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Analytical Analysis of Residual Chromatic Dispersion and Self-Phase Modulation Optimal Compensation in the Fractional Domain

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ABSTRACT We present an analytical expression of residual chromatic dispersion and self-phase modulation optimal compensation with a chirped Gaussian pulse in the fractional domain. The accuracy of the expression is confirmed by the simulation platform, which is based on the split-step Fourier method. Moreover, the simulation of a 28-GBd RZ-DQPSK optical fiber communication system is carried out to verify the correctness when the transmission distance is less than 800 km in a standard single-mode fiber and non-zero dispersion-shifted fiber.

INDEX TERMS Metrology, fiber nonlinear optics, fiber optics, fractional Fourier transform.

I. INTRODUCTION

Optimizing the link structure with the sweep method requires huge calculations and will be time consuming, so the theoretical analysis of optical links is an important issue for reliable routing decisions in dynamic optical networks [1]. White noise generated by amplifiers, chromatic dispersion (CD) and nonlinearity (NL) are three kinds of factors that limit the performance of the optical path [2], [3]. Additionally, NL contains self-phase modulation (SPM), cross-phase modulation (XPM) and four-wave mixing (FWM) [2]. It is known that the CD and SPM of an optical fiber cause additional chirps in the propagated optical signal in the frequency and time domains, respectively [4]. Therefore, the effect of CD and SPM can be considered as a chirp characteristic, which can be utilized for swift estimation of signal quality in routing decisions.

Fractional Fourier transform (FrFT) is the generalization of the conventional Fourier transform (FT) with transform order, has been widely used in optical signal processing and communications [5]–[7]. Compared with FT, FrFT represents the time-varying frequency distribution and can be used for non-stationary signals [8]–[10]. Therefore, the characteristics of FrFT are utilized to estimate a signal's chirp parameter [11]–[13], CD and SPM [3], [14]–[16] in an optical fiber transmission system.

In this paper, we present an analytical expression for residual CD and SPM optimal compensation by utilizing the chirp characteristic of a chirped Gaussian pulse in the fractional domain. The accuracy of the proposed expression is confirmed by the simulation, which is based on the split-step Fourier (SSF) method. At the same time, a 28 GBaud RZ-DQPSK optical fiber communication system is simulated by VPI software to verify the correctness of the proposed expression. For the optimal order search in the fractional domain, the multi-step search method is used to reduce the computation complexity, in which the search process is divided into a coarse stage and a fine stage to narrow the search scope gradually [17]. Compared with [15], the multi-step search method reduces the computation complexity while maintaining the same measurement accuracy.

This paper is organized as follows. In Section II, we introduce the chirp characteristic for CD and SPM with a chirped

Gaussian pulse in the fractional domain and calculate the analytical expression for optimal compensation. In Section III, we introduce the multi-step optimal order search method in the fractional domain to reduce the computation complexity while maintaining the same measurement accuracy. In Section IV, the accuracy of the analytical expression is confirmed by a SSF method-based simulation. Additionally, a 28 GBaud RZ-DQPSK optical fiber communication system is realized by VPI software to verify the correctness in optimizing the residual CD. In Section V, the conclusions are drawn.

II. CHIRP CHARACTERISTIC FOR CD AND SPM

FrFT is a linear transformation with transform order $\alpha \in [0, 2\pi)$. Mathematically, it maps a signal $x(t)$ onto $X_\alpha(u)$ [7] as

$$X_\alpha(u) = \int_{-\infty}^{+\infty} K_\alpha(u, t)x(t)\,dt \tag{1}$$

where $K_\alpha(u, t)$ is the transform kernel, α is the rotation angle, and the transform order p can be represented by $p = 2\alpha/\pi$.

A chirped Gaussian pulse is defined as

$$s(t) = exp[-\frac{(1+jC)t^2}{2T_0^2}] \tag{2}$$

where $s(t)$ is the chirped Gaussian pulse, T_0 is the half-width (at the 1/e-intensity point), and C is the chirp parameter [18]. It has an excellent energy convergence characteristic with optimal order in the fractional domain [14].

It is well known that the impact of CD and SPM on an optical signal can be considered as frequency and time domain chirps, respectively [3]. Therefore, the effects of CD and SPM can be regarded as additional chirps for a chirped Gaussian pulse in the fractional domain, and the transmitted signal will have rotation in the time-frequency distribution, as depicted in Figure 1. The transmitted signal has counterclockwise rotation for SPM and clockwise rotation for negative CD. Thus, optimal compensation between CD and SPM can eliminate the effects. The relationship between CD and the optimal order [14] is defined as

$$p_{\text{opt,CD}} = \frac{arctan(\frac{\lambda^2}{2\pi c}Dz\frac{df}{dt})}{\frac{\pi}{2}} \tag{3}$$

where $p_{\text{opt,CD}}$ is the optimal order in the fractional domain, λ is the wavelength of the signal, c is the speed of light, D is the dispersion parameter, z is the transmission distance, dt is the sampling interval in the time domain, and df is the sampling interval in the frequency domain.

And the relationship between SPM and the optimal order for a Gaussian pulse without considering CD effects [19] is defined as

$$p_{\text{opt,SPM}} = \frac{arccot(-\frac{2L_{\text{eff}}dt}{T_0^2 L_{\text{NL}}df})}{\frac{\pi}{2}} \tag{4}$$

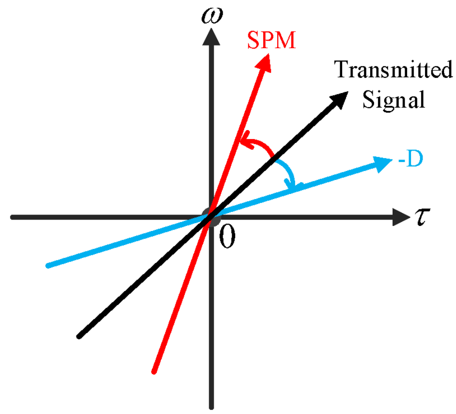

FIGURE 1. CD and SPM rotation direction in the time-frequency distribution.

where L_{eff} is the effective length of the optical fiber, and L_{NL} is the nonlinear length.

The projection of the signal in the time-frequency plane is depicted in Figure 2. To obtain the analytical expression for CD and SPM optimal compensation, it is essential to calculate the slope of the signal. Moreover, when the signal is transformed by FrFT with optimal order, the slope of the fractional domain is perpendicular to that of the signal in the time-frequency distribution; therefore, which can be replaced by calculating the $\tan(\frac{\pi}{2}p_{\text{opt}})$ of optimal rotation angle. The analytical expression for optimal compensation can be obtained by making Eq. (3) and Eq. (4) have the opposite value, which is expressed as

$$log(Dz) = -0.1P_{\text{dBm}} + b \tag{5}$$

where P_{dBm} is the launch power with dBm unit, b is the intercept of the expression, and Dz is the accumulated dispersion.

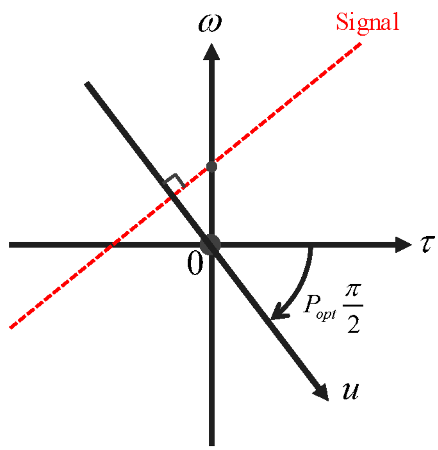

FIGURE 2. The projection of the signal in the time-frequency plane.

III. MULTI-STEP OPTIMAL ORDER SEARCH METHOD

When the chirped Gaussian pulse is transformed by FrFT with optimal order, convergence of the maximum energy can be realized, and the optimal order can be obtained using the extremum of the EC function [15], the EC function is defined as

$$EC(p) = \int_{-\infty}^{+\infty} |X_p(u)|^4 \, du. \tag{6}$$

where $X_p(u)$ is the FrFT of the signal with p order.

The optimal order will shift with different numbers of samples, as shown in Figure 3. The peak is narrower with more samples; thus, a smaller search step is required. Supposing that the measurement accuracy as peak decreases 3 dB, the relationship between the number of sampling points and the measurement accuracy is depicted in Figure 4, and the measurement results will be more accurate with more sampling points. The relationship between different numbers of samples and the related optimal order in the fractional domain is expressed as

$$(N_1 - 1)\tan(\frac{\pi}{2}p_{\text{opt1}}) = (N_2 - 1)\tan(\frac{\pi}{2}p_{\text{opt2}}) \tag{7}$$

where N_1 and N_2 are different numbers of samples, and p_{opt1} and p_{opt2} are the related optimal orders, respectively.

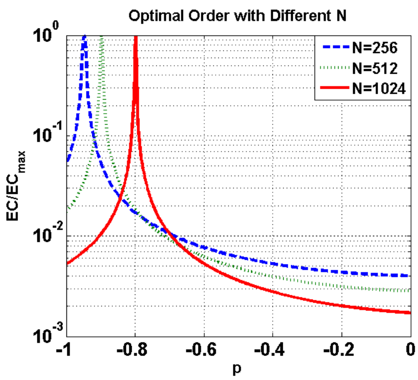

FIGURE 3. The shift of the optimal order in FrFT with different numbers of samples.

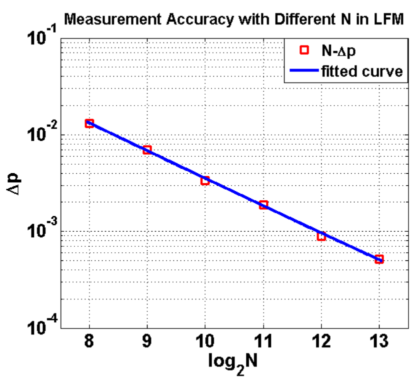

FIGURE 4. The relationship between the number of samples and measurement accuracy.

For the purpose of improving the measurement accuracy, more sampling points and a smaller search step are needed; however, these cause a higher computation complexity. For the multi-step search method, the search process is divided into a coarse stage and a fine stage to narrow the search scope gradually [17]. The flow chart of the multi-step search method for searching the optimal order in the fractional domain is shown in Figure 5. First, fewer sampling points and a larger search step are required. The EC function can be calculated with different fractional orders to obtain the optimal order using Eq. (6). To improve the measurement accuracy, the optimal order needs to be translated by Eq. (7) when utilizing more sampling points to obtain the new search interval. Then, the new EC function is calculated by repeating the previous process with the narrower search interval and more sampling points until the desired measurement accuracy is achieved. Finally, the precise optimal order will be obtained. Assume that N_i is the sampling points for loop i, Δp_i is the search step for loop i (Δp_i must be greater than the measurement accuracy obtained with N_i), and Δ_i is the search interval for i loop (Δ_i must be more than twice the previous search step, and the transformed optimal order is the center of the search interval). The computation complexity of the multi-step search method can be expressed as

$$O(\frac{N_1 log_2 N_1}{\Delta p_1/\Delta_1} + \frac{N_2 log_2 N_2}{\Delta p_2/\Delta_2} + \cdots + \frac{N_i log_2 N_i}{\Delta p_i/\Delta_i} + \cdots) \tag{8}$$

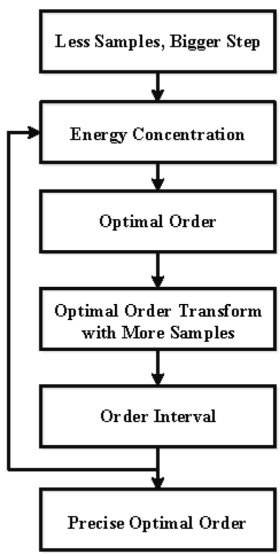

FIGURE 5. The flow chart of optimal order searching.

IV. DISCUSSION OF SIMULATION RESULTS

To verify the correctness of the analytical expression as expressed by Eq. (5), we simulate it with the SSF method. After splitting the optical fiber into small parts, the SSF method calculates the CD and SPM effects separately. The effects of CD and SPM in the SSF method are realized in the frequency and time domains, respectively, by multiplying a phase term [20]. The process of searching the optimal compensation between CD and SPM is depicted in Figure 6. First, the slope of the fractional domain for the transmitted signal with optimal order needs to be calculated. Then, the SPM

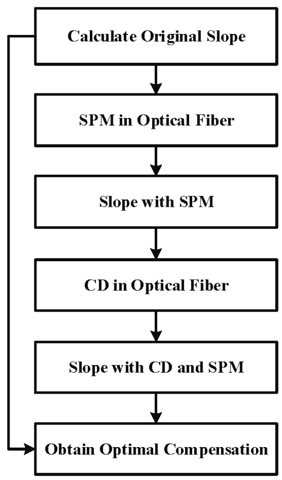

FIGURE 6. The process of searching the optimal compensation between CD and SPM.

effect is added to the optical fiber to calculate the slope. Next, the CD effect is added to the optical fiber with different transmission distances and a fixed dispersion parameter to compensate the SPM rotation. Finally, optimal compensation can be obtained by comparison with the original slope.

The simulation parameters are shown in Table 1, and the process of optimal searching is depicted in Figure 7. First, the number of sampling points is 256, the search step is 0.001, and the search interval is 2, between -1 and 1, to obtain the optimal order. Then, the optimal order is transformed to the case of 500 sampling points, 0.0001 search step, and 0.01 search interval to obtain the precise optimal order. Compared with the one-step search method, the computation complexity can be reduced by 20 times.

The simulation results are shown in Figure 8. First, the original slope is calculated to obtain the reference value.

TABLE 1. Simulation parameters.

T_0	$120ps$	Full width half wave
n_2	$2.6 \times 10^{-20} s/m^3$	Nonlinear index
D	$-16ps/(nm \times km)$	Dispersion parameter
P_0	$-3 \sim 3dBm$	Launch power

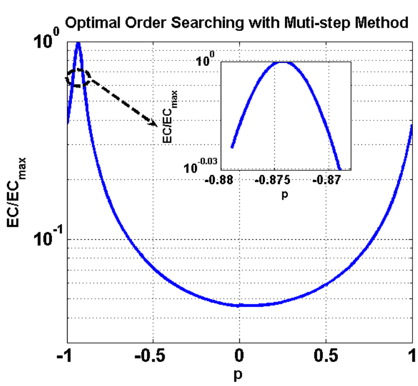

FIGURE 7. The process of optimal order searching.

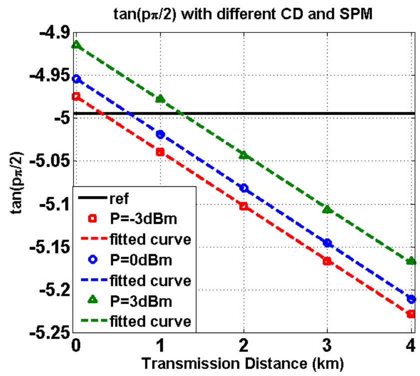

FIGURE 8. $\tan(p\pi/2)$ with different CD and SPM.

Then, the slope is calculated with different launch powers for different SPM and different transmission distances for different accumulated CD. Finally, the optimal compensation between CD and SPM is obtained by comparison with the reference. The optimal compensation between CD and SPM is shown in Figure 9, and the slope of the fitted curve is -0.1046, which is consistent with the analytical expression as expressed by Eq. (5). Therefore, the accuracy of the analytical expression is verified by the SSF-based simulation.

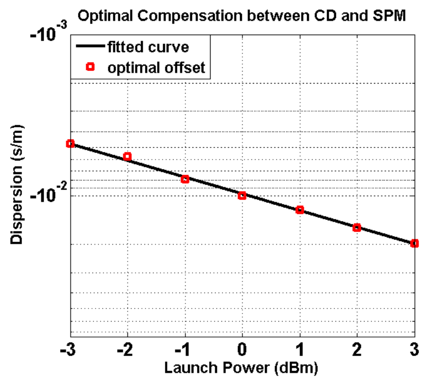

FIGURE 9. Optimal compensation between CD and SPM.

The simulation of a 28 GBaud RZ-DQPSK optical fiber communication system is conducted using VPI software as shown in Figure 10 with a periodically dispersion

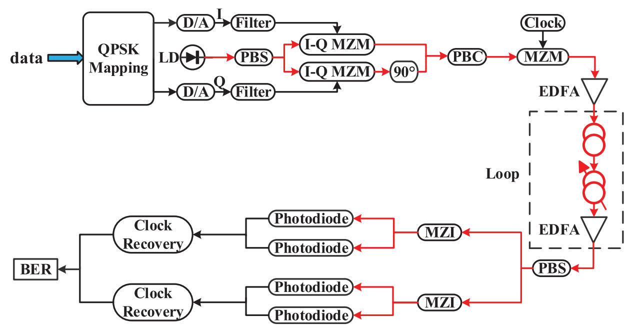

FIGURE 10. Diagram of the optical fiber transmission system.

compensated optical link. The blank arrow and red arrow represent electrical signals and optical signals, respectively. The Mach-Zehnder interferometer (MZI) is used for differential demodulation in the optical domain. The OSNR is set to 15 dB, the linewidth of the laser is set to 100 kHz, the dispersion parameter is set to 16 ps/(nm×km) and 4 ps/ (nm×km) for standard single mode fiber (SSMF) and non-zero dispersion-shifted fiber (NZDSF), respectively, the nonlinear index is set to 2.6×10^{-20} s/m^3, the fiber attenuation is set to 0.2 dB/km, the span length is 100 km, and the numbers of spans are 2, 4, 6, 8. The relationship between residual CD and the bit error rate (BER) is shown in Figure 11, which shows that the bit error rate (BER) changes with different residual CD, and the optimal residual CD occurs for the lowest BER.

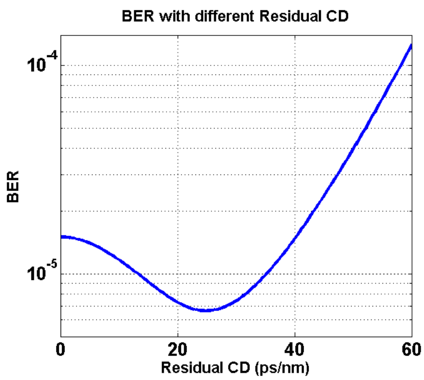

FIGURE 11. BER with different residual CD.

The absolute difference between the optimal residual CD with NL and without NL is represented by ΔCD. The relationship between the launch power and ΔCD is depicted in Figure 12 and Figure 13 for SSMF and NZDSF, respectively, by normalizing the intercept. The results show that the slope between the launch power and normalized ΔCD coincides with the analytical expression as expressed by Eq. (5). Moreover, the slope reduces with an increase of the transmission distance, as CD cannot compensate the irreversible effect. Therefore, the correctness of the analytical expression in optimizing residual CD with a periodically dispersion

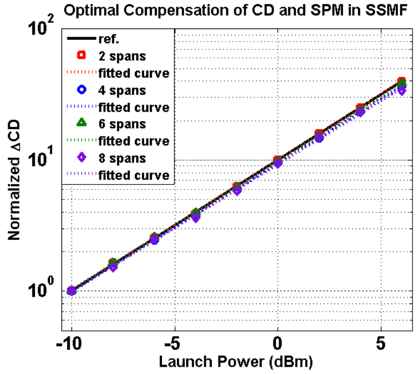

FIGURE 12. Optimal compensation of CD and SPM in SSMF.

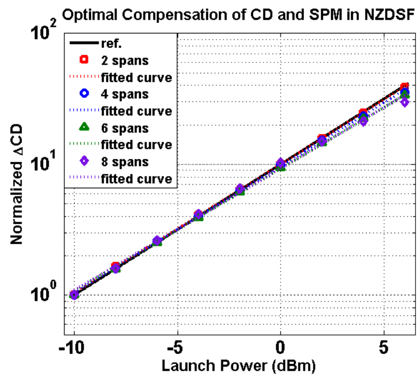

FIGURE 13. Optimal compensation of CD and SPM in NZDSF.

compensated optical link is verified when the transmission distance is less than 800 km in SSMF and NZDSF.

V. CONCLUSIONS

We present the analytical expression for residual CD and SPM optimal compensation by utilizing the chirp characteristic of a chirped Gaussian pulse in the fractional domain. The accuracy of the proposed expression is confirmed by the simulation, which is based on the SSF method. For the optimal order search in the fractional domain, a multi-step search method is used to reduce the computation complexity while maintaining the same measurement accuracy. The computation complexity can be reduced by 20 times with 500 sampling points and a 0.0001 search step. At the same time, the simulation of a 28 GBaud RZ-DQPSK optical fiber communication system is carried out to verify the correctness when the transmission distance is less than 800 km in SSMF and NZDSF.

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