IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received October 11, 2018, accepted October 24, 2018, date of publication October 30, 2018, date of current version November 30, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2878784

Underground Mining Method Selection With the Hesitant Fuzzy Linguistic Gained and Lost Dominance Score Method

ZIGUO FU[1], XINGLI WU[1], HUCHANG LIAO[1,2], (Senior Member, IEEE), AND FRANCISCO HERRERA[2,3], (Senior Member, IEEE)

[1]Business School, Sichuan University, Chengdu 610064, China
[2]Andalusian Research Institute on Data Science and Computational Intelligence, University of Granada, 18071 Granada, Spain
[3]Faculty of Computing and Information Technology, King Abdulaziz University, Jeddah 21589, Saudi Arabia

Corresponding author: Huchang Liao (liaohuchang@163.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 71501135 and Grant 71771156, in part by the 2019 Sichuan Planning Project of Social Science under Grant SC18A007, in part by the 2018 Key Project of the Key Research Institute of Humanities and Social Sciences in Sichuan Province under Grant Xq18A01 and Grant LYC18-02, in part by the Electronic Commerce and Modern Logistics Research Center Program, Key Research Base of Humanities and Social Science, Sichuan Provincial Education Department, under Grant DSWL18-2, in part by the Spark Project of Innovation, Sichuan University, under Grant 2018hhs-43, and in part by the Scientific Research Foundation for Excellent Young Scholars, Sichuan University, under Grant 2016SCU04A23.

ABSTRACT Underground mining method selection is a critical decision problem for available underground ore deposits in exploitation design. As many comprehensive factors, such as physical parameters, economic benefits, and environmental effects, are claimed to be established and a group of experts are involved in the issue, the underground mining method selection is deemed as a multiple experts multiple criteria decision making problem. Classical mining method assessment exists some gaps due to the way of representing opinions. To address this matter, a hesitant fuzzy linguistic gained and lost dominance score method is investigated in this paper. To enhance the flexibility and gain more information, mining planning engineers are allowed to convey their knowledge using hesitant fuzzy linguistic term sets in the underground mining method selection process. A novel score function of hesitant fuzzy linguistic term set is introduced to compare any hesitant fuzzy linguistic term sets. Then, based on the score function, a weight determining function is proposed to calculate the weights of criteria, which can magnify the "importance" and "unimportance" of criteria. To select the mining method, the hesitant fuzzy linguistic gained and dominance score method is developed. A case study concerning selecting a extraction method for a real mine in Yunnan province of China is presented to illustrate the applicability of the proposed method. The effectiveness of the proposed method is finally verified by comparing with other ranking methods.

INDEX TERMS Underground mining method selection, multiple criteria decision making, hesitant fuzzy linguistic term set, score function, gained and lost dominance score method, weight determination.

I. INTRODUCTION

Mining is an effective way to obtain potential underground mineral resources from the Earth. It provides basis materials for industry and thus directly affects the economic development of a country. Underground mining method selection, which determines how to use an available ore deposit, is a critical decision-making problem involved in mining design. In the past, mining planning engineers focused on technical and economic indicators but ignored environment criteria. The environment condition around the mining area has been damaged due to unreasonable mining and exploitation design [1]. Therefore, the undergound mining method should consider not only how to add economic values to enterprises, but also how to reduce the breakage to surrounding environment and thus achieve sustainable development. This makes it much harder for engineers to select the underground mining method.

Underground mining method selection generally involves two stages: preliminary selection and techno-economic index analysis. As for the first stage, several feasible methods are available toward the deposit according to the properties of the ore body [2]. At the second stage, many qualitative conflicting indicators (criteria), such as production, safety and environment, are discussed deeply by mining professionals based on their experience and intuition. Obviously, the undergound mining method selection process is a typical multiple experts multiple criteria decision making (MEMCDM) problem which involves uncertainty and ambiguity about the judgments. Generally, the MEMCDM involves three phases: (i) collecting evaluations, (ii) aggregating information, and (iii) ranking alternatives. Phase (i) is to collect information on alternatives evaluated by a group of experts over multiple criteria; phase (ii) is to aggregate the individual opinions into collective ones by some aggregation methods; phase (iii) is to select the best alternative by utilizing ranking methods. Nowadays, the MEMCDM has become a hot topic [3]–[5].

The underground mining method selection has attracted many scholars' attention. Regarding the information gathering (phase (i)), tools such as fuzzy set theory [6] and interval-valued fuzzy set theory [7] have been used to represent the uncertain information of experts over different undergound mining methods. For phase (ii), some weight determining methods [8] have been implemented to deduce the weight of criteria for aggregation. As for the ranking phase, some MEMCDM methods, such as fuzzy analytic hierarchy process (FAHP) [9], TOPISIS (Technique for Order Preference by Similarity to an Ideal Solution) [9], [10] and PROMETHEE (Preference Ranking Organization Methods for Enrichment Evaluations) [11], have been implemented to handle this issue. The application of the Nicholas technique [12] successfully provided a new way to select underground mining alternatives. In addition, researchers have combined two or more methods to handle the undergound mining method selection problem. For example, Yazdani-Chamzini *et al.* [8] presented the FAHP-TOPSIS method to select the mining alternatives in Angouran mine, in which the FAHP was used to determine the weights of criteria while the TOPSIS was used to rank alternatives.

There are some challenges for selecting underground mining methods:

(1) Most existing methods were conducted based on precise data, which cannot represent cognitive complex qualitative information. Even though few of them, such as [2] and [14] used fuzzy linguistic approaches to express qualitative information straightforwardly, these fuzzy linguistic approaches can only use singleton linguistic terms to represent experts' preferences but cannot represent hesitant and cognitive complex knowledge or expressions. This may lead to information loss. For example, when assessing the managerial complexity of a underground mining method, a mining professional may say "it is slightly high" and anther mining engineer may say "it is between medium and high," if they are uncertain and hesitant about their judgments.

(2) Most existing methods assumed that the weights of criteria are given in advance, which makes the decision-making process poor convincing.

(3) The existing ranking methods have some challenges which will be justified in-depth later.

To overcome these research gaps, in this paper, the hesitant fuzzy linguistic term sets (HFLTSs) [14] are used to represent the qualitative opinions in the underground mining method selection process. The HFLTS [14] was proposed based on the hesitant fuzzy set (HFS) [15] which uses a set of possible values to denote the membership degree of an element to a set. The HFS is a useful tool to manage the uncertainty information flexibly in that case that the decision maker is uncertain and hesitant about information due to the lack of knowledge. The HFS and its extensions have been widely used by many researchers from different fields [16]–[19]. However, the HFS and its extensions can only be used to represent quantitative information. In practical decision-making problems, people tend to use their natural language to express their preferences rather than numerical values. For this concern, the HFLTS is constructed by a set of possible linguistic terms. The HFLTS improves the flexibility of modeling uncertain and hesitant linguistic information since it allows to use more than one linguistic term to address opinions of experts [20]. The HFLTS has been successfully applied in many MEMCDM problems such as selecting fire resource plans [21], evaluating regional water resources [22], selecting EPR systems [23] and selecting the best shared-bike design [24]. It is worth mentioning that the linguistic approach has extended to several different forms after a few years research,, such as multi-granular hesitant linguistic approach [25], [26], interval-valued hesitant fuzzy linguistic approach [24], [27] and probability linguistic approach [28], [29], [30]. To the best of our knowledge, the adoption of HFLTSs in underground mining method selection is rare. In addtion, for phase (ii), we propose an objective method to determine the weighs of criteria to ensure the rationality of selecting underground mining methods. We define the priority degrees of criteria and establish a weight determining function to calculate the final weight vector, which can magnify the "importance" and "unimportance" of indicators.

In addition, in the case that the evaluation information is represented by HFLTSs, scholars proposed a number of ranking techniques, such as the HFL-TOPSIS method [31], the HFL-VIKOR (VlseKriterijumska Optimizacija Kompromisno Resenje) method [23], the HFL-TODIM (an acronym in Portuguese of interactive and multi-criteria decision-making) method [32] and the HFL-ELECTRE (Elimination Et Choix Traduisant la REalité-Elimination and Choice Expressing the Reality) method [33], to handle MEMCDM problems. However, these methods are all based on distance measures of HFLTSs and need to add new elements to the shorter HFLTSs when computing with HFLTSs by that measure. Adding artificial values is rude and definitely would change the original evaluation information. In this paper, we shall propose a new score function to compute with hesitant fuzzy linguistic evaluations. Such a new score function is based on the hesitancy degree of HFLTS and the mean and variance of its corresponding semantics, and we do not need to guarantee the same length of two HFLTSs.

What's more, the aforementioned ranking methods exist shortcomings. For example, the HFL-TOPSIS method ignored the weights of criteria in aggregating process; the subordinate orders of the "group utility" values and the "individual regret" values were not considered in the HFL-VIKOR method; the normalization process was not conducted in the HFL-TODIM method; too many parameters need to be established in the HFL-ELECTRE method. To tackle these drawbacks, in this paper, we develop a hesitant fuzzy linguistic gained and lost dominance score (HFL-GLDS) method and use it to select the underground mining methods.

In summary, this paper dedicates to achieving the following innovative contributions:

(1) The HFLTSs, as a powerful linguistic representation tool, is applied to express evaluations in the underground mining method selection process. It not only increases the flexibility of expressing knowledge but also preserves original information of mining professionals. Considering that the hesitancy degree can reflect uncertainty and fuzziness, and the mean and standard deviation of corresponding semantics depict the numerical characteristics of HFLTSs, two novel score functions of HFLTSs are proposed, respectively, to rank HFLTSs without adding elements.

(2) After defining the priority degrees of criteria by a programming model, a new weight determining method is established, which can magnify the "importance" and "unimportance" criteria. The effectiveness is confirmed by comparing the proposed method with other weight determination methods.

(3) By using the proposed score function to compute the dominance score between alternatives, a novel outranking method, named HFL-GLDS, is introduced for MEMCDM problems under the hesitant fuzzy linguistic context.

(4) A case study about selecting the mining methods for a real mine in China is analyzed by the proposed HFL-GLDS method. The effectiveness of the proposed method is highlighted by some comparative analyses.

The structure of the paper is demonstrated as follows: Section II recalls some concepts related to the HFLTS and the framework of the classical GLDS method. Section III proposes a novel score function of the HFLTS. The priority degrees of criteria are defined in Section IV and then a weight determining function is proposed. Section V develops a HFL-GLDS method to deal with MEMCDM problems under the hesitant fuzzy linguistic environment. In Section VI, we employ the HFL-GLDS technique to cope with a practical mining method selection problem of a mine in China, and the advantages in terms of the applicability and effectiveness of the proposed method are shown by comparing with other ranking methods. Section VII concludes the paper with some open discussions.

II. PRELIMINARIES

To facilitate further presentation, we recall some knowledge about the HFLTS and the classical GLDS method.

A. HESITANT FUZZY LINGUISTIC TERM SET

Rodríguez *et al.* [14] proposed the concept of the HFLTS considering more than one linguistic term to express hesitant linguistic opinions. A mathematical formalization of HFLTS is presented by Liao *et al.* [34] as follows:

Definition 1 [34]: Let $X = \{x_1, x_2, \ldots, x_N\}$ be a universe of discourse and $S = \{s_\alpha | \alpha = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ be a linguistic term set (LTS). A HFLTS on X is mathematically defined as

$$H_S = \{< x_i, h_S(x_i) > | x_i \in X\} \tag{1}$$

where $h_S(x_i) = \{s_{\varphi_l} | s_{\varphi_l} \in S, l = 1, 2, \ldots, L\}$ is a set of some values in S, denoting the possible degree of the linguistic variable x_i to the HFLTS H_S. L is the number of linguistic terms in $h_S(x_i)$. $h_S(x_i)$ is called the hesitant fuzzy linguistic element (HFLE).

Note 1: The HFLE is a continuous subset of the LTS S.

Definition 2 [14]: The interval type of a HFLE is defined as the envelope of the HFLE, which is constructed as:

$$\widetilde{h}_S = [s^-, s^+] \tag{2}$$

where s^-, s^+ are the lower and upper bounds of h_S, respectively.

Definition 3 [35]: Let $S = \{s_{-\tau}, \ldots, s_0, \ldots, s_\tau\}$ be a LTS. The negation of a linguistic term $s_i \in S$ is defined as:

$$Neg(s_i) = s_{-i} \tag{3}$$

It should be noted that the number of linguistic terms in different HFLEs may be different due to the flexibility of linguistic expressions. To facilitate the comparison of two HFLEs, Liao *et al.* [34] introduced a method to add the element s^* to the shorter HFLE till they have the same length.

To compare different HFLEs, Liao *et al.* [23] defined the score of the HFLE $h_S = \{s_{\varphi_l} | s_{\varphi_l} \in S; l = 1, \ldots, L\}$ as:

$$\rho(h_S) = s_{\overline{\alpha}},\ \overline{\alpha} = \frac{1}{L}(\sum_{l=1}^{L} \varphi_l) \tag{4}$$

B. THE CLASSICAL GLDS METHOD

The GLDS method was initially proposed by Wu and Liao [28] to handle comprehensive MEMCDM problems with probabilistic linguistic evaluations. It is a new outranking method based on both the gained and lost dominance relations between alternatives.

Consider a general decision matrix $D = (x_{ij})_{m \times n}$, where x_{ij} is the performance of alternative a_i on criterion c_j. The weights ω_j $(j = 1, 2, \ldots, n)$ of criteria are given by experts. Generally, the GLDS method includes the following steps:

Step 1: The dominance flow of alternative a_i over alternative a_v is measured in terms of their difference:

$$df_j(a_i, a_v) = \begin{cases} max\{\sigma(x_{ij})-\sigma(x_{vj}), 0\}, & \text{for benefit criterion } c_j \\ max\{\sigma(x_{vj})-\sigma(x_{ij}), 0\}, & \text{for cost criterion } c_j \end{cases} \tag{5}$$

where $\sigma(x)$ is a measure which translates the evaluation value of x into a crisp value. Then, we normalize the dominance flow by vector normalization (Eq. (6)) and obtain the normalized dominance flow $df_j^N(a_i, a_v)$.

$$df_j^N(a_i, a_v) = \frac{df_j(a_i, a_v)}{\sqrt{\sum_{v=1}^{m} \sum_{i=1}^{m} (df_j(a_i, a_v))^2}} \tag{6}$$

Step 2: Compute the overall gained dominance score of a_i under c_j by Eq. (7), and obtain a subordinate order set R_1 in descending order of the *OGDSs*.

$$OGDS(a_i) = \sum_{j=1}^{n} [\omega_j \sum_{v=1}^{m} df_j^N(a_i, a_v)] \tag{7}$$

Step 3: Compute the overall lost dominance score of a_i under c_j by Eq. (8), and obtain a subordinate order set R_2 in ascending order of the *OLDSs*.

$$OLDS(a_i) = \max_j [\omega_j \max_v df_j^N(a_i, a_v)] \tag{8}$$

Step 4: Normalize the *OGDSs* and *OLDSs* by the vector normalization formula that is similar to Eq. (6), and obtain $OGDS^N$s and $OLDS^N$s.

Step 5: The final ranks of alternatives are determined by an aggregation operator considering two subordinate order sets R_1 and R_2, and two score sets $OGDS^N$ and $OLDS^N$, simutaneously, shown as follows:

$$CS_i = OGDS^N(a_i) \cdot \frac{m - R_1(a_i) + 1}{m(m+1)/2} - OLDS^N(a_i) \cdot \frac{R_2(a_i)}{m(m+1)/2} \tag{9}$$

As we can see, the GLDS method has high robustness and effectiveness. On the one hand, the final aggregation operator considers not only the scores but also the subordinate orders; on the other hand, the "group utility" values calculated by Eq. (7) and the "individual regret" values calculated by Eq. (8) are taken into account, simultaneously. In addition, the GLDS method is flexible, and can be extended to different contexts, including both quantitative environment and qualitative circumstance. At last, the GLDS method uses the vector normalization two times, which can accelerate the speed of finding the solution and improve the accuracy of final result.

III. A NOVEL SCORE FUNCTION OF HFLES

In this section, we propose a novel score function to rank HFLEs. To do so, we first introduce the scale function which can transform linguistic variables into crisp values. Based on the concept of hesitancy degree of HFLE and the redefined mean and variance of HFLE, a new ranking technique for HFLEs is given.

The linguistic scale function provides us a powerful tool to convert linguistic terms to quantitative values, which is convenient to reflect the semantics of linguistic terms.

Definition 4 [27]: Let $S = \{s_\alpha | \alpha = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ be a LTS, $h_S = \{s_t | t \in [-\tau, \tau]\}$ be a HFLE, and $\xi(\xi \in [0, 1])$ be the semantic of a linguistic term s_t. The linguistic scale function to translate s_t into a crisp value ξ is defined as $g : s_t \rightarrow \xi$, where g is a strictly monotonically increasing function which maps the linguistic term s_t into a crisp value ξ in the interval [0,1].

For different types of LTSs, balanced and unbalanced, there are three kinds of linguistic scale functions [36], [37]. In this paper, to simplify the presentation, we address the balance situation where the semantics are uniformly distributed. In this case, the linguistic scale function can be defined as:

$$g(s_t) = \frac{t + \tau}{2\tau} \tag{10}$$

We can compute different HFLEs by using linguistic scale function rather than calculate their subscripts (which may lead to information loss). As we can see from Fig. 1, the linguistic scale function $g(s_t)$ maps discrete linguistic terms with both integer and non-integer subscripts to the vlaues in [0,1], which represent their semantics with precise number. In other words, the linguistic scale function can convert qualitative information to continuous numerical values within the interval [0,1].

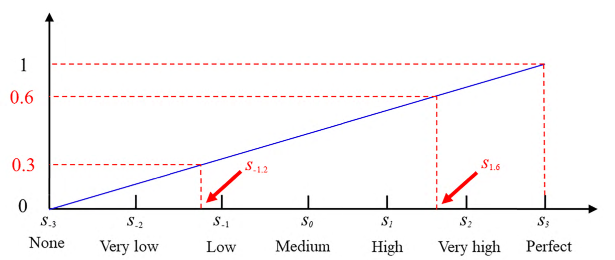

FIGURE 1. The mapping between linguistic terms and their corresponding semantics.

Definition 5: Let $h_S = \{s_{\varphi_l} | s_{\varphi_l} \in S, l = 1, 2, \ldots, L\}$ be a HFLE defined above. The mean of h_S is defined as:

$$\mu(h_S) = \frac{1}{L} \sum_{l=1}^{L} g(\varphi_l) \tag{11}$$

where g is a linguistic scale function. In this paper, we take Eq. (10) as an illustration for our further presentation.

Definition 6: The standard deviation of h_S is defined as:

$$\upsilon(h_S) = \frac{1}{L} \sqrt{\sum_{l=1}^{L} [g(\varphi_l) - \mu(h_S)]^2} \tag{12}$$

The meanings of $\mu(h_S)$ and $\upsilon(h_S)$ are similar to those of the mean and standard deviation in statistics. $\mu(h_S)$ reflects the overall scoring level of the linguistic terms in h_S; while $\upsilon(h_S)$

depicts the fluctuation value of scoring level of the linguistic terms in h_S.

To rank HFLEs, scholars have proposed comparison methods, such as the envelope-based method [14], the subscript-based score and variance method [23]. These two methods are based on the subscripts of linguistic terms and have some limitations [32]. In this regard, Wei *et al.* [38] proposed a probability theory-based method, which considers the possible degree of HFLE. But, this method also need to add new elements to shorter HFLEs and this may lead to the loss of original information. In addition, Wei *et al.* [32] presented a score function which considers both the mean and variance of HFLE. But this method is based on the calculation of subscripts and it may lead to inconsistent results against experts' cognition. Also, this method ignores the uncertainty of HFLE. Recently, Liao *et al.* [37] proposed a score function of HFLE by combining hesitancy degree and linguistic scale function, but such a score function ignores the difference degree of linguistic terms in the HFLE (Example 1 illustrates this point in detail). In view of the above analyses, we intend to propose a novel score function to rank HFLEs in the way of reflecting original information, considering the mean and standard deviation of semantics and integrating the hesitancy degree of the HFLE.

The hesitancy degree $\pi(h_S)$ of a HFLE is depicted by a monotonically increasing concave function with respect to the length of the HFLE, shown as [37]:

$$\pi(h_S) = \frac{LlnL}{(2\tau+1)ln(2\tau+1)} \tag{13}$$

where $2\tau+1$ is the length of S.

It can be seen from Eq. (13) that the more elements in a HFLE, the greater the hesitancy degree of the HFLE will be.

Definition 7: Let $S = \{s_\alpha|\alpha = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ be a LTS. The score of a HFLE $h_S = \{s_{\varphi_l}|\varphi_l \in [-\tau, \tau], l = 1, 2, \ldots, L\}$ can be defined as:

$$G(h_S) = (\mu(h_S) - \upsilon(h_S)) \times (1 - \pi(h_S)) \tag{14}$$

where $\mu(h_S)$, $\upsilon(h_S)$ and $\pi(h_S)$ are the mean, standard deviation and hesitancy degree of h_S, which can be computed by Eqs. (11), (12) and (13), respectively.

Proposition 1: Consider two HFLEs $h_S^1 = \{s_{\varphi_{l_1}}|\varphi_l \in [-\tau, \tau], l_1 = 1, 2, \ldots, L_1\}$ and $h_S^2 = \{s_{\varphi_{l_2}}|\varphi_l \in [-\tau, \tau], l_1 = 1, 2, \ldots, L_2\}$. The following properties hold:

1) $G(h_S) \geq 0$;
2) If $L_1 = L_2$, $\mu(h_S^1) > \mu(h_S^2)$ and $\upsilon(h_S^1) < \upsilon(h_S^2)$ or $\upsilon(h_S^1) = \upsilon(h_S^2)$, then $G(h_S^1) > G(h_S^1)$;
3) If $L_1 > L_2$, $\mu(h_S^1) < \mu(h_S^2)$ and $\upsilon(h_S^1) > \upsilon(h_S^2)$ or $\upsilon(h_S^1) = \upsilon(h_S^2)$, then $G(h_S^1) < G(h_S^1)$;
4) If and only if $L_1 = L_2$, $\mu(h_S^1) = \mu(h_S^2)$ and $\upsilon(h_S^1) = \upsilon(h_S^2)$, then $G(h_S^1) = G(h_S^1)$;
5) If $L = 2\tau + 1$, then $G(h_S) = 0$.

Proof: 1) Since $0 \leq \pi(h_S) \leq 1$, then $1-\pi(h_S) \geq 0$. To justify $G(h_S) \geq 0$, we only need to prove $\mu(h_S) - \upsilon(h_S) \geq 0$. Since $g(\varphi_l) \geq 0$, for $l = 1, 2, \ldots, L$, then $\mu(h_S) \geq 0$, and thus, $(g(\varphi_1) + g(\varphi_2) + \cdots + g(\varphi_L))^2 \geq g(\varphi_1)^2 + g(\varphi_2)^2 + \cdots + g(\varphi_L)^2 \geq (g(\varphi_1) - \mu(h_S))^2 + (g(\varphi_2) - \mu(h_S))^2 + \cdots + (g(\varphi_L) - \mu(h_S))^2$, i.e., $(\sum_{l=1}^{L} g(\varphi_l))^2 \geq \sum_{l=1}^{L}(g(\varphi_l))^2 \geq \sum_{l=1}^{L}(g(\varphi_l) - \mu(h_S))^2$. Then, it follows $(\frac{1}{L}\sum_{l=1}^{L} g(\varphi_l))^2 \geq \frac{1}{L^2}\sum_{l=1}^{L}(g(\varphi_l))^2 \geq \frac{1}{L^2}\sum_{l=1}^{L}(g(\varphi_l) - \mu(h_S))^2$. We can obtain $(\mu(h_S))^2 \geq (\upsilon(h_S))^2$ and $\mu(h_S), \upsilon(h_S) \geq 0$. Then, we obtain $\mu(h_S) \geq \upsilon(h_S)$. That is, $\mu(h_S) - \upsilon(h_S) \geq 0$. Hence, we have $G(h_S) \geq 0$.

2) If $L_1 = L_2$, by Eq. (13), we obtain $\pi(h_S^1) = \pi(h_S^1)$. In this case, if $\mu(h_S^1) > \mu(h_S^2)$, $\upsilon(h_S^1) < \upsilon(h_S^2)$ or $\upsilon(h_S^1) = \upsilon(h_S^2)$, then, we have $\mu(h_S^1) - \varphi(h_S^1) > \mu(h_S^2) - \varphi(h_S^2)$. Therefore, $G(h_S^1) > G(h_S^1)$;

3) It is similar to the proof of 2)

4) It is similar to the proof of 2)

5) If $L = 2\tau + 1$, by Eq. (13), we obtain $\pi(h_S) = 1$. Therefore, $G(h_S) = 0$ by Eq. (14).

In statistics, $\mu \pm \upsilon$ (where μ and υ denote the mean and standard deviation of a sample, respectively) is used to characterize the distribution of the mean value [39]. For a normal population, the statistical significance of $\mu \pm \upsilon$ is that the sample statistics fall into the interval $[\mu - \upsilon, \mu + \upsilon]$ with a large probability, while a small portion of data are outside the interval $[\mu - \upsilon, \mu + \upsilon]$. In practical decision-making problems, the evaluation information on a certain attribute basically obeys the Gauss distribution [24]. We devote to using $\mu(h_S) - \upsilon(h_S)$, which denotes the lowest level of the mean with a lager probability, to represent the final score of the HFLE h_S. Moreover, due to the uncertainty and ambiguity of the evaluation expressed in HFLE, we integrate the hesitancy degree of h_S into the score function to rank HFLEs reasonably.

With Eq. (14), we can rank any HFLEs:

1) If $G(h_S^1) > G(h_S^2)$, then $h_S^1 \succ h_S^2$;
2) If $G(h_S^1) = G(h_S^2)$, then $h_S^1 = h_S^2$.

Definition 8: The distance measure between two HFLEs h_S^1 and h_S^2 can be defined as:

$$d(h_S^1, h_S^2) = \frac{max[G(h_S^1), G(h_S^2)] - min[G(h_S^1), G(h_S^2)]}{max[G(h_S^1), G(h_S^2)]} \tag{15}$$

Definition 9: Let $X = \{x_1, x_2, \ldots, x_N\}$ be a reference set and $S = \{s_\alpha|\alpha = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ be a LTS. Suppose that $H_S = \{< x_i, h_S(x_i) > |x_i \in X\}$ is a HFLTS on X. The mean and the standard deviation of H_S are respectively defined as

$$U(H_S) = \frac{1}{N}\sum_{i=1}^{N} G(h_S(x_i)) \tag{16}$$

$$V(H_S) = \frac{1}{N}\sqrt{\sum_{i=1}^{N}[G(h_S(x_i)) - U(H_S)]^2} \tag{17}$$

Definition 10: The score for a HFLTS can be defined as:

$$\Phi(H_S) = U(H_S) - V(H_S) \tag{18}$$

Sometimes, for the same discourse set $X = \{x_i|i = 1, 2, \ldots, N\}$, the elements $h_S(x_i)$ $(i = 1, 2, \ldots, N)$ in H_S may be assigned different weights. Therefore, the score function should be redefined.

Definition 11: Let $X = \{x_1, x_2, \ldots, x_N\}$ be a reference set and $S = \{s_\alpha | \alpha = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ be a LTS. Suppose that $H_S = \{< x_i, h_S(x_i) > | x_i \in X\}$ is a HFLTS on X. The weighted score function for H_S is defined as

$$\widehat{\Phi}(H_S) = \sum_{i=1}^{N} G(h_S(x_i))\omega(h_S(x_i)) - \sqrt{\sum_{i=1}^{N} \omega(h_S(x_i))[G(h_S(x_i)) - \sum_{i=1}^{N} G(h_S(x_i))\omega(h_S(x_i))]^2} \tag{19}$$

where $\omega(h_S(x_i))$ is the weight of $h_S(x_i)$.

Definition 12: Suppose that H_S^1 and H_S^2 are two HFLTSs on X. The distance between H_S^1 and H_S^2 is defined as:

$$d(H_S^1, H_S^2) = \frac{max[\Phi(H_S^1), \Phi(H_S^2)] - min[\Phi(H_S^1), \Phi(H_S^2)]}{max[\Phi(H_S^1), \Phi(H_S^2)]} \tag{20}$$

In this regard, H_S^1 and H_S^2 can be ranked completely in the same way as that of the HFLEs.

Example 1: Let $H_S^i (i = 1, 2)$ be two HFLTSs for two alternatives under three criteria and $S = \{s_{-3}, \ldots, s_0, \ldots, s_3\}$ be a LTS. Suppose that $H_S^1 = \{h_S^{11} = \{s_{-2}\}, h_S^{12} = \{s_0, s_1, s_2\}, h_S^{13} = \{s_2, s_3\}\}$ and $H_S^2 = \{h_S^{21} = \{s_0, s_1, s_2\}, h_S^{22} = \{s_{-3}, s_{-2}, s_{-1}, s_0, s_1\}, h_S^{23} = \{s_2\}\}$ are given by experts. By Eqs. (10), (13) and (14), we obtain $G(h_S^{11}) = 0.167$, $G(h_S^{12}) = 0.447$, $G(h_S^{13}) = 0.772$, $G(h_S^{21}) = 0.447$, $G(h_S^{22}) = 0.103$ and $G(h_S^{23}) = 0.833$. Then, if using Eq. (16), we can obtain $U(H_S^1) = 0.462$ and $U(H_S^2) = 0.461$. They are very similar and thus we can hardly distinguish these two HFLTSs. By Eqs. (17) and (18), we obtain $\Phi(H_S^1) = 0.401$ and $\Phi(H_S^2) = 0.372$. Hence, we have $H_S^1 \succ H_S^2$. Suppose the weights of the criteria are 0.5, 0.2, 0.3, respectively. Then, by Eq. (19), we can obtain $\widehat{\Phi}(H_S^1) = 0.263$ and $\widehat{\Phi}(H_S^2) = 0.257$. Thus, $H_S^1 \succ H_S^2$. Therefore, the first alternative is a better choice.

In Example 1, the number of elements in the two HFLTSs are the same and they should have the same hesitancy degree by Eq. (13). According to the computation results, we obtain $U(H_S^1) \approx U(H_S^2)$, which denotes that the mean values of semantics of the two HFLTSs are equal in general. Then, according to the score function proposed in [37], the score values of there two HFLTSs are the same. We do not know which alternative is the better one. But in fact, there is a big difference between these two HFLTSs. The reason is stated that the score function proposed in [37] only considers the average level of a HFLTS, but ignores the distribution of the elements in the HFLTS. That is to say, it does not conseder the standard deviation of semantics. However, the score function proposed in this paper, revised on the basis of [37], can distinguish them very well.

IV. A SCORE FUNCTION-BASED WEIGHT-DETERMINING METHOD

Weight determination of criteria is a critical process for solving MEMCDM problems since different weight vectors may lead to different ranking results [40]. There are many methods to calculate criteria weights, which are mainly grouped into three categories: subjective, objective and combinative weight-determining methods. Practically, due to the complexity of the problem and the lack of knowledge, decision-making based on subjective weights may bring huge randomness. Therefore, investigating the internal relations of date is a proper way to compute the weights of criteria, which ensures the fairness of decision-making results. There are many approaches to compute the weights of criteria in objective way. For example, Farhadinia [41] presented the entropy measure of HFLTS based on a series of distance measures. Considering the interactive effect of evaluations, Gou *et al.* [42] established an objective programming model to compute the criteria weights based on entropy measure and cross-entropy measure. Li *et al.* [22] proposed a minimized divergence model based on hesitancy degrees to calculate the weights of criteria. In this section, we propose a novel objective weight-determining method based on the proposed score function of HFLE. We firstly define the priority degree of criterion, and then a weight function is presented. The effectiveness of the proposed method is verified by a numerical example.

A. WEIGHT-DETERMINING METHOD DESCRIPTION

A MEMCDM problem aims to rank a set of alternatives $A = \{a_1, a_2, \ldots, a_m\}$ under given criteria $C = \{c_1, c_2, \ldots, c_n\}$ by a group of experts $e_q = \{e_1, e_2, \ldots, e_Q\}$. Let $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the weight vector of criteria, where $\omega_j \in [0, 1], j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} \omega_j = 1$. In the context of hesitant fuzzy linguistic environment, a LTS $S = \{s_{-\tau}, \ldots, s_0, \ldots, s_\tau\}$ is given before evaluation. The experts freely express their assessments on the basis of their knowledge and experience using linguistic expressions. By the context-free grammar and translation function [14], the linguistic evaluations are converted into HFLEs. Suppose that $h_S^{ij(q)}$ denotes the HFLE determined by expert e_q on alternative a_i with respect to criterion c_j. In this way, an individual hesitant fuzzy linguistic decision matrix $D^{(q)}$ of expert e_q can be established:

$$D^{(q)} = \begin{bmatrix} h_S^{11(q)} & \ldots & h_S^{1j(q)} & \ldots & h_S^{1n(q)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ h_S^{i1(q)} & \ldots & h_S^{ij(q)} & \ldots & h_S^{in(q)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ h_S^{m1(q)} & \ldots & h_S^{mj(q)} & \ldots & h_S^{mn(q)} \end{bmatrix}$$

It is observed that there are two types of criteria: benefit and cost. Therefore, it is necessary to unify them and yield a normalized hesitant fuzzy linguistic decision matrix

$\overline{D}^{(q)} = (\overline{h}_S^{ij(q)})_{m\times n}$ by

$$\overline{h}_S^{ij(q)} = \begin{cases} h_S^{ij(q)}, & \text{for benefit criterion} \\ Neg(h_S^{ij(q)}), & \text{for cost criterion} \end{cases} \tag{21}$$

where $Neg(h_S^{ij(q)}) = \{Neg(s_{\varphi_l}) | s_{\varphi_l} \in h_S^{ij(q)}, l = 1, \ldots, L\}$ can be computed by Eq. (3).

As we know, the greater the otherness of a criterion, the more important role the criterion plays in ranking alternatives, and thus should be assigned with a larger weight. That is to say, if the overall score of a criterion, calculated by $\sum_{i=1}^{m} G(h_S^{ij(q)})$, is larger than other criteria, then it is supposed to assign a bigger weight to that criterion.

Meanwhile, the performance of each alternative a_i should have little otherness under a criterion. We use the distance measure shown as Eq. (15) to represent the otherness between alternatives under a criterion. The average distance of alternative a_i to all the other alternatives is established as $\frac{1}{m-1}\sum_{v=1,v\neq i}^{m} d(h_S^{ij(q)}, h_S^{vj(q)})$. For any criterion c_j, the overall distance of all alternatives, shown as $\sum_{i=1}^{m}\frac{1}{m-1}\sum_{v=1,v\neq i}^{m} d(h_S^{ij(q)}, h_S^{vj(q)})$, should be as small as possible to increase persuasiveness.

Integrating the overall score of a criterion and the overall distances among alternatives into one piece, we can obtain:

$$\sum_{i=1}^{m}[\frac{1}{m-1}\sum_{v=1,v\neq i}^{m}[1 - d(h_S^{ij(q)}, h_S^{vj(q)})] + G(h_S^{ij(q)})] \tag{22}$$

The bigger the value of Eq. (22) is, the more important the criterion c_j is. Therefore, we can establish a model to calculate weights based on Eq. (22).

However, limitations appear in the situation that some criteria are hard to distinguish. For example, suppose that there are four criteria with the values of 0.34, 0.21, 0.22, and 0.23, respectively, calculated by Eq. (22). If we take these four values as the weights of criteria, the latter three criteria with very similar weights shows no differences. In the case that a few criteria are especial important, sometimes we may need to assign higher weights to them to highlight their significance. To reflect such information, we propose the concept of the priority degree of criterion, which is depicted by

$$p_j^{(q)} = \frac{\sum_{i=1}^{m}[\frac{1}{m-1}\sum_{v=1,v\neq i}^{m}[1 - d(h_S^{ij(q)}, h_S^{vj(q)})] + G(h_S^{ij(q)})]}{\sum_{j=1}^{n}(\sum_{i=1}^{m}[\frac{1}{m-1}\sum_{v=1,v\neq i}^{m}[1 - d(h_S^{ij(q)}, h_S^{vj(q)})] + G(h_S^{ij(q)})])} \tag{23}$$

where p_j is the priority degree of criterion c_j and $p_j \in [0, 1]$.

The criterion with the small priority degree should be assigned a small weight to weaken its importance to alternatives; while the criterion with the large priority degree should be given a high weight to heighten its importance to alternatives. Based on this analysis, we can define a function to describe the relationship between weight and priority degree. This function must be a strictly monotonically increasing function, and it can map the priority degree to the interval [0,1]. In this sense, the weight function is defined as:

$$\omega_j^{(q)} = \frac{1}{1 + exp(-\eta_1 p_j^{(q)} + \eta_2)} \tag{24}$$

where η_1, η_2 are the optimization parameters which can guarantee the range of the weight function be within the interval [0,1] and the value of η_1, η_2 are determined according to practical situation.

Finally, we can normalize the absolute weights by

$$\omega_j^{(q)} = \frac{\omega^{(q)}(c_j)}{\sum_{j=1}^{n}\omega^{(q)}(c_j)} \tag{25}$$

Example 2: Suppose that we obtain the priority degree vector $(0.51, 0.23, 0.14, 0.12)^T$ by Eq. (23). Let $\eta_1 = 10$, $\eta_2 = 5$. By Eq. (24), we obtain the absolute weight vector $\omega = (0.52, 0.06, 0.03, 0.02)^T$ (see Fig. 2). Then, by Eq. (25), the normalized weight vector is $\omega = (0.82, 0.11, 0.04, 0.03)^T$. Comparing the priority degrees with the normalized weights of criteria, it is easy to see that the importance of criterion with larger priority degree is magnified through the proposed weight-determining technique, while the importance of criterion with smaller priority degree is deflated. As we can see from Fig. 2, the curve of the weight function takes $p_j = 0.5$ as the demarcation point. When $p_j > 0.5$, the growth rate of ω_j decreases with the increase of p_j; When $p_j < 0.5$, the growth rate of ω_j increases with the increase of p_j.

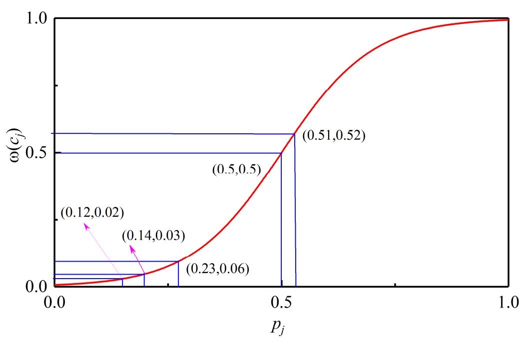

FIGURE 2. An example of the curve of the weight function.

B. AN ILLUSTRATIVE EXAMPLE

In this part, an example adopted from Liao *et al.* [22] is presented to illustrate the score function-based weight-determining method.

A company wants to give ratings on five movies $A = \{a_1, a_2, \ldots, a_5\}$ with respect to four criteria: story (c_1), act (c_2), visuals (c_3), and direction (c_4). Let $S = \{s_{-3} =$ terrible, $s_{-2} =$ very bad, $s_{-1} =$ bad, $s_0 =$ medium, $s_1 = well$, $s_2 =$ very well, $s_3 =$ perfect$\}$ be a LTS. Suppose that a group of experts are invited to assess the movies. The evaluations of the movies over the criteria are expressed by

HFLEs and thus the following matrix is obtained:

$$D = \begin{pmatrix} \{s_{-2}, s_{-1}, s_0\} & \{s_0, s_1\} & \{s_0, s_1, s_2\} & \{s_1, s_2\} \\ \{s_0, s_1, s_2\} & \{s_1, s_2\} & \{s_0, s_1\} & \{s_0, s_1, s_2\} \\ \{s_2, s_3\} & \{s_1, s_2, s_3\} & \{s_1, s_2\} & \{s_2\} \\ \{s_0, s_1, s_2\} & \{s_{-1}, s_0, s_1\} & \{s_1, s_2, s_3\} & \{s_1, s_2\} \\ \{s_{-1}, s_0\} & \{s_0, s_1, s_2\} & \{s_0, s_1, s_2\} & \{s_0, s_1\} \end{pmatrix}$$

The proposed weight-determining method is applied stepwise below:

Step 1: Calculate the score matrix. By Eq. (14), the score matrix is obtained as:

$$G = \begin{pmatrix} 0.253 & 0.524 & 0.505 & 0.674 \\ 0.505 & 0.674 & 0.524 & 0.505 \\ 0.823 & 0.632 & 0.674 & 0.833 \\ 0.505 & 0.379 & 0.632 & 0.674 \\ 0.374 & 0.505 & 0.505 & 0.524 \end{pmatrix}$$

Step 2: Compute of weights of criteria. Suppose that the information about criteria weights is completely unknown. Then by Eq. (23), we can obtain the priority degree vector $(0.251, 0.199, 0.248, 0.302)^T$. Plugging into Eqs. (24) and (25) and supposing $\eta_1 = 10, \eta_2 = 5$, the normalized weight vector is obtained as $\omega = (0.239, 0.147, 0.235, 0.379)^T$. Thus, their ranking orders are generated as: $R(c_1) = 2,\ R(c_2) = 4,\ R(c_3) = 3,\ R(c_4) = 1$.

C. COMPARISON WITH THE EXISTING WEIGHT-DETERMINING METHODS

We compare the proposed the score function-based weight-determining method with other existing approaches.

Farhadinia [41] introduced the entropy measure for HFLEs and suggested that the weights of criteria can be derived by means of information entropy of the evaluation ratings. Based on three kinds of distance measures, the entropy measure of a HFLE h_S is defined as follows [41]:

- The generalized distance-based entropy measure

$$K_{d_g}(h_S) = 1 - \frac{1}{N}\sum_{i=1}^{n}[(\frac{1}{L}\sum_{l=1}^{L}(\frac{|\varphi_l|}{2\tau})^{\varsigma})^{\frac{1}{\varsigma}}] \tag{26}$$

- The generalized Hausdorff distance-based entropy measure

$$K_{d_{gh}}(h_S) = 1 - \frac{1}{N}\sum_{i=1}^{n}[(\max_{l=1,2\ldots,L}(\frac{|\varphi_l|}{2\tau})^{\varsigma})^{\frac{1}{\varsigma}}] \tag{27}$$

- The generalized hybird Hamming distance-based entropy measure

$$K_{d_{ghh}}(h_S) = 1 - \frac{1}{N} \times \sum_{i=1}^{n}[(\frac{\frac{1}{L}\sum_{l=1}^{L}(\frac{|\varphi_l|}{2\tau})^{\varsigma} + \max_{l=1,2\ldots,L}(\frac{|\varphi_l|}{2\tau})^{\varsigma}}{2})^{\frac{1}{\varsigma}}] \tag{28}$$

where N is the number of all HFLEs in the HFLTS and $\varsigma > 0$.

For the above example, we use Eq. (26) with $\varsigma = 1$ as an example to deduce the weights of criteria.

Step 1: Construct the entropy matrix. By Eq. (26), the entropy matrix is computed as:

$$K = \begin{pmatrix} 5/6 & 11/12 & 5/6 & 3/4 \\ 5/6 & 3/4 & 11/12 & 5/6 \\ 7/12 & 2/3 & 3/4 & 2/3 \\ 5/6 & 8/9 & 2/3 & 3/4 \\ 11/12 & 5/6 & 5/6 & 11/12 \end{pmatrix}$$

Step 2: Compute of weights of criteria. Using Eq. (29), we can obtain the weight vector as $\omega = (0.248, 0.235, 0.248, 0.269)^T$ and their ranking order is achieved as $R(c_1) = 2,\ R(c_2) = 4,\ R(c_3) = 3,\ R(c_4) = 1$.

$$\omega_j = \frac{\sum_{i=1}^{m} k_{ij}}{\sum_{j=1}^{n}\sum_{i=1}^{m} k_{ij}} \tag{29}$$

Although the weights of criteria calculated by above two methods are different, the ranking orders of criteria are the same. This verifies the effectiveness of the proposed method. Compared with the entropy-based weight-determining method, our method can magnify the weight of the most important criterion and reduce the weights of the least important criteria. This feature will speed up our search for the best solution in the ranking process.

Besides, the maximizing deviation method [43] was proposed to determine the criteria weights under isolationistic fuzzy environment. But, to the best of our knowledge, no one uses this method to cope with the MEMCDM problems with completely unknown weights of criteria within the hesitant fuzzy linguistic context. To extend the maximizing deviation method to the HFLTS circumstance, the score function needs to predefine. To highlight the merits of the proposed score function (Eq. (14)) base on the mean and standard deviation of semantics and hesitancy degree, we use Eq. (4) to transform the HFLEs into crisp values before employing the maximizing deviation method. A general procedure for determining the weights of criteria by utilizing the maximizing deviation approach (we called Max. deviation-1) is as follows:

Step 1: Construct the score matrix. By Eq. (4), the score matrix is computed as:

$$S_\rho = \begin{pmatrix} -1 & 0.5 & 1 & 1.5 \\ 1 & 1.5 & 0.5 & 1 \\ 2.5 & 2 & 1.5 & 2 \\ 1 & 0 & 2 & 1.5 \\ -0.5 & 1 & 1 & 0.5 \end{pmatrix}$$

Step 2: Compute of weights of criteria. By Eqs. (30) and (31), we can obtain the weight vector as $\omega = (0.414, 0.244, 0.171, 0.171)^T$ and the ranking orders as $R(c_1) = 1,\ R(c_2) = 2,\ R(c_3) = 3,\ R(c_4) = 4$.

$$\omega_j^* = \frac{\sum_{i=1}^{m}\sum_{1}^{m}|\rho_{ij} - \rho_{kj}|}{\sqrt{\sum_{j=1}^{n}[\sum_{i=1}^{m}\sum_{1}^{m}|\rho_{ij} - \rho_{kj}|]^2}} \tag{30}$$

$$\omega_j = \frac{\omega_j^*}{\sum_{j=1}^{n}\omega_j^*} \tag{31}$$

TABLE 1. Weight values and ranking results by different methods.

Methods	Weights of criteria	ranking of criteria
K_{d_g}	$\omega = (0.248, 0.235, 0.248, 0.269)^T$	$c_4 > c_1 > c_3 > c_2$
$K_{d_{gh}}$	$\omega = (0.263, 0.237, 0.263, 0.237)^T$	$c_3 > c_1 > c_4 > c_2$
$K_{d_{ghh}}$	$\omega = (0.257, 0.236, 0.257, 0.249)^T$	$c_3 > c_1 > c_4 > c_2$
K_{AM}	$\omega = (0.257, 0.236, 0.257, 0.249)^T$	$c_4 > c_1 > c_3 > c_2$
Max. deviation-1	$\omega = (0.414, 0.244, 0.171, 0.171)^T$	$c_1 > c_2 > c_3 > c_4$
Max. deviation-2	$\omega = (0.247, 0.143, 0.221, 0.389)^T$	$c_4 > c_1 > c_3 > c_2$
Proposed method	$\omega = (0.239, 0.147, 0.235, 0.379)^T$	$c_4 > c_1 > c_3 > c_2$

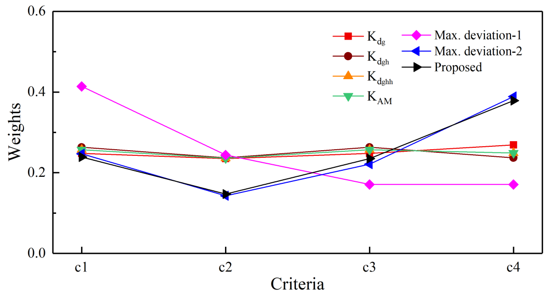

FIGURE 3. The weights of criteria calculated by different methods.

The results obtained from the above-mentioned approaches are shown in Table 1 and Fig. 3. As we can see from Fig. 3, the differences of criteria weights calculated by the entropy methods with respect to three distance measures are relatively small. The criteria are almost assigned the same weights, but the ranking orders of criteria vary by utilizing different entropy measures. To eliminate the inconsistency of the ranking orders, we can use the average method defined in Eq. (32) to derive the final ranking result. From Table 1, we find that the ranking orders of criteria obtained by the average method (Eq. 32) and the generalized distance-based entropy method (Eq. (26)) are the same as that derived by the proposed method. Meanwhile, it is worth mentioning that the results may be different when ς takes different values and it gives the uncertainty to determine the weights of criteria. Therefore, it is necessary to make further research on which entropy measure should be adopted and what value of ς should be used to achieve the most reasonable weights of criteria.

$$K_{AM} = \frac{1}{3}(K_{d_g} + K_{d_{gh}} + K_{d_{ghh}}) \tag{32}$$

From Fig. 3, we find that the ranking orders of criteria obtained by the Max. deviation-1 method are quite inconsistency with those derived by the other methods due to the negative values in the decision matrix. In other words, adopting improper score function which cannot reflect the original evaluations with linguistic expressions will lead to bias in final results. Based on this consideration, we use Eq. (14) to replace Eq. (4) and then use the maximizing deviation method (called Max. deviation-2) to calcualte the weights. The ranking orders of criteria are the same as those obtained by our proposed weight-determining method. This verfies the rationality of the new score function proposed in this paper.

V. THE HFL-GLDS METHOD TO DEAL WITH MEMCDM PROBLEMS

In this section, we propose a HFL-GLDS method to solve MEMCDM problems with hesitant fuzzy linguistic information. Before we conduct this process, it is very necessary to introduce a method to aggregate individual hesitant fuzzy linguistic decision matrices into a collective one.

A. AGGREGATING INDIVIDUAL HESITANT FUZZY LINGUISTIC DECISION MATRICES FOR MEMCDM PROBLEMS

To rank alternatives or obtain a reliable and appropriate result, we need to adopt methods to aggregate the normalized individual decision matrices $\overline{D}^{(q)}, q = 1, 2, \ldots, Q$ into group matrix $\widetilde{D} = (\widetilde{h}_S^{ij})_{m\times n}$ in MEMCDM. Considering that the evaluations of experts are sets of discrete linguistic terms, it is hard to integrate them. Inspired by the aggregation method of the continuous interval-valued linguistic terms set [24], we propose a method to cope with this problem.

Firstly, we transform each expert' evaluations into interval type. Then we integrate the individual interval judgments to group values by Eq. (33) (suppose that the experts have the same weight). Finally, we transform the group decision matrix which consists of interval linguistic terms into the hesitant fuzzy linguistic decision matrix so as to facilitate the sequencing of ranking alternatives.

$$\widetilde{h}_S^{ij} = [s_L^{ij}, s_U^{ij}] = [\frac{1}{Q}\sum_{q=1}^{Q} s_L^{ij(q)}, \frac{1}{Q}\sum_{q=1}^{Q} s_U^{ij(q)}] \tag{33}$$

Example 3: Let $S = \{s_{-3}, \ldots, s_0, \ldots, s_3\}$ be a LTS. Suppose that there are four inspectors $e_q, q = 1, \ldots, 4$ who are invited to evaluate the quality of products. Suppose that the inspectors' judgments are given in HFLEs as: $h_S^1 = \{s_{-1}, s_0, s_1, s_2\}$, $h_S^2 = \{s_{-2}, s_{-1}, s_0, s_1, s_2, s_3\}$, $h_S^3 = \{s_0, s_1, s_2, \}$, $h_S^4 = \{s_1, s_2, s_3\}$. First, we transform the HFLEs into their envelops by Eq. (2), and obtain $\widetilde{h}_S^1 = [s_{-1}, s_2]$, $\widetilde{h}_S^2 = [s_{-2}, s_3]$, $\widetilde{h}_S^3 = [s_0, s_2]$, $\widetilde{h}_S^4 = [s_1, s_3]$, respectively. Then, we aggregate these envelops by Eq. (33) and the result is $\widetilde{h}_S = [s_{-0.5}, s_{2.5}]$. Finally, we can obtain the collective opinion in HFLTS, $h_S = \{s_{-0.5}, s_0, s_1, s_2, s_{2.5}\}$. It is observed that some virtual linguistic terms with non-integer subscripts are generated in the calculation process.

B. ALTERNATIVE RANKING WITH THE HFL-GLDS METHOD

With the aggregation method, we can get the group decision matrix for MEMEDM problems. Then, we use the ranking method, HFL-GLDS, to deduce the ranking of alternatives.

The core of the GLDS method is to compute the unicriterion gained dominance scores and the unicriterion lost dominance scores of alternatives, and then integrates them with associated weights of criteria to obtain the overall gained dominance score and the overall lost dominance score of each alternative. Finally, by employing an aggregation function considering both the subordinate orders and the the overall gained and lost dominance scores, we obtain the collective score of each alternative to determine the final ranking of the alternatives. The specific implementation is justified in what follows.

For the collective hesitant fuzzy linguistic decision matrix, suppose that h_S^{ij} and h_S^{vj} are the evaluations of alternative a_i and a_v under criterion c_j, respectively. To reflect that alternative a_i is superior to alternative a_v under criterion c_j, we use the proposed score function to measure the difference between a_i and a_v. Then, the dominance flow associated to criterion c_j can be mathematically noted as:

$$d\theta_j(\alpha_i, \alpha_v) = \begin{cases} G(h_S^{ij}) - G(h_S^{vj}), & \text{if } G(h_S^{ij}) \geq G(\alpha^{vj(k)}) \\ 0, & \text{if } G(h_S^{ij}) < G(\alpha^{vj(k)}) \end{cases} \quad (34)$$

Considering the different standard in evaluating different criteria, we apply the vector normalization shown as Eq. (35) to normalize the dominance flow matrix:

$$d\theta_j^N(\alpha_i, \alpha_v) = \frac{d\theta_j(\alpha_i, \alpha_v)}{\sqrt{\sum_{v=1}^m \sum_{i=1}^m (d\theta_j(\alpha_i, \alpha_v))^2}} \quad (35)$$

The unicriterion gained dominance score on criterion c_j denotes that alternative a_i dominates all other alternatives a_v, $v = 1, 2, \ldots, m$ under the criterion c_j, which is mathematically defined as:

$$gd_j(a_i) = \sum_{v=1}^{m} d\theta_j^N(a_i, a_v) \quad (36)$$

The overall gained dominance score of alternative a_i under all criteria c_j $(j = 1, 2, \ldots, n)$ is the weighted sum of the unicriterion gained dominance scores of a_i:

$$DS_1(a_i) = \sum_{j=1}^{n} \omega_j gd_j(a_i) \quad (37)$$

where ω_j is the weight of criterion c_j.

The overall gained dominance score is similar to the "group utility" value of each alternative. In this regard, we can obtain a subordinate rank set $R_1 = \{r_1(a_1), r_1(a_2), \ldots, r_1(a_m)\}$, which is in descending order of $DS_1(a_i)$ $(i = 1, 2, \ldots, m)$.

Given that the performance of a_i is not always better than a_v on all criteria, to describe the "negative flow" of an alternative over other alternatives under criterion c_j, we use the unicriterion lost dominance score to reflect this feature by employing the maximizing operator:

$$ld_j(a_i) = \max_v d\theta_j^N(a_i, a_v) \quad (38)$$

The overall lost dominance score is calculated by:

$$DS_2(a_i) = \max_j \omega_j ld_j(a_i) \quad (39)$$

where ω_j is the weight of criterion c_j.

The overall lost dominance score is similar to the "individual regret" value of each alternative. In this regard, we can obtain another subordinate rank set $R_2 = \{r_2(a_1), r_2(a_2), \ldots, r_2(a_m)\}$, which is in ascending order of $DS_2(a_i)$ $(i = 1, 2, \ldots, m)$.

We normalize $DS_1(a_i)$ and $DS_2(a_i)$ by the vector normalization formula as:

$$DS_y^N(a_i) = \frac{DS_y(a_i)}{\sqrt{\sum_{i=1}^m (DS_y(a_i))^2}}, \quad y = 1, 2 \quad (40)$$

Finally, we integrate the overall gained dominance score and the net lost dominance score into a collective score. Meanwhile, two subordinate rank sets are also included in such aggregation function. The collective score (CS) of each alternative is calculated by

$$CS_i = DS_1^N(a_i) \cdot \frac{m - r_1(a_i) + 1}{m(m+1)/2} - DS_2^N(a_i) \cdot \frac{r_2(a_i)}{m(m+1)/2} \quad (41)$$

The final rank set $R = \{r(a_1), r(a_2), \ldots, r(a_m)\}$ can be determined in descending order of $CS_i (i = 1, 2, \ldots, m)$.

C. PROCEDURE OF THE HFL-GLDS METHOD FOR MEMCDM PROBLEMS

The general procedure of the HFL-GLDS method to deal with the MEMCDM problems is as follows. The flowchart of the HFL-GLDS method is shown as Fig. 4.

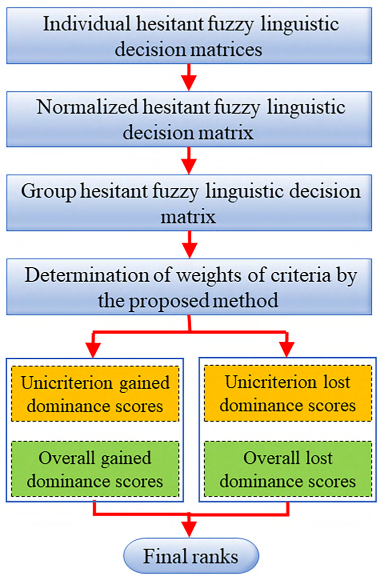

FIGURE 4. The flowchart of the HFL-GLDS technique.

Step 1 (Data Collection): Collect the linguistic expressions from experts and use the translation function to convert them into HFLEs. Then, establish the individual hesitant fuzzy linguistic decision matrices $D^{(q)} = (h_S^{ij(q)})_{m \times n}, q = 1, 2, \ldots, Q$. Go to the next Step.

Step 2 (Normalization): Normalize the individual decision matrices by Eqs. (3) and (21) and obtain the normalized individual decision matrices $\overline{D}^{(q)} = (\overline{h}_S^{ij(q)})_{m \times n}, q = 1, 2, \ldots, Q$. Go to the next Step.

Step 3 (Aggregation): Aggregate the normalized individual decision matrices into the group decision matrix $\widetilde{D} = (\widetilde{h}_S^{ij})_{m \times n}$ by Eq. (33). Go to the next Step.

Step 4 (Weight Determination): Transform each HFLE h_S^{ij} into its score by the score function given as (14) and thus establish the score decision matrix $S = (E(h_S^{ij}))_{m\times n}$. Then, the weights of criteria can be computed by Eqs. (23), (24) and (25). Go to the next Step.

Step 5 (Calculating Dominance Scores): Compute the dominance flows by Eq. (34) and normalize them by Eq. (35). The unicriterion gained dominance scores and the unicriterion lost dominance scores are calculated by Eq.(36) and Eq. (38), respectively. Go to the next Step.

Step 6 (Elicitation): Compute the overall gained dominance scores by Eq. (37) and the overall lost dominance scores by Eq. (39). Then, the subordinate sets $R_1 = \{r_1(a_1), r_1(a_2), \ldots, r_1(a_m)\}$ and $R_2 = \{r_2(a_1), r_2(a_2), \ldots, r_2(a_m)\}$ are obtained. Integrate them by Eq. (40) and Eq. (41) and determine the final rank set. Ends.

Below we highlight the contributions of our HFL-GLDS method.

(1) In the case that two HFLEs have different lengths, most existing operations [23], [31]–[33] extended the shorter one by adding some elements. The subjective randomness of such extension procedures may lead to the loss of original information. However, in our method, when calculating the dominance degree between two alternatives whose performances are represented in HFLEs, we use the scores of HFLEs and thus do not need to extend the HFLEs with equal length. The proposed score function reflects the fuzziness and uncertainty of original qualitative linguistic information by integrating the mean, standard deviation and hesitancy degree of original HFLEs as well as the linguistic scale function.

(2) With the HFL-GLDS method, the selected solution dominates all other alternatives. The solution selected by the reference pointed-based methods, such as TOPSIS and VIKOR, is the closest one to the ideal solution but does not always dominate others. The gained and lost dominance functions based on the corresponding semantics of linguistic terms and the score function of HFLEs, can reflect the dominant degree of an alternative over the others under each criterion more precisely than the distance measure-based method.

(3) We conduct a normalization process before integrating the dominance flows under different criteria. However, such a process is ignored in other MEMCDM methods, such as PROMRTHEE [44] and TODIM [45].

(4) The final integration function given as Eq. (41) considers the "group utility" and "individual regret" values at the same time, which guarantees that the selected solution not only performs excellently in total but is not bad under each criterion. Most importantly, as we take into account the "group utility" value, the "individual regret" value and the corresponding subordinate sets, a robust result is obtained by such a novel aggregation formula. In this regard, the GLDS is superior to the MULTIMOORA (Multi-Objective Optimization on the basis of a Ratio Analysis plus the full MULTIplicative form) [46] method which just considers the subordinate ranks when deriving the final ranking of alternatives.

VI. A CASE STUDY: SELECTING THE MIMING METHOD

In this section, we solve an engineering example concerning the selection of miming methods by the HFL-GLDS approach. The feasibility and effectiveness of the proposed method is further illustrated by some comparative analyses.

A. CASE DESCRIPTION

Determination of a technically feasible and economically reasonable extraction approach is a complex task that requires to consider many influencing factors. A lead-zinc mine is located in Huidong town, Huize City, Yunnan Province, China. This mine area is composed of six irregular ore bodies, shown as Fig. 5. A company want to select an appropriate mining method to extract No.3 ore body whose total ore reserve is calculated as 300 million tons, with an average grade of 33.28 per cent Zn. The ore body is 123 meters along the direction with a thickness of 3-8 meters, and the inclination angle is 59°-79°. The rock substance strength of ore is 7.9, which belongs to a moderately stable rock mass. According to the comprehensive consideration and analysis over these mining technical parameters, four feasible extraction schemes are picked out by mining engineers, including the shallow-length hole shrinkage mining method (a_1), the upward drift cemented filling mining method (a_2), the upward horizontal stratified cemented filling mining method (a_3) and the sub-level caving stoping method (a_4).

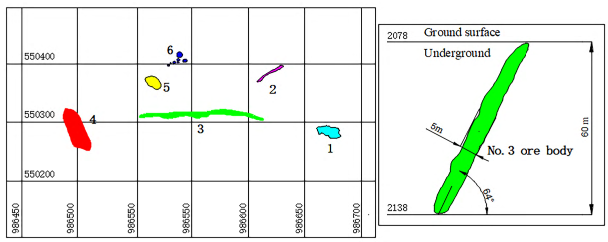

FIGURE 5. Plane layout and occurrence conditions of No. 3 ore body.

As we know, each candidate method has unique advantages, but fails to overcome the challenges related to management complexity and environment protection. For example, the sub-level caving stoping method is a simple process which requires low work intensity, but it may bring some safety and environmental risks, such as the waste of resources, the surface subsidence and the geological disasters. Besides, the tailings produced by beatification will not only occupy large tracts of land but also destroy ecological environment. Conversely, the upward horizontal stratified cemented filling mining method can deal with these problems. Nevertheless, this method requires expensive production costs and complex technology. As we can see, to select an appropriate mining method is difficult and we need to consider various factors comprehensively.

According to the proposal of mining engineers, the economic benefit c_1, technical feasibility c_2, management complexity c_3, security status c_4 and environmental benefit c_5

TABLE 2. Criteria for selecting underground mining methods.

Primary indicators	Benefit/Cost	Description
Economic benefit (c_1)	Benefit	The indexes of economic efficiency, including mining cost, production capacity, ore loss rate, ore dilution rate, grade and types of ore, and main material consuption
Technical feasibility (c_2)	Benefit	The physical parameters, including the occurrence conditions of ore deposits, the stability of ore and surrounding rock, and the possiblity to cave in the surface
Management complexity(c_3)	Cost	The complexity of production management, including labor productivity, complexity of mining process, acquisition and operation of equipment, and difficulty and flexibility of organization management
Security status (c_4)	Benefit	The safety of mining production, including conditions for workplace, ventilation and dust-proof conditions, and construction of safety protection facilities
Environmental benefit (c_5)	Benefit	The damage to the environment of mining methods, including degree of surface subsidence, waste rock and tailings discharge mode, waste water treatment, the recovery of environment around the mine

TABLE 3. The judgments on mining methods given by expert e_1.

Evaluations	c_1	c_2	c_3	c_4	c_5
a_1	QH	Between H and QH	M	M	Between M and H
a_2	Between M and SH	H	Between SL and M	H	Between M and SH
a_3	SH	H	H	QH	H
a_4	H	M	Between SH and H	QH	Between SL and M

TABLE 4. The judgments on mining methods given by expert e_2.

Evaluations	c_1	c_2	c_3	c_4	c_5
a_1	QH	H	Between SL and M	Between M and SH	Between M and SH
a_2	Between M and SH	H	M	H	SH
a_3	SH	Between SH and H	H	Between H and QH	Between SH and H
a_4	Between SH and H	M	Between SH and H	QH	SL

TABLE 5. The judgments on mining methods given by expert e_3.

Evaluations	c_1	c_2	c_3	c_4	c_5
a_1	At least SH	At least M and less than QH	M	Between M and SH	SL
a_2	Between M and SH	Between SH and H	SL	Between SH and H	QH
a_3	Between SL and SH	Between SH and H	SH	H	SH
a_4	Between M and H	Between M and SH	Between SH and H	Between SH and H	L

TABLE 6. The judgments on mining methods given by expert e_4.

Evaluations	c_1	c_2	c_3	c_4	c_5
a_1	At least SH	Between M and H	Between L and M	Between M and SH	Between L and SL
a_2	Between M and H	Between SH and H	Between L and SL	More than M	Between M and H
a_3	Between SL and SH	Between M and H	Between SH and H	Between H and QH	H
a_4	Between L and H	Between M and SH	H	QH	Less than SL

are taken as evaluation criteria. Detail descriptions of these criteria are shown in Table 2. Four experts $e_q (q = 1, 2, \dots, 4)$ are invited to give judgments about the above four mining methods over the five criteria. The experts tend to express their opinions with linguistic expressions. Let $S = \{s_{-3}, \dots, s_0, \dots, s_3\}$ be a LTS and the specific meanings of the linguistic terms are uniformly expressed as: s_{-3} = Quite Low(QL), s_{-2} = Low(L), s_{-1} = Slightly Low(SL), s_0 = Medium(M), s_1 = Slightly High(SH), s_2 = High(H), s_3 = Quite High(QH). Based on the LTS S, the linguistic expressions on alternatives over the criteria given by the four experts are shown in Tables 3-6, respectively.

B. SOLVING THE CASE BY THE HFL-GLDS METHOD

It is apparent that choosing an appropriate extraction method for Huidong mine of No.3 ore body is a typical MEMCDM problem. Below we use the proposed HFL-GLDS method to solve this problem.

Step 1: By the translation function [14], the linguistic judgments in Tables 3-6 are translated into HFLEs, and then four individual decision matrices are established as:

$$D^{(1)} = \begin{bmatrix} \{s_3\} & \{s_2, s_3\} & \{s_0\} & \{s_0\} & \{s_0, s_1, s_2\} \\ \{s_0, s_1\} & \{s_2\} & \{s_{-1}, s_0\} & \{s_2\} & \{s_0, s_1\} \\ \{s_1\} & \{s_2\} & \{s_2\} & \{s_3\} & \{s_2\} \\ \{s_{-2}\} & \{s_0\} & \{s_1, s_2\} & \{s_3\} & \{s_{-1}, s_0\} \end{bmatrix}$$

$$D^{(2)} = \begin{bmatrix} \{s_3\} & \{s_2\} & \{s_{-1}, s_0\} & \{s_0, s_1\} & \{s_0, s_1\} \\ \{s_0, s_1\} & \{s_2\} & \{s_0\} & \{s_2\} & \{s_1\} \\ \{s_1\} & \{s_1, s_2\} & \{s_2\} & \{s_2, s_3\} & \{s_1, s_2\} \\ \{s_1, s_2\} & \{s_0\} & \{s_1, s_2\} & \{s_3\} & \{s_{-1}\} \end{bmatrix}$$

$$D^{(3)} = \begin{bmatrix} \{s_1, s_2, s_3\} & \{s_0, s_1, s_2\} & \{s_0\} & \{s_0, s_1\} & \{s_{-1}\} \\ \{s_0, s_1\} & \{s_1, s_2\} & \{s_{-1}\} & \{s_1, s_2\} & \{s_3\} \\ \{s_{-1}, s_0, s_1\} & \{s_1, s_2\} & \{s_1\} & \{s_2\} & \{s_1\} \\ \{s_0, s_1, s_2\} & \{s_0, s_1\} & \{s_1, s_2\} & \{s_1, s_2\} & \{s_{-2}\} \end{bmatrix}$$

$$D^{(4)} = \begin{bmatrix} \{s_1, s_2, s_3\} & \{s_0, s_1, s_2\} & \{s_{-2}, s_{-1}, s_0\} & \{s_0, s_1\} & \{s_{-2}, s_{-1}\} \\ \{s_0, s_1, s_2\} & \{s_1, s_2\} & \{s_{-2}, s_{-1}\} & \{s_1, s_2, s_3\} & \{s_0, s_1, s_2\} \\ \{s_{-1}, s_0, s_1\} & \{s_0, s_1, s_2\} & \{s_1, s_2\} & \{s_2, s_3\} & \{s_2\} \\ \{s_{-2}, s_{-1}, s_0, s_1, s_2\} & \{s_0, s_1\} & \{s_2\} & \{s_3\} & \{s_{-3}, s_{-2}, s_{-1}\} \end{bmatrix}$$

Step 2: Normalizing the above four individual decision matrices by Eqs. (3) and (21), we obtain the normalized individual decision matrices as

$$\overline{D}^{(1)} = \begin{bmatrix} \{s_3\} & \{s_2, s_3\} & \{s_0\} & \{s_0\} & \{s_0, s_1, s_2\} \\ \{s_0, s_1\} & \{s_2\} & \{s_0, s_1\} & \{s_2\} & \{s_0, s_1\} \\ \{s_1\} & \{s_2\} & \{s_{-2}\} & \{s_3\} & \{s_2\} \\ \{s_{-2}\} & \{s_0\} & \{s_{-2}, s_{-1}\} & \{s_3\} & \{s_{-1}, s_0\} \end{bmatrix}$$

$$\overline{D}^{(2)} = \begin{bmatrix} \{s_3\} & \{s_2\} & \{s_0, s_1\} & \{s_0, s_1\} & \{s_0, s_1\} \\ \{s_0, s_1\} & \{s_2\} & \{s_0\} & \{s_2\} & \{s_1\} \\ \{s_1\} & \{s_1, s_2\} & \{s_{-2}\} & \{s_2, s_3\} & \{s_1, s_2\} \\ \{s_1, s_2\} & \{s_0\} & \{s_{-2}, s_{-1}\} & \{s_3\} & \{s_{-1}\} \end{bmatrix}$$

$$\overline{D}^{(3)} = \begin{bmatrix} \{s_1, s_2, s_3\} & \{s_0, s_1, s_2\} & \{s_0\} & \{s_0, s_1\} & \{s_{-1}\} \\ \{s_0, s_1\} & \{s_1, s_2\} & \{s_1\} & \{s_1, s_2\} & \{s_3\} \\ \{s_{-1}, s_0, s_1\} & \{s_1, s_2\} & \{s_{-1}\} & \{s_2\} & \{s_1\} \\ \{s_0, s_1, s_2\} & \{s_0, s_1\} & \{s_{-2}, s_{-1}\} & \{s_1, s_2\} & \{s_{-2}\} \end{bmatrix}$$

$$\overline{D}^{(4)} = \begin{bmatrix} \{s_1, s_2, s_3\} & \{s_0, s_1, s_2\} & \{s_0, s_1, s_2\} & \{s_0, s_1\} & \{s_{-2}, s_{-1}\} \\ \{s_0, s_1, s_2\} & \{s_1, s_2\} & \{s_1, s_2\} & \{s_1, s_2, s_3\} & \{s_0, s_1, s_2\} \\ \{s_{-1}, s_0, s_1\} & \{s_0, s_1, s_2\} & \{s_{-2}, s_{-1}\} & \{s_2, s_3\} & \{s_2\} \\ \{s_{-2}, s_{-1}, s_0, s_1, s_2\} & \{s_0, s_1\} & \{s_{-2}\} & \{s_3\} & \{s_{-3}, s_{-2}, s_{-1}\} \end{bmatrix}$$

Step 3: Suppose that the four experts have equal importance. Then we aggregate the four normalized matrices into the collective decision matrix by Eq. (33). Thus, the group decision matrix is obtained as:

$$\widetilde{D} = \begin{bmatrix} \{s_2, s_3\} & \{s_1, s_2, s_{2.25}\} & \{s_0, s_{0.75}\} & \{s_0, s_{0.75}\} & \{s_{-1.25}, s_{-1}, s_{0.25}\} \\ \{s_0, s_1, s_{1.25}\} & \{s_{1.5}, s_2\} & \{s_{0.5}, s_1\} & \{s_{1.5}, s_2, s_{2.25}\} & \{s_1, s_{1.75}\} \\ \{s_0, s_1\} & \{s_1, s_2\} & \{s_{-1.75}, s_{-1.5}\} & \{s_{2.25}, s_{2.75}\} & \{s_{1.5}, s_{1.75}\} \\ \{s_{-0.75}, s_0, s_1\} & \{s_0, s_{0.5}\} & \{s_{-2}, s_{-1.75}\} & \{s_{2.5}, s_{2.75}\} & \{s_{-1.75}, s_{-1.5}\} \end{bmatrix}$$

Step 4: Calculate the score of each HFLE in $\widetilde{D}$ by Eqs. (10), (13) and (14), and then we obtain the score matrix:

$$\widetilde{S} = \begin{pmatrix} 0.823 & 0.600 & 0.505 & 0.505 & 0.337 \\ 0.474 & 0.674 & 0.561 & 0.621 & 0.655 \\ 0.524 & 0.626 & 0.206 & 0.823 & 0.699 \\ 0.390 & 0.486 & 0.168 & 0.823 & 0.206 \end{pmatrix}$$

By Eq. (23), the priority degree vector of criteria is computed as $(0.215, 0.04, 0.246, 0.150, 0.350)^T$. Let $\eta_1 = 4$, $\eta_2 = 2$. By Eqs. (24) and (25), the weighs of criteria can be obtained as $\omega_1 = 0.203$, $\omega_2 = 0.114$, $\omega_3 = 0.222$, $\omega_4 = 0.165$, $\omega_5 = 0.293$.

Step 5: Based on the weights of criteria and the group decision matrix $\widetilde{D}$, we can obtain the gained and lost dominance scores of each alternative by Eqs. (34), (35), (36), (38), (37) and (39), which are shown in Table 7 and Table 8, respectively.

TABLE 7. The gained dominance scores of each alternative.

Alternatives	Unicriterion gained dominance scores					Overall gained dominance score
	c_1	c_2	c_3	c_4	c_5	
a_1	1.556	0.713	0.823	0.000	0.071	0.601
a_2	0.055	0.559	1.151	0.252	0.873	0.630
a_3	0.255	0.574	0.005	0.805	1.074	0.569
a_4	0.000	0.000	0.000	0.881	0.000	0.145

TABLE 8. The lost dominance scores of each alternative.

Alternatives	Unicriterion lost dominance scores					Overall lost dominance scores
	c_1	c_2	c_3	c_4	c_5	
a_1	0.523	0.086	0.253	0.000	0.063	0.191
a_2	0.030	0.070	0.344	0.089	0.449	0.236
a_3	0.168	0.078	0.003	0.225	0.534	0.240
a_4	0.000	0.000	0.000	0.239	0.000	0.040

Step 6: Using Eqs. (40) and (41), we obtain the collective scores of all alternatives as $CS_1 = 0.123$, $CS_2 = 0.181$, $CS_3 = 0.178$, $CS_4 = 0.012$, and the final ranking is $a_2 \succ a_3 \succ a_1 \succ a_4$. Therefore, a_2 is the selected mining method.

C. COMPARATIVE ANALYSES WITH OTHER MEMCDM METHODS

To further illustrate the reliability of the HFL-GLDS method, we handle the case by two widely used MEMCDM approaches: the HFL-VIKOR method [23] and the HFL-TOPSIS method [31], and then compare them with the HFL-GLDS method.

1) SOLVING THE CASE BY THE HFL-VIKOR METHOD

Below we deal with the case by the HFL-VIKOR method [23].

Steps 1-3: They are the same as Steps 1-4 of the HFL-GLDS method. The weights of criteria are also calculated as $\omega_j = (0.203, 0.114, 0.222, 0.165, 0.293)^T$.

Step 4: Based on the score function of HFLE, the positive ideal solution and the negative ideal solution are found out as $f^+ = (\{s_2, s_3\}, \{s_{1.5}, s_2\}, \{s_{0.5}, s_1\}, \{s_{2.5}, s_{2.75}\}, \{s_{1.5}, s_{1.75}\})$, $f^- = (\{s_{-0.75}, s_0, s_1\}, \{s_0, s_{0.5}\}, \{s_{-2}, s_{-1.75}\}, \{s_0, s_{0.75}\}, \{s_{-1.75}, s_{-1.5}\})$, respectively. Then, calculate the HFL group utility measure $HFLGU_i$, the HFL individual regret measure $HFLIR_i$ and the HFL compromise measure $HFLC_i$ for each alternative by Eqs. (42), (43) and (44). The results are shown in Table 9.

$$HFLGU_i = \sum_{j=1}^{n} \omega_j \frac{d_{ed}(h_j^+, h_j^i)}{d_{ed}(h_j^+, h_j^-)} \tag{42}$$

$$HFLIR_i = max(\omega_j \frac{d_{ed}(h_j^+, h_j^i)}{d_{ed}(h_j^+, h_j^-)}) \tag{43}$$

TABLE 9. Computation results of the HFL-VIKOR method.

Alternatives	Criteria					Measures		
	$c_1(0.203)$	$c_2(0.114)$	$c_3(0.222)$	$c_4(0.165)$	$c_5(0.296)$	$HFLGU_i$	$HFLIR_i$	$HFLC_i$
a_1	$\{s_2, s_3\}$	$\{s_1, s_2, s_{2.25}\}$	$\{s_0, s_{0.75}\}$	$\{s_0, s_{0.75}\}$	$\{s_{-1.25}, s_{-1}, s_{0.25}\}$	0.4438	0.2259	0.4561
a_2	$\{s_0, s_1, s_{1.25}\}$	$\{s_{1.5}, s_2\}$	$\{s_{0.5}, s_1\}$	$\{s_{1.5}, s_2, s_{2.25}\}$	$\{s_1, s_{1.75}\}$	0.2286	0.1469	0.0000
a_3	$\{s_0, s_1\}$	$\{s_1, s_2\}$	$\{s_{-1.75}, s_{-1.5}\}$	$\{s_{2.25}, s_{2.75}\}$	$\{s_{1.5}, s_{1.75}\}$	0.4000	0.2001	0.3307
a_4	$\{s_{-0.75}, s_0, s_1\}$	$\{s_0, s_{0.5}\}$	$\{s_{-2}, s_{-1.75}\}$	$\{s_{2.5}, s_{2.75}\}$	$\{s_{-1.75}, s_{-1.5}\}$	0.7915	0.2960	1.0000
h_j^+	$\{s_2, s_3\}$	$\{s_{1.5}, s_2\}$	$\{s_{0.5}, s_1\}$	$\{s_{2.5}, s_{2.75}\}$	$\{s_{1.5}, s_{1.75}\}$			
h_j^-	$\{s_{-0.75}, s_0, s_1\}$	$\{s_0, s_{0.5}\}$	$\{s_{-2}, s_{-1.75}\}$	$\{s_0, s_{0.75}\}$	$\{s_{-1.75}, s_{-1.5}\}$			
$d_{ed}(h_j^+, h_j^-)$	0.3481	0.2945	0.3754	0.3234	0.4643			
$d_{ed}(h_j^+, h_j^1)$	0.0000	0.0505	0.0565	0.3234	0.3543			
$d_{ed}(h_j^+, h_j^2)$	0.2517	0.0000	0.0000	0.0973	0.0505			
$d_{ed}(h_j^+, h_j^3)$	0.2857	0.0505	0.3398	0.0253	0.0000			
$d_{ed}(h_j^+, h_j^4)$	0.3481	0.1821	0.3754	0.0000	0.4643			

$$HFLC_i = \delta \frac{HFLGU_i - HFLGU^+}{HFLGU^- - HFLGU^+} + (1-\delta)\frac{HFLIR_i - HFLIR^+}{HFLIR^- - HFLIR^+} \quad (44)$$

where d_{ed} is the Euclidean distance given as [23], ω_j is the weight of criterion c_j, $HFLGU^+ = \min_i HFLGU_i$, $HFLGU^- = \max_i HFLGU_i$, $HFLIR^+ = \min_i HFLIR_i$, $HFLIR^- = \max_i HFLIR_i$, and δ is the weight of the maximum overall utility. Without loss of generality, let $\delta = 0.5$.

Step 5: From Table 9, we can obtain $HFLGU_2 < HFLGU_3 < HFLGU_1 < HFLGU_4$, $HFLIR_2 < HFLIR_3 < HFLIR_1 < HFLIR_4$, $HFLC_2 < HFLC_3 < HFLC_1 < HFLC_4$, which means that a_2 reaches the minimum value on these three measures, simultaneously. Hence, a_2 is the selected mining method.

By the HFL-VIKOR method, we obtain the same ranking result as that derived by the HFL-GLDS method. This shows the correctness of the HFL-GLDS method. In Table 9, $HFLC_2 = 0$ means that a_2 is an appropriate alternative, but it does not mean that a_2 is an ideal solution. In this sense, the HFL-VIKOR method has some limitations. We can only get the final ranking of alternatives, but fail to perceive the performances of alternatives over the criteria. It is not conducive for improvement. In addition, it is also not conducive for optimization on alternatives since no ideal solution exists. However, by the HFL-GLDS method, from Tables 7 and 8, we can clearly see the specific performance of alternatives under the criteria. This is the advantage of our proposed method over the HFL-VIKOR method.

2) SOLVING THE CASE BY THE HFL-TOPSIS METHOD

Tackling the case by the HFL-TOPSIS method [31] involves the following steps:

Steps 1-3: They are the same as Steps 1-4 of the HFL-GLDS method.

Step 4: To better illustrate the validity of the score function-based distance measure defined in this paper, we use the distance measure shown as Eq. (15) to calculate the positive ideal matrix D^+ and the negative ideal matrix D^- as:

$$D^+ = \begin{bmatrix} 0.000 & 0.074 & 0.056 & 0.318 & 0.362 \\ 0.349 & 0.000 & 0.000 & 0.202 & 0.044 \\ 0.299 & 0.048 & 0.355 & 0.000 & 0.000 \\ 0.433 & 0.188 & 0.393 & 0.000 & 0.493 \end{bmatrix}$$

$$D^- = \begin{bmatrix} 0.433 & 0.114 & 0.337 & 0.000 & 0.131 \\ 0.084 & 0.188 & 0.393 & 0.116 & 0.449 \\ 0.134 & 0.140 & 0.038 & 0.318 & 0.493 \\ 0.000 & 0.000 & 0.000 & 0.318 & 0.000 \end{bmatrix}$$

Step 5: The relative closeness (RC) of each alternative to the ideal solution can be calculated by Eq. (45).

$$RC(a_i) = \frac{D_i^-}{D_i^+ + D_i^-}, \quad i = 1, 2, \ldots, m \quad (45)$$

where $D^- = \sum_{j=1}^n d(h_j^i, h_j^-)$ and $D^+ = \sum_{j=1}^n d(h_j^i, h_j^+)$, d is the proposed distance measure between two HFLEs. The computation results are $RC(a_1) = 0.556$, $RC(a_2) = 0.674$, $RC(a_3) = 0.615$, $RC(a_4) = 0.174$.

Step 6: The alternatives can be ranked as $a_2 \succ a_3 \succ a_1 \succ a_4$. Hence, a_2 is the selected mining method.

By the HFL-TOPSIS method, we get the same result as that obtained by the HFL-GLDS method, which verifies the effectiveness of the HFL-GLDS method. Although the HFL-TOPSIS method is easy for us to select the best alternative due to its simple calculation process, it ignores the influence of criteria weights.

3) COMPARATIVE ANALYSES

All the above three ranking methods show that the second mining method is the best choice, which verifies the effectiveness of the HFL-GLDS approach. The ranking of alternatives derived by these three approaches is the same, i.e., $a_2 \succ a_3 \succ a_1 \succ a_4$. However, the weighs of criteria are not considered in the HFL-TOPSIS method. That means even if we assign different weights to the criteria, we will obtain similar results. But in the most real-life MEMCDM problems, the criteria may have different importance. In addition, it is noted that we use the distance measure proposed in this paper to establish ideal matrices and obtain the compromise solution, which shows the validity of the score-function-based distance formula.

The HFL-GLDS method and the HFL-VIKOR method are similar in some places. They are all excellent approach in handling the qualitative MEMCDM problems with conflicting criteria. These two methods both take into account the "group utility" values and the "individual regret" values, and weights of criteria are considered. However, there are some differences between the HFL-GLDS method and the HFL-VIKOR approach. First, the compromise measure of the HFL-VIKOR method ignores the subordinate ranks, which may causes the results with low robustness. Second, the HFL-VIKOR method does not have the normalization process. Third, the HFL-VIKOR method only considers the distance between evaluations with HFLEs, but ignores the corresponding semantics of the HFLEs. While in the HFL-GLDS method, the scores are on the basis of the dominance flow, which consider both the hesitancy degree and the numerical features of the corresponding semantics of judgments. It eliminates bad scores of the HFLES which may appear in the aggregation process and makes the result more reliable. Form Table 8, we can see that alternative a_1 is better than a_3, but it may perform badly under some criteria. To avoid this mistake, we need to find out the worst value of each alternative with respect to the criteria by the weighted maximum operator. The overall lost score (individual regret value), listed in Table 9, shows that a_1 is bad under criterion c_1, and a_3 is moderate under all criteria. Therefore, the final score of a_3 is close to the score of a_1.

In addition, inspired by [47], we can compare the proposed method with the HFL-VIKOR [23] method and the HFL-TOPSIS [31] method from some tangible angles, such as time complexity, modeling uncertainty and last aggregation function. Regarding the modeling uncertainty, these three methods are able to handle imprecise information under hesitant fuzzy linguistic context. However, the proposed method has less information loss than the other two since it is conducted based on a score function. The score function we proposed considers both hesitancy degrees of evaluations and numerical characters of the corresponding semantics. It can decrease the complexity of computing with linguistic information. As for last aggregation function, our proposed method considers more information than the other two. Therefore, the results deduced by our method is more reliable and convincing. As for the time complexity, it is related to the consumption of time and the complexity of computation. Since the weights calculation of the proposed method needs to compute the priority degree of criteria firstly and two aspects of information about the alternatives are required to compute, the proposed method has more time complexity than the other two.

To show the robustness of our proposed method, we conduct a sensitivity analysis about the ranking results. The experiment is designed by changing the importance of criteria. Since the criterion c_1 is of medium importance among all criteria, we increase the weights of criteria c_5 and c_3 by 10% of the weight of criterion c_1, while decrease the weihts of criteria c_2 and c_4 in the amount of increasing of criteria c_5 and c_3. In this way, ten experiments are generated. The text results are displayed in Fig. 6. From Fig. 6, we can see that the alternative a_2 possesses the highest collective score among all experiments, which denotes that the HFL-GLDS method is not sensitive to the change of criteria weiths as long as the ranking orders of criteria are the same. The reason is that the aggregation function of the proposed method considers two subordinate orders and two dominance scores. The sensitivity analysis implies that the proposed method has high robustness.

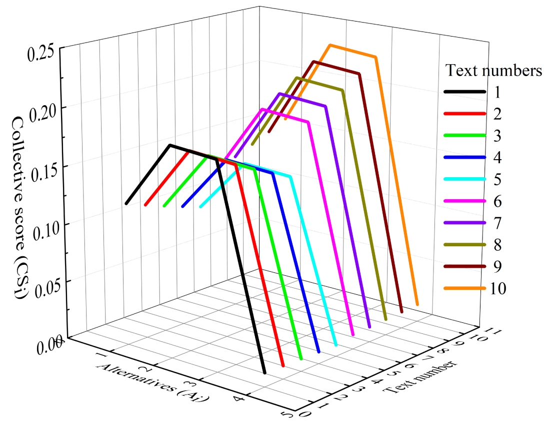

FIGURE 6. The results of the sensitive analysis.

In conclusion, the proposed HFL-GLDS method is better than the HFL-VIKOR method and the HFL-TOPSIS method. It avoids the limitation of non-normalization. Further, the new aggregation operator considering both the subordinate dominance scores and the subordinate ranks guarantees the result with high robustness. This improves the reliability of the HFL-GLDS method in dealing with MEMCDM problems.

VII. CONCLUSIONS

In this paper, the HFL-GLDS method was developed to solve the qualitative MEMCDM problems under the hesitant fuzzy linguistic environment. We proposed new ranking method for HFLEs based on a novel score function which considers both the uncertainty and fuzziness of the evaluation presented by hesitant fuzzy linguistic approach and the numerical characteristics of the corresponding semantics of linguistic terms. Based on the proposed score function, we established a model to define the priority degree of criterion, and then a weight function, which can magnify the "importance" and "unimportance" of criteria, was proposed to determine the weights of criteria with hesitant fuzzy linguistic judgments. The HFLTS was used by the mining professionals to express their evaluations, which increased the flexibility of expressing knowledge. Furthermore, we used the proposed score function to compute dominance flows and extended the GLDS approach to the hesitant fuzzy linguistic context.

Finally, we used the HFL-GLDS to select an appropriate mining method of a real mine in China, and obtained that the alternative a_2, i.e, the upward horizontal stratified cemented filling mining method is an appropriate extraction method for the mine. The effectiveness and applicability were illustrated by some comparison analysis. The comparative analysis suggested that the result was reliable.

However, as we can see, the appearance of virtual linguistic terms increases the hesitancy degrees of HFLEs when aggregating individuals' evaluations into a collective one, which may reduce the score of a HFLE. This challenge will be overcome in future study. Further, combining the GLDS method with other decision-making methods would find more useful information in the decision-making process. It would be also very interesting to implement the HFL-GLDS technique to handle other practical and complex MEMCDM problems.

REFERENCES

[1] N. Nguyen, B. Boruff, and M. Tonts, "Fool's gold: Understanding social, economic and environmental impacts from gold mining in Quang Nam Province, Vietnam," *Sustainability*, vol. 10, no. 5, pp. 1951–2071, Oct. 2018.

[2] F. S. Namin, K. Shahriar, A. Bascetin, and S. H. Ghodsypour, "Practical applications from decision-making techniques for selection of suitable mining method in Iran," *Gospodarka Surowcami Mineralnymi*, vol. 25, no. 3, pp. 57–77, Mar. 2009.

[3] Z.-P. Tian, R.-X. Nie, J.-Q. Wang, and H.-Y. Zhang, "A two-fold feedback mechanism to support consensus-reaching in social network group decision-making," *Knowl.-Based Syst.*, to be published, doi: 10.1016/j.knosys.2018.09.030.

[4] C. Bousoño-Calzón, H. Molina-Bulla, J. J. Escudero-Garzás, and F. J. Herrera-Gálvez, "Expert selection in prediction markets with homological invariants," *IEEE Access*, vol. 6, pp. 32226–32239, 2018.

[5] M.-Y. Quan, Z.-L. Wang, H.-C. Liu, and H. Shi, "A hybrid MCDM approach for large group green supplier selection with uncertain linguistic information," *IEEE Access*, vol. 6, pp. 50372–50383, 2018.

[6] A. Karadogan, A. Kahriman, and U. Ozer, "Application of fuzzy set theory in the selection of underground mining method," *J. Sothern Afr. Inst. Mining Metall.*, vol. 108, no. 2, pp. 73–79, Mar. 2008.

[7] C. Samantra, S. Datta, and S. S. Mahapatra, "A risk-based decision support framework for selection of appropriate safety measure system for underground coal mines," *Int. J. Injury Control Saf. Promotion*, vol. 24, no. 1, pp. 54–68, Jul. 2015.

[8] A. Yazdani-Chamzini, S. H. Yakchali, and E. K. Zavadskas, "Using a integrated MCDM model for mining method selection in presence of uncertainty," *Econ. Res.-Ekonomska Istraživanja*, vol. 25, no. 4, pp. 869–904, Dec. 2012.

[9] R. Mikaeil, M. Z. Naghadehi, M. Ataei, and R. Khalokakaie, "A decision support system using fuzzy analytical hierarchy process (FAHP) and TOPSIS approaches for selection of the optimum underground mining method," *Arch. Mining Sci.*, vol. 54, no. 2, pp. 349–368, Feb. 2009.

[10] G. Zou and S. Cai, "The application of fuzzy comprehensive evaluation and topsis approach to selection of optimum underground mining method," in *Proc. 2nd Int. Conf. Inf. Sci. Eng.*, Dec. 2010, pp. 6233–6237.

[11] B. C. Balusa and J. Singam, "Underground mining method selection using WPM and PROMETHEE," *J. Inst. Eng. D*, vol. 99, no. 1, pp. 165–171, Apr. 2018.

[12] A. Azadeh, M. Osanloo, and M. Ataei, "A new approach to mining method selection based on modifying the Nicholas technique," *Appl. Soft Comput.*, vol. 10, no. 4, pp. 1040–1061, Sep. 2010.

[13] W. Liang, G. Zhao, and C. Hong, "Selecting the optimal mining method with extended multi-objective optimization by ratio analysis plus the full multiplicative form (MULTIMOORA) approach," *Neural Comput. Appl.*, to be published, doi: 10.1007/s00521-018-3405-5.

[14] R. M. Rodríguez, L. Martínez, and F. Herrera, "Hesitant fuzzy linguistic term sets for decision making," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 1, pp. 109–119, Feb. 2012.

[15] V. Torra, "Hesitant fuzzy sets," *Int. J. Intell. Syst.*, vol. 25, no. 6, pp. 529–539, Jun. 2010.

[16] S. M. Mousavi, B. Vahdani, H. Gitinavard, and H. Hashemi, "Solving robot selection problem by a new interval-valued hesitant fuzzy multi-attributes group decision method," *In. J. Ind. Math.*, vol. 8, no. 3, pp. 231–240, 2016.

[17] M. P. Borujeni and H. Gitinavard, "Evaluating the sustainable mining contractor selection problems: An imprecise last aggregation preference selection index method," *J. Sustain. Mining*, vol. 16, no. 4, pp. 207–218, 2017.

[18] R. Tavakkoli-Moghaddam, H. Gitinavard, S. M. Mousavi, and A. Siadat, "An interval-valued hesitant fuzzy TOPSIS method to determine the criteria weights," in *Proc. Int. Conf. Group Decis. Negotiation*, Jun. 2015, pp. 157–169.

[19] H. Gitinavard, A. Makui, and A. Jabbarzadeh, "Interval-valued hesitant fuzzy method based on group decision analysis for estimating weights of decision makers," *J. Ind. Syst. Eng.*, vol. 9, no. 3, pp. 96–110, Jul. 2016.

[20] H. C. Liao, Z. S. Xu, E. Herrera-Viedma, and F. Herrera, "Hesitant fuzzy linguistic term set and its application in decision making: A state-of-the-art survey," *Int. J. Fuzzy Syst.*, vol. 20, no. 7, pp. 2084–2110, 2018.

[21] H. C. Liao, G. S. Si, Z. S. Xu, and H. Fujita, "Hesitant fuzzy linguistic preference utility set and its application in selection of fire rescue plans," *Int. J. Environ. Res. Public Health*, vol. 15, no. 4, p. 664, Apr. 2018.

[22] Z. M. Li, Q. Zhang, and H. C. Liao, "Efficient-equitable-ecological evaluation of regional water resource coordination considering both visible and virtual water," *Omega,* to be published, doi: 10.1016/j.omega.2018.06.003.

[23] H. C. Liao, Z. S. Xu, and X. J. Zeng, "Hesitant fuzzy linguistic VIKOR method and its application in qualitative multiple criteria decision making," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 5, pp. 1343–1355, Oct. 2015.

[24] H. C. Liao, X. L. Wu, X. D. Liang, J.-B. Yang, D.-L. Xu, and F. Herrera, "A continuous interval-valued linguistic ORESTE method for multi-criteria group decision making," *Knowle.-Based Syst.*, vol. 153, no. 1, pp. 65–77, Aug. 2018.

[25] Z.-P. Tian, J.-Q. Wang, H.-Y. Zhang, and T.-L. Wang, "Signed distance-based consensus in multi-criteria group decision-making with multi-granular hesitant unbalanced linguistic information," *Comput. Ind. Eng.*, vol. 124, pp. 125–138, Oct. 2018.

[26] R.-X. Nie, Z.-P. Tian, X.-K. Wang, J.-Q. Wang, and T.-L. Wang, "Risk evaluation by FMEA of supercritical water gasification system using multi-granular linguistic distribution assessment," *Knowle.-Based Syst.*, to be published, doi: 10.1016/j.knosys.2018.05.030.

[27] J.-Q. Wang, J.-T. Wu, J. Wang, H.-Y. Zhang, and X.-H. Chen, "Interval-valued hesitant fuzzy linguistic sets and their applications in multi-criteria decision-making problems," *Inf. Sci.*, vol. 288, no. 1, pp. 55–72, Dec. 2014.

[28] X. L. Wu and H. C. Liao, "A consensus-based probabilistic linguistic gained and lost dominance score method," *Eur. J. Oper. Res.*, vol. 272, no. 3, pp. 1017–1027, 2019.

[29] H. C. Liao, L. S. Jiang, Z. S. Xu, J. P. Xu, and F. Herrera, "A linear programming method for multiple criteria decision making with probabilistic linguistic information," *Inf. Sci.*, vols. 415–416, pp. 341–355, 2017.

[30] X. L. Wu and H. C. Liao, "An approach to quality function deployment based on probabilistic linguistic term sets and ORESTE method for multi-expert multi-criteria decision making," *Inform. Fusion*, vol. 43, pp. 13–26, 2018.

[31] I. Beg and T. Rashid, "TOPSIS for hesitant fuzzy linguistic term sets," *Int. J. Intell. Syst.*, vol. 28, no. 12, pp. 1162–1171, Dec. 2013.

[32] C. P. Wei, Z. L. Ren, and R. M. Rodríguez, "A hesitant fuzzy linguistic TODIM method based on a score function," *Int. J. Comput. Intell. Syst.*, vol. 8, no. 4, pp. 701–712, Aug. 2015.

[33] J.-Q. Wang, J. Wang, Q.-H. Chen, H.-Y. Zhang, and X.-H. Chen, "An outranking approach for multi-criteria decision-making with hesitant fuzzy linguistic term sets," *Inf. Sci.*, vol. 280, no. 1, pp. 338–351, Oct. 2014.

[34] H. C. Liao, Z. S. Xu, and X.-J. Zeng, "Distance and similarity measures for hesitant fuzzy linguistic term sets and their application in multi-criteria decision making," *Inf. Sci.*, vol. 271, no. 1, pp. 125–142, Jul. 2014.

[35] F. Herrera and L. Martinez, "A 2-tuple fuzzy linguistic representation model for computing with words," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 6, pp. 746–752, Dec. 2000.

[36] G. Y. Bao, X. L. Lian, M. He, and L. L. Wang, "Improved two-tuple linguistic representation model based on new linguistic evaluation scale," *Control Decis.*, vol. 25, no. 5, pp. 780–784, May 2010.

[37] H. C. Liao, R. Qin, C. Y. Gao, X. L. Wu, A. Hafezalkotob, and F. Herrera, "Score-HeDLiSF: A score function of hesitant fuzzy linguistic term set based on hesitant degrees and linguistic scale functions: An application to unbalanced hesitant fuzzy linguistic MULTIMOORA," *Inf. Fusion*, vol. 48, pp. 39–54, Aug. 2019.

[38] C. P. Wei, N. Zhao, and X. Tang, "Operators and comparisons of hesitant fuzzy linguistic term sets," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 3, pp. 575–585, Jun. 2014.

[39] F. D. J. Dunstan and A. B. J. Nix, "Estimation of the mean and standard deviation using order statistics," *Stat. Med.*, vol. 10, no. 6, pp. 855–869, Jun. 2010.

[40] S. Das, B. Dutta, and D. Guha, "Weight computation of criteria in a decision-making problem by knowledge measure with intuitionistic fuzzy set and interval-valued intuitionistic fuzzy set," *Soft Comput.*, vol. 20, no. 9, pp. 3421–3442, Sep. 2016.

[41] B. Farhadinia, "Multiple criteria decision-making methods with completely unknown weights in hesitant fuzzy linguistic term setting," *Knowl.-Based Syst.*, vol. 93, pp. 135–144, Feb. 2016.

[42] X. J. Gou, Z. S. Xu, and H. C. Liao, "Hesitant fuzzy linguistic entropy and cross-entropy measures and alternative queuing method for multiple criteria decision making," *Inf. Sci.*, vols. 388–389, pp. 225–246, May 2017.

[43] G. W. Wei, "Maximizing deviation method for multiple attribute decision making in intuitionistic fuzzy setting," *Knowl.-Based Syst.*, vol. 21, no. 8, pp. 833–836, 2008.

[44] M. Behzadiana, R. B. Kazemzadeh, A. Albadvi, and M. Aghdasi, "PROMETHEE: A comprehensive literature review on methodologies and applications," *Eur. J. Oper. Res.*, vol. 200, no. 1, pp. 198–215, Jan. 2010.

[45] J. Qin, X. Liu, and W. Pedrycz, "An extended TODIM multi-criteria group decision making method for green supplier selection in interval type-2 fuzzy environment," *Eur. J. Oper. Res.*, vol. 258, no. 2, pp. 626–638, 2017.

[46] W. K. M. Brauers and E. K. Zavadskas, "Project management by MULTIMOORA as an instrument for transition economies," *Tech. Econ. Develop. Econ.*, vol. 16, no. 1, pp. 5–24, Oct. 2010.

[47] H. Gitinavard, H. Ghaderi, and M. S. Pishvaee, "Green supplier evaluation in manufacturing systems: A novel interval-valued hesitant fuzzy group outranking approach," *Soft Comput.*, vol. 22, no. 19, pp. 6441–6460, Oct. 2018.

ZIGUO FU is currently pursuing the Ph.D. degree in industrial engineering with Sichuan University, Sichuan, China. He has published several papers in international journals. His current research interests include multiple criteria group decision making and information fusion.

XINGLI WU is currently pursuing the master's degree in industrial engineering with Sichuan University, Sichuan, China. She has published several peer reviewed papers, many in high-quality international journals, including the *European Journal of Operational Research*, the IEEE TRANSACTIONS ON FUZZY SYSTEMS, *Information Fusion*, and *Knowledge-Based Systems*. Her current research interest includes decision analysis under uncertainty and computing with words. She was a recipient of the National Encouragement Scholarship for Undergraduates of China Award both in 2014 and 2015 and National Scholarship for Graduates of China Award in 2018. She wins the First Prize Scholarship for graduates of SCU Award in 2016 and 2018.

HUCHANG LIAO (M'13–SM'17) received the Ph.D. degree in management science and engineering from Shanghai Jiao Tong University, Shanghai, China, in 2015. He has been an Academic Visitor with The University of Manchester, the University of Technology Sydney, and the University of Granada since 2013. He is currently a Research Fellow with the Business School, Sichuan University, Chengdu, China.

He has published three monographs, one chapter, and over 100 peer-reviewed papers, many in high-quality international journals, including the *European Journal of Operational Research*, *Omega*, the IEEE TRANSACTIONS ON FUZZY SYSTEMS, *Information Sciences*, *Information Fusion*, and *Knowledge-Based Systems*. He was ranked top 959 in the ESI top 1% of the world's 3193 most influential scientists in the field of computer science, and ranked top 6371 in the ESI top 1% of the world's 8571 most influential scientists in the field of engineering, in 2018. His current research interests include multiple criteria decision analysis under uncertainty, business intelligence and data science, cognitive computing, fuzzy set and systems, healthcare management, and evidential reasoning theory with applications in big data analytics.

Dr. Liao has received numerous honors and awards, including the Thousand Talents Plan for Young Professionals in Sichuan Province, the Candidate of Academic and Technical Leaders in Sichuan Province, the 2017 Outstanding Scientific Research Achievement Award in higher institutions (First class in Natural Science), the 2016 Dongtu Academic Innovation Award granted by the Sichuan University, the 2015 Endeavour Research Fellowship award granted by the Australia Government, and among others. He is an Associate Editor, a Guest Editor, or an Editorial Board Member for over 17 international journals, including *Information Fusion* (SCI, impact factor: 6.639), the *International Journal of Fuzzy Systems* (SCI, impact factor: 2.396), and *Mathematical Problems in Engineering* (SCI, impact factor: 1.145). He also serves as a reviewer for over 100 international journals.

FRANCISCO HERRERA (SM'15) received the M.Sc. and Ph.D. degrees in mathematics from the University of Granada, Spain, in 1988 and 1991, respectively. He is currently a Professor with the Department of Computer Science and Artificial Intelligence, University of Granada. He has been the supervisor of 41 Ph.D. students. He has published over 350 journal papers that have received more than 57,000 citations (Scholar Google and H-index 118). He has co-authored the books *Genetic Fuzzy Systems* (World Scientific, 2001) and *Data Preprocessing in Data Mining* (Springer, 2015), *The 2-tuple Linguistic Model: Computing with Words in Decision Making* (Springer, 2015), *Multilabel Classification: Problem Analysis, Metrics and Techniques*" (Springer, 2016), and among others. He currently acts as an Editor-in-Chief of the international journals *Information Fusion* (Elsevier) and *Progress in Artificial Intelligence* (Springer). His current research interests include among others, soft computing, computing with words, information fusion and decision making, biometrics, and data science. He received numerous honors and awards, including ECCAI Fellow 2009, IFSA Fellow 2013, the 2013 AEPIA Award to a scientific career in artificial intelligence, and the 2017 Andalucía Medal, among others. He has been selected as a Highly Cited Researcher in the fields of computer science and engineering since 2014. He acts as an editorial member of a dozen of journals.