenges, due to the following reasons. Firstly, in the view of the unpredictable activities of PUs, the available channels for SUs are time varying, and hence so are the number of affected SUs. Secondly, several SUs access to the same channel of a CRN for efficient spectrum utilization, so that a considerable number of SUs will be affected by PUs especially when multiple channels are reclaimed simultaneously. Finally, the current transmissions between adjacent SUs may result in the packet collisions, and hence the interference in CRN occurs.

Concentrating on the topology control, great efforts have been made to prevent the CRN from being disconnected and alleviate the interference between SUs. The channel-assignment-based topology control algorithms were proposed in [13]–[15], where the underlying topology is fixed due to the constant transmission power of SUs. To further improve the fault-tolerant capabilities and reduce the energy consumption of CRNs, Liu *et al.* [16], Wang *et al.* [17], and Sheng *et al.* [18] jointly considered the power control and channel assignment to minimize the number of required channels. As such, the probability that SUs' channels are reclaimed by PUs can be also minimized. Note that, all aforementioned works are confined to ensure the 2-channel-connected property of a CRN, i.e., the CRN could remain connected given that only one channel is reclaimed. When it comes to the case that multiple channels are reclaimed by PUs simultaneously, the network connectivity could hardly be guaranteed by existing topology control algorithms.

In this paper, we introduce *k-channel-connectivity* to evaluate the robustness of CRNs. A CRN is referred to as $k$-channel-connectivity if it could remain connected whenever any $k-1$ $(k = 2, 3, 4, \ldots)$ channels are occupied by PUs simultaneously. Apparently, $k$-channel-connected CRNs are more tolerant to the activities of PUs than 2-channel-connected ones. In addition to the channel interruptions from PUs, the interference between SUs must be coped with as well. Conflict-free property is of vital importance to CRNs, with which, no transmission collisions occur in the network, i.e., SUs transmitting on the same channel do not interfere with each other. Therefore, subject to both $k$-channel-connected and conflict-free constraints, we aim at controlling the topology of a CRN, to minimize the number of required channels. Nevertheless, two major issues must be tackled.

On one hand, there exists a tradeoff between guaranteeing $k$-channel-connectivity and minimizing the number of required channels in a conflict-free CRN. For instance, consider a $k$-vertex-connected[1] topology. By assigning each SU one exclusive channel, the topology will be $k$-channel-connected since only $k-1$ SUs could be affected by the reclamation of $k-1$ channels. Moreover, the conflict-free constraint is also satisfied since SUs in different channels do not interfere with each other. Obviously, this approach cannot be applied into dense CRNs due to its induced numerous required channels. On the contrary, the number of required channels can be minimized through a greedy coloring algorithm [19]. Nevertheless, the CRN may be partitioned when any channel is reclaimed, because only conflict-free property is ensured by this strategy. Therefore, a proper topology control algorithm that enables $k$-channel-connected and conflict-free CRN with few channels is highly desirable, especially in a spectrum-scarce CRN.

One the other hand, it is also challenging to guarantee $k$-channel-connectivity with an acceptable computation complexity in a spectrum-scarce CRN. Take a CRN with $K$ channels as an example, if 2-channel-connectivity (i.e., only a channel of CRN is reclaimed by PU) is considered, there are totally $K$ possible combinations of channel occupation. However, for the $k$-channel-connected topology, the number of possible occupation cases will reach $\frac{K!}{(k-1)!(K-(k-1))!}$, whenever any $k-1$ channels are occupied simultaneously. It follows that the $k$-channel-connectivity topology control problem is much more complex than the 2-channel-connectivity one. An intuitive approach is to check the $k$-channel-connectivity of a CRN when assigning a channel to each SU, if the CRN remains connected after any $k-1$ channels are reclaimed, then assign the SU with the channel; if not, choose another available channel and repeat the connectivity test. Obviously, the computation complexity of this approach is very high and the global information of network is necessitated, so that it is unlikely to be implemented in the distributed systems.

In a conflict-free CRN, we note that the SUs transmitting on the same channel do not interfere with each other, i.e., they form an independent set of the network. When multiple channels are reclaimed by PUs, multiple independent sets will be removed. Note that, if a connected graph has been partitioned into separate isolated sub-topologies, i.e., disconnected, after removing special set of vertexes and their related edges, then the resulted set of vertexes is termed as a vertex-cut set of graph. Therefore, if a CRN is desired to satisfy both $k$-channel-connected and conflict-free requirements, it is essential to ensure any $k-1$ independent sets are not any vertex-cut one of the network and this constitutes the basic principle of this paper. Towards this end, we consider a joint power control and channel assignment scheme, which adjusts the SUs' transmission power to acquire an appropriate topology, and then assign channels for this topology to achieve both $k$-channel-connected and conflict-free properties simultaneously.

The main contributions of this paper are as follows.

- The definition of $k$-channel-connectivity topology control problem is first proposed, and then both the sufficient and necessary conditions for its feasibility are provided. The topology control problem is proved to be NP-hard as well.

[1] A network with $k$-vertex-connectivity indicates that it can remain connected after removing any $k-1$ nodes of it.

- For the sufficient condition of the topology control problem, both centralized and distributed algorithms are proposed to enable the $k$-channel-connected and conflict-free CRN. Then, to validate the effectiveness of both algorithms, not only the theoretical analysis is conducted to show the algorithms' correctness and acceptable computation complexity, but also extensive simulations are performed to reveal their advantages in terms of the reduction of required channels and power consumption.
- In particular, the distributed algorithm with the local information is proposed to be a counterpart of the centralized one. Firstly, its correctness and message complexity are given through theoretical analysis. Then, from the extensive simulation results, it can be revealed that, the distributed algorithm's performance approaches that of the centralized one, and only the information between 2-hop neighbors is required for most SUs to ensure $k$-channel-connectivity.

The remainder of this paper is organized as follows. Section II presents the related works. Network model and the problem definition are given in Section III. In Section IV and V, the details and analysis of the proposed centralized and distributed topology control algorithm are provided, respectively. Section VI demonstrates the simulation results. Finally, concluding remarks are given in Section VII.

## II. RELATED WORKS

Connectivity is essential for users to exchange the information in wireless networks. As an efficient approach to enable the connectivity, topology control has been widely studied. The related works can be classified into the following two types.

### A. TOPOLOGY CONTROL IN AD HOC NETWORKS

Considerable existing works on topology control have been conducted in ad hoc networks. Long *et al.* [20], Marina *et al.* [21], Tang *et al.* [22], Burkhart *et al.* [23], Moaveni-Nejad and Li [24], and Li *et al.* [25] concentrated on ensuring the connectivity of ad hoc networks by mitigating the interference between users. Particularly, power control was employed in [23] and [24], and channel assignment was considered in [20]–[22]. In [25], a joint power control and channel assignment scheme was designed to further improve the spectral efficiency. Although sufficiently high spectrum utilization was enabled in these works, only the 1-vertex-connectivity has been considered, i.e., the network may become disconnected whenever any user is invalid. Nevertheless, in practical wireless networks, several users may be invalided simultaneously due to the battery depletion or weather factors. On this account, the topology control algorithms to guarantee $k$-vertex-connectivity [5]–[12], or $k$-edge-connectivity [5], [6] were proposed, where $k$-vertex or $k$-edge-connectivity indicates that the removal of any $k-1$ nodes or links leaves the network connected. In a CRN, multiple SUs may be affected by the unpredictable activities of PUs, such that these algorithms can hardly be applied to CRNs directly.

### B. TOPOLOGY CONTROL IN CRNS

The concept of connectivity in CRNs is distinct from that of ad hoc networks due to the dynamic availability of channels. In view of this, Thomas *et al.* [26] and Komali *et al.* [27] minimized both the maximum transmission power and the number of required channels to achieve connected and conflict-free CRNs. Moreover, a methodology to evaluate the $k$-vertex-connectivity of CRNs was provided in [28], while the relationship between network parameters (e.g., different transmission ranges and positions of SUs) and connectivity was investigated in [29]. These works, nevertheless, neglect either the unpredictable activities of PUs or the mutual interference among SUs.

To guarantee the 2-channel-connected and conflict-free requirements concurrently, several topology control algorithms were proposed in [13]–[17]. In particular, Robust Topology Control Algorithm (RTCA) [13], Minimum Interference Robust Topology Construction (MIRTC) [14] and Resource-Minimized Channel Assignment (RMCA) [15] were designed to satisfy these two requirements via the appropriate channel assignment mechanisms. Liu *et al.* [16] further jointly considered power control and channel assignment to minimize the number of required channels while constructing 2-channel-connected and conflict-free topologies. However, the topology control algorithm proposed in [16] requires the global information of a CRN. To this end, Wang *et al.* [17] and Sheng *et al.* [18] proposed distributed algorithms to optimize the topologies of CRNs. Notice that the topologies constructed via existing topology control algorithms are with limited fault tolerant capability, since they can only maintain the connectivity of CRNs when at most one channel is reclaimed.

To this end, Yadav and Misra [30] proposed a topology control algorithm to assign SUs with different channels in a global way, but the conflict in CRNs cannot be alleviated effectively because the topology is fixed during the operation processing. In [31], Shi *et al.* further developed the work of [18], the coexistence conditions between SUs and multiple PUs were defined, and the SUs that can coexist with PUs were utilized to construct $k$-channel-connected topology. Note that it is assumed that each SU can detect the information of PUs to aware the coexistence relationship. In practice, perfect channel state is hard to achieve and inaccurate detection may cause the partition of resulting topologies. Therefore, in the case that multiple channels of a CRN are reclaimed by PUs simultaneously, a topology control algorithm that can enable both $k$-channel-connected and conflict-free topology in a more general scenario is highly desirable.

In the conference version of this paper [32], a centralized topology control algorithm joint power control and channel assignment was proposed to achieve $k$-channel-connectivity. In this work, we further present both the sufficient and necessary conditions for $k$-channel-connectivity topology control problem, followed by a more practical distributed realization.

## III. NETWORK MODEL AND PROBLEM DEFINITION

### A. NETWORK MODEL

As shown in Fig. 1, a cognitive radio network consists of $n$ SUs and $K$ available channels. Each SU $u$ can adjust its transmission power, $P_u$, from 0 to $P_{max}$. In this paper, We consider the broadcast operation in CRNs. It is assumed that each SU has a specific transmission range and can transmit packets to other SUs within its transmission range. Each SU can transmit on one channel and receive on all channels, yet cannot transmit and receive simultaneously on the same channel due to the half-duplex constraint. Moreover, additive white Gaussian noise channels is considered and the unit disk graph model [33] is employed.

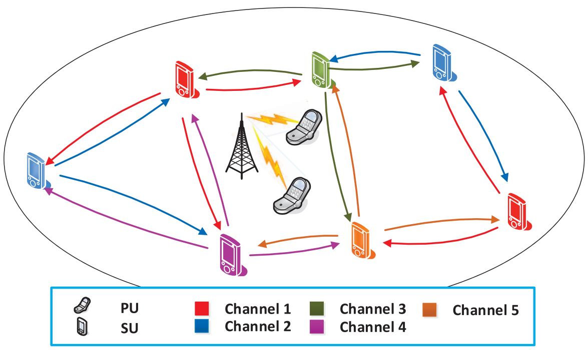



**FIGURE 1.** A CRN with 7 SUs (all SUs transmitting on the same channel are marked in the same color).

*Definition 1 (Topology):* Let the network topology be represented by a directed graph $G = (V(G), E(G))$, where vertex set $V(G)$ denotes the set of SUs and $E(G)$ is the set of directed edges indicating the wireless links between SUs.

*Definition 2 (Link):* A wireless link exists from SU $u$ to $v$, only when the packet transmitted by SU $u$ can be correctly received by SU $v$, i.e., SU $v$ locates in the transmission range of SU $u$. That is to say, the received signal power must exceed the receiver's sensitivity $\beta$, i.e., $P_u d_{u,v}^{-\alpha} \geq \beta$, where $d_{u,v}$ is the Euclidean distance between SU $u$ and SU $v$, and $\alpha$ is the path loss exponent. Accordingly, each SU has a maximum transmission range $R_{max} = (\frac{P_{max}}{\beta})^{\frac{1}{\alpha}}$.

*Definition 3 (Weight):* Each directed edge $(u, v) \in E(G)$ has a weight $\overrightarrow{w}(u, v) = \beta d_{u,v}^{\alpha}$, indicating the minimum transmission power of SU $u$ to enable the correct reception of packets by $v$. Note that $\overrightarrow{w}(u, v) = \overrightarrow{w}(v, u)$.

*Definition 4 (Neighbor):* If there exists a directed edge from SU $u$ to SU $v$, i.e., $(u, v) \in E(G)$, we define $v$ as one neighbor of $u$ and denote such relationship by $u \rightarrow v$. When both $(u, v) \in E(G)$ and $(v, u) \in E(G)$ hold, a bi-directed edge exists between $u$ and $v$, denoted by $u \leftrightarrow v$.

*Definition 5 (Connectivity):* $G$ is connected if and only if any two SUs in $G$ are connected via either a bi-directed link or a bi-directed path, such that the message can be exchanged among any pair of SUs in the CRN.

*Definition 6 (k-Channel-Connectivity):*. Assume multiple PUs active in the primary network and any PU could interfere with all SUs. If a channel is reclaimed by a PU, all the SUs have to cease their transmissions on this channel to protect the PU's transmission. When PUs switch to multiple channels of a CRN simultaneously, a possible network partition may occur. Thus, in a CRN with $K$ licensed channels, $G$ is referred to as *k-channel-connected* if the remaining network is connected even when any $k-1$ ($k = 2, 3, \ldots, K$) channels are reclaimed by PUs simultaneously.

*Definition 7 (Conflict Graph):* In our model, not only the impact of PUs' activities on SUs, but also the interference between SUs is considered. Accordingly, a conflict graph $C_G = (V(C_G), E(C_G))$ is transformed by $G$, where $V(C_G) = V(G)$, and $E(C_G)$ is the set of undirected edges representing the potential interference between any two SUs in $G$.

*Definition 8 (Conflict):* $(u, v) \in E(C_G)$ indicates that SU $u$ and $v$ interfere with each other if they transmit on same channel. In this case, either of the following conditions should be satisfied: i) $u$ is a neighbor of $v$, or $v$ is a neighbor of $u$, i.e., $u \rightarrow v$ or $v \rightarrow u$; ii) $u$ and $v$ have a common neighbor, $z$, i.e., $u \rightarrow z$ and $v \rightarrow z$.

*Definition 9 (Conflict-Free):* $G$ is said to be conflict-free if any two conflicting SUs are assigned different channels.

*Definition 10 (Vertex-Cut Set):* If $G$ should be partitioned by removing special set of vertexes and their related edges, then the set of such vertexes is called a vertex-cut set of graph.

### B. PROBLEM DEFINITION

In this work, we circumvent the partition of a CRN and avoid the transmission collisions among SUs, with the corresponding topology control problem defined as follows.

#### 1) k-CHANNEL-CONNECTIVITY TOPOLOGY CONTROL PROBLEM

Given a set of $K$ licensed channels and a CRN $G = (V(G), E(G))$, the topology control problem turns out to be assigning transmission power and channels to different SUs, such that the induced subgraph $S = (V(S), E(S))$ preserves both $k$-channel-connected and conflict-free properties. In $S$, $V(S) = V(G)$, $E(S) \subseteq E(G)$ and our objective is to minimize the number of channels required by $S$.

Before solving the topology control problem, the feasibility of the problem should be discussed. Along this line, the sufficient and necessary feasibility conditions of the problem are given as follows.

#### 2) SUFFICIENT FEASIBILITY CONDITION

If the topology of a CRN, $G$, is $k$-vertex-connected, and $K \geq n$, then the $k$-channel-connected and conflict-free topology can be constructed via a feasible topology control algorithm.

*Proof:* Upon the assumption that $G$ is $k$-vertex-connected, intuitively, consider one topology control algorithm that assigns each SU a unique channel. It is obvious that the conflict-free CRN is available. Moreover, when PUs reclaim any $k-1$ channels, $G$ remains connected since only $k-1$ SUs are invalid. Therefore, $k$-channel-connectivity is acquired. □

#### 3) NECESSARY FEASIBILITY CONDITION

If the resulting topology of a topology control algorithm is $k$-channel-connected and conflict-free, then the given topology of the CRN should be at least $k$-vertex-connected, and $K \geq k+1$.

*Proof:* If the induced topology by a topology control algorithm is $k$-channel-connected and conflict-free, then all the 1-hop neighbors of each SU have been assigned with different channels from it. Therefore, the degree (i.e., the number of 1-hop neighbors of an SU) of each SU must be at least $k$. Each SU and its 1-hop neighbors form a clique in the distance-2 conflict subgraph, and the clique number (i.e., the number of SUs in the clique) is at least $k+1$ because any two SUs in a clique are interfering with each other. It is worth mentioning that the number of channels required to achieve conflict-free property is lower bounded by the clique number of the distance-2 conflict graph, so the number of licensed channels in a CRN should be more than $k+1$. □

From the aforementioned feasibility conditions, it follows that $n$ and $k+1$ are the maximum and minimum value of the number of required licensed channels, respectively. Therefore, the feasibility conditions provide that the given topology $G$ is at least $k$-vertex connected and $K \geq k+1$ holds in this paper, then a topology control algorithm, which enables $k$-channel-connected and conflict-free properties with less than $n$ licensed channels, is anticipated.

*Theorem 1: The topology control problem is NP-hard.*

*Proof:* The $k$-channel-connectivity topology control problem reduces to the 2-channel-connectivity one if $k = 2$, the NP-hardness of which has been proved in the previous work [17]. It is straightforward that, the $k$-channel-connectivity topology control problem is also NP-hard. □

## IV. CENTRALIZED TOPOLOGY CONTROL ALGORITHM

In this section, firstly, a simple example is introduced to illustrate the design philosophy. Next, the details of the centralized $k$-channel-connectivity topology control algorithm (CKCC) are elaborated, followed by its correctness proof and complexity analysis.

### A. DESIGN PHILOSOPHY

Before depicting the details of the centralized algorithm, an example utilizing as few required channels as possible to achieve $k$-channel-connected and conflict-free properties is shown in Fig. 2, where the number associated with each SU indicates the index of its assigned channel.

Fig. 2(a) gives a 3-vertex-connected CRN consisting of 9 SUs. On one hand, obviously, if each SU is assigned a unique channel, then the 3-channel-connected and conflict-free topology can be acquired. However, this approach requires considerable number of channels (which is equal to that of SUs actually), and hence cannot be applied in spectrum-scarce or dense CRNs. On the other hand, the number of required channels can be minimized by the greedy coloring algorithm provided that the conflict-free property is guaranteed [19]. As such, by lowering the transmission power of SUs, the topology in Fig. 2(a) will turn out to be the one in Fig. 2(b). We see that only 4 channels are required for the CRN to ensure the connectivity and conflict-free property. Nonetheless, the partition of the topology occurs whenever any channel is reclaimed.

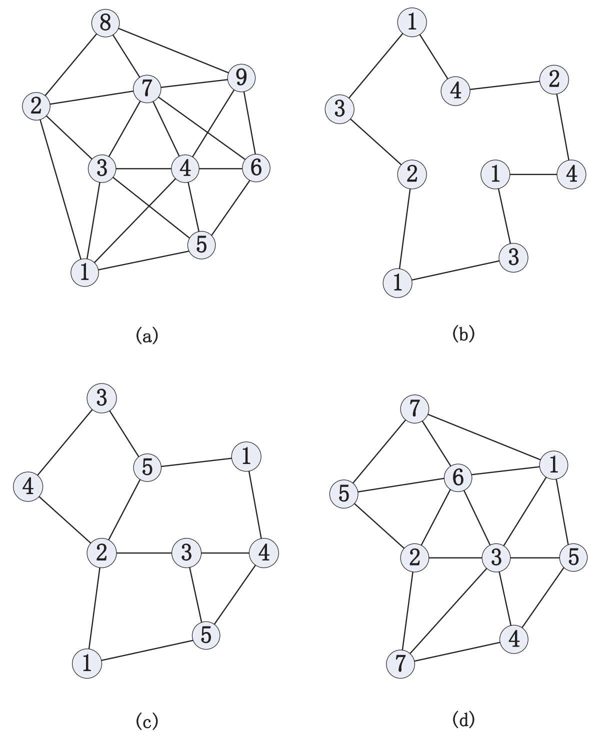



**FIGURE 2.** **(a) 3-vertex-connected CRN. (b) 1-channel-connected CRN. (c) 2-channel-connected CRN. (d) 3-channel-connected CRN. Illustration of different topologies of the network, and the number associated with each SU indicates its requested channel.**

It follows that there exists a tradeoff between $k$-channel-connectivity guarantee and required channels minimization. To balance the tradeoff, we will first show that all the SUs assigned the same channel constitute a independent set in the conflict graph, and then ensure that any $k-1$ independent sets are not any vertex-cut one of the topology. With this guidance, $k$-channel-connected and conflict-free properties can be achieved simultaneously with fewer channels, and the examples in Fig. 2(c) and (d) can illustrate this. Also originating from the topology in Fig. 2(a), a 2-channel-connected and conflict-free CRN is available in Fig. 2(c) with this guidance. It can been seen that each independent set is not the vertex-cut one of the CRN. However, when two channels (e.g., channel 1 and 3) are reclaimed, the SU transmitting on channel 1 and 3 should be removed out of the topology, and a network partition occurs. Thus, for a higher robust topology, we ensure that any two independent sets do not constitute a vertex-cut one in Fig. 2(d). As such, the CRN in Fig.3(d) is

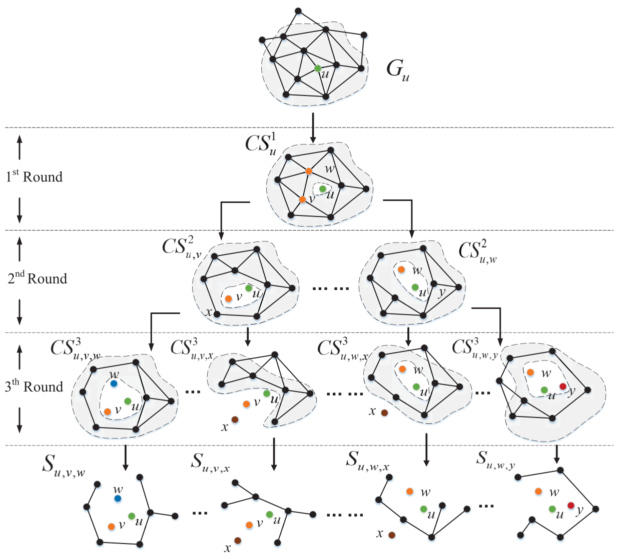



**FIGURE 3.** An example of subgraph construction for SU $u$ to achieve 4-channel-connectivity.

3-channel-connected, and only 7 channels are consumed, which is fewer than the approach in Fig. 2(a).

As illustrated in Fig. 2, our design philosophy is, 1) to ensure that the SUs in each vertex-cut set are assigned at least $k$ different channels, i.e., $k$-channel-connectivity guarantee; 2) to tailor the given network topology by lowering the transmission power so that conflict-free property can be achieved with fewer required channels.

### B. ALGORITHM DESCRIPTION

The centralized topology control algorithm consists of three phases, namely, topology construction, power adjustment as well as channel assignment. In the first phase, to mitigate the interference between SUs, the maximum power topology[2] $G_{max} = (V(G_{max}), E(G_{max}))$ is tailored to a subtopology $S = (V(S), E(S))$, where $V(S) = V(G_{max})$ and $E(S) \subseteq E(G_{max})$. In particular, through a sequential topology construction with $k-1$ rounds, it can be ensured that any $k-1$ independent sets in the corresponding conflict graph are not vertex-cut one of $S$. In the second phase, based on the induced topology $S$, the transmission power of each SU is adjusted. Finally, a greedy graph coloring algorithm is adopted to build the $k$-channel-connected and conflict-free topology with the minimum number of required channels. These three phases will be described in detail as follows, and be summarized in Algorithm 1.

In the topology construction phase, there exist a total of three steps: 1) minimum-power subgraph construction; 2) sequential topology construction; and 3) subgraph combination.

*Step 1):* We first find the minimum power paths between each pair of SUs in $G_{max}$ via Floyd-Warshall algorithm, and thus a minimum-power subgraph $S = (V(S), E(S))$ can be acquired by adding all the edges in the minimum power paths to $E(S)$. Next, all SUs are sorted in non-descending order of degree in $G_{max}$, and the degree of an SU is the number of SUs connected with it directly. Following this order, we obtain the conflict neighbor set of SU $u$ in $S$, $CN_u$, which is the set of SUs that are connected with $u$ in the conflict graph $C_S$. Note that in the conflict-free CRN, all SUs in $CN_u$ have to request different channels from $u$.

*Step 2):* Following Step 1), we construct a local conflict-neighbor subgraph for each SU $u$, $CS_u^1 = (V(CS_u^1), E(CS_u^1))$,[3]

**Algorithm 1** Centralized $k$-Channel-Connectivity Topology Control Algorithm (CKCC)

**Require:**
  The maximum power topology $G_{max}$;
**Ensure:**
  The induced topology $S$ and the channel assignment $\mathcal{A}$;
$V(S) \Leftarrow V(G_{max})$, $E(S) \Leftarrow \varnothing$;
Find the minimal power paths between each pair of SUs in $\{G_{max} \setminus u\}$;
Construct the subgraph $T$ by including all the undirected links in the obtained paths;
$E(S) \Leftarrow E(T)$;
Sort all SUs in the order of non-descending degree;
**for** each SU $u \in V(S)$ **do**
  $\{CS_u\} \Leftarrow \varnothing$;
  Construct a $(k-1)$-vertex-connected conflict-neighbor subgraph $CS_u^1$ in $G_{max}$;
  $\{CS_u\} \Leftarrow CS_u^1$;
  **for** $i = 2 : (k-1)$ **do**
    **for** $j = 1 : |\{CS_u\}|$ **do**
      $CS_u^{i-1} \Leftarrow \{CS_u\}_j$;
      Sort all SUs in $CS_u^{i-1}$ in the order of non-descending degree;
      **for** each SU $v \in CS_u^{i-1}$ **do**
        Construct a $(k-i)$-vertex-connected conflict-neighbor subgraph in $\{CS_u^{i-1} \setminus v\}$;
        Sort the induced conflict-neighbor subgraph into the set $\{CS_u\}$;
      **end for**
    **end for**
  **end for**
  **for** each subgraph in $\{CS_u\}$ **do**
    Construct a spanning subgraph via CBCC in [17];
    Sort the edges of the induced spanning subgraph into $E(S_u)$;
  **end for**
  $E(S) \Leftarrow E(S) \cup E(S_u)$; $LCN_u \Leftarrow V(S_u)$;
**end for**
$P_u \Leftarrow \max\{\beta d_{u,v}^{\alpha} \mid (u,v) \in S, v \in V(S)\}$, for $u \in V(S)$;
Call the greedy coloring algorithm to assign the channels to SUs and output the channel assignment $\mathcal{A}$.

[2]When all SUs transmit with the maximum power, the topology of the CRN refers to as the maximum power topology.

[3]For easy-to-understand, the superscript "1" denotes the 1st round of the topology construction phase via Algorithm 1.

where $V(CS_u^1) = CN_u$, $E(CS_u^1) \Leftarrow \{(x, y)|x, y \in CN_u, (x, y) \in E(G_{max})$, and $u \notin V(CS_u^1)$. Then, the connectivity of $CS_u^1$ should be checked; if $CS_u^1$ is not $(k-1)$-vertex-connected, the SUs and edges in $G_{max} \backslash u$ are added into $CS_u^1$ to built a $(k-1)$-vertex-connected one. Particularly, FGSS$_k$ is a typical algorithm to acquire $k$-vertex-connected topology [9], which is adopted to construct the $i$-vertex-connected graph $(i = 1, 2, \ldots, k-1)$. Note that the $h(h \geq 3)$-hop neighbor SUs in $V(G_{max}) \setminus (CN_u \& u)$ are added into $CS_u^1$ sequentially until it reaches $(k-1)$-vertex-connectivity. Next, all SUs in $CS_u^1$ are sorted in non-descending order of degree (referred to as the 1st round); following this order, for each SU $v \in CN_u$, both its conflict neighbor set in $CS_u^1$, i.e., $CN_{u,v}$, and local conflict neighbor subgraph $CS_{u,v}^2 = (V(CS_{u,v}^2), E(CS_{u,v}^2))$ can be detected, where $V(CS_u^2) = CN_{u,v}$, $E(CS_u^2) \Leftarrow \{(x, y)|x, y \in CN_{u,v}, (x, y) \in E(CS_u^1)$. Note that $u, v \notin V(CS_{u,v}^2)$ and $CS_{u,v}^2 \subseteq CS_u^1$. Afterwards, in the 2nd round, the connectivity of $CS_{u,v}^2$ should also be checked; if $CS_{u,v}^2$ is not $(k-2)$-vertex-connected, a $(k-2)$-vertex-connected $CS_{u,v}^2$ should be constructed in $CS_{u,v}^1 \setminus v$ (similar to the 1st round). Sequentially, for each SU $w \in CN_{u,v}$, the execution proceeds until the $(k-1)$-th round. After that, all the induced conflict-neighbor subgraphs are sorted into the induced subgraph set of SU $u$, $\{CS_u\}$. Finally, for each conflict-neighbor subgraph in $\{CS_u\}$, a local spanning subgraph is built over it via the 2-channel-connectivity topology control algorithm (referred to as CBCC [17]). Above all, through the sequential topology construction, all possible occupation cases can be traversed.

*Step 3):* In this step, all the local spanning subgraphs are combined as the resulting topology of SU $u$, i.e., $S_u$. Particularly, the bi-directed edges of the local spanning subgraphs are supplemented to $E(S_u)$, and SUs in $V(S_u)$ are recorded in $LCN_u$ (the set of SU $u$'s logical conflict neighbors). Finally, for each SU $u \in V(G_{max})$, its resulting topology $S_u$ are added into the induced topology $S$. As such, a $k$-channel-connected topology could be eventually achieved, and the corresponding proof will be given later.

In the power adjustment phase, with the induced topology $S$, all the 1-hop neighbors of each SU $u$ in $S$ are sorted as its logical neighbors, and then its transmission power is adjusted to reach the logical neighbor with the furthest distance away from itself.

In the channel assignment phase, a greedy coloring algorithm is employed to assign channels to SUs. Particularly, the conflict graph $C_S$ is firstly updated upon the edges between each SU $u$ and its logical conflict neighbors $LCN_u$. Then, the SU with the largest conflict degree [4] will be assigned the channel with the lowest occupancy probability. Next, for the $j$-th $(j = 2, 3, \ldots, n)$ SU, we find the channels unutilized by its conflict neighbors, and assign it the unutilized channel that with the lowest occupancy probability. Finally, after the sequential channel assignment, not only the conflict-free property is achieved, but also the $k$-channel-connectivity.

[4]The conflict degree of an SU indicates the number of SUs connected with it directly in $C_S$.

### C. THEORETICAL ANALYSIS

To prove that $k$-channel-connectivity can be achieved by CKCC, Lemma 1 is given firstly.

*Lemma 1: A k-vertex-connected topology, $G_{max}$, can be transferred to be k-channel-connected by Algorithm 1.*

*Proof:* If $G_{max}$ is desired to be transfered from $k$-vertex-connectivity to $k$-channel-connectivity, then power adjustment should not be required; otherwise, $G_{max}$ may be changed. Accordingly, without the power adjustment phase, CKCC reduces to a two-stage topology control algorithm. Firstly, each SU finds its conflict neighbors in $G_{max}$, and constructs a $k$-vertex-connected subgraph, consisting of both itself and its conflict neighbors. Then, the greedy coloring algorithm is executed to assign channels to SUs. In the worst case, all the SUs in $G_{max}$ conflict with each other, and each SU has to be assigned a unique channel. Obviously, in this case, $G_{max}$ turns out to be $k$-channel-connected. Therefore, the case that multiple SUs assigned the same channel should be discussed in detail in the following.

Consider any two SUs $u$ and $v$, which are connected by a bi-directed path $P_a(u \leftrightarrow v) = (u \leftrightarrow q_0 \leftrightarrow, \ldots, \leftrightarrow q_m \leftrightarrow v)$ in $G_{max}$. There exists one or more SU in $P_a(u \leftrightarrow v)$ influenced by PUs, the set of which can be denoted as $V'_{P_a(u \leftrightarrow v)}$. For any SU $q_i \in V'_{P_a(u,v)}$, its neighbor $q_{i-1}$ and $q_{i+1}$ in the path are assigned different channels from $q_i$, and $q_{i-1}, q_{i+1} \notin V'_{P_a(u,v)}$. Hence, if $q_{i-1}$ and $q_{i+1}$ are connected given that $q_i$ is influenced by PU, then $u$ and $v$ can be deemed as being connected. Meanwhile, as mentioned above, each SU constructs a $k$-vertex-connected subgraph in $G_{max}$, and sort all the other SUs in the subgraph as its conflict neighbors. Therefore, there exist other $k-1$ paths from $q_{i-1}$ to $q_{i+1}$, which internally disjoint with the path $(q_{i-1}, q_i, q_{i+1})$, and all SUs in the other $k-1$ paths belong to $LCN_{q_i}$ that are assigned different channels from $q_i$. Therefore, when PUs reclaim any $k-1$ channels, there still exists a bi-directed path from $u$ to $v$, i.e., $G_{max}$ is $k$-channel-connected. □

*Theorem 2: The topology generated by CKCC, S, is k-channel-connected if $G_{max}$ is k-vertex-connected.*

*Proof:* We first define the resulting topology $S = \cup_{u \in V(G_{max})} S_u$ when the $k$-channel-connected topology is desired $(k = 2, 3, \ldots)$. In the following, we prove Theorem 2 by induction.

If $k = 2$, then CKCC reduces to the algorithm proposed in [17], i.e., the sequential topology construction is executed for only one round, where the induced topology has been proved to be 2-channel-connected. Therefore, $S$ is also 2-channel-connected.

Next, assuming that $S$ is $(k-1)$-channel-connected after $k-2$ rounds of sequential topology construction, we need to prove $S$ is $k$-channel-connected after $k-1$ rounds. In other words, it should be shown that $S$ remains connected whenever any $k-1$ channels are occupied by PUs. Assume that PUs reclaim any $k-1$ channels and split $S$ into two parts, $S^{'}$ and $S^{''}$, by removing the set of affected SUs (i.e., $\{v_1, v_2, \ldots, v_p\}$). From Lemma 1, it follows that the occupation of any $k-1$ channels will not split $G_{max}$ via CKCC. Hence, there exists a set of SUs, $\{v_{p+1}, v_{p+2}, \ldots, v_q\} \in \{V(G_{max})\backslash\{v_1, v_2, \ldots, v_p\}\}$, connecting $S^{'}$ and $S^{''}$. It should be noted that, through the sequential topology construction after $(k-1)$ rounds, there exists at least one SU in $\{v_{p+1}, v_{p+2}, \ldots, v_q\}$ that belongs to the local resulting topologies $V(S_u)$, where $u \in V(G_{max})$, it contradicts the assumption that $\{v_1, v_2, \ldots, v_p\}$ is the vertex-cut set of $S$.

Therefore, $S$ remains connected whenever any $k-1$ channels are reclaimed by PUs, i.e., $S$ is $k$-channel-connected. □

*Theorem 3: The computation complexity of CKCC reaches $O(|V||E|\rho^k + |V|^3)$, where $|V|$ and $|E|$ are the number of SUs and links in $G_{max}$, respectively, and $\rho$ is the maximum vertex's degree in the conflict graph.*

*Proof:* In the topology construction phase, the operation in line 3 (i.e., Floyd-Warshall algorithm) costs $O(|V|^3)$. Then, in line 8 and 15, $FGSS_k$ is employed to construct a $i$-vertex-connected subgraph $CS_u^i$, $(i = 1, 2, \ldots, k-1)$ with the distance weight, and its complexity is $O(|E(CS_u^i)||V(CS_u^i)|^2)$, where $|V(CS_u^i)|$ and $|E(CS_u^i)|$ are the numbers of SUs and links in $CS_u^i$, respectively. It is worth noting that CKCC needs to construct $O(|CN_u|^{k-2})$ $i$-vertex-connected subgraphs; therefore, in the worst case, the complexity reaches $O(|E|\rho^k)$, considering that $V(CS_u^i)| \leq |CN_u| \leq \rho$ and $|E(CS_u^i)| \leq |E|$. Moreover, since the spanning subgraph construction via CBCC takes the complexity of $O(|E|log|V|)$, the complexity from line 20 to 23 reaches $O(\rho^{k-2}|E|log|V|)$. Furthermore, the procedure from line 6 to 23 must be executed by each SU, resulting in the complexity of $O(|V||E|\rho^k + |V|^3)$ for the topology construction. In addition, in line 27, the complexity with respect to the greedy coloring algorithm is $O(|V|^2)$. Above all, the total computation complexity of CKCC is $O(|V||E|\rho^k + |V|^3)$. □

## V. DISTRIBUTED TOPOLOGY CONTROL ALGORITHM

In this section, our attention is shifted to solve the $k$-channel-connectivity topology control problem via a distributed approach. In particular, each SU make topology control decisions with its local information independently. Firstly, we present the description of the distributed algorithm. Sequentially, its correctness proof and message complexity are given, respectively.

### A. ALGORITHM DESCRIPTION

The distributed $k$-channel-connectivity topology control algorithm (DKCC) consists of four phases, specifically, neighbor discovery, topology construction, power adjustment and channel assignment. The details are given as follows.

#### 1) NEIGHBOR DISCOVERY

In this stage, each SU $u$ collects the available information, such as IDs and locations, by exchanging the HELLO messages. Particularly, two time slots are required for each SU $u$ to aware of its distance-2 neighbors. In the first slot, each SU $u$ seeks an idle channel in a backoff time and broadcasts the HELLO message with its maximum transmission power $P_{max}$, its ID and location information are included in the HELLO message. Therefore, each SU can obtain the locations of its 1-hop neighbors and the edges between them by receiving the HELLO messages. In the second slot, each SU $u$ chooses a backoff time and broadcasts the neighbor list with its maximum transmission power $P_{max}$ on an idle channel, the IDs and locations of its 1-hop neighbors are deposited in the neighbor list. After two slots, each SU can gather the neighbor lists from the 1-hop neighbors, and thus could acquire the locations of the 2-hop neighbors, as well as edges between neighbors. Moreover, it is assumed that any two messages in two successive time slots are independent and conflict-free with each other.

Next, each SU $u$ builds a local subgraph $G_u = (V(G_u), E(G_u))$, where the vertex set $V(G_u)$ consists of SU $u$ and its 2-hop neighbors, and the edge set $E(G_u)$ involves all the links of $V(G_u)$ in $G_{max}$. Meanwhile, given the neighbors' locations, the weight $\overrightarrow{w}(u, v)$ of each edge $(u, v) \in G_u$ is readily available for SU $u$. Furthermore, to avoid the unsuccessful reception by packet collisions, an ALOHA-like protocol is employed in the MAC layer.

#### 2) TOPOLOGY CONSTRUCTION

In this stage, by tailoring the edges of local 2-hop subgraph $G_u$, each SU $u$ constructs its local spanning subgraph $S_u = (V(S_u), E(S_u))$ independently. In particular, each SU $u$ first applies the Dijkstra's algorithm to find the minimum power paths from itself to its neighbors in the 2-hop distance, i.e., all the SUs in $\{V(G_u)/u\}$. After that, all the edges of the paths are added into $E(S_u)$. Meanwhile, a local conflict graph $C_{S_u}$ can be transformed from $S_u$. Based on $C_{S_u}$, each SU $u$ finds the set of the SUs are connected with SU $u$ in $C_{S_u}$, i.e., the conflict neighbor set of $u$ in $S_u$, which is denoted by $CN_u$. Note that $CN_u \subset V(S_u)$.

Next, SU $u$ will make the topology control decisions based on the information between the SUs in $CN_u$. Particularly, SU $u$ constructs a local subgraph of SU $u$, $CS_u^1 = (V(CS_u^1), E(CS_u^1))$, where $V(CS_u^1) = CN_u$ and $E(CS_u^1) \Leftarrow \{(x, y)|x, y \in CN_u, (x, y) \in E(G_u)$. If $CS_u^1$ is not $(k-1)$-vertex-connected, then SU $u$ sequential gathers the information of $h(h \geq 3)$-hop neighbors in $G_{max}/G_u$ via the HELLO messages exchanging approach, the information dissemination runs periodically until a local $(k-1)$-vertex-connected graph can be successfully constructed by $FLSS_k$, which is an efficient local topology control algorithm in [9].

Sequentially, SU $u$ starts the second round of the topology construction if $k > 2$. Following the non-descending order of degree, for each SU $v \in CN_u$, SU $u$ finds the conflict neighbor set of SU $v$ in $CS_u^1$, $CN_{u,v}$, and constructs a local subgraph of SU $v$, $CS_u^2 = (V(CS_u^2), E(CS_u^2))$, where $V(CS_u^2) = CN_{u,v}$ and $E(CS_u^2) \Leftarrow \{(x, y)|x, y \in CN_{u,v}, (x, y) \in E(CS_u^1)$. If $CS_{u,v}$ is not $(k-2)$-vertex-connected, SU $u$ builds a

$(k-2)$-vertex-connected subgraph $CS^2_{u,v}$ in $CS^1_u \setminus v$. This process has to execute until the $(k-1)$-th round, and the induced subgraphs are sorted into the subgraph set $\{CS_u\}$. For each induced subgraphs in $\{CS_u\}$, SU $u$ calls the distributed 2-channel-connectivity algorithm in [17], which refers to DBCC, to generate a local spanning tree over it. Afterwards, SU $u$ sorts the edges and SUs of the local spanning subgraphs into $E(S_u)$ and logical conflict neighbor set $LCN_u$.

For illustration, an example of Topology Construction Stage is shown in Fig. 3, which generates a series of subtopologies after 3 rounds to achieve 4-channel-connectivity. It can be seen that, e.g., $CS^3_{w,u,v}$, in the case that PUs reclaim the channels of SU $u$, $v$ and $w$, the removal of the SUs leaves the remaining the remaining network connected. This is because after the sequential topology construction, the common conflict neighbors of SU $u$, $v$ and $w$ would be assigned different channels from them in a conflict-free CRN.

Finally, the information of the edges in $E(S_u)$ and logical conflict neighbor in $LCN_u$ are broadcast by each SU $u$ to inform the other SUs in $S_u$. Particularly, the local flooding approach is adopted. After that, each SU updates its logical conflict neighbors set and local subgraph. Note that the minimum-power paths and the local subgraphs are constructed independently by each SU's local information, and hence the induced subgraph $S$ by the distributed topology control algorithm may be over-connected than the centralized one.

#### 3) POWER ADJUSTMENT

On the basis of the induced local subgraph $S_u$, each SU $u$ sorts all its 1-hop neighbors as the logical neighbors. Then, SU $u$ finds the furthest logical neighbor and adjusts its transmission power, e.g., if SU $w$ is the furthest neighbor of $u$, then $P_u = \beta d^{\alpha}_{u,w}$.

#### 4) CHANNEL ASSIGNMENT

In this stage, a distributed coloring-based channel assignment algorithm is adopted by each SU. The algorithm executes in a sequential order. the conflict degree of the SUs in $S_u$ is readily available for each SU $u$, and larger conflict degree implies higher priority to request the channel. The SU with largest conflict degree requests a channel with the lowest occupancy probability, and then broadcasts its ID and the channel ID. After that, other SUs find the channels that are not yet requested by its conflict neighbor and chooses the one with lowest occupancy probability.

### B. THEORETICAL ANALYSIS

*Theorem 4: The topology generated by DKCC, $S$, is $k$-channel-connected if $G_{max}$ is $k$-vertex-connected.*

*Proof:* The main difference between CKCC and DKCC lies in that whether the minimum-power-path subgraph and the local subgraph $S_u$ are constructed by each SU independently whether or not. In reality, reminding the proof of Theorem 2, the construction of $k$-channel-connected topology is independent of whether or not both subgraphs are built by SU $u$. Therefore, the correctness proof of CKCC can be also applied to demonstrate the validity of DKCC on $k$-channel-connectivity guarantee. □

*Theorem 5: The message complexity of DKCC is $O(4|V| + 2|V|\varphi)$, where $|V|$ is the number of SUs in $G_{max}$, and $\varphi$ is the maximum degree of SUs in the resulting topology $S$.*

*Proof:* In the neighbor discovery stage, each SU $u$ broadcasts its local information twice to build the local subgraph $G_u$, resulting in the message complexity of $O(2|V|)$. In the topology construction stage, each SU may require the information of its $h$-hop ($h \geq 3$) neighborhood since its conflict neighbors cannot constitute a $i$-vertex-connected subgraph, ($i = 1, 2, \ldots, k-1$), in $G_u$. As a result, each SU $u$ only needs to broadcast the ID of its 1-hop neighbors in the induced graph after the topology construction, which leading to a message complexity of $O(|V| + |V|\varphi)$. It is worth declaring that although each SU $u$ needs aware its neighbors in $G_u$ with $P_{max}$, the topology decisions are made by the information between SUs in its 2-hop range of the minimum-power subgraph $S_u$, which has a much smaller number of SUs than that of $G_u$, and we will further verify it in the following simulation results. Next, in the power adjustment stage, there exists no message exchange since each SU could calculates its transmission power independently. Finally, similar to the topology construction stage, each SU $u$ informs the 1-hop neighbors of its request channel, resulting in the message complexity of $O(|V| + |V|\varphi)$ likewise. Above all, the total message complexity of DKCC reaches $O(4|V| + 2|V|\varphi)$. □

## VI. SIMULATION RESULTS

In this section, the performance of our centralized and distributed topology control algorithms, i.e., CKCC and DKCC, is evaluated via extensive simulations. Consider a network in a $1000\times1000$ m$^2$ region, where the locations of SUs are generated randomly. The path loss exponent $\alpha$ and the receive sensitivity $\beta$ are set to be 4 and -80 dBm, respectively. The maximum transmission power $P_{max}$ of each SU is 256 mW and the corresponding maximum transmission range $R_{max}$ is 400 m. In a CRN with a total of $K = 50$ licensed channels, the number of required channels through our proposed algorithms is expected to be less than $K$.

To construct a 3-channel-connected and conflict-free topology, i.e., the occupation of any two channels can be tolerable, we first generate a CRN with 20 SUs in the region. The topology in Fig. 4(a) induces the most severe interference since $P_{max}$ is employed by SUs. From Fig. 4 (b) (i.e., the topology induced by CKCC), we can see that the degree of each SU could be reduced substantially, leading to the interference mitigation between SUs. In other words, our algorithm can guarantee $k$-channel-connected and conflict-free properties with fewer required channels. In particular, the removal of any two channels still leaves the remaining network connected, e.g., the removal of channel 4 and 10 in Fig. 4(c), channel 9 and 10 in Fig. 4(d), channel 7 and 8 in Fig. 4(e). More particularly, the network can only be partitioned if three or more channels are reclaimed by PUs

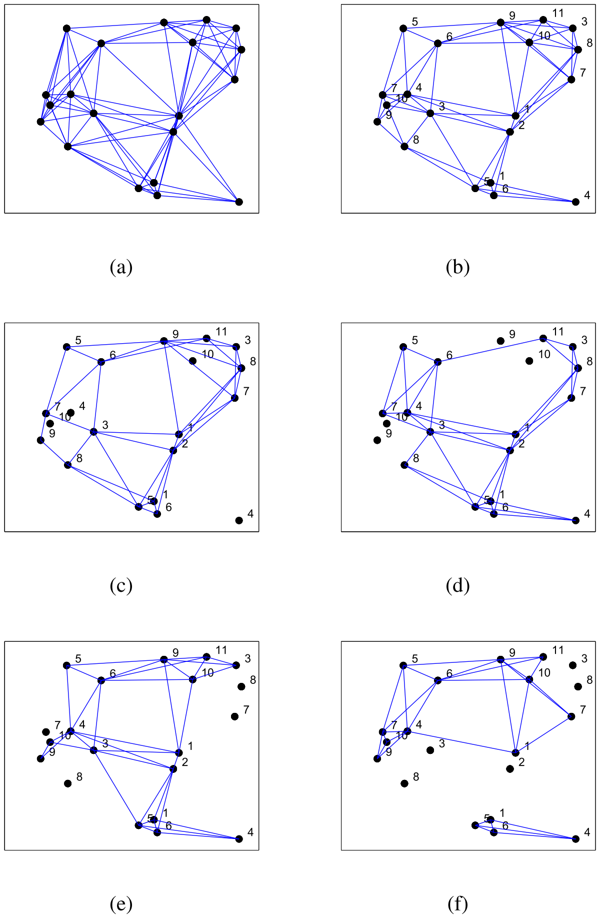



**FIGURE 4.** **(a) Maximum power topology. (b) The topology derived by CKCC (the number associated with each SU represents the assigned channel ID). (c) The topology when channel 4 and 10 are reclaimed by PUs. (d) The topology when channel 9 and 10 are reclaimed by PUs. (e) The topology when channel 7 and 8 are reclaimed by PUs. (f) The topology when channel 2, 3 and 8 are reclaimed by PUs.**

simultaneously, e.g., the removal of channel 2, 3 and 8 in Fig. 4(f).

Next, to demonstrate the effectiveness of our proposed algorithms in denser networks, we vary the number of SUs from 50 to 150, and compare our algorithms with several existing ones, i.e., $\text{GBC}_k$, $\text{GBC}_k + \text{DC}$ and $\text{FGSS}_k + \text{PC}$. The simulation results are averaged over 400 runs. In particular, as a modified version of GBC (i.e., the 2-channel-connectivity topology control algorithm proposed in [16]). The basic principle behind $\text{GBC}_k$ is to construct a $k$-vertex-connected topology at first, and then assign channels to each SU sequentially to ensure the $k$-channel-connected and conflict-free properties. It should be noted that in the channel assignment stage, a connectivity test has to be performed to check whether or not the CRN remains connected whenever any $k-1$ channels are reclaimed. Therefore, $\text{GBC}_k$ can hardly be implemented in distributed systems due to the requirement for global information and the high computational complexity. Moreover, $\text{GBC}_k + \text{DC}$ attaches the degree control to $\text{GBC}_k$ to minimize the maximum SU's degree. In addition, $\text{FGSS}_k + \text{PC}$ constructs a $k$-vertex-connectivity with minimum power paths, and then assigns channels to SUs in a greedy way; however, it can only preserve the conflict-free property.

Fig. 5 compares the average transmission ranges of different algorithms with $k = 3$. The transmission range of an SU indicates the length of the link between itself and its farthest logical neighbor. Note that the larger the average transmission range, the larger the power consumption. In Fig. 5, Maxpower refers to the algorithm that assigns channels to the maximum power topology with the greedy coloring algorithm. It can be seen that the average transmission ranges of other algorithms are lower than those of Maxpower, and decrease as the number of SUs grows. This is because the paths with lower power weight are maintained in above algorithms, thus facilitating the interference alleviation. Note that both CKCC and DKCC require larger transmission range than $\text{FGSS}_k + \text{PC}$ since only conflict-free constraint needs to be satisfied for $\text{FGSS}_k + \text{PC}$. Nonetheless, our algorithms acquire less average transmission range than other algorithms, and thus could be more energy efficient in the lifetime prolonging of energy-limited networks.

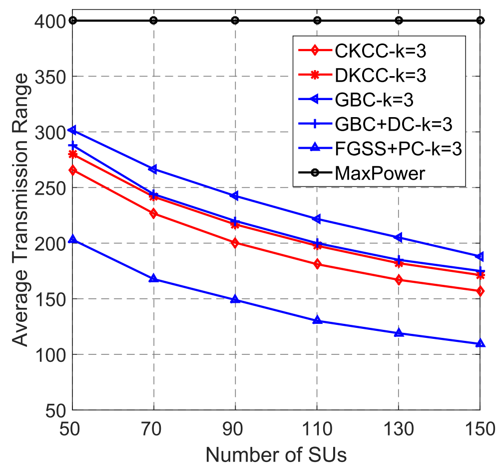



**FIGURE 5.** **Comparison of average transmission ranges between CKCC, DKCC, $\text{GBC}_k$, $\text{GBC}_k + \text{DC}$, $\text{FGSS}_k + \text{PC}$ and Maxpower, as a function of the numbers of SUs when $k = 3$.**

In Fig. 6, the average transmission ranges of CKCC and DKCC with different values of $k$ are given. The performance of DKCC is inferior to CKCC since its induced topology by the local information could be over-connected. Meanwhile, the power consumption grows with the increase of $k$ since more SUs are required for higher robustness. Moreover, under different values of $k$, the average transmission ranges of SUs are much smaller than the network region, especially in dense networks. This exhibits that the information exchange between 2-hop neighbors typically lies in a small region with a low message complexity.

In Fig. 7, we compare the average number of channels required by our centralized algorithm (CKCC) and the one in [30] (CCSS). With the increase of the SUs' density, CKCC outperforms CCSS under different requirements of

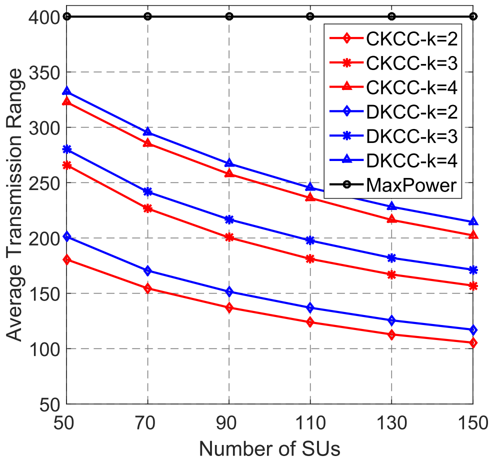



**FIGURE 6.** Comparison of average transmission ranges between CKCC, DKCC, and Maxpower, as a function of the numbers of SUs.

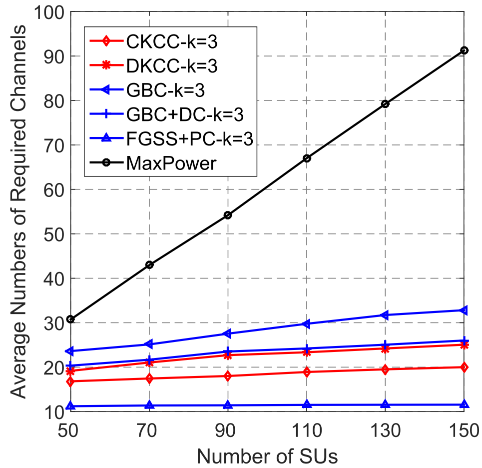



**FIGURE 8.** Comparison of average number of required channels between CKCC, DKCC, $GBC_k$, $GBC_k$ + DC, $FGSS_k$ + PC and Maxpower, as a function of the numbers of SUs when $k = 3$.

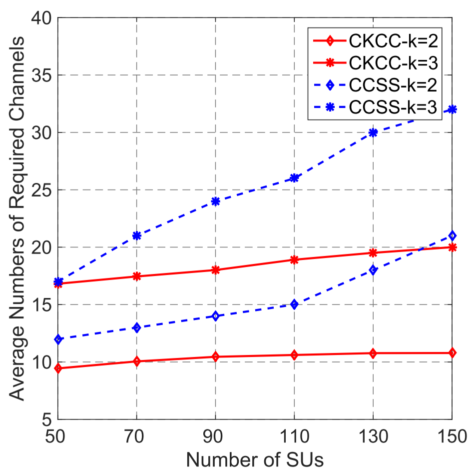



**FIGURE 7.** Comparison of average number of required channels between CKCC and CCSS, as a function of the numbers of SUs.

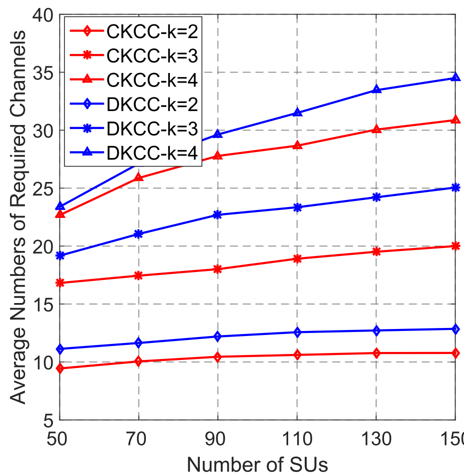



**FIGURE 9.** Comparison of average number of required channels between CKCC and DKCC, as a function of the numbers of SUs.

constructing 2-channel-connected and 3-channel-connected topologies, because less channels are request by CKCC. Mean while, the average number of channels of CCSS grow linearly, while CKCC only with a slight increase. The reason is that the combination of power control and channel assignment is considered in CKCC, so that the induced $k$-channel-connected topology can be more concisely, and the interference between SUs can be alleviated significantly as well.

Fig. 8 illustrates the average number of channels required by different algorithms with $k = 3$, respectively. It can be seen that DKCC requires more channels than CKCC due to the over-connected derived topology. However, DKCC and CKCC still outperform $GBC_k$ + DC and $GBC_k$, with the reasons as follows: 1) In the topology construction stage, as shown in Fig. 5, our algorithms can alleviate potential interference between SUs by constructing the topology with minimum power paths, so that both the transmission range and the number of each SU's conflict neighbors can be reduced as well. 2) In the channel assignment stage, given the topology that involves all possible occupation cases, only conflict-free property needs to be satisfied, thus, a greedy coloring algorithm can be implemented to acquire as fewer channels as possible. $GBC_k$ + DC and $GBC_k$ incur a performance loss, because without considering to reduce the consumption of available channels while constructing $k$-channel-connected and conflict-free topology. Furthermore, only conflict-free property is guaranteed under $FGSS_k$ + PC, so it can offer a lower bound on the number of required channels. It can be seen that the performance of CKCC approximates that of $FGSS_k$ + PC.

Fig. 9 and 10 give the average and maximum number of channels required by CKCC and DKCC with different values of $k$, respectively. In particular, the performance gaps between different $k$ is almost equal, while the gaps between the average and maximum values are relatively small, which also validate the robustness of our algorithms. Moreover, the maximum number of required channels is much smaller than the number of SUs, which can be deemed as the upper

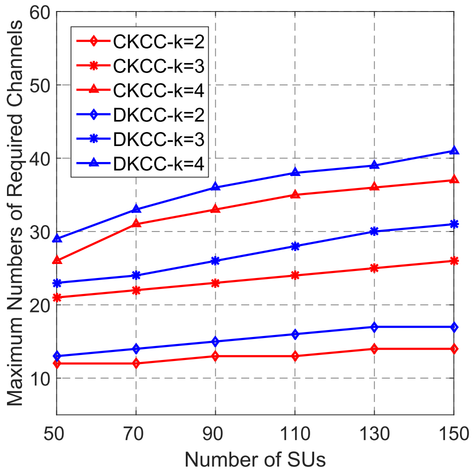
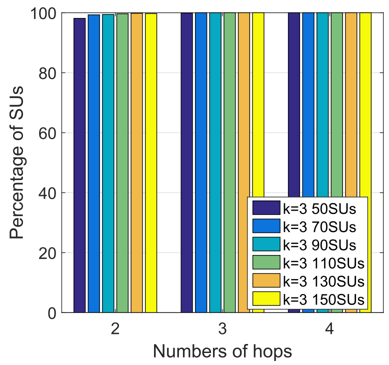
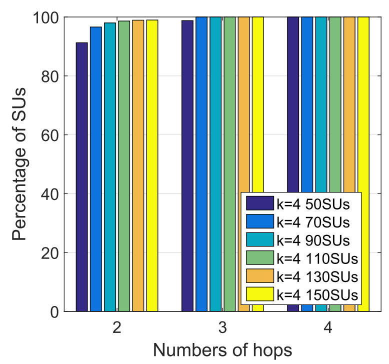



**FIGURE 10.** Comparison of maximum number of required channels between CKCC and DKCC, as a function of the numbers of SUs.




(a)




(b)

**FIGURE 11.** (a) k = 3. (b) k = 4. Percentage of SUs that require the knowledge of no more than $m$-hop ($m = 2, 3, 4$) neighbors in the topology control decision ($\alpha = 4$, $\beta = -80$dBm, $R_{max} = 400$m, $1000 \times 1000\text{m}^2$ region).

bound of the maximum conflict degree $\rho$ in the resulting topology via CKCC and DKCC. This could also indicate that the computation complexity of our algorithms is much lower than the that of $\text{GBC}_k + \text{DC}$ and $\text{GBC}_k$ (i.e., the global connectivity examination way).

At last, we will show how much information each SU needs to construct the $k$-channel-connected topology with DKCC. In Fig. 11(a) and (b), we present the percentage of SUs that require the number of hop ($m = 2, 3, 4$) neighbors in constructing the 3-channel-connected and 4-channel-connected topology. With the increase of $k$, more neighbors are involved to reach the higher robustness of topology. It can be seen that, the percentage of SUs that require the information of more than 2-hop neighbors grows with the increase of $k$. However, it should be noticed that, almost all SUs only require the knowledge of 2-hop neighbors to make the topology control decisions, which indicates that our algorithm operate with low-overhead and high-performance.

## VII. CONCLUSION

In this paper, to avoid the topology partition induced by the unpredicted activities of PUs, we studied the $k$-channel-connected topology control problem in CRNs; meanwhile, both the sufficient and necessary conditions are provided. To solve the problem, we guaranteed that any $k - 1$ independent sets are not any vertex-cut one of the CRN. Following the guidance, both centralized and distributed topology control algorithms are proposed to minimize the number of required channels. Theoretical analysis and simulation results verified the proposed algorithms can maintain $k$-channel-connected topologies with an acceptable computation complexity. Moreover, extensive simulation results are conducted to show the effectiveness of our algorithms, in terms of fault tolerance, connectivity guarantee as well as energy efficiency.

## REFERENCES

[1] J. Mitola and G. Q. Maguire, Jr., "Cognitive radio: Making software radios more personal," *IEEE Pers. Commun.*, vol. 6, no. 4, pp. 13–18, Apr. 1999.
[2] Z. Junhui, Y. Tao, G. Yi, W. Jiao, and F. Lei, "Power control algorithm of cognitive radio based on non-cooperative game theory," *China Commun.*, vol. 10, no. 11, pp. 143–154, Nov. 2013.
[3] R. W. Thomas, D. H. Friend, L. A. DaSilva, and A. B. MacKenzie, "Cognitive networks: Adaptation and learning to achieve end-to-end performance objectives," *IEEE Commun. Mag.*, vol. 44, no. 12, pp. 51–57, Dec. 2006.
[4] E. Ahmed, A. Gani, S. Abolfazli, L. J. Yao, and S. U. Khan, "Channel assignment algorithms in cognitive radio networks: Taxonomy, open issues, and challenges," *IEEE Commun. Surveys Tuts.*, vol. 18, no. 1, pp. 795–823, 1st Quart., 2016.
[5] L. Li, J. Y. Halpern, P. Bahl, Y.-M. Wang, and R. Wattenhofer, "A cone-based distributed topology-control algorithm for wireless multi-hop networks," *IEEE/ACM Trans. Netw.*, vol. 13, no. 1, pp. 147–159, Feb. 2005.
[6] K. Miyao, H. Nakayama, N. Ansari, and N. Kato, "LTRT: An efficient and reliable topology control algorithm for ad-hoc networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 12, pp. 6050–6058, Dec. 2009.
[7] A. K. Das and M. Mesbahi, "K-node connected power efficient topologies in wireless networks: A semidefinite programming approach," in *Proc. IEEE GLOBECOM*, vol. 1, Nov./Dec. 2005, pp. 1–6.
[8] X. Jia, D. Kim, S. Makki, P.-J. Wan, and C.-W. Yi, "Power assignment for K-connectivity in wireless ad hoc networks," in *Proc. IEEE 24th Annu. Joint Conf. Comput. Commun. Societies*, vol. 3, Mar. 2005, pp. 2206–2211.
[9] N. Li and J. C. Hou, "Localized fault-tolerant topology control in wireless ad hoc networks," *IEEE Trans. Parallel Distrib. Syst.*, vol. 17, no. 4, pp. 307–320, Apr. 2006.
[10] M. Hajiaghayi, N. Immorlica, and V. S. Mirrokni, "Power optimization in fault-tolerant topology control algorithms for wireless multi-hop networks," *IEEE/ACM Trans. Netw.*, vol. 15, no. 6, pp. 1345–1358, Dec. 2007.

[11] F. Wang, M. T. Thai, Y. Li, X. Cheng, and D. Z. Du, "Fault-tolerant topology control for all-to-one and one-to-all communication in wireles networks," *IEEE Trans. Mobile Comput.*, vol. 7, no. 3, pp. 322–331, Mar. 2008.

[12] X. Wang, M. Sheng, M. Liu, D. Zhai, and Y. Zhang, "RESP: A K-connected residual energy-aware topology control algorithm for ad hoc networks," in *Proc. IEEE WCNC*, Shanghai, China, Apr. 2013, pp. 1009–1014.

[13] J. Zhao and G. Cao, "Robust topology control in multi-hop cognitive radio networks," *IEEE Trans. Mobile Comput.*, vol. 13, no. 11, pp. 2634–2647, Nov. 2014.

[14] P.-K. Tseng, W.-H. Chung, and P.-C. Hsiu, "Minimum interference topology construction for robust multi-hop cognitive radio networks," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Shanghai, China, Apr. 2013, pp. 101–105.

[15] R. E. Irwin, A. B. MacKenzie, and L. A. DaSilva, "Resource-minimized channel assignment for multi-transceiver cognitive radio networks," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 3, pp. 442–450, Mar. 2013.

[16] H. Liu, Y. Zhou, X. Chu, Y.-W. Leung, and Z. Hao, "Generalized-bi-connectivity for fault tolerant cognitive radio networks," in *Proc. 21st Int. Conf. Comput. Commun. Netw. (ICCCN)*, Jul./Aug. 2012, pp. 1–8.

[17] X. Wang, M. Sheng, D. Zhai, J. Li, G. Mao, and Y. Zhang, "Achieving bi-channel-connectivity with topology control in cognitive radio networks," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 11, pp. 2163–2176, Nov. 2014.

[18] M. Sheng, X. Li, X. Wang, and C. Xu, "Topology control with successive interference cancellation in cognitive radio networks," *IEEE Trans. Commun.*, vol. 65, no. 1, pp. 37–48, Jan. 2017.

[19] M. M. Halldórsson and J. Radhakrishnan, "Greed is good: Approximating independent sets in sparse and bounded-degree graphs," *Algorithmica*, vol. 18, no. 1, pp. 145–163, 1997.

[20] Y. Long, H. Li, M. Pan, Y. Fang, and T. F. Wong, "A fair QoS-aware resource-allocation scheme for multiradio multichannel networks," *IEEE Trans. Veh. Technol.*, vol. 62, no. 7, pp. 3349–3358, Sep. 2013.

[21] M. K. Marina, S. R. Das, and A. P. Subramanian, "A topology control approach for utilizing multiple channels in multi-radio wireless mesh networks," *Comput. Netw.*, vol. 54, no. 2, pp. 241–256, Feb. 2010.

[22] J. Tang, G. Xue, and W. Zhang, "Interference-aware topology control and QoS routing in multi-channel wireless mesh networks," in *Proc. ACM Mobihoc*, Urbana-Champaign, IL, USA, May 2005, pp. 68–77.

[23] M. Burkhart, P. von Rickenbach, R. Wattenhofer, and A. Zollinger, "Does topology control reduce interference?" in *Proc. ACM Mobihoc*, Ropongi, Japan, May 2004, pp. 9–19.

[24] K. Moaveni-Nejad and X.-Y. Li, "Low-interference topology control for wireless ad hoc networks," *Ad Hoc Sensor Wireless Netw.*, vol. 1, nos. 1–2, pp. 41–64, Mar. 2005.

[25] W. Li, P. Fan, and K. B. Lataief, "Joint processing of topology control and channel assignment in wireless ad hoc networks," *Wireless Commun. Mobile Comput.*, vol. 9, pp. 269–281, Apr. 2008.

[26] R. W. Thomas, R. S. Komali, A. B. MacKenzie, and L. A. DaSilva, "Joint power and channel minimization in topology control: A cognitive network approach," in *Proc. IEEE ICC*, Glasgow, U.K., Jun. 2007, pp. 6538–6543.

[27] R. S. Komali, R. W. Thomas, L. A. DaSilva, and A. B. MacKenzie, "The price of ignorance: Distributed topology control in cognitive networks," *IEEE Trans. Wireless Commun.*, vol. 9, no. 4, pp. 1434–1445, Apr. 2010.

[28] A. Abbagnale and F. Cuomo, "Gymkhana: A connectivity-based routing scheme for cognitive radio ad hoc networks," in *Proc. IEEE INFOCOM*, San Diego, CA, USA, Mar. 2010, pp. 1–5.

[29] Y. Li, Z. Wang, B. Cao, and W. Huang, "Impact of spectrum allocation on connectivity of cognitive radio ad-hoc networks," in *Proc. IEEE GLOBECOM*, Houston, TX, USA, Dec. 2011, pp. 1–5.

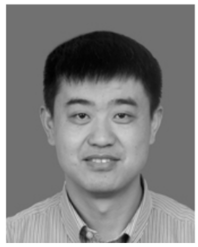
[30] R. N. Yadav and R. Misra, "K-channel connected topology control algorithm for cognitive radio networks," in *Proc. IEEE COMSNETS*, Bangalore, India, Jan. 2016, pp. 1–8.

[31] Y. Shi, H. Sun, M. Sheng, J. Li, and X. Li, "Constructing a robust topology for reliable communications in multi-channel cognitive radio ad hoc networks," *IEEE Commun. Mag.*, vol. 56, no. 4, pp. 172–179, Apr. 2018.

[32] B. Liu, X. Li, H. Sun, and M. Sheng, "Achieving K-channel-connectivity with topology control in cognitive radio networks," in *Proc. IEEE ICCC*, Chengdu, China, Jul. 2016, pp. 1–5.

[33] P. Cardieri, "Modeling interference in wireless ad hoc networks," *IEEE Commun. Surveys Tuts.*, vol. 12, no. 4, pp. 551–572, 4th Quart., 2010.

**XUAN LI** receivedthe B.S. degree in electronic and information Engineering from the Beijing Electronic Science and Technology Institute, Beijing, China, in 2008, and the Ph.D. degree in telecommunications engineering from Xidian University, Xi'an, China, in 2017. He is currently a Lecturer with the School of Information Engineering, East China Jiaotong University. His research interests include interference management, topology control in graph theory, and cognitive radio networks.



**JUNHUI ZHAO** (S'00–M'04–SM'09) received the M.S. and Ph.D. degrees from Southeast University, Nanjing, China, in 1998 and 2004, respectively. Since 2016, he has been with East China Jiaotong University as a Professor. He is currently a Professor with Beijing Jiaotong University. His current research interests include wireless and mobile communications and the related applications, which contain 5G mobile communication technology, high-speed railway communications, vehicle communication network, wireless localization, and cognitive radio. He received the IEEE WCSP 2017 Best Paper Award.

**YU YAO** received the B.S. degree from Nanchang Hangkong University, Nanchang, China, in 2007, and the M.S. and Ph.D. degrees from Southeast University, Nanjing, China, in 2015. He is currently an Assistant Professor with East China Jiaotong University. His research focuses on the communication and radar signal processing and cognitive radio, particularly with the radar-communication integration.

**TIANQING ZHOU** received the Ph.D. degree in information and communication engineering from Southeast University, Nanjing, China, in 2016. He joined the School of Information Engineering, East China Jiaotong University, Nanchang, China, in 2016. His current research interests include load balancing, cell selection and activation, wireless resource allocation, and interference management.

**YI GONG** (S'99–M'03–SM'07) received the Ph.D. degree in electrical engineering from The Hong Kong University of Science and Technology, Hong Kong, in 2002. He was with the Hong Kong Applied Science and Technology Research Institute, Hong Kong, and Nanyang Technological University, Singapore. He is currently a Professor with the Southern University of Science and Technology, Shenzhen, China. His research interests include cellular networks, mobile computing, and signal processing for wireless communications and related applications. From 2006 to 2017, he served on the Editorial Board of the IEEE Transactions on Wireless Communications and the IEEE Transactions on Vehicular Technology.

**LEI XIONG** was born in 1978. He received the B.S. and Ph.D. degrees from Beijing Jiaotong University in 2000 and 2007, respectively. He is currently an Associate Professor with the State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University. He has authored over 40 papers and two books. His current research interests include wireless channel modeling and simulation, software-defined radio, and network architecture.