# Study on Installation Error Analysis and Calibration of Acoustic Transceiver Array Based on SINS/USBL Integrated System

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**ABSTRACT** The installation error of USBL acoustic transceiver array is one of the important error sources for the SINS/USBL integrated navigation system. The installation error of USBL transceiver array can be divided into coordinate shift error and angle rotation error. The angle rotation error (i.e., installation error angle) has a great influence on the positioning accuracy of USBL. In this paper, the traditional surround sailing method is used to study the relationship between the installation error angle and the USBL positioning accuracy. The influence on USBL positioning is analyzed from the component of heading, pitch, and roll of installation error angle, and the simulation results show that the horizontal positioning errors of USBL are mainly affected by the heading and rolling installation errors. The horizontal positioning error of USBL caused by heading or rolling installation error of $1°$ will result in 17.45 m, under the circumstance that the depth of transponder is 100 m and the radius of surround sailing is 1000 m, without considering the influence of other positioning errors. 1.75 m horizontal positioning error of USBL caused by pitching positioning error of $1°$ pitching positioning error and the horizontal positioning error of USBL caused by heading and pitching positioning error is also related to the horizontal distance between the transmitter and transponder. A dynamic calibration algorithm of installation error angle based on incremental iteration is proposed in this paper, which could realize the dynamic on-line estimation of installation error angle of USBL transceiver array. The installation error angle increment module can accurately estimate the installation angle error by iterating more than 10 times. In the case of ignoring the other error terms, the horizontal positioning error caused by the installation error angle can be reduced from 8 to 2 m in the positioning range of 1000 m.

**INDEX TERMS** SINS/USBL, installation error angle, navigation, on-line, increment iteration, dynamic calibration.

## I. INTRODUCTION

High-precision underwater positioning system is needed for marine resources exploration, exploitation of seabed oil and gas resources, laying and repairing of submarine pipelines and the salvage of underwater sediments [1]. The development of marine resources and military needs have promoted the development of underwater high-precision positioning technology. The electromagnetic wave is not suitable for long-distance transmission under water because of its serious loss during propagation. Sound wave is the only information carriers that can be transmitted over long distances underwater currently. Underwater acoustic positioning system can be divided into Long Base Line (LBL), Short Base Line (SBL) and Ultra-Short Base Line (USBL) according to the base line length of acoustic array. USBL positioning system is widely used in underwater vehicle navigation and positioning domain for its small baseline array size, portable use, easy operation, and low cost [2].

With the development of underwater engineering construction and development of underwater resources, a single underwater positioning method can not to meet the requirements of underwater long-distance and high-precision positioning. The technology of SINS/GNSS integrated system which based on vehicle, air-borne, missile-borne and

carrier-borne [3], [4] (Including loosely integrated, tightly integrated [5] and ultra-tightly integrated) have been matured. Hu *et al.* [6] and Gao *et al.* [7] applied the nonlinear filtering technology in the SINS/GNSS integrated navigation system. Underwater navigation usually adopts the scheme of SINS/LBL, SINS/SBL, SINS/USBL or SINS/DVL integrated navigation [8]. The related technical research of SINS/DVL integrated navigation mainly focuses on the DVL aiding SINS initial alignment [9]. In recent years, the SINS/USBL integrated navigation system has been widely used in underwater positioning projects. Morgado *et al.* [10] first proposed the concept of tight coupled SINS/USBL navigation system in 2007, and systematically demonstrated the method of SINS/USBL ultra-tight integrated navigation system in 2013 [11]. In 2015, Bingbing [12] put forward new scheme which based on SINS/USBL/DVL integrated navigation. In 2015, Yawen [13] proposed SINS/USBL tightly integrated model which used three transponders. Heqing of Southeast University put forward a integrated model based on the oblique distance and arrival distance difference in 2017 [14]. The theoretical research of SINS/USBL integrated navigation is mature, and some research results have been widely applied in engineering application. The main factors affect the positioning accuracy of high precision ultra-short baseline positioning system based SINS/USBL are system model error, ocean environment parameter measurement error, time delay measurement error and transceiver array installation error [15], [16]. The installation error of transceiver array is the main error source for USBL positioning system, and it should be corrected accurately before it is used in practice [17]. The installation error calibration of USBL acoustic array can be accomplished by acoustic calibration method at sea [18]. The existing literatures and technical documents show that the current USBL installation error calibration requires the carrier-ship to navigate strictly according to the established trajectory, while the carrier-ship is affected by wind, sea waves, automatic maneuver of the carrier-ship and other interference, the carrier-ship cannot strictly execute the expected trajectory [19]. The installation error angle will be changed, and the calibration needs to be re-calibrated when the installation position of USBL sensor is changed, or the USBL sensor is disassembled and reinstalled because of maintenance. The cost of calibration is high and the calibration process is complex, so an efficient and portable USBL installation error calibration is the key technology to high-precision positioning of USBL.

Li *et al.* of Harbin Engineering University mentioned in their paper in 2013 that the installation error of 1° can produce the positioning error of 1.7% oblique distance [20]. McEwen *et al.* [21] found that the average installation error between the USBL underwater acoustic sensor and the attitude sensor in the discrete structure design was about 2.5° and the standard deviation was about 0.5° in 2005. Professor Hsin-hung Chen National Sun Yat-sen University of China Taiwan analyzed and demonstrated the principle of USBL alignment error in 2008 and 2013, proposed a method of

calibration at sea by surround sailing method, and fully verified the feasibility of the calibration algorithm by simulation [19], [22], [23]. However, it is necessary to measure the horizontal distance between transceiver acoustic center and transponder continuously. The measurement accuracy of the distance will influence the estimation accuracy of the installation error directly. The specific measurement method is not given in this paper, so it is difficult to be realized in engineering practices. In addition, Chen's method, considering measurement errors, comprehensively analyzed the oscillation phenomena occurring in estimating heading and pitching installation angle, but did not give an effective solution.

This paper adopts the design scheme of USBL/SINS integrated structure as the background of demand, aiming at solving the problem of high precision estimation of installation error angle between USBL transceiver array and installation base, and improving the positioning accuracy of USBL acoustic positioning system. Aiming at the problem of Chen's method analyzed above, an improved incremental iteration algorithm is adopted in this paper. In the process of USBL installation error increment estimation, the increment is estimated by the precise length value of the double transponder (rolling direction increment estimation), so the measurement error caused by the depth measurement of transponder can be eliminated; The increment is estimated by selecting a single observation of the transponder (heading and pitching direction increment estimation), so that the measurement of horizontal distance between transceiver and transponder isn't needed, so the measurement error caused by the distance measurement is eliminated.

The structure of the paper is as follows:

The first section is the introduction, introducing the research status. The second section introduces the basic principle of the USBL positioning system, mainly including: the basic working principle of the USBL positioning system, the coordinate system and its mutual conversion involved in this paper, and the error analysis of USBL positioning system. The third section analyzes the influence of installation error angle component on USBL positioning. The forth section introduces the principle of new installation error angle calibration algorithm, which is the main contribution of this paper. The fifth section is experiment and verification. The sixth section is summary.

## II. PRINCIPLE AND ERROR ANALYSIS OF USBL POSITIONING SYSTEM
### A. THE BASIC POSITIONING PRINCIPLE OF USBL POSITIONING SYSTME
In most literatures, the underwater acoustic positioning system is divided into Long Baseline (LBL), Short Baseline (SBL) and Ultra-Short Baseline (USBL) according to the length of baseline. USBL is a new underwater acoustic positioning system developed from SBL in 1970s, which is a variant of SBL [24]. The algorithm base on 'Incidence angle

and Distance' is usually used in the USBL positioning calculation. In order to illustrate the positioning principle of USBL, the simplified structure of receiving array was adopted in this paper. Fig.1 is a schematic diagram of USBL positioning principle based on the structure of binary array elements vertical receiving base array.
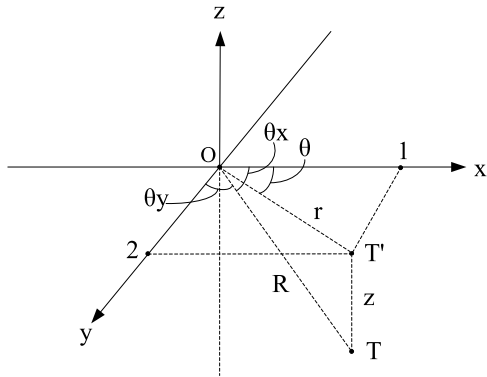


**FIGURE 1.** The schematic diagram of USBL positioning principle based on the binary array elements vertical structure receiving base array.

The array base element 1 on the X axis and element 2 on Y axis of USBL acoustic array coordinate system (x, y, z) form a vertical underwater acoustic receiving array structure. The underwater acoustic transmitter located at the coordinate origin. The main task of the acoustic transmitter is to send a positioning pulse signal to the underwater transponder, the transponder sends back a response signal with different frequency after receiving the underwater acoustic signal from the transmitter. The transmitter is located in the geometric center of the acoustic array. The point $T$ shown in fig.1 is the location of the transponder. $R$ represents the distance between the transponder and the center of the USBL acoustic array, where $T'$ represents the projection of the transponder $T$ on the plane $xoy$, $r$ represents the horizontal slant distance of the transponder, that is, the projection of $R$ on the plane $xoy$.

The coordinate vector $T$ of the target transponder is $\overline{OT}$, the length of the slanting distance from the transponder $T$ to the origin of the coordinate is,

$$R = \sqrt{x^2 + y^2 + z^2} \qquad (1)$$

Where $\angle\theta_x$ and $\angle\theta_y$ are the angles between R and X, Y axis respectively, $\angle\theta$ is the angle between R and X axis. The direction cosine of $\angle\theta_x$, $\angle\theta_y$ are respectively

$$\cos\theta_x = \frac{x}{R} \qquad (2)$$

$$\cos\theta_y = \frac{y}{R} \qquad (3)$$

$$\theta = \arctan\frac{y}{x} \qquad (4)$$

According to Eq. (2), (3), it can be obtained that

$$x = R\cos\theta_x \qquad (5)$$

$$y = R\cos\theta_y \qquad (6)$$

$$\theta = \arctan\frac{\cos\theta_y}{\cos\theta_x} \qquad (7)$$

$$r = \sqrt{x^2 + y^2} \qquad (8)$$

$$z = \sqrt{R^2 - r^2} \qquad (9)$$

In engineering application, the oblique distance of the target transponder is generally calculated by the acoustic round-trip time delay. Other more accurate methods are mentioned in other literatures [25] and relevant research are not involved in this paper. In this paper, we assumed that the water quality is uniform, and the environmental factors have little influence on the acoustic speed of the water. The Eq. (10) will be used to calculate the slanting distance between the target transponder and the USBL acoustic center.

$$R = \frac{C\Delta t}{2} \qquad (10)$$

Where $C$ represents the underwater acoustic equivalent speed, $\Delta t$ is the time period between the underwater acoustic signal transmitted by USBL sound transmitter and the receiving array received the return signal from the target transponder, which includes sending signal time $T_s$ and receiving signal time $T_r$.

In Eq. (5) and (6), the directional cosine of $\angle\theta_x$ and $\angle\theta_y$ of the target transponder $T$ are needed. In this paper, the directional cosine of the target transponder is measured by phase difference method.

Fig. 2 is the diagram of the phase difference between the four receiving elements of the USBL array. The acoustic emitter is located in the acoustic geometry center of the four receiving elements. The receiving elements 1 and 3 are located on the x-axis of the USBL acoustic array coordinate system, and the receiving elements 2 and 4 are located on the Y-axis of the coordinate system. The length between elements 1 and 3 is d. Because the size of the acoustic array is very small, on the condition that the far-field plane wave is approximated, it can be considered that all the underwater acoustic lines incident to the array elements are parallel.
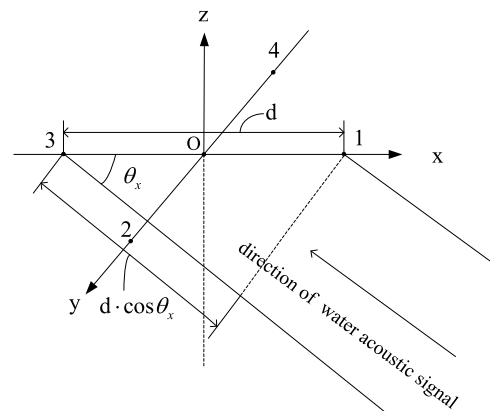


**FIGURE 2.** The phase difference between two receiving elements.

The phase difference of received signals between hydrophone array 1 and 3 is

$$\varphi_{1,3} = \frac{2\pi d \cos\theta_x}{\lambda} \tag{11}$$

Similarly, the phase difference of the received signals between the hydrophone array 2 and 4 is

$$\varphi_{2,4} = \frac{2\pi d \cos\theta_y}{\lambda} \tag{12}$$

Where $d$ represents the distance between two hydrophone elements, $\varphi_{1,3}$ is the phase difference between the two hydrophone elements receiving the underwater acoustic signal on the $x$ axis, $\varphi_{2,4}$ is the phase difference between the two hydrophone elements receiving the underwater acoustic signal on the $y$ axis. When the frequency of underwater acoustic signals is $f_0$, the wavelength is

$$\lambda = \frac{c}{f_0} \tag{13}$$

Rewriting Eq. (11) and (12) it can be obtained that,

$$\cos\theta_x = \frac{\lambda\varphi_{1,3}}{2\pi d} \tag{14}$$

$$\cos\theta_y = \frac{\lambda\varphi_{2,4}}{2\pi d} \tag{15}$$

Substitute Eq. (14) and (15) into Eq. (5) and (6),

$$x = \frac{R\lambda\varphi_{1,3}}{2\pi d} \tag{16}$$

$$y = \frac{R\lambda\varphi_{2,4}}{2\pi d} \tag{17}$$

Eq. (9), (16) and (17) are the basic equations for acoustic positioning of USBL. The above formulas can be used to calculate the position of the transponder based on the acoustic coordinate system.

$$P_t = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \dfrac{R\lambda\varphi_{1,3}}{2\pi d} \\ \dfrac{R\lambda\varphi_{2,4}}{2\pi d} \\ \sqrt{R^2 - r^2} \end{bmatrix} \tag{18}$$

With measuring the underwater acoustic velocity $C$, the delay difference of underwater acoustic signal $\Delta t$, the distance $d$ between USBL array elements, and the phase difference $\varphi_{i,j}$ accurately ($i, j$ is the identifier number of the array elements on the same coordinate axis), the location value of the target transponder in the USBL acoustic array coordinate system can be precisely calculated.

Horizontal azimuth is often used in engineering practice, and can be obtained by Eq. (4), (14) and (15).

$$\theta = \arctan\frac{y}{x} = \arctan\frac{R\cos\theta_y}{R\cos\theta_x} = \arctan\frac{\cos\theta_y}{\cos\theta_x}$$

$$= \arctan\frac{\frac{\lambda\varphi_{2,4}}{2\pi d}}{\frac{\lambda\varphi_{1,3}}{2\pi d}} = \arctan\frac{\varphi_{2,4}}{\varphi_{1,3}} \tag{19}$$
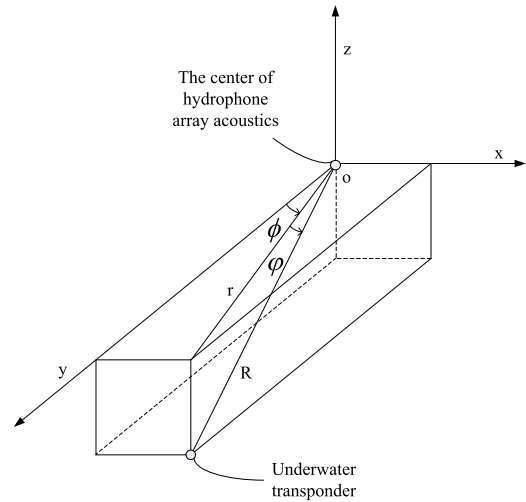


**FIGURE 3.** Schematic diagram of USBL acoustic positioning system.

In the literature [16], another method was adopted to calculate the target transponder, which is shown in Fig3.

The fig.3 and Eq. (18) are consistent in the principle. The slant range ($R$), azimuth ($\phi$) and pitch ($\varphi$) from the target transponder to the USBL acoustics center are used in the position calculation. The location vector of the target responder is shown in formula (20).

$$P_t = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} R\cos\varphi\sin\phi \\ R\cos\varphi\cos\phi \\ -R\sin\varphi \end{bmatrix} \tag{20}$$

With this method, the position of the target transponder in the USBL acoustic array coordinate system can be determined only need measuring the underwater acoustic velocity $C$, the delay difference $\Delta t$ of the underwater acoustic signal, the horizontal azimuth angle $\phi$ and the pitch angle $\varphi$.

### B. COORDINATE SYSTEM

Several coordinate systems are used in this paper. As shown in Fig.3, the original solutions of the target transponder are calculated by the USBL system in the USBL acoustic array coordinate system. The USBL acoustic array coordinate system is defined as fig.4 (Four elements receiver array as an example).

USBL acoustic array coordinate system (abbreviated as $U$ coordinate system ($O_U X_U Y_U Z_U$), 1, 3 receiving array elements are located on the X axis, 2, 4 elements are located on the Y axis. Set the position of underwater acoustic transmitter as the origin of coordinates, the origin of coordinates is located at the USBL array acoustic center. The USBL acoustic array is mounted on the ship rigidly and its attitude can be measured by the strap down vertical reference unites ($SVRU$).

Base array coordinate system (or carrier coordinate system, abbreviated as $B$ coordinate system $O_B X_B Y_B Z_B$), The USBL acoustic array is mounted below the base. In the
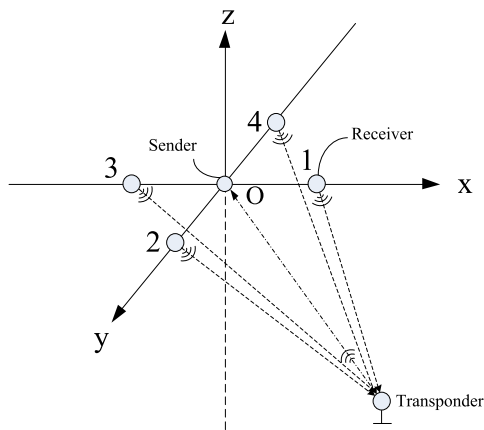
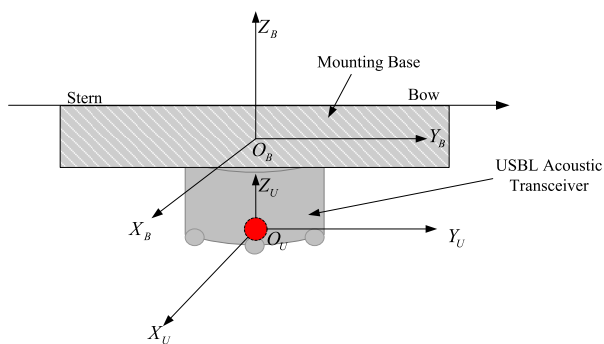**FIGURE 4.** USBL acoustic array coordinate system.



**FIGURE 5.** Schematic diagram of base coordinate system.

ideal case, the base coordinate system and USBL acoustic array coordinate system are completely coincide. But there is the misalignment installation angle between B coordinate system and U coordinate system caused by installation error angle exists in engineering practice.

If the absolute coordinate position is needed, the global coordinate system (G system for short, $O_G X_G Y_G Z_G$) is required. G coordinate system using the general WGS-84 coordination system, that is the 'East, North, Up' coordinate system, X axis refers to the east, Y axis refers to the north, Z axis refers to the up.

In order to simplify the analysis, the auxiliary coordinate system (A system for short, $O_A X_A Y_A Z_A$) is introduced as the reference coordinate system. As shown in Fig.6, the vertical line is drawn from the transponder to the horizontal plane, and the intersection point between the vertical line and the horizontal plane is the origin of the coordinate system A. The X-axis points to the east, the Y-axis points to the north, and the Z-axis points to the sky. The coordinate values of transponder in the A coordinate system are $P_T^A = (0, 0, -h)$

## C. ERROR ANALYSIS OF USBL POSITIONING SYSTEM

Taking the *x*-direction error component as an example, the positioning error of USBL system is analyzed, and the full
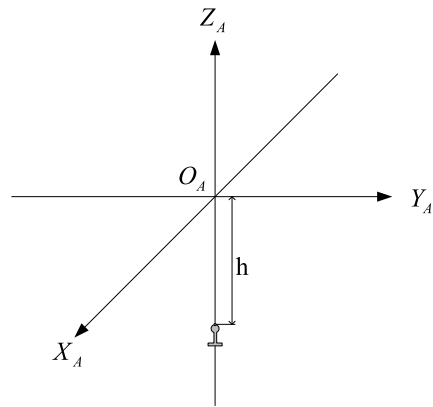


**FIGURE 6.** Auxiliary coordinate system.

differential is obtained for equation (16),

$$\Delta x = \frac{R\lambda}{2\pi d}\Delta\varphi_{1,3} + \frac{R\lambda\varphi_{1,3}}{2\pi d}\Delta\lambda + \frac{\lambda\varphi_{1,3}}{2\pi d}\Delta R - \frac{R\lambda\varphi_{1,3}}{2\pi}\frac{\Delta d}{d^2} \tag{21}$$

The relative error is,

$$\frac{\Delta x}{x} = \frac{\Delta\varphi_{1,3}}{\varphi_{1,3}} + \frac{\Delta\lambda}{\lambda} + \frac{\Delta R}{R} - \frac{\Delta d}{d} \tag{22}$$

The analysis of the third items on the right side of Eq. (22) is carried out.

$$R = \frac{CT}{2} \tag{23}$$

In the Eq. (23), R is the distance between the center of USBL acoustics transceiver and the target transponder. *C* is the equivalent underwater acoustic velocity and *T* is the time includes the underwater acoustic signal receiving and sending back and forth.

$$\Delta R = \frac{\Delta C \cdot T}{2} + \frac{\Delta T \cdot C}{2} \tag{24}$$

$$\frac{\Delta R}{R} = \frac{\Delta C}{C} + \frac{\Delta T}{T} \tag{25}$$

As $\lambda$, C and $f_0$ have the following relationship

$$\lambda = \frac{C}{f_0} \tag{26}$$

Then

$$\Delta\lambda = \frac{\Delta C}{f_0} \tag{27}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta C}{C} \tag{28}$$

Rewrite the Eq. (22), obtained that,

$$\frac{\Delta x}{x} = \frac{\Delta\varphi_{1,3}}{\varphi_{1,3}} + 2\frac{\Delta C}{C} + \frac{\Delta T}{T} - \frac{\Delta d}{d} \tag{29}$$

The above are the relative positioning accuracy. In USBL positioning system, the error is usually related with the distance of the target transponder. The oblique distance relative

positioning accuracy is usually used to analyze the positioning accuracy of the system in engineering practice. It can be obtained by Eq. (16) and (29) that,

$$\frac{\Delta x}{R} = \frac{\lambda \varphi_{1,3}}{2\pi d}\left(\frac{\Delta \varphi_{1,3}}{\varphi_{1,3}} + 2\frac{\Delta C}{C} + \frac{\Delta T}{T} - \frac{\Delta d}{d}\right) \quad (30)$$

When the location error items are independent of each other, the mean square error of relative positioning accuracy of slant range is analyzed as follow.

$$\delta_x^2 = \left(\frac{\Delta x}{R}\right)^2 = \left(\frac{\lambda \Delta \varphi_{1,3}}{2\pi d}\right)^2 + \left(\frac{\lambda \varphi_{1,3}}{2\pi d}\right)^2$$
$$\times \left[4\left(\frac{\Delta C}{C}\right)^2 + \left(\frac{\Delta T}{T}\right)^2 + \left(\frac{\Delta d}{d}\right)^2\right] \quad (31)$$

It can be seen from Eq. (31) that the oblique relative positioning error of USBL mainly includes the phase measurement error of the receiving element, the velocity measurement error, the delay measurement error, the distance error between the elements and so on. In the errors mentioned above, the velocity error and the delay measurement error can be controlled in a very small range [17]. The error of the second term in Eq. (31) can be omitted when the measurement of sound velocity, time delay and element spacing are accurate. It can be seen that the measurement error of phase difference between two elements is the major error source of USBL positioning system. According to Eq. (14) and Eq. (31), the phase error is essentially caused by the angle $\Delta \theta_x$ which between the acoustic array coordinate system axis and the oblique distance, which is also the installation error angle of USBL. It is mentioned in related literature that in the USBL positioning system with positioning accuracy of 5% oblique distance, installation error of 1° can cause positioning error of 1.7% times of slant distance.

## III. THE INFLUENCE OF INSTALLATION ERROR ANGLE ERROR ANGLE OF USBL SYSTEM ON POSITIONING

### A. USBL ACOUSTIC ARRAY COORDINATE SYSTEM AND THE DEFINITION OF INSTALLATION ERROR ANGLE

The coordinate system shown in Fig.7 is the USBL acoustic array coordinate system $O_U X_U Y_U Z_U$. The origin of the coordinate is the acoustic center of the array and coincides with the acoustic transmitter which emits the underwater acoustic signal. The Y-axis is pointing in the forward direction of the bow and stern line of the carry-ship. The Z axis is perpendicular to the $XOY$ plane and its positive direction is upward.

Ideally, there is only coordinate translation between USBL acoustic array coordinate system and carrier coordinate system without angular rotation deviation, as shown in figure 5. In fact, there are error angles $(\alpha, \beta, \gamma)$ between the two coordinate systems in the heading, pitching and rolling direction. The USBL installation error studied in this paper includes the fabrication of the elements of USBL acoustic array itself and the angle deviation during the installation. The angle deviation of the installation error is shown as fig.5. The installation error angle mentioned in this paper refers to the

angle deviation of the three axes between the USBL acoustic array coordinate system and the carrier coordinate system.

USBL installation error angles include heading, pitching and rolling installation error angle, as shown in Figure 8, 9, 10 respectively. The installation error angles defined in this paper are as follow: The positive direction of the heading is
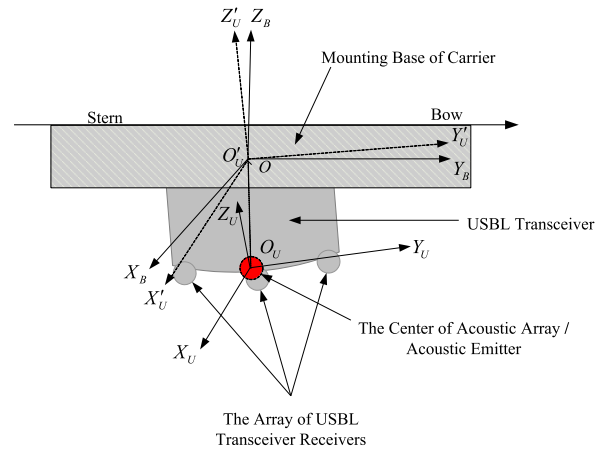


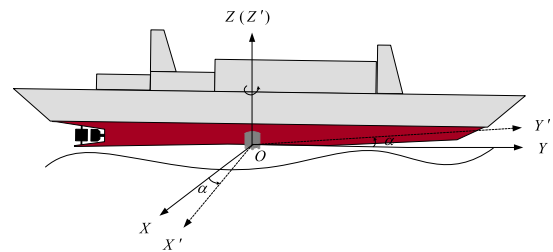**FIGURE 7.** Schematic diagram of USBL installation error angle.



**FIGURE 8.** The diagram of heading installing error angle.
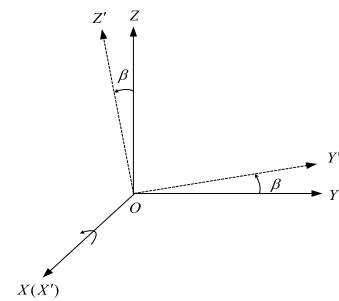


**FIGURE 9.** The diagram of pitching installing error angle.
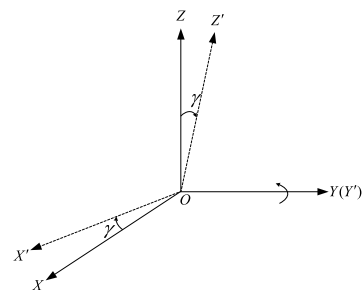


**FIGURE 10.** The diagram of rolling installing error angle.

defined that the positive direction of the Y axis to the negative direction of the X axis; The positive direction of the pitching is defined that the positive direction of the Y axis to the positive direction of the Z axis; The positive direction of the Rolling is defined that the positive direction of the X axis to the positive direction of the Z axis, as shown in Fig. 7, 8, 9, 10.

Formula (32), (33) and (34) transform formulas between two coordinates.

$$C_B^U(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (32)$$

$$C_B^U(\beta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix} \quad (33)$$

$$C_B^U(\gamma) = \begin{bmatrix} \cos\gamma & 0 & -\sin\gamma \\ 0 & 1 & 0 \\ \sin\gamma & 0 & \cos\gamma \end{bmatrix} \quad (34)$$

The $C_i^j$ represents coordinate transformation matrix of coordinate system from i to j. Equations (32), (33) and (34) are represent the coordinate transformation caused by the in installation error angles in the direction of heading, pitching and rolling respectively. $C_B^U$ represents the transformation from the carrier coordinate system B to the ultra-short baseline acoustic coordinate system U. Considering both of the simultaneous existence of Z axis rotation and X axis translation, which was shown in fig.11,
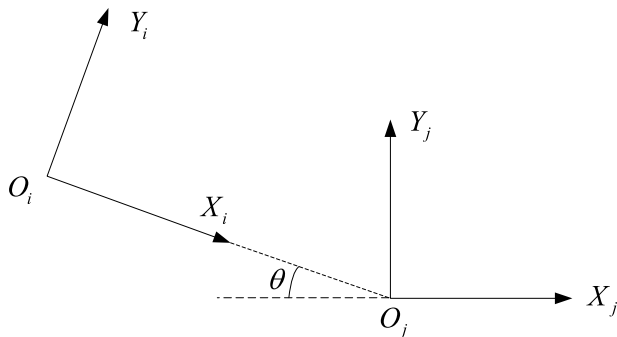


**FIGURE 11.** The diagram of the rotation and translation simultaneous existence.

Its coordinate transformation equation is

$$\mathbf{C}_j^i(L,\theta) = \begin{bmatrix} \mathbf{C} & \mathbf{T} \\ \mathbf{0}^T & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & T_x \\ -\sin\theta & \cos\theta & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (35)$$

The matrix $\mathbf{C}_j^i$ represents the transformation from the coordinate system j to the coordinate system i, the matrix C is the rotation transformation matrix around the Z axis, and T is the translation of coordinates.
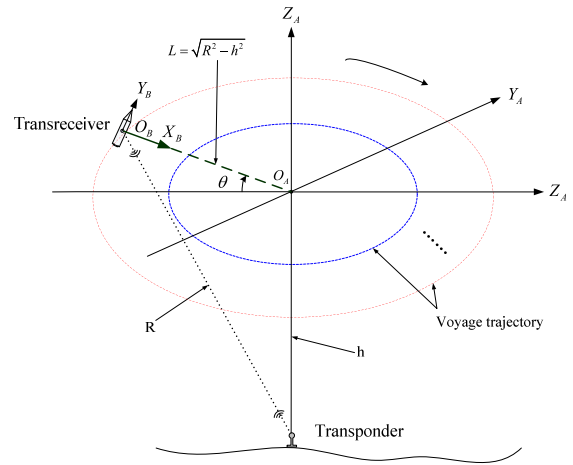
**FIGURE 12.** Schematic diagram of surround sailing method.

## B. INFLUENCE ANSLYSIS OF USBL INSTALLATION ERROR ANGLE ON POSITIONING

Transponder fixed on the underwater seabed was located by the method of multiple surround sailing in this paper. By this method, the effect of circumferential radius on USBL positioning accuracy has been studied. The transponder is fixed on the seabed, and the ship which carried transceiver of USBL was centered with the intersection of the vertical line of transponder and the water surface and sailed around the center with a radius of L. The influence of the installation error angle on the positioning accuracy of the USBL positioning system has been discussed. The influence of the installation error angle on the positioning accuracy of the USBL horizontal positioning system has been analyzed, as shown in figure12:

Where $L$ is the distance between the center of acoustics of transceiver and the origin of coordinates $O_A$,

$$L = \sqrt{R^2 - h^2} \quad (36)$$

Where $R$ is the distance between transceiver and transponder, which was obtained by underwater acoustic ranging accurately and $h$ is the depth of Transponder, which can be measured by depth meter accurately.

### 1) THEORETICAL DERIVATION

Firstly, the influence of USBL pitching installation error on transponder positioning was studied. The coordinate value of transponder in the auxiliary coordinate system $O_A X_A Y_A Z_A$ is $P_T^A$,

$$P_T^A = \begin{bmatrix} 0 & 0 & -h & 1 \end{bmatrix}^T \quad (37)$$

$$P_T^B = C_A^B P_T^A$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 & L \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cdot \begin{bmatrix} 0 & 0 & -h & 1 \end{bmatrix}^T = \begin{bmatrix} L \\ 0 \\ -h \\ 1 \end{bmatrix} \quad (38)$$

Where $C_A^B$ represents the transformation matrix from auxiliary coordinates system A to carrier coordinates system B. If $C_A^B$ is reversible, $C_B^A$ represents the transformation matrix from carrier coordinate system B to auxiliary coordinate system A, then

$$C_B^A = \left(C_A^B\right)^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & -L\cos\theta \\ -\sin\theta & \cos\theta & 0 & L\sin\theta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{39}$$

Ideally, when there is no pitching installation error,

$$P_T^U = P_T^B \tag{40}$$

Where $P_T^U$ represents the coordinate value of transponder in the USBL acoustic array coordinate system U, and $P_T^B$ represents the coordinates value of transponder in the carrier coordinate system B. When there is a heading installation error angle $\alpha$ between the USBL acoustic base coordinate system and the carrier base coordinate system,

$$P_T^U(\alpha) = C_B^U(\alpha) \cdot P_T^B$$
$$= \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} L \\ 0 \\ -h \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} L\cos\alpha \\ -L\sin\alpha \\ -h \\ 1 \end{bmatrix} \tag{41}$$

Similarly, when there is a pitching installation error angle $\beta$ between the USBL acoustic base coordinate system and the carrier base coordinate system,

$$P_T^U(\beta) = C_B^U(\beta) \cdot P_T^B$$
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & \sin\beta & 0 \\ 0 & -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} L \\ 0 \\ -h \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} L \\ -h\sin\beta \\ -h\cos\beta \\ 1 \end{bmatrix} \tag{42}$$

When there is pitching installation error angle $\gamma$ between the USBL acoustic base coordinate system and the carrier base coordinate system,

$$P_T^U(\gamma) = C_B^U(\gamma) \cdot P_T^B$$
$$= \begin{bmatrix} \cos\gamma & 0 & -\sin\gamma & 0 \\ 0 & 1 & 0 & 0 \\ \sin\gamma & 0 & \cos\gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} L \\ 0 \\ -h \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} L\cos\gamma + h\sin\gamma \\ 0 \\ L\sin\gamma - h\cos\gamma \\ 1 \end{bmatrix} \tag{43}$$

In order to analyze the influence of heading installation error angle on the transponder positioning, the location error of transponder caused by heading installation error angle $\alpha$ in coordinate system U is calculated as follow,

$$\varepsilon_{BU}(\alpha) = P_T^B - P_T^U(\alpha)$$
$$= \begin{bmatrix} L \\ 0 \\ -h \\ 1 \end{bmatrix} - \begin{bmatrix} L\cos\alpha \\ -L\sin\alpha \\ -h \\ 1 \end{bmatrix} = \begin{bmatrix} L(1-\cos\alpha) \\ L\sin\alpha \\ 0 \\ 0 \end{bmatrix} \tag{44}$$

Similarly, the positioning error of transponder caused by the pitching installation error angle $\beta$ under the coordinate system U is,

$$\varepsilon_{BU}(\beta) = P_T^B - P_T^U(\beta)$$
$$= \begin{bmatrix} L \\ 0 \\ -h \\ 1 \end{bmatrix} - \begin{bmatrix} L \\ -h\sin\beta \\ -h\cos\beta \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ h\sin\beta \\ h(\cos\beta - 1) \\ 0 \end{bmatrix} \tag{45}$$

The positioning error of transponder caused by rolling installation error angle $\gamma$ under the coordinate system U is,

$$\varepsilon_{BU}(\gamma) = P_T^B - P_T^U(\gamma)$$
$$= \begin{bmatrix} L \\ 0 \\ -h \\ 1 \end{bmatrix} - \begin{bmatrix} L\cos\gamma + h\sin\gamma \\ 0 \\ L\sin\gamma - h\cos\gamma \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} L(1-\cos\gamma) - h\sin\gamma \\ 0 \\ h(\cos\gamma - 1) - L\sin\gamma \\ 0 \end{bmatrix} \tag{46}$$

### 2) SIMULATION ANSLYSIS OF INFLUENCE OF INSTALLATION ERROR ANGLE ON POSITIONING ACCURACY

In order to analyze the influence of each installation error angle components on the positioning accuracy of transponder more intuitively, the following simulation work has been carried out. As Fig12 shows, the underwater depth of the transponder is 100m, the mother-ship equipped with USBL transceiver sails around the transponder. The horizontal distances of the USBL transceiver and the transponder are 100m, 500m, 1000m, and 1500m respectively. The heading error angle, the pitching error angle and the rolling error angle are all set to 1°. (In order to analysis the influence of the error angle, this value usually greater or less than actual value in engineering practice)

From the Eq. (44), it can be known that the X-direction positioning error caused by USBL heading installation error angle is

$$\varepsilon_{BU\_x}(a) = L(1 - \cos\alpha) \tag{47}$$

The positioning error in the Y-direction is

$$\varepsilon_{BU\_y}(a) = Lsia\alpha \tag{48}$$

The horizontal positioning error is

$$
\begin{aligned}
\varepsilon_{BU\_h}(a) &\\
&= \sqrt{\varepsilon_{BUx}(a)^2 + \varepsilon_{BUy}(a)^2} \\
&= \sqrt{(L(1 - \cos \alpha))^2 + (Lsia\alpha)^2} = L\sqrt{2(1 - \cos a)}
\end{aligned} \quad (49)
$$

Eq. (44) shows that the positioning error in Z direction caused by heading installation error is 0, which means there is no positioning error in Z direction. The simulation results are shown in Fig.13 and 14.
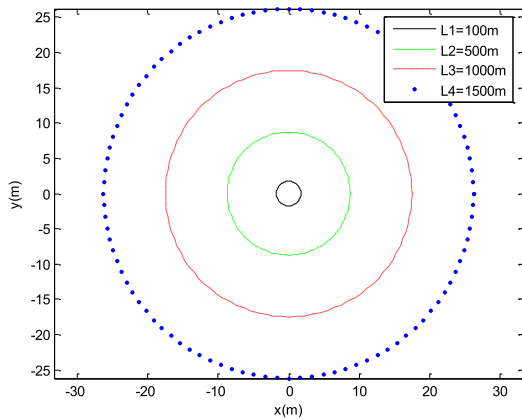


**FIGURE 13.** USBL horizontal positioning error caused by heading installation error angle.
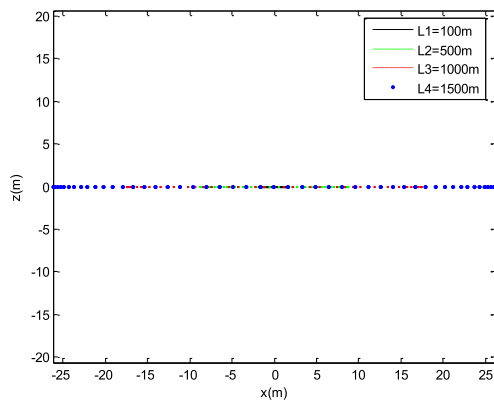


**FIGURE 14.** USBL Z-direction positioning error caused by heading installation error angle.

As can be seen from Fig. 13, the horizontal positioning error increases linearly with the increase of the horizontal distance L which between the transceiver and transponder of USBL system. It can be seen that the error angle of heading installation has great influence on long distance positioning of USBL. If it has not been calibrated accurately in advance, the USBL system will bring a large horizontal positioning error when it is used for remote positioning.

Eq. (45) shows that the X-direction component of the positioning error is 0, therefore, the pitching installation error does not influence the positioning accuracy of the X-axis direction, and the positioning error of the direction Y and Z

are related to the size of the installation error angle and the depth of the transponder.

The Y-direction position error caused by USBL pitching installation error angle is

$$
\varepsilon_{BU\_y}(\beta) = h \sin \beta \quad (50)
$$

The positioning error in the Z direction is

$$
\varepsilon_{BU\_z}(\beta) = h(\cos \beta - 1) \quad (51)
$$

Its horizontal positioning error is mainly caused by installation error of Y axis. Fig.14 and 15 show the influence of pitching installation error angle on the positioning accuracy of USBL.
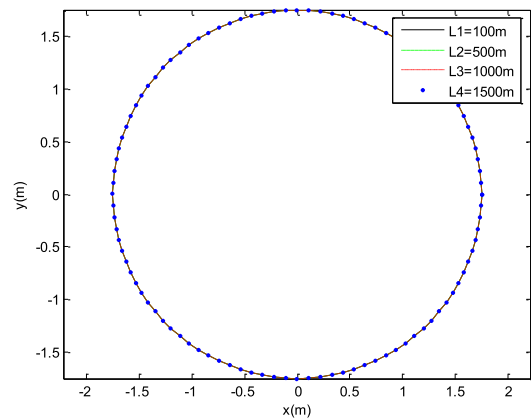


**FIGURE 15.** Horizontal positioning error caused by pitching installation error angle.

It can be seen from fig.15 and fig.16 that the positioning error caused by pitching installation error angle (including horizontal and Z -direction positioning error) is a constant value, which is irrelevant to the horizontal distance L between the USBL transceiver acoustic array and the transponder, and it is related to the depth of transponder and the size of pitching installation error angle.
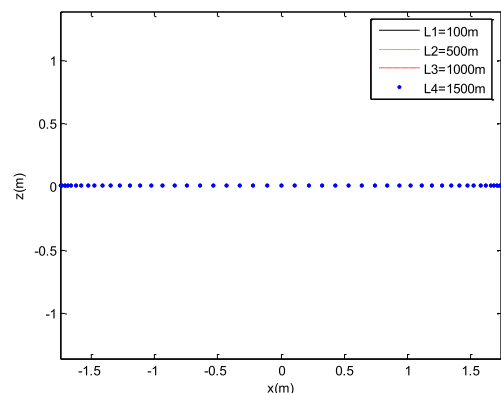


**FIGURE 16.** Z-direction positioning error caused by pitching installation error angle.

Eq.46 shows that the component of the positioning error on the Y-axis caused by the rolling installation error angle

**TABLE 1.** Influence of heading installation error on positioning accuracy.

| Heading Error Angle (deg.) | Absolute Error $\varepsilon_i$ (m) | | | | Relative Error $\varepsilon_i$ /L (%) | | | |
|---|---|---|---|---|---|---|---|---|
| | $\varepsilon_x$ | $\varepsilon_y$ | $\varepsilon_z$ | $\varepsilon_h$ | $\varepsilon_x$ / R | $\varepsilon_y$ / R | $\varepsilon_z$ / R | $\varepsilon_h$ / R |
| 0.1 | 0.0015 | 1.7453 | 0 | 1.7453 | 1.5231e-006 | 0.0017 | 0 | 0.0017 |
| 0.14 | 0.0030 | 2.4435 | 0 | 2.4435 | 2.9852e-006 | 0.0024 | 0 | 0.0024 |
| 0.15 | 0.0034 | 2.6180 | 0 | 2.6180 | 3.4269e-006 | 0.0026 | 0 | 0.0026 |
| 0.2 | 0.0061 | 3.4907 | 0 | 3.4907 | 6.0923e-006 | 0.0035 | 0 | 0.0035 |
| 0.5 | 0.0381 | 8.7265 | 0 | 8.7266 | 3.8077e-005 | 0.0087 | 0 | 0.0087 |
| 1 | 0.1523 | 17.4524 | 0 | 17.4531 | 1.5230e-004 | 0.0175 | 0 | 0.0175 |
| 2 | 0.6092 | 34.8995 | 0 | 34.9048 | 6.0917e-004 | 0.0349 | 0 | 0.0349 |
| 5 | 3.8053 | 87.1557 | 0 | 87.2388 | 0.0038 | 0.0872 | 0 | 0.0872 |

is 0, therefore, the rolling installation error will not influence the positioning accuracy of the $Y$-axis direction, and the positioning error in the $X$-axis direction caused by the rolling installation error angle $\gamma$ of USBL is that

$$\varepsilon_{BU\_x}(\gamma) = L(1 - \cos\gamma) - h\sin\gamma \qquad (52)$$

The positioning error in the Z-axis direction is

$$\varepsilon_{BU\_z}(\gamma) = h(\cos\gamma - 1) - L\sin\gamma \qquad (53)$$

The horizontal positioning error is mainly caused by X-axis direction positioning error. The positioning errors in X and Z axis directions are related to the pitching installation error angle, the horizontal distance L and the depth of the transponder.

Fig. 17 and equation (46) show that, when the rolling installation error angle is small, the distance L has little effect on the horizontal positioning errors. Therefore, the horizontal positioning error of USBL caused by the rolling installation error has little correlation with the distance of USBL positioning. Fig.18 shows that the Z positioning error increases linearly with the increase of distance value L.
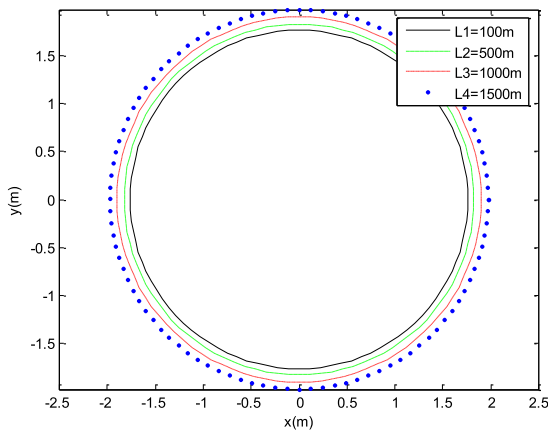


**FIGURE 17.** Schematic diagram of horizontal positioning error caused by rolling installation error angle.

The above analysis has studied the effect of each component of the installation error angle on the positioning accuracy
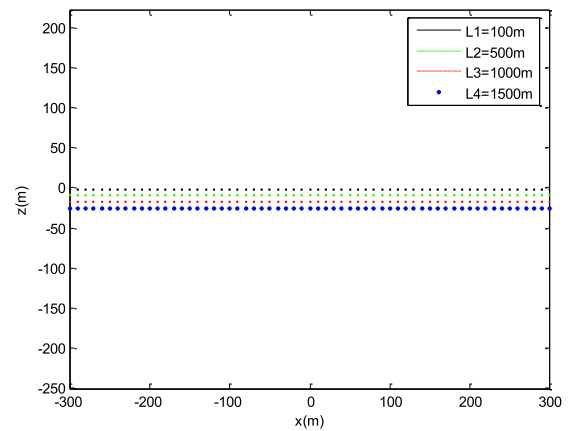


**FIGURE 18.** Schematic diagram of Z-direction positioning error caused by rolling installation error angle.

in different directions under different horizontal distance L. The following simulation has analyze the effect of different installation error angle value on the each axis-direction accuracy under the same horizontal distance L. Simulation environment: Horizontal distance L = 1000m, depth h = 100m, heading error angles are 0.1°, 0.14°, 0.15°, 0.2°, 0.5°, 1°, 2° and 5°, and pitching and rolling installation error angles were set to 0. A total of 8 sets of simulation experiments were done, as shown in Table 1.

The influence of pitching installation error on positioning accuracy was simulated. The simulation environment parameters was set as follow: horizontal distance L = 1000m, depth h = 100m, heading error angles were 0.1°, 0.14°, 0.15°, 0.2°, 0.5°, 1°, 2°, 5°, heading and rolling installation error angles are set to 0, a total of 9 sets of simulation experiments were done, as shown in Table 2.

Some simulation research on the influence of positioning accuracy with the rolling installation error has also been carried out. The simulation environment parameters were set as follow: horizontal distance L = 1000m, depth h = 100m, heading error angles are 0.1°, 0.14°, 0.15°, 0.2°, 0.5°, 1°, 2° and 5°, heading and longitudinal installation error angles are set to 0, a total of 8 groups of simulation experiments were done, as shown in Table 3.

**TABLE 2.** Influence of pitching installation error on positioning accuracy.

| Pitching Error Angle (deg.) | Absolute Error $\varepsilon_i$ (m) | | | | Relative Error $\varepsilon_i/L$ (%) | | | |
|---|---|---|---|---|---|---|---|---|
| | $\varepsilon_x$ | $\varepsilon_y$ | $\varepsilon_z$ | $\varepsilon_h$ | $\varepsilon_x/R$ | $\varepsilon_y/R$ | $\varepsilon_z/R$ | $\varepsilon_h/R$ |
| 0.1 | 0 | 0.1745 | 1.5231e-004 | 0.1745 | 0 | 1.7453e-004 | 1.5231e-007 | 1.7453e-004 |
| 0.14 | 0 | 0.2443 | 2.9852e-004 | 0.2443 | 0 | 2.4435e-004 | 2.9852e-007 | 2.4435e-004 |
| 0.15 | 0 | 0.2618 | 3.4269e-004 | 0.2618 | 0 | 2.6180e-004 | 3.4269e-007 | 2.6180e-004 |
| 0.2 | 0 | 0.3491 | 6.0923e-004 | 0.3491 | 0 | 3.4907e-004 | 6.0923e-007 | 3.4907e-004 |
| 0.5 | 0 | 0.8727 | 0.0038 | 0.8727 | 0 | 8.7265e-004 | 3.8077e-006 | 8.7265e-004 |
| 1 | 0 | 1.7452 | 0.0152 | 1.7452 | 0 | 0.0017 | 1.5230e-005 | 0.0017 |
| 1.4 | 0 | 2.4432 | 0.0299 | 2.4432 | 0 | 0.0024 | 2.9851e-005 | 0.0024 |
| 2 | 0 | 3.4899 | 0.0609 | 3.4899 | 0 | 0.0035 | 6.0917e-005 | 0.0035 |
| 5 | 0 | 8.7156 | 0.3805 | 8.7156 | 0 | 0.0087 | 3.8053e-004 | 0.0087 |

**TABLE 3.** Influence of rolling installation error on positioning accuracy.

| Rolling Error Angle (deg.) | Absolute Error $\varepsilon_i$ (m) | | | | Relative Error $\varepsilon_i/L$ (%) | | | |
|---|---|---|---|---|---|---|---|---|
| | $\varepsilon_x$ | $\varepsilon_y$ | $\varepsilon_z$ | $\varepsilon_h$ | $\varepsilon_x/R$ | $\varepsilon_y/R$ | $\varepsilon_z/R$ | $\varepsilon_h/R$ |
| 0.1 | 0.1761 | 0 | 1.7452 | 0.1761 | 1.7606e-004 | 0 | 0.0017 | 1.7606e-004 |
| 0.14 | 0.2473 | 0 | 2.4432 | 0.2473 | 2.4733e-004 | 0 | 0.0024 | 2.4733e-004 |
| 0.15 | 0.2652 | 0 | 2.6176 | 0.2652 | 2.6523e-004 | 0 | 0.0026 | 2.6523e-004 |
| 0.2 | 0.3552 | 0 | 3.4900 | 0.3552 | 3.5516e-004 | 0 | 0.0035 | 3.5516e-004 |
| 0.5 | 0.9107 | 0 | 8.7227 | 0.9107 | 9.1073e-004 | 0 | 0.0087 | 9.1073e-004 |
| 1 | 1.8975 | 0 | 17.4372 | 1.8975 | 0.0019 | 0 | 0.0174 | 0.0019 |
| 2 | 4.0991 | 0 | 34.8386 | 4.0991 | 0.0041 | 0 | 0.0348 | 0.0041 |
| 5 | 12.5209 | 0 | 86.7752 | 12.5209 | 0.0125 | 0 | 0.0868 | 0.0125 |

According to transponder positioning error expressions (44), (45), (46), the influence of installation error angle on positioning accuracy is analyzed. The following conclusions can be drawn from tables 1, 2 and 3.

a) The heading installation error angle of USBL does not affect the Z-direction positioning accuracy of USBL positioning system. It has little effect on the X-direction positioning accuracy, brings a larger positioning error to the Y-direction, and has a greater influence on the horizontal positioning accuracy. The heading installation error angle has a positive correlation with the positioning distance of USBL. If the relative positioning accuracy of the USBL system (namely, the ratio of the positioning error to the horizontal distance L between transceiver and transponder) needs to reach 2.5%, the heading installation error angle should be controlled within 0.14°, without considering other error sources.

b) The pitching installation error angle of USBL will not affect the X-direction positioning accuracy of USBL positioning system, and it has little effect on the Z-direction positioning accuracy. But it brings a larger positioning error to the Y-direction. However, compared with the heading and rolling errors, the pitching installation error angle has less influence on the positioning accuracy (The horizontal positioning error caused by the pitching error angle is only 1/10 of that

caused by the same heading or rolling installation error angle.) If the relative positioning accuracy of the USBL system needs to reach 2.5%, the pitching installation error angle should be controlled within 1.4°, without considering other error sources.

c) The rolling installation error angle of USBL does not affect the Y-direction positioning accuracy, but has little effect on the X-direction positioning accuracy, and brings a larger positioning error to Z-direction. If the relative positioning accuracy of USBL system needs to reach 2.5%, the rolling installation error angle should be controlled within 0.14°, without considering other errors.

It can be seen from Fig.13 that the influence of the heading installation error angle on the positioning accuracy of USBL positioning system is related to the radius of surround sailing. The larger the radius of surround sailing, the greater the error of underwater acoustic positioning is. It is necessary to measure the circumferential radius many times to adopt the surround sailing method in [12]. The measurement error of the radius will introduce a new positioning error. In the existing literature, the calibration process of installation error angle requires the mother-ship to navigate strictly according to the set trajectory, which puts forward higher requirements for the operation of calibration test, and it has some difficulties in engineering application. In this paper, a new dynamic

calibration algorithm is proposed to solve the problem of high precision and fast installation error angle calibration of USBL.

## IV. A DYNAMIC CALIBRATION ALGORITHM FOR USBL INSTALLATION ERROR ANDGLE BASED ON INCREMENTAL ITERATION

In the literature [19], the depth parameters $h$ could be eliminated by the surround sailing method. But the process is complicated. This method has stringent requirements for the trajectory of the carrier ship, and the carrier needs to be circumnavigated many times. In this paper, a fast and accurate estimating method under arbitrary tracks about USBL installation error angle calibration is studied, which is based on the influence of the installation error angle component on the positioning accuracy of the transponder.

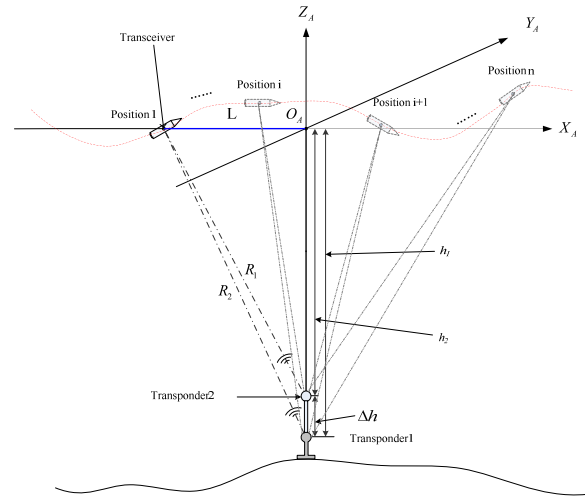### A. ESTIMATION OF THE ROLL-DIRECTION INSTALLATION ERROR ANGLE

The auxiliary coordinate system A is selected as the reference coordinate system, and the underwater transponder is designed with a dual transponder with the vertical structure. The depth of Transponder1 is $h_1$, the depth of Transponder2 is $h_2$, the distance of Transponder1 and Transponder2 is $\Delta h$. The distance from the acoustics central of USBL to the original point of the reference coordinate system is L.

Whether X-direction positioning error component or Z-direction positioning error component is selected to estimate the installation error angle, it is necessary to measure the depth of the transponder and the horizontal distance between transceiver and transponder accurately. The measurement error of these two parameters will affect the estimation accuracy of the installation error angle. In the literature [19], the depth parameter $h$ could be eliminated by the method of surround sailing by different radius. This method has strict requirements for the trajectory of the carrier-ship, and the carrier-ship needs to be sail surround many times, which process is complicated. In order to reduce the influence of multiple times of surround sailing and radius measurement error, the method of double transponder is designed to eliminate the depth parameter $h$. As shown in Figure 19, in the negative direction of the Z axis in the auxiliary coordinate system A, two transponder 1 and 2 of the same type are located at depths h1 and h2 meters respectively, and the coordinates value of transponder 1 and 2 in the coordinate system A are as follow,

$$P_{T_1} = \begin{bmatrix} 0 \\ 0 \\ -h_1 \end{bmatrix} \quad (54)$$

$$P_{T_2} = \begin{bmatrix} 0 \\ 0 \\ -h_2 \end{bmatrix} \quad (55)$$

Studying the influence of transverse installation error on horizontal positioning error, it can be seen from equation (46) and table 3 that rolling installation error angle mainly affects



**FIGURE 19.** The schematic diagram of double Transponders work principle.

X-direction positioning accuracy. In this paper, the error component of X direction is used to solve the rolling installation error angle. It can be seem from Eq. (43), the X-axis orientation values of the transpnder1 and transponder2 are obtained when the carrier-ship sailing around the double transponders array.

$$P_{T_1\_x}^U(\gamma) = L \cos \gamma + h_1 \sin \gamma \quad (56)$$
$$P_{T_2\_x}^U(\gamma) = L \cos \gamma - h_2 \sin \gamma \quad (57)$$

$P_{T_i\ x}^U(\gamma)$, (i=1, 2) represent the X coordinate values of the Transponder1, 2 in the presence of the rolling installation error.

From Eq. (56)-(57), we can get the following equation

$$P_{T_1\_x}^U(\gamma) - P_{T_2\_x}^U(\gamma) = (h_2 - h_1) \sin \gamma \quad (58)$$

Then

$$\gamma = \sin^{-1}\left(\frac{P_{T_1\_x}^U(\gamma) - P_{T_2\_x}^U(\gamma)}{h_2 - h_1}\right)$$
$$= \sin^{-1}\left(\frac{P_{T_1\_x}^U(\gamma) - P_{T_2\_x}^U(\gamma)}{\Delta h}\right) \quad (59)$$

In equation (57), $P_{T_1\_x}^U(\gamma)$, $P_{T_2\_x}^U(\gamma)$ represent the X-axis coordinate values of transonder1 and transonder2 in USBL array coordinate system respectively, in the presence of the rolling installation error. Where $\Delta h$ is the distance between the Transponder1 and Transponder2, which can be accurately measured when the transponders are machined.

### B. ESTIMATION OF THE PITCH-DIRECTION INSTALLATION ERROR ANGLE

It can be seen from Eq.(42) that it is necessary to measure the depth $h$ of Transponder1 precisely for the estimation of the increment of installation error angle accurately, by choosing either Y component or Z component of the pitch-direction installation error angle. To study the accuracy of estimation

**TABLE 4.** Influence of depth measurement error on pitch-direction installation error angle.

| Order of experiment | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Depth (m) | 100.01 | 100.05 | 100.1 | 100.2 | 100.5 | 101 | 102 | 105 |
| The Error of Depth (m) | 0.01 | 0.05 | 0.1 | 0.2 | 0. 5 | 1 | 2 | 5 |
| The estimation value of pitching installation error angle (deg) | 0.8103 | 1.8115 | 2.5613 | 3.6207 | 5.7177 | 8.0693 | 9.8625 | 11.3649 |
| The estimation Error of pitching installation error angle (deg) | -1.6897 | -0.6885 | 0.0613 | 1.1207 | 3.2177 | 5.5693 | 7.3625 | 8.8649 |

installation error angle affected by the measurement error of the depth of transponder1, the following simulation experiments were carried out.

The carrier-ship equipped with USBL transceiver is on the water surface stationary, and the underwater transponder located by USBL system continuously. The simulation parameters are as follows: the installation error angle is 0.25°, the depth of Transponder 1 is 100 meters, the depth measurement error are 0.01m, 0.05m, 0.1m,0.2m, 0.5m, 1m, 2m and 5m.

It can be seen from Table 4 that 0.2 m depth measurement error will bring estimation error of 1.12° pitch-direction installation error angle. 1 m depth measurement error will bring estimation error of 5.56° error angle. Therefore, the precise measurement of the depth $h$ is very important to the accurate estimation of installation error angle. This paper proposes to adopt a new estimation method, which can accurately estimate the pitch-direction installation error angle of USBL system without accurately measurement of the depth of transponder.

From the equation (42), it can be seen that

$$P_{T_{1\_y}}^U(\beta) = -h_1 \sin \beta \tag{60}$$

$$P_{T_{1\_z}}^U(\beta) = -h_1 \cos \beta \tag{61}$$

Among them, $P_{T_1 y}^U(\beta)$ and $P_{T_1 z}^U(\beta)$ are the value y and z components of USBL system's location solution to Transponder 1 under the influence of pitch-direction installation error angle separately. When the observation value $P_{T_1 z}^U(\beta)$ is not equal to zero, the both sides of Eq. (60) divided by both sides of Eq. (61) separately, it can be obtained that,

$$\frac{P_{T_{1\_y}}^U(\beta)}{P_{T_{1\_z}}^U(\beta)} = \frac{-h_1 \sin \beta}{-h_1 \cos \beta} = \tan \beta \tag{62}$$

Therefore

$$\beta = \tan^{-1} \frac{P_{T_1 y}^U(\beta)}{P_{T_1 z}^U(\beta)} \tag{63}$$

It can be seen from equation (63) that the pitch-direction installation error $\beta$ of USBL array can be accurately estimated by the Y and Z-direction value of transponder 1

position value calculated by USBL system, and the influence resulted from the measurement error of transponder depth can be avoided.

## C. ESTIMATION OF THE HEAD-DIRECTION INSTALLATION ERROR ANGLE

From Fig13, it can be seen that the heading installation error angle has a great influence on the horizontal positioning error. Eq. 41 shows that the heading installation error angle can be deduced from the x direction and y direction components of the positioning value of Transponder1 by USBL. However, to complete the estimation of heading angle, the existing methods in literature [19] require precise measurements of the horizontal distance between transponder and transceiver, many times of surround sailing by the carrier equipped the USBL transceiver and the trajectory of ship is also strictly required. As shown in Fig.12. A new estimation method is proposed in this paper, which does not require the horizontal distance between transceiver and transponder or the depth of transponder, and also can accurately estimate the heading installation error angle of USBL systems.

Equations (64)-(65) could be obtained from the equation (41)

$$P_{T_{1\_x}}^U(\alpha) = L \cos \alpha \tag{64}$$

$$P_{T_{1\_y}}^U(\alpha) = -L \sin \alpha \tag{65}$$

Among them, $P_{T_{1\_x}}^U(\alpha)$ and $P_{T_{1\_y}}^U(\alpha)$ are the X and Y components of positioning solution value of USBL system to Transponder1 under the influence of heading installation error angle. When the observation value is not equal to zero, the two side of equation (64) are divided by two side of Eq. (65) separately.

$$\frac{P_{T_{1\_y}}^U(\alpha)}{P_{T_{1\_x}}^U(\alpha)} = \frac{-L \sin \alpha}{L \cos \alpha} = -\tan \alpha \tag{66}$$

Therefore,

$$\alpha = ar \tan\left(-\frac{P_{T_{1\_y}}^U(\alpha)}{P_{T_{1\_x}}^U(\alpha)}\right) \tag{67}$$
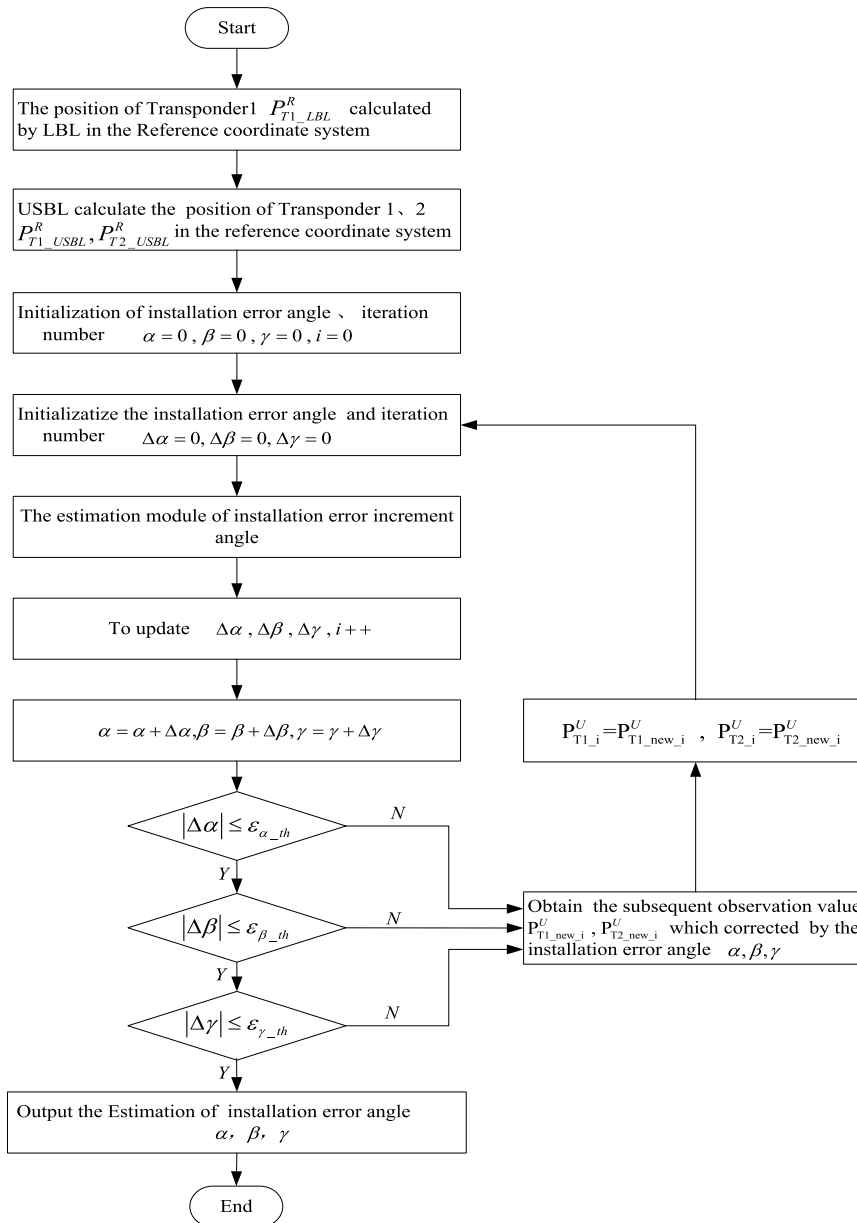
**FIGURE 20.** Flow chart of installation error angle correction.

From equation (67), it can be seen that using the X and Y orientation position value of Transponder 1 calculated by USBL can accurately estimate the heading installation error angle, without measuring the horizontal distance between Transponder and Transceiver and the depth of transponder, and the influence of these measurement error on the heading installation error angle estimation can be avoided. However, the analysis above is based on the assumption that the installation error angles in three directions affect the positioning accuracy of the target transponder separately, in the coordinate system U. However in engineering practice, the installation error angles in three directions affect the positioning accuracy of the target transponder together.

Therefore, the method above cannot used to estimate the error angle directly. In this paper, the incremental iteration method is used to estimate the increment of the installation error angle after several iterations, and the new estimated installation error angle is used to correct the subsequent positioning observations. Finally, all the incremental installation errors are accumulated to obtain the installation errors. Its working principle is shown in Fig. 20.

## V. EXPERIMENT AND VERIFICATION

The rolling, pitching and heading components of the installation error angle are all set to 2 °, set the distance between Transponder 1 and Transponder 2 to 1 meter. Set $\varepsilon_{\alpha_t h} = 10^{-5}$,

**TABLE 5.** The value of installation error angle increment estimation.

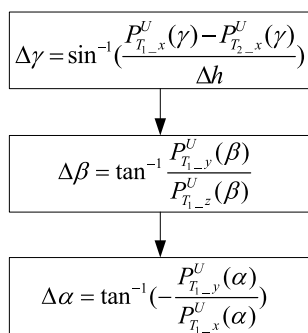| Number of iterations | $\Delta\gamma$ | $\Delta\beta$ | $\Delta\alpha$ |
|---|---|---|---|
| 1 | 2.0010321420 | 1.9719271381 | 1.9933735665 |
| 2 | 0.0064730108 | 0.0101023965 | 0.0000333647 |
| 3 | 0.0000630198 | 0.0002397212 | 0.0000311360 |
| 4 | 0.0001811806 | 0.0001600507 | 0.0000309358 |
| 5 | 0.0002099131 | 0.0001539743 | 0.0000306991 |
| 6 | 0.0001336396 | 0.0001421570 | 0.0000295948 |
| 7 | 0.0001153239 | 0.0001336960 | 0.0000291248 |
| 8 | 0.0000106990 | 0.0001253414 | 0.0000288082 |
| 9 | 0.0001715838 | 0.0001124860 | 0.0000279907 |
| 10 | 0.0002002948 | 0.0001106556 | 0.0000278736 |
| 11 | 0.0000187749 | 0.0001084011 | 0.0000277512 |
| 12 | 0.0000573090 | 0.0001046323 | 0.0000272822 |
| 13 | 0.0001143500 | 0.0000856289 | 0.0000272700 |
| 14 | 0.0000590110 | 0.0000758851 | 0.0000269084 |
| 15 | 0.0001555989 | 0.0000381411 | 0.0000262685 |
| 16 | 0.0001285783 | 0.0000278705 | 0.0000246520 |
| 17 | 0.0000553810 | 0.0000193140 | 0.0000228505 |
| 18 | 0.0000314085 | 0.0000040484 | 0.0000215508 |
| 19 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 |

$$\Delta\gamma = \sin^{-1}\left(\frac{P_{T_1\_x}^U(\gamma) - P_{T_2\_x}^U(\gamma)}{\Delta h}\right)$$

$$\Delta\beta = \tan^{-1}\frac{P_{T_1\_y}^U(\beta)}{P_{T_1\_z}^U(\beta)}$$

$$\Delta\alpha = \tan^{-1}\left(-\frac{P_{T_1\_y}^U(\alpha)}{P_{T_1\_x}^U(\alpha)}\right)$$

**FIGURE 21.** The order of Installation error angle increment calibration sequence diagram.
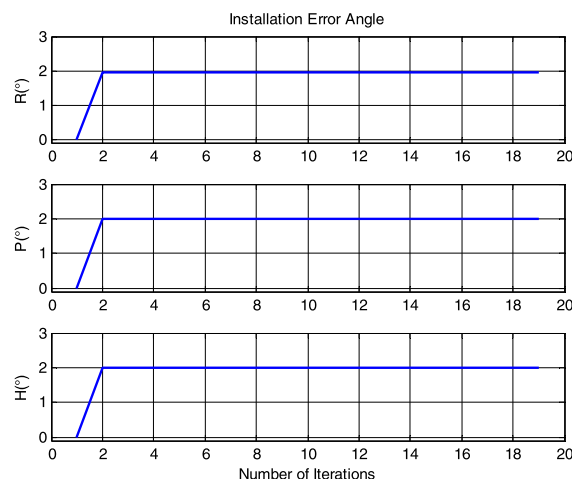


**FIGURE 22.** The Schematic diagram of installation error Angle estimation.

$\varepsilon_{\beta_t h} = 10^{-6}$, $\varepsilon_{\gamma_t h} = 10^{-5}$, The estimation of installation error Angle incremental is shown in Fig.22, The installation error angles can be accurately estimated after more than ten iterations.

It can be seen from table 5 that the estimation of the installation error angle of more than 99.9% can be completed in the first three incremental iterations of the installation error Angle. It can be seen that the dynamic calibration algorithm based on incremental iteration in this paper has the characteristics of rapid calibration. The final estimated installation error angles are 1.993848, 1.983672 and 2.009211 in heading, pitching and rolling respectively.

The installation error angle calculated by a new calibration algorithm proposed in this paper is simulated and verified. The specific parameters are: the design of four receiving elements which receiving and receiving combined sonar array structure is adopted, the diagonal element spacing $d = 0.3$m, the double transponder spacing is $\Delta h = 1$m, the depth of transponder1 $h_{transponder\,1} = 10$m, the depth of transponder2 $h_{transponder\,2} = 11$m, the installation error angle of heading, pitching and rolling directions are set to $2°$. The mothership carrying RTK receiving equipment is set to sail around the underwater transponder. The RTK horizontal positioning accuracy is $\pm(10 + 1\times10^{-6}\times D)$ mm, and the elevation accuracy is $\pm(20+1\times10\text{-}6\times D)$mm. TDOA method is used to measure the distance between the emitter of USBL transceiver and underwater transponder, and the antenna

position of the RTK at the time of TDOA measurement is recorded. The position of the transmitter in USBL transceiver is obtained by position conversion. The position of the underwater transponder calculated by LBL method are (0.02, 0.08, 10.03), as shown in Fig.23. This value is used as the reference true value of transponder.
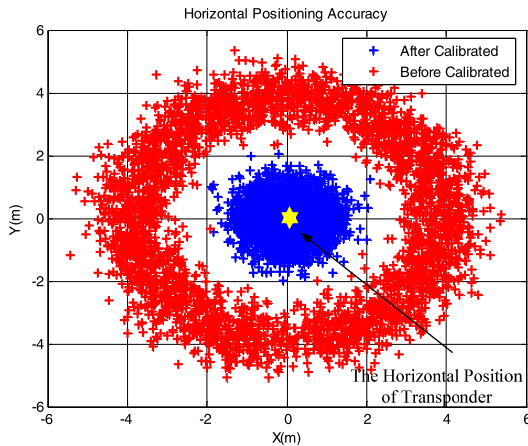


**FIGURE 23.** Comparison of horizontal positioning accuracy of transponder1 located by USBL system before and after the installation error angle correction.

The mother-ship carrying USBL transceiver navigates along any track in the range of underwater acoustic signals. During the voyage, responder 1 is positioned 3500 times by USBL. The horizontal position of transponder1 in the auxiliary coordinate system is shown as fig.23 (red dot), It can be seen from the point cloud distribution that there is a large horizontal positioning error before compensation. The horizontal position of transponder1 is calculated by compensating the installation error angle (1.993848, 1.983672, 2.009211), as shown in fig.24 (blue dot). Fig.24 is a schematic diagram of Z positioning accuracy before and after installation error compensation. There is no strict requirement for the ship's
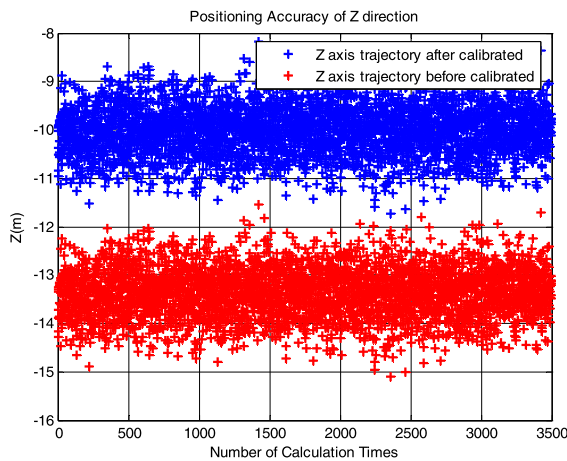


**FIGURE 24.** Comparison of Z-direction positioning accuracy of transponder1 located by USBL system before and after the installation error angle correction.

trajectory during the calibration process. From Fig. 23, it can be seen that the horizontal positioning accuracy of USBL has been distinctly improved, and the position error is reduced from 8 m to 2 m after the error angle correction of USBL installation.

## VI. CONCLUSION

In this paper, the influence of USBL installation error angle on the positioning accuracy of USBL positioning system is analyzed by using the traditional method of water surface carrier surround sailing. The simulation experimental results show that the error angle of course installation will cause a large USBL positioning error. Among them, the horizontal positioning error is mainly affected by the heading and rolling installation error. 2° heading installation error could result in 35 m position error, under the horizontal distance between transceiver and transponder of 1000 m. With the same installation error angle, the horizontal positioning accuracy of the USBL is also related to the distance between transceiver and transponder, the greater the distance is, the bigger the positioning error angle will be. In this paper, a dynamic calibration algorithm of installation error Angle based on incremental iteration is proposed. In traditional installation error angle estimation method, there are special requirements on trajectory of the carrier-ship, the algorithm only requires the USBL transceiver to work within the effective working range, and there is no strict requirement on the trajectory of the carrier-ship. The method does not need to measure the depth of the transponder and the horizontal distance between the transceiver and the transponder, which effectively avoids the effect of measurement error on the installation error angle calibration. This paper presents a method of using double transponder to estimate the rolling installation error angle, which improves the estimation accuracy of the error angle greatly. The experimental results show that the installation error angle increment module in the new method can accurately estimate the installation error angle only by iterating more than ten times. Without considering other error terms, the horizontal positioning error caused by the installation error angle can be reduced from 8 m to 2 m within 1000 m positioning range. This method can greatly improve the positioning accuracy of USBL position system, which has certain engineering practical value.

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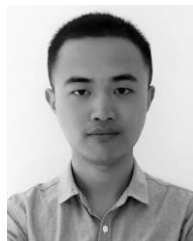
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