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Exponential Synchronization of Chaotic Lur'e Systems Using an Adaptive Event-Triggered Mechanism

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ABSTRACT This paper investigates event-triggered exponential synchronization of master-slave chaotic Lur'e systems (CLSs). First, in order to save communication resources, a novel event-based transmission strategy is developed using continuous-time measurements while a positive minimum inter-event time can be ensured. Second, an adaptive law is adopted to adjust dynamically the event-triggered threshold parameter. Compared with some existing static event-triggered mechanisms, the proposed event-triggered mechanism can provide a better tradeoff between communication resource saving and desired synchronization performance. Third, a switched Lyapunov–Krasovskii functional (LKF) is introduced, which is continuous at the switching instants but not necessarily positive definite at sampling intervals. This LKF is employed to derive a less conservative synchronization criterion for CLSs, based on which, the synchronization controller gain and the event-triggered parameters can be co-designed in terms of linear matrix inequalities. Finally, numerical simulations of Chua's circuit and neural network are provided to illustrate the efficiency of the proposed method.

INDEX TERMS Master-slave CLSs, event-triggered communication strategy, exponential synchronization, LMIs approach.

I. INTRODUCTION

Synchronization of chaotic systems has been attracting great attention in the past decades [1]–[3]. This is due to its potential applications in information science, image processing and secure communication. In [4], the idea of chaotic synchronization was first reported. Since then, researchers have proposed a number of effective control methods to achieve synchronization of chaotic systems, such as nonlinear control [5], intermittent control [6], and adaptive sliding mode control [7]. In fact, some actual nonlinear systems can be modeled as a Lur'e system, such as Chua's circuit [8] and neural networks [9]. Therefore, the master-slave synchronization problem of CLSs has been an important research topic, and numerous results on this problem have been published. For example, in [10], a time-varying-delay feedback controller was designed for master-slave synchronization of CLSs, and sufficient delay-dependent synchronization criteria were formulated in form of LMIs. In [11], by introducing a novel piecewise differentiable LKF, the master-slave synchronization problem was investigated for CLSs. Recently, in [12], a sampled-based control scheme was presented to address the exponential synchronization problem for master-slave CLSs.

Note that the control tasks for CLSs in the aforementioned references are executed in a periodic way. This may lead to unnecessary utilization of the communication bandwidth when there is very little fluctuation of the sampled data. In many real engineering applications, communication resources are limited [13]. Therefore, it is more preferable to design an appropriate control strategy to keep the desirable system performance while saving communication resources as much as possible. In [14], an event-triggered scheduling strategy was proposed. Under this scheduling strategy, whether the control task needs to be executed depends on a preselected event condition. A novel sampled-based

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event-triggered control strategy was developed in [15], where a minimum time between two adjacent events is ensured to be not less than a sampling period [16], [17]. Compared with the traditional control schemes, the event-triggered control scheme typically require less network bandwidth. Motivated by this observation, the master-slave synchronization problem for CLSs using event-triggered control scheme has been studied in [18]–[22]. In [19]–[21], some sampled-based event-triggered control schemes were developed to study the master-slave synchronization problem for CLSs. Recently, a hybrid event-triggered scheme was proposed in [22] for master-slave synchronization of CLSs with time-varying communication delays. These works improve our understanding on how to design appropriate event-triggered control strategy to ensure master-slave synchronization of CLSs.

However, for the static output-feedback continuous event-triggered control strategy, infinite events may occur within a finite time interval (Zeno phenomenon). Sampled-based event-triggered control strategy avoids this phenomenon but it can not utilize continuous-time measurement information. Hence, it is meaningful to introduce a novel event-based control scheme that exploits the advantages of the continuous-time measurements and ensures a positive minimum inter-event time. This is the motivation of this work.

In this paper, the exponential synchronization problem for master-slave CLSs with an event-triggered control scheme is investigated. Compared with the existing literature, the proposed scheme has the following features:

1) A novel event-triggered control scheme is proposed to save the limited communication resources. Different from the existing control schemes in [18]–[22], the proposed method exploits the advantage of the continuous-time measurements and ensures a positive minimum inter-event time.
2) The event-triggered threshold parameter can be dynamically adjusted according to an adaptive law. Compared with the event-triggered mechanism with a constant threshold parameter, it can provide more flexibility in scheduling data transmission.
3) A switched LKF is employed, which is continuous at the switching instants but not necessarily positive definite inside the sampling intervals. Based on this LKF, a less conservative synchronization criterion for CLSs can be obtained.
4) A co-design method for determining the synchronization controller gain and the event-triggered parameters is given.

The outline of this paper is organized as follows. In Section II, we state the control objective and the adaptive event-triggered communication mechanism for master-slave CLSs. The main results are presented in Section III. Simulation examples are given in Section IV to show the effectiveness of the proposed results, and Section V concludes the paper.

II. PRELIMINARIES

Consider the following master-slave CLSs:

$$\mathcal{M}:\begin{cases}\dot{m}(t)=Am(t)+Wf(Lm(t))\\ v(t)=Cm(t)\end{cases}\tag{1}$$

$$\mathcal{S}:\begin{cases}\dot{s}(t)=As(t)+Wf(Ls(t))+u(t)\\ w(t)=Cs(t)\end{cases}\tag{2}$$

which consists of the master-system $\mathcal{M}$ and slave-system $\mathcal{S}$. When $u(t)=0$, $\mathcal{M}$ and $\mathcal{S}$ are identical CLSs with system states $m(t), s(t)\in\mathbb{R}^n$, outputs of subsystems $v(t), w(t)\in\mathbb{R}^l$, respectively. $u(t)\in\mathbb{R}^n$ is the slave-system control input. $A\in\mathbb{R}^{n\times n}$, $C\in\mathbb{R}^{l\times n}$, $L\in\mathbb{R}^{n_h\times n}$ and $W\in\mathbb{R}^{n\times n_h}$ are known constant matrices. $f(\cdot):\mathbb{R}^{n_h}\to\mathbb{R}^{n\times n_h}$ belonging to the sector $[0,\rho_i]$ is assumed to be a diagonal nonlinearity, $i=1,2,\cdots,n_h$. The system states $m(t)$ and $s(t)$ are unmeasured. One can only use output measurements $v(t)$ and $w(t)$ to construct control input $u(t)$.

We define $r(t)=m(t)-s(t)$ as the synchronization error. Then, the following error system can be obtained:

$$\begin{cases}\dot{r}(t)=Ar(t)+Wg(Lr(t),s(t))-u(t)\\ y(t)=Cr(t)\end{cases}\tag{3}$$

where $g(Lr(t),s(t))=f(L(r(t)+s(t)))-f(Ls(t))$. Since $f_i(\cdot)$ belongs to the sector $[0,\rho_i]$, one can obtain

$$0\le\frac{g_i(l_i^Tr,s)}{l_i^Tr}=\frac{f_i(l_i^T(r+s))-f_i(l_i^Ts)}{l_i^Tr}\le\rho_i,$$
$$\forall r,s,\quad l_i^Tr\neq 0,\ i=1,2,\cdots,n_h,\tag{4}$$

where l_i^T is the ith row vector of L. From (4), we can get

$$g_i(l_i^Tr,s)(g_i(l_i^Tr,s)-\rho_il_i^Tr)\le 0,\quad i=1,2,\cdots,n_h.\tag{5}$$

Obviously, for any matrix $\Lambda=diag\{\lambda_1,\lambda_2,\cdots,\lambda_{n_h}\}\ge 0$ ($diag\{\cdots\}$ stands for a diagonal or block-diagonal matrix), the following inequality holds:

$$-\sum_{i=1}^{n_h}\lambda_ig_i(l_i^Tr,s)(g_i(l_i^Tr,s)-\rho_il_i^Tr)\ge 0,\tag{6}$$

which implies that

$$r^T(t)L^T\rho\Lambda g(Lr(t),s(t))$$
$$-g(Lr(t),s(t))^T\Lambda g(Lr(t),\quad s(t))\ge 0,\tag{7}$$

where $\rho=diag\{\rho_1,\rho_2,\cdots,\rho_{n_h}\}$.

The overall goal is to design a novel event-based control scheme to reduce the amount of sent measurements while guaranteeing the synchronization of the master-slave CLSs. For this purpose, we introduce an adaptive event-triggered transmission strategy, whose framework is illustrated in Fig. 1. It is assumed that the output measurement $y(t)$ is available for the synchronization purpose only at discrete time instant $s_k (k\in\mathbb{N})$,

$$0=s_0<s_1<\cdots,\quad \lim_{k\to+\infty}s_k=+\infty.\tag{8}$$

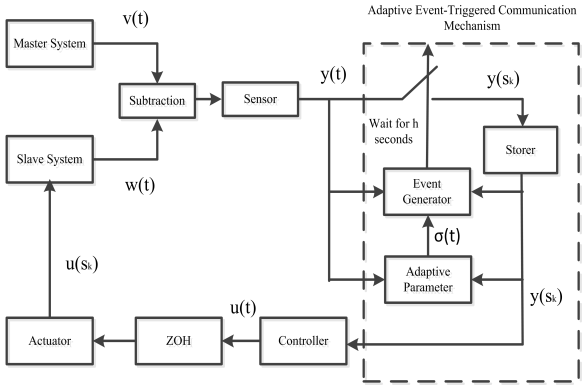

FIGURE 1. **An adaptive event-triggered transmission strategy.**

From Fig. 1, we can see that the sensor is used to continuously measure the output signal $y(t)$. Whether the current measurement data should be transmitted to the storer rely on the event generator. In the framework of our proposed event-based control scheme, after the measurement data $y(s_k)$ has been sent, the event generator waits for h seconds, then the event generator begins to continuously check the event-triggered condition. Once the event-triggered condition is satisfied, the output measurement $y(s_{k+1})$ is transmitted to the storer. Then, the controller and event generator update their input by using the received data from the storer. Moreover, the event-triggered threshold parameter $\sigma(t)$ is adaptively adjusted according to the current measurement data $y(t)$ and past measurement data $y(s_k)$. The considered event-triggered condition is formulated as

$$s_{k+1} = \min\{\, t \geq s_k + h \mid \mathcal{H}(t) \geq 0 \,\} \tag{9}$$

$$\mathcal{H}(t) = \epsilon^T(t)\Psi\epsilon(t) - \sigma(t)y^T(t)\Omega y(t) \tag{10}$$

$$\dot{\sigma}(t) = -\mu\sigma^2(t)\epsilon^T(t)\Psi\epsilon(t) \tag{11}$$

where $\epsilon(t) = y(s_k) - y(t)$. $\mu > 0$ is a prescribed constant. $\sigma(t)$ is the dynamic trigger parameter and $\sigma(0) = \sigma_0 > 0$, $\Psi \geq 0$ and $\Omega \geq 0$ are two weighting matrices.

Remark 1: Different from the previous works [18]–[22], the novel event-triggered mechanism (9)-(11) can exploit the advantage of the continuous-time measurements while avoiding Zeno phenomenon. Moreover, compared with the state-dependent event-triggering conditions in [21] and [22], the output-based event-triggered mechanism (9)-(11) is more practical.

Remark 2: Note that the threshold parameter of the triggering condition (9) can be dynamically adjusted in accordance with the adaptive law (11). When the parameter $\sigma(t)$ closely approaches to zero, the event-triggered mechanism (9) reduces to the sampled-data control scheme in [12]. Hence, the proposed event-triggered mechanism may provide flexibility in scheduling data transmission than static event-triggered mechanism and sampled-data control scheme.

Now, an event-based synchronization controller is given as

$$u(t) = Ky(s_k) = KCr(s_k), \quad t \in [s_k, s_{k+1}). \tag{12}$$

Then, the system (3) can be presented as a switched system

$$\begin{aligned}\dot{r}(t) &= Ar(t) + Wg(Lr(t), s(t)) - KCr(s_k)\\ &= (A - KC)r(t) + \chi(t)KC\int_{t-\tau(t)}^{t}\dot{r}(\theta)d\theta\\ &\quad - (1-\chi(t))K\epsilon(t) + Wg(Lr(t), s(t))\end{aligned} \tag{13}$$

where

$$\tau(t) = t - s_k \leq h, \quad t \in [s_k, s_k + h),$$

$$\chi(t) = \begin{cases}1, & t \in [s_k, s_k + h),\\ 0, & t \in [s_k + h, s_{k+1}).\end{cases}$$

Definition 1: If there exist two constant $\alpha > 0$ and $\beta > 0$ such that

$$\|r(t)\| \leq \beta e^{-\alpha t}\|r_0\|, \quad \forall t \geq 0, \tag{14}$$

then, we call that the master-slave CLSs are exponentially synchronous, and α is the convergence rate of the synchronization error $r(t)$.

Next, some useful lemmas are introduced as follows.

Lemma 1: [24] Consider a differentiable signal $r : [a, b] \rightarrow \mathbb{R}^n$. For a vector $\xi \in \mathbb{R}^m$, symmetric matrices $U(\in \mathbb{R}^{n\times n}) > 0$, and any matrices $N_1, N_2 \in \mathbb{R}^{n\times m}$, we have:

$$\begin{aligned}&-\int_a^b \dot{r}^T(\theta)U\dot{r}(\theta)d\theta\\ &\leq (b-a)\xi^T[N_1^TU^{-1}N_1 + \frac{(b-a)^2}{3}N_2^TU^{-1}N_2]\xi\\ &\quad + 2\xi^T[N_1^T(r(b) - r(a)) - 2N_2^T\int_a^b r(\theta)d\theta]\\ &\quad + 2(b-a)\xi^TN_2^T[r(b) + r(a)].\end{aligned} \tag{15}$$

Lemma 2: For the error system (13). We can get the following relationship

$$\|r(t)\|^2 \leq \nu\|r(s_k)\|^2, \quad t \in [s_k, s_k + h), \tag{16}$$

where

$$\nu = 4(1 + h^2\|KC\|^2)e^{4h^2(\|A\|^2+\|W\|^2\|\rho L\|^2)}.$$

Proof: For any $t \in [s_k, s_k + h)$, it is easily found from (13) that

$$\begin{aligned}\|r(t)\| &\leq \|r(s_k)\| + \|\int_{s_k}^t Ar(\theta)d\theta\|\\ &\quad + \|\int_{s_k}^t KCr(s_k)d\theta\| + \|\int_{s_k}^t Wg(Lr(\theta), s(\theta))d\theta\|.\end{aligned} \tag{17}$$

On the other hand, it can be found from (4) that

$$\|g(Lr(\theta), s(\theta))\|^2 \leq \|\rho L\|^2\|r(\theta)\|^2. \tag{18}$$

Then, from (17), (18) and applying the Cauchy-Schwarz inequality, one can obtain

$$\begin{aligned}\|r(t)\|^2 &\leq 4\|r(s_k)\|^2 + 4\|\int_{s_k}^t Ar(\theta)d\theta\|^2\\ &\quad + 4\|\int_{s_k}^t KCr(s_k)d\theta\|^2\end{aligned}$$

$$
\begin{aligned}
&+4\|\int_{s_k}^{t} Wg(Lr(\theta), s(\theta))d\theta\|^2 \\
&\le 4\|r(s_k)\|^2 + 4h\int_{s_k}^{t}\|Ar(\theta)\|^2 d\theta \\
&\quad +4h\int_{s_k}^{t}\|KCr(s_k)\|^2 d\theta \\
&\quad +4h\int_{s_k}^{t}\|Wg(Lr(\theta), s(\theta))\|^2 d\theta \\
&\le 4h(\|A\|^2 + \|W\|^2\|\rho L\|^2)\int_{s_k}^{t}\|r(\theta)\|^2 d\theta \\
&\quad +4(1+h^2\|KC\|^2)\|r(s_k)\|^2. \qquad (19)
\end{aligned}
$$

By using the Gronwall-Bellman Lemma to (19), we have

$$\|r(t)\|^2 \le \nu\|r(s_k)\|^2, \quad t \in [s_k, s_k + h). \qquad (20)$$

This completes the proof.

Lemma 3: For given an initial condition $\sigma_0 > 0$, then

$$0 < \sigma(t) \le \sigma_0, \quad t \in [0, \infty), \qquad (21)$$

and

$$\epsilon^T(t)\Psi\epsilon(t) \le \sigma_0 y^T(t)\Omega y(t), \quad t \in [s_k + h, s_{k+1}). \qquad (22)$$

Proof: It is easily found from (11) that $\dot{\sigma}(t) \le 0$, which implies that $\sigma(t)$ is monotone decreasing. Thus, for any $\sigma_0 > 0$, we have $\sigma(t) \le \sigma_0$ holds all the time. In the following, proofs by contradiction. Assume that there exist one time instant t_1 satisfies $\sigma(t_1) = 0$ and $\sigma(t_1) > 0$ for $\forall t \in (0, t_1)$. Under this assumption, for $\forall t \in (0, t_1)$, (11) can be rewritten as

$$-\frac{\dot{\sigma}(t)}{\sigma^2(t)} = \mu\epsilon^T(t)\Psi\epsilon(t). \qquad (23)$$

From (23), one can see that

$$\sigma(t) = \frac{1}{\frac{1}{\sigma_0} + \int_0^t \mu\epsilon^T(\theta)\Psi\epsilon(\theta)d\theta}. \qquad (24)$$

Let $t \to t_1$, according to the continuity property of $\sigma(t)$, we can obtain

$$\sigma(t) \to \sigma(t_1) = \frac{1}{\frac{1}{\sigma_0} + \int_0^{t_1} \mu\epsilon^T(\theta)\Psi\epsilon(\theta)d\theta} > 0, \qquad (25)$$

which is contrary to the assumption that $\sigma(t_1) = 0$. So the assumption does not hold, which implies that $\sigma(t) > 0$ holds all the time.

According to the property of event-triggered communication scheme (9), the following condition holds

$$\epsilon^T(t)\Psi\epsilon(t) - \sigma(t)y^T(t)\Omega y(t) \le 0, \quad t \in [s_k + h, s_{k+1}). \qquad (26)$$

Since $\sigma(t)$ is monotone decreasing, one can obtain

$$\epsilon^T(t)\Psi\epsilon(t) \le \sigma_0 y^T(t)\Omega y(t), \quad t \in [s_k + h, s_{k+1}). \qquad (27)$$

This completes the proof.

Problem: In this paper, the main objective is to design event-triggered mechanism (9)-(11) and static output-feedback controller (12) for the CLSs (1) and (2), such that the master-systems $\mathcal{M}$ and slave-systems $\mathcal{S}$ are exponentially synchronous.

III. MAIN RESULTS

Before presenting the main results, for brevity, we have the following notations:

$$
\begin{aligned}
\varpi(t) &= \int_{s_k}^{t} r(\theta)d\theta, \\
\eta(t) &= [r^T(t), r^T(s_k), \varpi^T(t)]^T, \\
\xi_1(t) &= [r^T(t), r^T(s_k), \dot{r}^T(t), g^T(Lr, s), \varpi^T(t)]^T, \\
\xi_2(t) &= [r^T(t), \dot{r}^T(t), g^T(Lr, s), \epsilon^T(t)]^T, \\
e_i &= [0_{n\times(i-1)n}, I_n, 0_{n\times(5-i)n}], \quad i = 1, \cdots, 5, \\
\bar{e}_i &= [0_{n\times(i-1)n}, I_n, 0_{n\times(3-i)n}, 0_{n\times l}], \quad i = 1, \cdots, 3, \\
\bar{e}_4 &= [0_{l\times 3n}, I_l],
\end{aligned}
$$

and $sym(N)$ denotes $N + N^T$ for any matrix N; The symbol " $*$ " represents the symmetric term; $\mathbb{S}^n$ is the set of $n \times n$ symmetric matrices.

For the master-slave CLSs (1) and (2), we give the following theorem.

Theorem 1: Given scalars $\alpha > 0$, $h > 0$, $\sigma_0 > 0$, ε and γ, suppose that there exist positive definite matrices $P \in \mathbb{S}^n$, $U \in \mathbb{S}^n$, $\Psi \in \mathbb{S}^l$, $\Omega \in \mathbb{S}^l$ and any matrices $Q \in \mathbb{S}^n$, $X_i \in \mathbb{R}^{n\times n}$, $(i = 1, 2, 3, 4)$, $X_5 \in \mathbb{S}^n$, $N_j \in \mathbb{R}^{n\times 5n}$, $(j = 1, 2)$, $G \in \mathbb{R}^{n\times n}$, $T \in \mathbb{R}^{n\times m}$ and diagonal positive definite matrix $\Lambda \in \mathbb{S}^n$ such that

$$\Xi_1 = \Phi_1 + h\Phi_3 + \Phi_4 < 0 \qquad (28)$$

$$\Xi_2 = \begin{bmatrix} \Phi_1 + h\Phi_2 + \Phi_4 & hN_1^T & h^2N_2^T \\ * & -he^{-2\alpha h}U & 0 \\ * & * & -3he^{-2\alpha h}U \end{bmatrix} < 0 \qquad (29)$$

$$\Xi_3 = \Phi_5 + \Phi_6 < 0 \qquad (30)$$

where

$$
\begin{aligned}
\Phi_1 &= sym(e_1^TPe_3 + N_1^T\Pi_4 - 2N_2^Te_5) - \Pi_1^TX\Pi_1 \\
&\quad + 2\alpha e_1^TPe_1, \\
\Phi_2 &= sym(N_2^T\Pi_3) - e_2^TQe_2, \\
\Phi_3 &= sym(\Pi_1^TX\Pi_2) + e_3^TUe_3 + e_2^TQe_2 + 2\alpha\Pi_1^TX\Pi_1, \\
\Phi_4 &= sym(\Pi_5^TG\Pi_6 - \Pi_5^TTCe_2 + e_1^TL^T\rho\Lambda e_4) \\
&\quad - 2e_4^T\Lambda e_4, \\
\Phi_5 &= sym(\bar{e}_1^TP\bar{e}_2) + 2\alpha\bar{e}_1^TP\bar{e}_1, \\
\Phi_6 &= sym(\Pi_7^TG\Pi_8 - \Pi_7^TTC\bar{e}_1 - \Pi_7^TT\bar{e}_4 \\
&\quad + \bar{e}_1^TL^T\rho\Lambda\bar{e}_3) + \sigma_0\bar{e}_1^TC^T\Omega C\bar{e}_1 - \bar{e}_4^T\Psi\bar{e}_4 \\
&\quad - 2\bar{e}_3^T\Lambda\bar{e}_3, \\
\Pi_1 &= [e_1^T, e_2^T, e_5^T]^T, \quad \Pi_2 = [e_3^T, 0, e_1^T]^T, \\
\Pi_3 &= e_1 + e_2, \quad \Pi_4 = e_1 - e_2, \\
\Pi_5 &= \varepsilon e_1 + e_3 + \gamma e_2,
\end{aligned}
$$

$$\Pi_6 = -e_3 + Ae_1 + We_4,$$
$$\Pi_7 = \varepsilon\bar{e}_1 + \bar{e}_2,$$
$$\Pi_8 = -\bar{e}_2 + A\bar{e}_1 + W\bar{e}_3.$$

Then, under the event-trigger mechanism (9), the error system (3) is exponentially stable. Furthermore, the desired controller gain matrix can be obtained by

$$K = G^{-1}T. \tag{31}$$

Proof : Under the event-triggered mechanism (9), we construct different LKF for the switched system (13). For (13) with $\chi(t) = 0$ we consider

$$V(t) = V_P(r) = r^T(t)Pr(t), \quad P > 0. \tag{32}$$

For (13) with $\chi(t) = 1$ we apply the functional form

$$V(t) = V_P(r) + V_X(t, r_t) + V_Q(t, r_t) + V_U(t, \dot{r}_t), \tag{33}$$

where $r_t(\vartheta) = r(t + \vartheta)$ for $\vartheta \in [-h, 0]$,

$$V_X(t, r_t) = (h - \tau(t))\eta^T(t)X\eta(t),$$
$$V_Q(t, r_t) = \tau(t)(h - \tau(t))r^T(s_k)Qr(s_k),$$
$$V_U(t, \dot{r}_t) = (h - \tau(t)) \int_{s_k}^{t} e^{2\alpha(\theta - t)}\dot{r}^T(\theta)U\dot{r}(\theta)d\theta,$$

and

$$X = \begin{bmatrix} X_1 + X_1^T & -X_1 + X_2 & X_3 \\ * & -X_2 - X_2^T & X_4 \\ * & * & X_5 \end{bmatrix}, \quad U > 0.$$

Note that the values of $V(t)$ coincide at the switching instants s_k and $s_k + h$.

Case I: We firstly consider the case $\chi(t) = 1$. Taking the derivative of $V(t)$ along the trajectories of system (13) gives

$$\dot{V}_P(r) = 2\xi_1^T(t)e_1^TPe_3\xi_1(t) \tag{34}$$
$$\dot{V}_X(t, r_t) = \xi_1^T(t)[-\Pi_1^TX\Pi_1 + 2(h - \tau(t)) \times \Pi_1^TX\Pi_2]\xi_1(t) \tag{35}$$
$$\dot{V}_Q(t, r_t) = \xi_1^T(t)[(h - \tau(t))e_2^TQe_2 - \tau(t)e_2^TQe_2]\xi_1(t) \tag{36}$$

Moreover, we find

$$\begin{aligned} &\frac{d}{dt}V_U(t, \dot{r}_t) + 2\alpha V_U(t, \dot{r}_t) \\ &\quad = (h - \tau(t))\dot{r}^T(t)U\dot{r}(t) - \int_{s_k}^{t} e^{2\alpha(\theta - t)}\dot{r}^T(\theta)U\dot{r}(\theta)d\theta \\ &\quad \leq (h - \tau(t))\dot{r}^T(t)U\dot{r}(t) - e^{-2\alpha h}\int_{s_k}^{t} \dot{r}^T(\theta)U\dot{r}(\theta)d\theta \end{aligned} \tag{37}$$

Using Lemma 1, for any matrices N_1, N_2, one can obtain

$$\begin{aligned} &-e^{-2\alpha h}\int_{s_k}^{t} \dot{r}^T(\theta)U\dot{r}(\theta)d\theta \\ &\leq \xi_1^T(t)[\tau(t)(N_1^Te^{2\alpha h}U^{-1}N_1 + \frac{\tau(t)^2}{3}N_2^Te^{2\alpha h} \\ &\quad \times U^{-1}N_2 + 2N_2^T\Pi_3) + 2(N_1^T\Pi_4 - 2N_2^Te_5)]\xi_1(t) \\ &\leq \xi_1^T(t)[\tau(t)(N_1^Te^{2\alpha h}U^{-1}N_1 + \frac{h^2}{3}N_2^Te^{2\alpha h}U^{-1}N_2 \\ &\quad + 2N_2^T\Pi_3) + 2(N_1^T\Pi_4 - 2N_2^Te_5)]\xi_1(t) \end{aligned} \tag{38}$$

Then, we have

$$\begin{aligned} &\dot{V}(t) + 2\alpha V(t) \\ &\leq \xi_1^T(t)(2e_1^TPe_3 - \Pi_1^TX\Pi_1 + 2\alpha e_1^TPe_1 \\ &\quad + (h - \tau(t))(2\Pi_1^TX\Pi_2 + e_3^TUe_3 + 2\alpha\Pi_1^TX\Pi_1 \\ &\quad + e_2^TQe_2) + \tau(t)(N_1^Te^{2\alpha h}U^{-1}N_1 \\ &\quad + \frac{h^2}{3}N_2^Te^{2\alpha h}U^{-1}N_2 + 2N_2^T\Pi_3 - e_2^TQe_2) \\ &\quad + 2(N_1^T\Pi_4 - 2N_2^T\Pi_5))\xi_1(t) \\ &\leq \xi_1^T(t)(\Phi_1 + \tau(t)\hat{\Phi}_2 + (h - \tau(t))\Phi_3)\xi_1(t) \end{aligned} \tag{39}$$

where $\hat{\Phi}_2 = \Phi_2 + N_1^Te^{2\alpha h}U^{-1}N_1 + \frac{h^2}{3}N_2^Te^{2\alpha h}U^{-1}N_2$.

It follows from (13) that for any appropriately dimensioned matrix G, and scalars ε and γ, we have

$$\begin{aligned} 0 &= 2[\varepsilon r^T(t)G + \dot{r}^T(t)G + \gamma r^T(s_k)G][-\dot{r}(t) \\ &\quad + Ar(t) + Wg(Lr(t), s(t)) - KCr(s_k)] \\ &= 2\xi_1^T(t)(\Pi_5^TG\Pi_6 - \Pi_5^TGKCe_2)\xi_1(t). \end{aligned} \tag{40}$$

It is noted that, based on (7), for any matrix $\Lambda = diag\{\lambda_1, \lambda_2, \cdots, \lambda_n\} > 0$ the following inequality holds:

$$\begin{aligned} 0 &\leq 2[r^T(t)L^T\rho\Lambda g(Lr, s) - g(Lr, s)^T\Lambda g(Lr, s)] \\ &= 2\xi_1^T(t)(e_1^TL^T\rho\Lambda e_4 - e_4^T\Lambda e_4)\xi_1(t). \end{aligned} \tag{41}$$

Then, from (39)-(41) and letting $T = GK$, we obtain that

$$\dot{V}(t) + 2\alpha V(t) \leq \xi_1^T(t)(\frac{h - \tau(t)}{h}\Xi_1 + \frac{\tau(t)}{h}\hat{\Xi}_2)\xi_1(t) \tag{42}$$

where $\hat{\Xi}_2 = \Phi_1 + h\hat{\Phi}_2 + \Phi_4$.

Case II: We consider the case $\chi(t) = 0$. From Lemma 3, we have

$$\begin{aligned} 0 &\leq \sigma_0 r^T(t)C^T\Omega Cr(t) - \epsilon^T(t)\Psi\epsilon(t) \\ &= \xi_2^T(t)(\sigma_0\bar{e}_1^TC^T\Omega C\bar{e}_1 - \bar{e}_4^T\Psi\bar{e}_4)\xi_2(t). \end{aligned} \tag{43}$$

On the other hand, similar to (40) and (41), and letting $T = GK$, one can obtain

$$\begin{aligned} 0 &= 2[\varepsilon r^T(t)G + \dot{r}^T(t)G][-\dot{r}(t) + Ar(t) \\ &\quad + Wg(Lr(t), s(t)) - KCr(t) - K\epsilon(t)] \\ &= 2\xi_2^T(t)(\Pi_7^TG\Pi_8 - \Pi_7^TTC\bar{e}_1 - \Pi_7^TT\bar{e}_4)\xi_2(t). \end{aligned} \tag{44}$$

$$\begin{aligned} 0 &\leq 2[r^T(t)L^T\rho\Lambda g(Lr, s) - g(Lr, s)^T\Lambda g(Lr, s)] \\ &\leq 2\xi_2^T(t)(\bar{e}_1^TL^T\rho\Lambda\bar{e}_3 - \bar{e}_3^T\Lambda\bar{e}_3)\xi_2(t). \end{aligned} \tag{45}$$

Then, adding (43)-(45) to $\dot{V}_P + 2\alpha V_P$, we obtain that

$$\begin{aligned} &\dot{V}_P + 2\alpha V_P \\ &\leq \xi_2^T(t)(2\bar{e}_1^TP\bar{e}_2 + 2\alpha\bar{e}_1^TP\bar{e}_1 + \sigma_0\bar{e}_1^TC^T\Omega C\bar{e}_1 \\ &\quad - \bar{e}_4^T\Psi\bar{e}_4 + 2\Pi_7^TG\Pi_8 - 2\Pi_7^TTC\bar{e}_1 - 2\Pi_7^TT\bar{e}_4 \\ &\quad + 2\bar{e}_1^TL^T\rho\Lambda\bar{e}_3 - 2\bar{e}_3^T\Lambda\bar{e}_3)\xi_2(t) \\ &\leq \xi_2^T(t)\Xi_3\xi_2(t) \end{aligned} \tag{46}$$

Based on the above two cases, if (28)-(30) are satisfied, we obtain that

$$\dot{V}(t) + 2\alpha V(t) \leq 0, \quad t \in [s_k, s_{k+1}) \tag{47}$$

Thus, it follows that

$$\begin{aligned} V(s_k) &\le e^{-2\alpha(s_k-s_{k-1})}V(s_{k-1}) \\ &\le \cdots \le e^{-2\alpha(s_k-s_0)}V(s_0) \end{aligned} \tag{48}$$

By using Lemma 2 and (48), for $t \in [s_k, s_k+h)$, we have

$$\begin{aligned} \|r(t)\|^2 &\le \nu\|r(s_k)\|^2 \\ &\le \frac{\nu}{\lambda_{min}(P)}V(s_k) \\ &\le \frac{\nu e^{2\alpha h}}{\lambda_{min}(P)}e^{-2\alpha t}V(0) \end{aligned} \tag{49}$$

On the other hand, from (32), (47) and (48), one can conclude that for $t \in [s_k+h, s_{k+1})$

$$\begin{aligned} \|r(t)\|^2 &\le \frac{1}{\lambda_{min}(P)}V(t) \\ &\le \frac{1}{\lambda_{min}(P)}e^{-2\alpha(t-s_k)}V(s_k) \\ &\le \frac{1}{\lambda_{min}(P)}e^{-2\alpha t}V(0) \end{aligned} \tag{50}$$

From (49) and (50), we can conclude that for $t \in [s_k, s_{k+1})$

$$\|r(t)\|^2 \le \frac{max\{\nu e^{2\alpha h}, 1\}}{\lambda_{min}(P)}e^{-2\alpha t}V(0) \tag{51}$$

Moreover, it is easy to see

$$V(0) = r^T(0)Pr(0) \le \lambda_{max}(P)\|r(0)\|^2 \tag{52}$$

Using (51) and (52), we can get

$$\|r(t)\|^2 \le \frac{\lambda_{max}(P)\cdot max\{\nu e^{2\alpha h}, 1\}}{\lambda_{min}(P)}e^{-2\alpha t}\|r(0)\|^2 \tag{53}$$

Thus, according to Definition 1, the error system (3) is exponentially stable, i.e., the master-slave systems $\mathcal{M}$ and $\mathcal{S}$ are exponentially synchronous. This completes the proof.

Remark 3: We should point out that the proposed LKF is continuous in time, and it may be not positive definite at sampling intervals. Different from the LKF introduced in [8], [12], and [18], two new functions $V_X(t, r_t)$ and $V_Q(t, r_t)$ are introduced in this paper, which makes it possible to deduce less conservative synchronization criterion.

Remark 4: Theorem 1 provides an effective method to design the desired controller under the event-triggered transmission strategy (9). In fact, given scalars α, h, σ_0, ε and γ, one can obtain the desired controller gain K and the event-triggered parameters (Ψ, Ω) by solving LMIs (28)-(30).

Next, we consider the sampled-based event-triggered control strategy by choosing

$$\begin{aligned} s_{k+1} = min\{&s_k+ih, i\in\mathbb{N}|\sigma y(s_k+ih)^T\Omega \\ &\times y(s_k+ih) \le (y(s_k+ih)-y(s_k))^T\Psi \\ &\times(y(s_k+ih)-y(s_k))\} \end{aligned} \tag{54}$$

Correspondingly, the synchronization error system can be rewritten as

$$\dot{r}(t) = Ar(t) + Wg(Lr(t), \quad s(t)) - KCr(s_k) \tag{55}$$

Consider the following LKF

$$V(t) = V_P(r) + V_X(t, r_t) + V_Q(t, r_t) + V_U(t, \dot{r}_t), \tag{56}$$

where $V_P(r)$, $V_X(t, r_t)$, $V_Q(t, r_t)$ and $V_U(t, \dot{r}_t)$ are given in (33). For brevity, we have the following definition:

$$\begin{aligned} &\varsigma(t) = y(s_k) - y(s_k+ih), \\ &\xi_3(t) = [\xi_1^T(t), \varsigma^T(t)]^T, \\ &\quad \hat{e}_i = [0_{n\times(i-1)n}, I_n, 0_{n\times(5-i)n}, 0_{n\times l}], \quad i = 1, \cdots, 5, \\ &\quad \hat{e}_6 = [0_{l\times 5n}, I_l]. \end{aligned}$$

Then, we can get the following corollary.

Corollary 1: Given scalars $\alpha > 0$, $h > 0$, $\sigma > 0$, ε and γ, suppose that there exist positive definite matrices $P \in \mathbb{S}^n$, $U \in \mathbb{S}^n$, $\Psi \in \mathbb{S}^l$, $\Omega \in \mathbb{S}^l$ and any matrices $Q \in \mathbb{S}^n$, $X_i \in \mathbb{R}^{n\times n}$, $(i = 1, 2, 3, 4)$, $X_5 \in \mathbb{S}^n$, $N_j \in \mathbb{R}^{n\times 5n}$, $(j = 1, 2)$, $G \in \mathbb{R}^{n\times n}$, $T \in \mathbb{R}^{n\times m}$ and diagonal positive definite matrix $\Lambda \in \mathbb{S}^n$ such that

$$\Xi_4 = \Phi_7 + h\Phi_9 + \Phi_{10} < 0 \tag{57}$$

$$\Xi_5 = \begin{bmatrix} \Phi_7 + h\Phi_8 + \Phi_{10} & hN_1^T & h^2N_2^T \\ * & -he^{-2\alpha h}U & 0 \\ * & * & -3he^{-2\alpha h}U \end{bmatrix} < 0 \tag{58}$$

where

$$\begin{aligned} \Phi_7 &= sym(\hat{e}_1^TP\hat{e}_3 + N_1^T\Pi_{12} - 2N_2^T\hat{e}_5) - \Pi_9^TX\Pi_9 \\ &\quad + 2\alpha\hat{e}_1^TP\hat{e}_1, \\ \Phi_8 &= sym(N_2^T\Pi_{11}) - \hat{e}_2^TQ\hat{e}_2, \\ \Phi_9 &= sym(\Pi_9^TX\Pi_{10}) + \hat{e}_3^TU\hat{e}_3 + \hat{e}_2^TQ\hat{e}_2 \\ &\quad + 2\alpha\Pi_9^TX\Pi_9, \\ \Phi_{10} &= sym(\Pi_{13}^TG\Pi_{14} - \Pi_{13}^TTC\hat{e}_2 + \hat{e}_1^TL^T\rho\Lambda\hat{e}_4) \\ &\quad + \sigma\hat{e}_1^TC^T\Omega C\hat{e}_1 - \hat{e}_6^T\Psi\hat{e}_6 - 2\hat{e}_4^T\Lambda\hat{e}_4, \\ \Pi_9 &= [\hat{e}_1^T, \hat{e}_2^T, \hat{e}_5^T]^T, \quad \Pi_{10} = [\hat{e}_3^T, 0, \hat{e}_1^T]^T, \\ \Pi_{11} &= \hat{e}_1 + \hat{e}_2, \quad \Pi_{12} = \hat{e}_1 - \hat{e}_2, \\ \Pi_{13} &= \varepsilon\hat{e}_1 + \hat{e}_3 + \gamma\hat{e}_2, \quad \Pi_{14} = -\hat{e}_3 + A\hat{e}_1 + W\hat{e}_4. \end{aligned}$$

Then the synchronization error system (3) under the event-trigger mechanism (54) is exponentially stable. Furthermore, the desired output-based event-triggered controller gain matrix can be obtained by (31).

Proof: The proof process is similar to the theorem 1, thus it is omitted.

Remark 5: Compared with literature [18]–[22], a new LKF (56) is constructed, which makes full use of the characteristic information of actual sampling pattern. Thus the proposed control scheme is less conservative. The proposed method can be extended to synchronization of discrete-time chaotic systems using some recent results in [25] and [26].

Remark 6: For the same h, σ, Ω and Ψ, the amount of sent measurements under sampled-based event-triggered control strategy (54) is less than under event-triggered mechanism (9). However, the event-triggered mechanism (9) may provide a better tradeoff between saving computation resources and

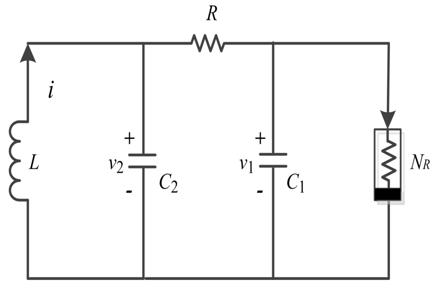

FIGURE 2. Chua's circuit.

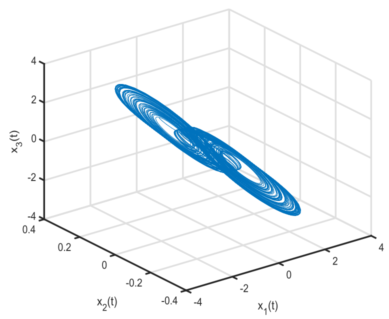

FIGURE 3. Master system $\mathcal{M}$.

achieving better synchronization performance compared with sampled-based event-triggered control strategy (54).

IV. NUMERICAL EXAMPLES

Next, we provide two simulation examples to demonstrate the effectiveness of the proposed synchronization method:

Example 1: Consider the following Chua's circuit system:

$$\begin{cases} \dot{z}_1(t) = a(z_2(t) - c_1 z_1(t) + h(z_1(t))) \\ \dot{z}_2(t) = z_1(t) - z_2(t) + z_3(t) \\ \dot{z}_3(t) = -bz_2(t) \end{cases}$$

where $h(z_1(t)) = \frac{1}{2}(c_1 - c_0)(|z_1(t) + 1| - |z_1(t) - 1|)$, and choose $a = 9$, $b = 14.28$, $c_0 = -1/7$, $c_1 = 2/7$. Fig. 2 is the standard Chua's circuit.

We can represent the Chua's circuit system in the chaotic Lur'e form with

$$A = \begin{bmatrix} -ac_1 & a & 0 \\ 1 & -1 & 1 \\ 0 & -b & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
$$W = \begin{bmatrix} a(c_1 - c_0) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

and $f_1(z_1(t)) = \frac{1}{2}(c_1 - c_0)(|z_1(t)+1| - |z_1(t) - 1|)$ belonging to sector $[0, 1]$, $f_2(z_2(t)) = f_3(z_3(t)) = 0$. Choosing the initial conditions of the master-system $\mathcal{M}$ and the slave-system $\mathcal{S}$ as $m(0) = [0.2\ 0.3\ 0.2]^T$ and $s(0) = [-0.3\ \ -0.1\ 0.4]^T$, respectively. Figs. 3 shows the master Chua's circuit system states $m(t)$.

For $\sigma = 0$, the proposed event-based communication mechanism (54) reduces to the sampled-data communication mechanism. Therefore, Corollary 1 can be used to obtain the maximum sampling period h. To show the reduced conservatism of the proposed method, we choose $C = [1\ 0\ 0]$, $\varepsilon = 2$ and $\gamma = 0$. Applying Corollary 1, one can obtain the different maximum sampling period h for different α, as shown in Table I. One can see that Corollary 1 can provide larger maximum sampling period h compared with [12]. On the other hand, we can choose a smaller sampling period h to make the master-slave CLSs reach the synchronization faster.

TABLE 1. Calculated maximum h for given α.

α	0.1	0.2	0.3	0.4	0.5
[12] (d=1)	0.3247	0.2941	0.2658	0.2396	0.2154
Corollary 1	0.3339	0.3123	0.2930	0.2757	0.2599

TABLE 2. Comparison of DTTs with different (a, b) for $t = 5s$.

(a,b)	(8.5,11.28)	(9,12.28)	(9,14.28)	(10,15.28)
DTTs	84	89	92	99

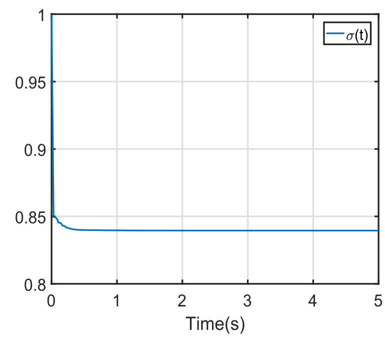

FIGURE 4. Adaptive threshold parameter $\sigma(t)$.

Choosing $\alpha = 0.1$, $h = 0.03$, $\sigma_0 = 1$, $\mu = 5$, $C = [1\ 0\ 0]$, $\varepsilon = 2$, $\gamma = 0$ and using Theorem 1, one can obatin the following controller gain and event-triggered matrix

$$K = [8.7459\ \ 2.0026\ \ -6.0295]^T, \tag{59}$$
$$\Omega = 0.2516, \quad \Psi = 3.6952. \tag{60}$$

For the controller gain (59) and event-triggered matrix (60), the adaptive threshold parameter $\sigma(t)$ is illustrated in Fig. 4, and the release instants under the event-triggered mechanism (9) is illustrated in Fig. 5. Moreover, the data transmission times (DTTs) based on event-triggered mechanism (9) is 92 in the time interval $[0, 5s]$. Compared with periodic sampling strategy [11], [12], [30], one can conclude that the event-trigger mechanism (9) can reduce the average amounts of sent measurements by almost 44.91%. From Fig. 6, one can see that the synchronization error finally converges to zero. Thus, the proposed event-based control scheme can reduce the amount of sent measurements while preserving the desired synchronization performance. Compared with the time-triggered sampling synchronization mechanism in [11], [12], and [30], the proposed event-triggered control scheme is more practical.

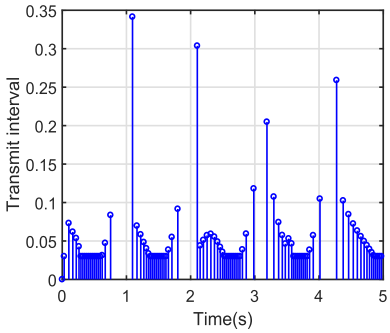

FIGURE 5. Release instants under the event-triggered (9).

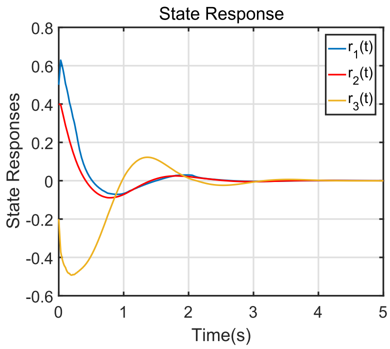

FIGURE 6. State response of error system (3).

Next, we shall show the effect of A, C, L and W variations on the response of the proposed adaptive event-triggered control mechanism. Choosing $\alpha = 0.1, h = 0.03, \sigma_0 = 1, \mu = 5$, $C = [1\ 0\ 0]$, $\varepsilon = 2$, $\gamma = 0$ and using Theorem 1, the DTTs with different (a, b) are given in Table II, which shows that the DTTs decreases with the decrease of (a, b). Therefore, in the case of other parameters have been determined, by choosing smaller (a, b), the proposed adaptive event-triggered control mechanism can save more communication resources.

Example 2: Consider the master-system $\mathcal{M}$ and slave-system $\mathcal{S}$ with the following parameters:

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
$$W = \begin{bmatrix} 1.2 & -1.6 & 0 \\ 1.24 & 1 & 0.9 \\ 0 & 2.2 & 1.5 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

which implies that the CLSs reduces to a neural network with three neurons, and $f_i(z_i(t)) = \frac{1}{2}(|z_i(t) + 1| - |z_i(t) - 1|)$ $(i = 1, 2, 3)$ are the neuron activation functions. One can see that $\rho_1 = \rho_2 = \rho_3 = 1$. Choosing the initial conditions of the master-system $\mathcal{M}$ and the slave-system $\mathcal{S}$ as $m(0) = [0.4\ 0.3\ 0.8]^T$ and $s(0) = [0.2\ 0.4\ 0.9]^T$, respectively. Figs. 7 shows the trajectory of the master-system $\mathcal{M}$.

Choosing $\alpha = 0.1$, $h = 0.05$, $\sigma_0 = 0.5$, $\mu = 50$ and using Theorem 1, one can obtain the following controller gain and

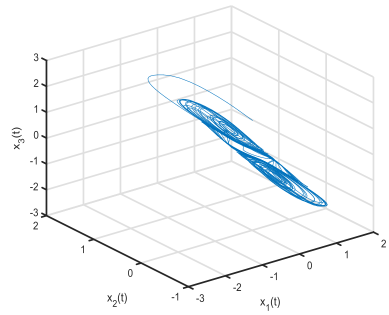

FIGURE 7. Master system $\mathcal{M}$.

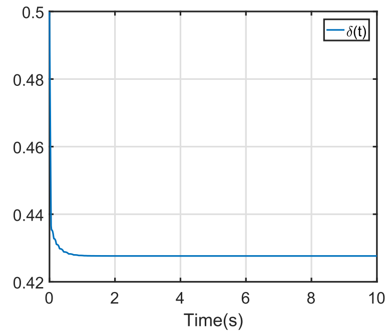

FIGURE 8. Adaptive threshold parameter $\sigma(t)$.

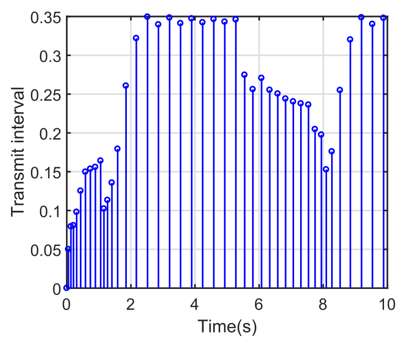

FIGURE 9. Release instants under the event-triggered (9).

event-triggered matrix

$$K = \begin{bmatrix} 2.2461 & 0.0570 & -1.0177 \\ 0.3564 & 2.0777 & 1.8940 \\ -1.0406 & 2.1457 & 3.4098 \end{bmatrix} \tag{61}$$
$$\Omega = \begin{bmatrix} 0.7301 & -0.0872 & 0.1165 \\ -0.0872 & 0.7661 & -0.1715 \\ 0.1165 & -0.1715 & 0.6829 \end{bmatrix} \tag{62}$$
$$\Psi = \begin{bmatrix} 1.7739 & 0.1290 & -0.2928 \\ 0.1290 & 1.6233 & 0.6026 \\ -0.2928 & 0.6026 & 1.9290 \end{bmatrix} \tag{63}$$

That is, under event-triggered mechanism (9), there exists a output-feedback controller (61) such that the slave-system $\mathcal{S}$ can asymptotical synchronize the master-system $\mathcal{M}$. For the above gain matrix, the adaptive threshold parameter $\sigma(t)$ is illustrated in Fig. 8, and the release instants under the

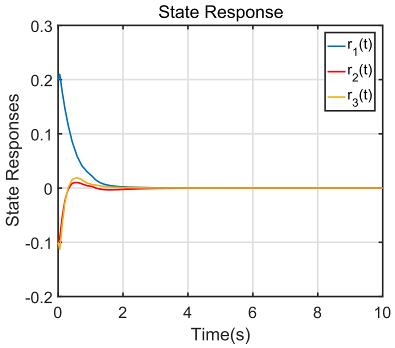

FIGURE 10. State response of error system (3).

event-triggered mechanism (9) is illustrated in Fig. 9. Over the time interval $[0, 10s]$, the event-trigger mechanism (9) can reduce the average amounts of sent measurements by almost 78.6% compared to periodic sampling. From Fig. 10, one can see that the synchronization error finally converges to zero.

V. CONCLUSION

The synchronization problem has been investigated for master-slave CLSs. By introducing a novel adaptive event-triggered control mechanism, the workload of the communication network can be reduced. Different from some existing event-triggered schemes, the threshold parameter of the proposed event-triggered mechanism can be dynamically adjusted. Based on the input delay analysis method, a novel LKF has been employed to derive a less conservative exponentially synchronization criterion for the considered CLSs. This criterion has then been used to design suitable controller gains in terms of solutions to a number of LMIs. We have finally illustrated the effectiveness of the proposed event-based control scheme through numerical examples. Our future research will focus on the distributed event-triggered control for multi-agent systems with uncertain Lur'e-type nonlinear dynamics.

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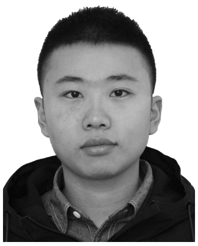

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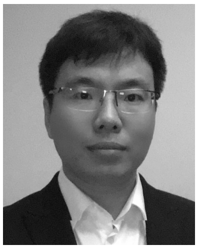

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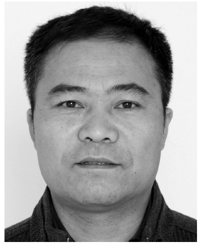

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