IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received August 28, 2018, accepted October 7, 2018, date of publication October 16, 2018, date of current version November 9, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2876398

Generalized-Extended-State-Observer-Based Repetitive Control for MIMO Systems With Mismatched Disturbances

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This work was supported by the National Natural Science Foundation of China under Grant 61673167 and Grant 61473313.

ABSTRACT This paper concerns the problem of designing a modified repetitive-control system based on a generalized extended state observer (GESO) for a class of multi-input, multi-output systems in the form of a nonintegral chain and with mismatched disturbances. A GESO is constructed to estimate a lumped disturbance in a real-time fashion. Then, an improved repetitive control law is developed to attenuate the influence of the lumped disturbance from the output channel. A correction to the amount of the delay of the repetitive controller is introduced to further reduce the steady-state tracking error. A stability criterion and a design algorithm are presented. Finally, simulations and comparisons with other methods demonstrate the validity of this method.

INDEX TERMS Repetitive control, generalized extended state observer, disturbance rejection, mismatched disturbance.

I. INTRODUCTION

In many engineering applications, reference inputs or disturbances are periodic signals, e.g., repeated tasks of a robotic manipulator [1], disturbances in a track-following servo system of optical disk drives [2], and position-dependent disturbances in a mechatronic rotary system [3]. Based on the internal model principle [4], repetitive control provides an effective way to deal with periodic signals. A repetitive controller contains a delay line in a positive-feedback loop, which results in the infinite dimension of the system [5]. In a repetitive-control system (RCS), self-learning is performed by periodically updating of the control input through the pure-delay line. However, since a RCS is a neutral-type delay system, it is difficult to stabilize. To derive a relaxed stability condition, a modified RCS (MRCS) was devised by inserting a low-pass filter in the delay line [6]. During the last three decades, numerous researches have been made on the analysis and synthesis of an MRCS [7]–[10]. But most articles only considered single-input, single-output (SISO) systems.

On the other hand, repetitive control cannot reject and even may amplify aperiodic disturbances [11]. A number of strategies have been proposed to deal with this problem, such as adaptive repetitive control [12], sliding-mode-based repetitive control [13], and H_∞ repetitive control [14]. However, they mainly focus on the system stability. In general, robustness is achieved at the price of sacrificing nominal tracking performance [15]–[20].

One intuitive idea to deal with a disturbance is first to estimate it from measurable variables, and then, taking a control action by making use of the estimate to compensate for the influence of the disturbance [21]. This motivated the development of active disturbance rejection methods for MRCS such as disturbance-observer-based repetitive control (DOB-RC) [22], equivalent-input-disturbance (EID)-based repetitive control (EID-RC) [11], [23]–[25], and extended-state-observer-based repetitive control (ESOB-RC) [26]. Comparing with single-degree-of-freedom control methods, such as extreme-learning-machine-based control [27] and neuro-adaptive-observer-based control [28], the active disturbance rejection methods proactively estimate and compensate for system disturbances. However, DOB-RC and ESOB-RC require that disturbances satisfy a matching condition, i.e., the disturbances can only be added on the same channel as that of control input. In addition, ESOB-RC can only deal with a class of SISO integral chain systems. While the EID-RC method can handle this kind of problems, the stability condition is strict and may fail in finding a feasible controller [11].

Mismatched disturbances and uncertainties widely exist in practical applications. They make the design much more challenging than the matched one. In this paper, an active mismatched disturbance attenuation method is first presented for an MRCS for a class of multi-input, multi-output (MIMO) systems in the form of a nonintegral chain. It is based on a generalized extended state observer (GESO) that estimates both the system states and the lumped disturbance including external disturbances and the unknown dynamics of the system. An integrated control law is constructed by properly choosing a disturbance compensation gain and incorporating the GESO into the MRCS. It ensures the perfect rejection of periodic disturbances and removes mismatched disturbances from the output channel. First, the formulation of a GESO is described and the configuration of a GESO-based MRCS is presented. Next, an integrated control law that combines a repetitive-control law and a disturbance-compensation feedback is devised to attenuate the influences of disturbances on the output. Then, a stability criterion and a design algorithm of the system are presented. Finally, a numerical example exhibits the design procedure. The comparisons with conventional MRCS and EID-based MRCS demonstrate the superiority of this method.

II. PROBLEM DESCRIPTION

This section gives the model of an MIMO system with a mismatched disturbance, the description of the GESO, and the configuration of a GESO-based MRCS.

A. PLANT MODEL

Consider an MIMO system affected by a mismatched disturbance:

$$\begin{cases} \dot{x}_p(t) = A_p x_p(t) + B_u u(t) + B_d f(x_p(t), w(t), t) \\ y(t) = C_p x_p(t) \end{cases} \tag{1}$$

where $x_p(t)$ ($\in \mathbb{R}^n$) is the state, $u(t)$ ($\in \mathbb{R}^m$) is the input, $w(t)$ ($\in \mathbb{R}^l$) is an external disturbance, $y(t)$ ($\in \mathbb{R}^p$) is the output, and $f(x_p(t), w(t), t)$ ($\in \mathbb{R}^s$) is a lumped disturbance that is an unknown nonlinear function of $x_p(t)$ and $w(t)$. This term contains external disturbances, parameter variation, unmodeled dynamics, and unknown nonlinearities. A_p is the system matrix, B_u and B_d are the input matrices, and C_p is the output matrix. They are of appropriate dimensions.

Remark 1: The model (1) represents a general class of systems that is not limited to the form of an integral chain and is subject to a mismatch disturbance [29]. The matching condition implies that $B_u = B_d$, or more precisely $B_u = B_d\Gamma$ for some invertible matrix Γ [21], which means that the disturbances can be transformed to the control input channel by coordinate transformation [21], [30].

Take the following second-order system as an example

$$\begin{cases} \dot{x}_1(t) = x_1(t) - x_2(t) + u(t) \\ \dot{x}_2(t) = 2x_1(t) + x_2(t) + f(x_1, x_2, w, t). \end{cases} \tag{2}$$

In (2), the disturbance, $f(x_1, x_2, w, t)$, enters the system via a different channel from the one of the control input, $u(t)$. Therefore, the matching condition is not satisfied.

Remark 2: The standard extended-state-observer based control (ESOBC) method cannot be applied to the system (1) because it requires that the system has the form of an integral chain and satisfies the matching condition [29]. The sliding-mode observer technique cannot be adapted as well due to the requirement for the observer matching condition [31]. Thus, it is imperative to develop a new control method for system (1). Motivated by this situation, this study devises a GESO-based control method.

B. GESO

Choose a new state as

$$\begin{cases} x_{n+1}(t) = d(t) = f(x_p(t), w(t), t) \\ h(t) = \dot{f}(x_p(t), w(t), t). \end{cases} \tag{3}$$

System (1) can be formulated in an augmented state space form

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Eh(x, w(t), t) \\ y(t) = Cx(t) \end{cases} \tag{4}$$

where $x(t) = \begin{bmatrix} x_p^T(t) & x_{n+1}^T(t) \end{bmatrix}^T$ and

$$A = \begin{bmatrix} A_p & B_d \\ 0_{s\times n} & 0_{s\times s} \end{bmatrix}, \quad B = \begin{bmatrix} B_u \\ 0_{s\times m} \end{bmatrix}$$

$$E = \begin{bmatrix} 0_{n\times 1} \\ 1_{s\times 1} \end{bmatrix}, \quad C = \begin{bmatrix} C_p & 0_{p\times s} \end{bmatrix}.$$

The following assumptions are made.

Assumption 1: $(A_p,\ B_u)$ is controllable and $(A,\ B)$ is observable.

Assumption 2: $B_u B_u^{\mathrm{T}}$ is invertible.

Assumption 3: The lumped disturbance $d(t)$ is bounded and satisfies the following conditions: 1) $d(t) = f(x_p, w, t) \approx \bar{f}(w, t)$; 2) it is constant in steady state, i.e., $\lim_{t\to\infty} \dot{d}(t) = \lim_{t\to\infty} h(t) = 0$.

Regarding Assumption 1, a necessary condition of $(A,\ B)$ observable is that $(A_p,\ B_u)$ is observable [29]. Assumption 2 is used to design the disturbance-compensation gain, and it holds for many practical control systems. Assumption 3 is used for stability analysis. The lumped disturbance may contain some state uncertainties and it is difficult to prove the stability for this case. Meanwhile, in many practical engineering systems, such state uncertainties are relatively weak and will not affect the system stability, i.e., the dominated dynamics can be stabilized by the feedback control [29]. The effectiveness of our proposed method in such a case has been demonstrated by numerical examples in Section IV.

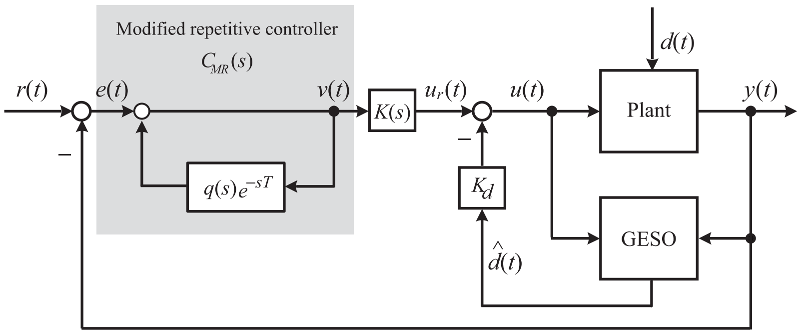

FIGURE 1. Configuration of GESO-based MRCS.

A GESO

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y - \hat{y}) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{5}$$

reproduces the state variables for system (4), where

$$\hat{x}(t) = \left[\hat{x}_p^T(t)\ \hat{x}_{n+1}^T(t) \right]^T$$

$\hat{x}_p(t)$ and $\hat{x}_{n+1}(t)$ estimate $x_p(t)$ and $x_{n+1}(t)$, respectively; and L is the observer gain to be determined.

Remark 3: In the presence of mismatched disturbances, the conventional ESOBC law [$u(t) = u_f(t) - \hat{d}(t)$] in [32] cannot effectively compensate for the lumped disturbance in (1), where $\hat{d}(t)$ represents the estimate of the lumped disturbance, and $u_f(t)$ is a feedback control law.

C. CONFIGURATION OF GESO-BASED MRCS

The configuration of the GESO-based MRCS is shown in Fig. 1. The external disturbance contains a periodic signal with the period of T_r. The fundamental angular frequency of the periodic signal is

$$\omega_0 = 2\pi/T_r. \tag{6}$$

$e(t) = r(t) - y(t)$ is the error to be suppressed. The transfer function of the repetitive controller is

$$C_{MR}(s) = \frac{1}{1 - q(s)e^{-Ts}}. \tag{7}$$

$q(s)$ is a low-pass filter. It is chosen to be a first-order one in this study

$$q(s) = \frac{\omega_c}{s + \omega_c} \tag{8}$$

where ω_c is the cutoff angular frequency of the filter, which satisfies the following frequency characteristics [6]:

$$\begin{cases} |q(j\omega)| \approx 1, & \omega \le \omega_r \\ |q(j\omega)| < 1, & \omega > \omega_r \end{cases} \tag{9}$$

where $[0,\ \omega_r] (\subset [0,\ \infty))$ is a selected frequency band for tracking and/or rejecting periodic signals. Thus, the state-space model of the repetitive controller is

$$\begin{cases} \dot{v}(t) = -\omega_c v(t) + \omega_c v(t-T) + \omega_c e(t) + \dot{e}(t) \\ y_r(t) = v(t) \\ v(t) = 0, \quad -T \le t \le 0 \end{cases} \tag{10}$$

where $v(t)$ is the output variable. The delay time T was set to be the period of the external periodic signal, T_r, in many articles on repetitive control. However, a slight adjustment of the amount of the delay

$$T = T_r - 1/\omega_c \tag{11}$$

leads to a higher control precision. The details is given in Appendix A.

In this paper, an integrated control law is designed as

$$u(t) = u_r(t) - K_d\hat{d}(t) \tag{12}$$

$$u_r(t) = L^{-1}\{K(s)V(s)\} \tag{13}$$

where

$$\hat{d}(t) = \hat{x}_{n+1}(t) \tag{14}$$

and $K(s)$ is the matrix-valued transfer function of the compensator that stabilizes the feedback system and $V(s)$ is the Laplace transform of $v(t)$. In (12), K_d is the disturbance compensation gain, designed as

$$K_d = B_u^+ B_d \tag{15}$$

where

$$B_u^+ := B_u^{\mathrm{T}} \left(B_u B_u^{\mathrm{T}} \right)^{-1}. \tag{16}$$

Remark 4: In (12), $u_r(t)$ focuses on system stability and dynamic performance, and the compensation loop focuses on disturbance rejection and robustness against the lumped disturbance. The calculation formula (15) for the disturbance compensation gain K_d is adaptable for both matching and mismatching cases. For the matching case, i.e., $B_u = B_d\Gamma$, it can be obtained from (15) that $K_d = \Gamma^{-1}$, which is the same as that for ESOBC [32].

For the GESO-based MRCS in Fig. 1, we aim to design a modified repetitive controller $C_{MR}(s)$, a GESO, and a compensator $K(s)$ so that both the bounded stability and the disturbance-rejection performance can be guaranteed. In this paper, we only consider the disturbance attenuation problem for the MIMO system (1) and we let $r(t) = 0$.

Substituting the composite control law (12) into system (1) and considering (15) and (16), the compensated plant (1) can be formulated as

$$\dot{x}_p(t) = A_p x_p(t) + B_u u_r(t) + B_d[f(x_p(t), w(t), t) - \hat{d}(t)]. \quad (17)$$

It can be observed from (17) that the effects of the lumped disturbance can be removed from the output channel in steady state provided that the lumped disturbance has been adequately estimated.

Remark 5: As pointed out in [21], [29], [33], the influence of the mismatched disturbance cannot be eliminated completely on the system state no matter what controller is designed. The main objective of the proposed GESO-based MRCS in this paper is to attenuate the influence of the lumped disturbance on the output.

III. STABILITY ANALYSIS AND DESIGN OF GESO-BASED MRCS

The estimation error is defined as

$$x_e(t) = \hat{x}(t) - x(t) \quad (18)$$

Combining (4), (5), and (18) yields

$$\dot{x}_e(t) = (A - LC)x_e(t) - Eh(t). \quad (19)$$

In Fig. 1, the transfer function matrix from $u_r(t)$ to $y(t)$ is

$$\begin{aligned} P(s) &= \begin{bmatrix} C & 0 \end{bmatrix} (sI - \bar{A})^{-1} \begin{bmatrix} B^T & 0 \end{bmatrix}^T \\ &= C(sI - A + B\tilde{K}_d)^{-1} B \end{aligned} \quad (20)$$

where the matrices

$$\bar{A} = \begin{bmatrix} A - B\tilde{K}_d & B\tilde{K}_d \\ 0 & A - LC \end{bmatrix}, \quad \tilde{K}_d = \begin{bmatrix} 0_{m\times n} & K_d \end{bmatrix}.$$

Remark 6: Since the transfer function matrix, $P(s)$, in (20) from $u_r(t)$ to $y(t)$ does not contain the observer gain, L, the GESO can be designed independent of the original MRCS and the compensator, as long as stability is the only concern.

Let

$$\begin{aligned} G(s) &= P(s)K(s) \\ \|G\|_\infty &:= \sup_{0\le\omega<\infty} \sigma_{max}[G(j\omega)] \end{aligned} \quad (21)$$

where $\sigma_{max}[\cdot]$ is the maximum singular value.

In Fig. 1, the transfer function matrix of the basic closed-loop system (the one without the delay loop) is

$$\tilde{G}(s) = G_0(s)G(s) \quad (22)$$

where

$$G_0(s) = (I + G(s))^{-1}. \quad (23)$$

Definition 1 ([34]): For a vector-valued function u defined on $[0, \infty)$, we denote by $P_\tau u$ its restriction to $[0, \tau]$. The system Σ is called *well-posed*, if on any finite time interval $[0, \tau]$, the operators from the initial state $x(0)$ and the input function $P_\tau u$ to the final state $x(\tau)$ and the output function $P_\tau y$ are bounded.

Lemma 1 ([32]): If $A - LC$ in (19) is Hurwitz, then the estimation error x_e for the GESO is bounded for any bounded $h(t)$.

Lemma 2 ([29]): The following single-input linear system

$$\dot{x}(t) = Mx(t) + gu(t) \quad (24)$$

is asymptotically stable if M is a Hurwitz matrix, u is bounded and satisfies $\lim_{t\to\infty} u(t) = 0$.

Proof. Since $\lim_{t\to\infty} u(t) = 0$, all poles of $sU(s)$ lie in the left half s plane. Also, all poles of $(sI - M)^{-1}$ lie in the left half s plane since M is Hurwitz. From the final value theorem [35], all poles of $sX(s)$ lie in the left half s plane and

$$\begin{aligned} \lim_{t\to\infty} x(t) &= \lim_{s\to 0} sX(s) = \lim_{s\to 0}(sI - M)^{-1} \cdot g \lim_{s\to 0} sU(s) \\ &= \lim_{s\to 0}(sI - M)^{-1} \cdot g \lim_{t\to\infty} u(t) = 0. \end{aligned}$$

We obtain the following theorem from the above definition and lemma.

Theorem 1: Suppose that Assumptions 1-3 are satisfied and $K(s)$ and $P(s)$ are proper rational matrix-valued transfer functions. The bounded stability of the MIMO system (1) under the integrated GESO-based repetitive-control law (12) with (15) is guaranteed if the following conditions are satisfied:

(a) $A - LC$ is Hurwitz;
(b) there are no unstable pole-zero cancelations in $P(s)K(s)$;
(c) $\tilde{G}(s) \in H_\infty$ and $\tilde{G}(s)$ is well posed;
(d) $\|qG_0\|_\infty < 1$

where H_∞ is the Hardy space, which consists of all complex-valued functions $\tilde{G}(s)$ of a complex variable s, and $\tilde{G}(s)$ is analytic and bounded in the open right half-plane, $Re(s) > 0$.

Proof. Since the GESO (5) can be designed independently of the original MRCS, the stability of the whole system is equivalent to that of these two subsystems.

Applying Lemmas 1 and 2 to (19), Condition (a) ensures that system (19) is stable and $\lim_{t\to\infty} x_e(t) = 0$ especially for single-input case. Meanwhile, as explained in [6], if the basic closed-loop system $\tilde{G}(s)$ in (22) is stable and $\|qG_0\|_\infty < 1$, then the original MRCS without disturbance is bounded-input bounded-output (BIBO) stable. In other words, Conditions (b), (c), and (d) imply that the original MRCS is BIBO stable.

So, it can be concluded that if L and $K(s)$ are selected such that $A - LC$ is Hurwitz and the original MRCS is stable, then the GESO-based MRCS is BIBO stable for any bounded $h(t)$ and $d(t)$. □

Summarizing the above results yields the following design algorithm for the GESO-based MRCS in Fig. 1.

Algorithm for designing the GESO-based MRCS:

Step 1 Design a low-pass filter, $q(s)$, satisfying the frequency characteristic (9).

Step 2 Design the observer gain, L, using the pole placement method hence to ensure $A - LC$ is Hurwitz.

Step 3 Calculate K_d from (15).

Step 4 Design the feedback compensator, $K(s)$, so that Conditions (b) and (c) in Theorem 1 hold. A PID or a lead-lag compensator is usually feasible [2], [11].

Step 5 Check if Condition (d) in Theorem 1 holds. If not, go to Step 2, and redesign L and $K(s)$.

IV. NUMERICAL EXAMPLE

In this section, we use a numerical example to explain the design procedure and demonstrate the superiority of the proposed GESO-based MRCS through comparisons with conventional PID-based repetitive control and EID-RC.

A second-order nonlinear system with mismatched disturbances is considered

$$\begin{cases} \dot{x}_1(t) = x_2(t) + u_2(t) + f(x(t), w(t), t) \\ \dot{x}_2(t) = -x_1(t) - 2x_2(t) + u_1(t) \\ y(t) = x_1(t) \end{cases} \tag{25}$$

where $u_2(t) = w_r(t) = \sin(2\pi t)$ is a known periodic external disturbance to be rejected. Thus, $T_r = 1\ s$.

By denoting

$$\begin{cases} u(t) = \begin{bmatrix} u_1(t) & u_2(t) \end{bmatrix}^T \\ f(x(t), w(t), t) = 3e^{x_1} + w(t) \\ w(t) = \begin{cases} 1.5 \arctan t, & t \geq 5 \\ 0, & t < 5 \end{cases} \\ A_p = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \ B_u = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ B_d = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ C_p = \begin{bmatrix} 1 & 0 \end{bmatrix} \end{cases} \tag{26}$$

it can be observed that system (25) has the expression of (1).

A. INTEGRATED CONTROL LAW DESIGN

The algorithm in Section III is used to design the GESO-based MRCS in Fig. 1.

For the repetitive controller, the cutoff angular frequency in (8) is chosen to be

$$\omega_c = 100\ rad/s. \tag{27}$$

From (11), we choose the delay of the repetitive controller to be

$$T = T_r - 1/\omega_c = 0.99. \tag{28}$$

The observer gain vector in (5) is chosen as

$$L = \begin{bmatrix} 28 & -257 & 500 \end{bmatrix}^T \tag{29}$$

and the related GESO poles are

$$p_{geso} = \begin{bmatrix} -10 & -10 & -10 \end{bmatrix}^T. \tag{30}$$

According to (15), the disturbance compensation gain is calculated, giving as

$$K_d = \begin{bmatrix} 0 & 1 \end{bmatrix}^T. \tag{31}$$

From (20), (26), and (31) yields

$$P(s) = \begin{bmatrix} \dfrac{1}{s^2 + 2s + 1} & \dfrac{s+2}{s^2 + 2s + 1} \end{bmatrix}. \tag{32}$$

The compensator, $K(s)$, is designed as

$$K(s) = \begin{bmatrix} 0 & 500 \times (1 + \dfrac{180}{s} + \dfrac{s}{s+1}) \end{bmatrix}^T. \tag{33}$$

Combining (21), (22), and (32), with (33), and using MATLAB Robust Control Toolbox, the transfer function of the basic closed-loop system (the one with $C_{MR}(s) = 1$) in (22) is given by

$$\tilde{G}(s) = \frac{1000s^3 + 92500s^2 + 271000s + 180000}{s^4 + 1003s^3 + 92503s^2 + 271001s + 180000} \in H_0^\infty. \tag{34}$$

Moreover,

$$\|q(s)G_0(s)\|_\infty = 5.5556 \times 10^{-6}. \tag{35}$$

Then, from Theorem 1 in Section III, it can be concluded from (30), (34), and (35) that the closed-loop GESO-based MRCS is stable.

B. SIMULATIONS

The time responses of the actual and estimated states and their estimation errors (Fig. 2) show that the GESO (5) is very effective in tracking the system (4) for not only the states, x_1 and x_2, but also the extended state (lumped disturbance) x_3. The estimation errors of GESO converge to zero for all states in the presence of mismatched disturbances.

The control objective is to attenuate the lumped disturbances from the output channel. Here the setpoint of the output is zero. The simulation results (Figs. 2-3) show that the GESO-based MRCS attenuates the disturbances $f(x_p, w(t), t)$ and $w_r(t)$; and suppresses the transient tracking error caused by $w(t)$. Clearly, the GESO-based MRCS is robustly stable and produces both satisfactory disturbance-rejection and control performance.

In practical applications, nonlinearities, such as a dead zone, seriously degrade control performance and may destabilize the control system. To verify the effectiveness of the method, a dead zone in the range of $[-0.2\ s\ \ 0.2\ s]$ was added to the input. The simulation result (Fig. 4) shows that the resulting system remains stable and the largest peak-to-peak (PTP) steady-state error is as small as 7.8950×10^{-5}. This implies that the GESO-based MRCS satisfactorily compensates for the dead zone.

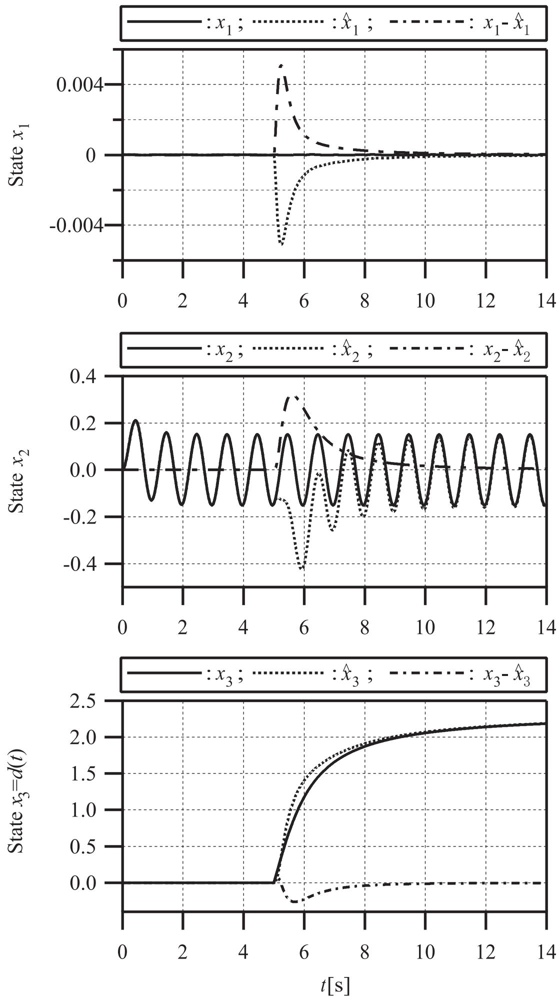

FIGURE 2. Time responses of the actual and estimated states, and their estimate errors.

C. COMPARISONS

The steady-state errors of the GESO-based MRCS and the conventional PID-based MRCS with the parameter (33) are compared (Fig. 5). The largest PTP steady-state error caused by the disturbances is 2.5681×10^{-3} for the PID-based MRCS, but it was only 3.4891×10^{-5} for the GESO-based MRCS. Clearly, the GESO automatically produced an offset as an estimate of the total disturbance, $d(t)$, and the effect of the disturbances on the output is 98.64% smaller for the GESO-based MRCS than for the conventional PID-based MRCS. This demonstrates that the incorporation of the GESO into an MRCS enhances the disturbance-rejection performance.

We also verify that the correction in (11) leads to a smaller steady-state error by redoing the simulation with the compensator (33) but with $T = T_r = 1$ s. Simulation results (Fig. 6) show that at the beginning, the result is similar to that with the correction in the time-delay; but the largest steady-state PTP error increases by 6.0654% than our method.

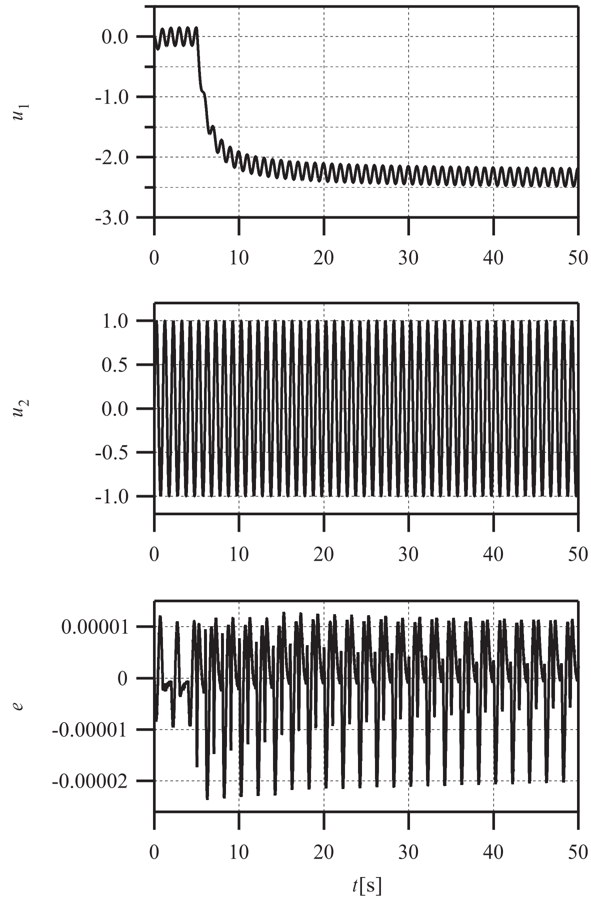

FIGURE 3. Simulation results for $f(x_p, w(t), t)$ and $w_r(t)$.

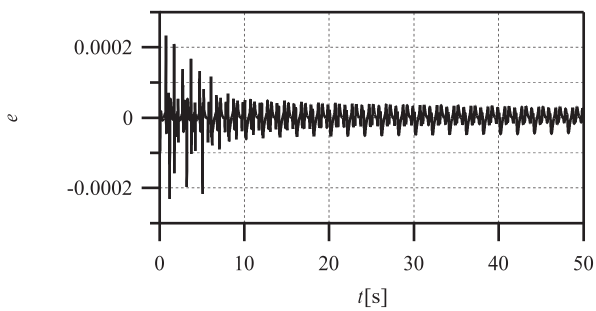

FIGURE 4. Simulation result with dead zone (range: [−0.2 s 0.2 s]) in the input.

To better assess the performance of the GESO-based MRCS, we design an EID-based MRCS (Fig. 7, [11]) and compare the disturbance rejection performance of the system with ours. Denoting

$$\bar{d}(t) = w_r(t) + f(x, w(t), t), \quad B_p = \begin{bmatrix} 0 & 1 \end{bmatrix}^T \tag{36}$$

and reformulating the system (25) gives the following SISO plant:

$$\begin{cases} \dot{x}_p(t) = A_p x_p(t) + B_p u(t) + B_d \bar{d}(t), \\ y(t) = C_p x_p(t). \end{cases} \tag{37}$$

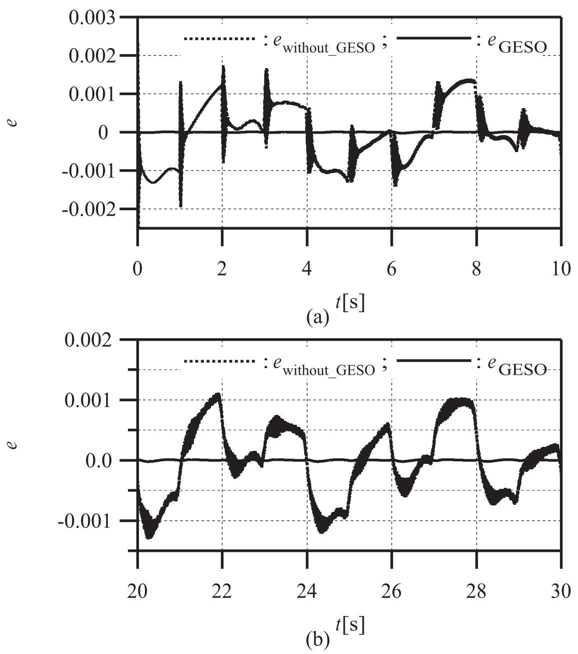

FIGURE 5. Tracking errors with and without GESO.

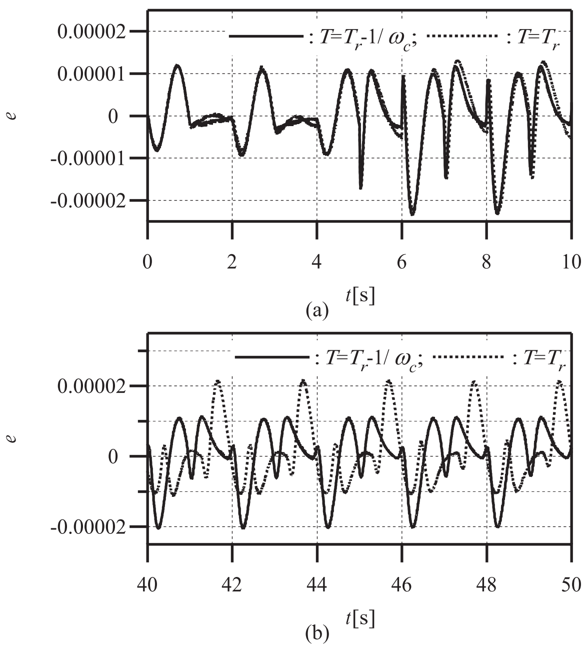

FIGURE 6. Comparisons between $T = T_r - \frac{1}{\omega_c}$ and $T = T_r$ as in [11], [23], [24].

Combining (8) and (33) and using the algorithms in [11], we obtain the parameters of the EID-based MRCS:

$$\begin{cases} T = 1\ s,\ F(s) = \dfrac{200}{s+200} \\ B^+ = (B_p^T B_p)^{-1} B_p^T = \begin{bmatrix} 0 & 1 \end{bmatrix} \\ L = [18\ \ 63]^T . \end{cases} \tag{38}$$

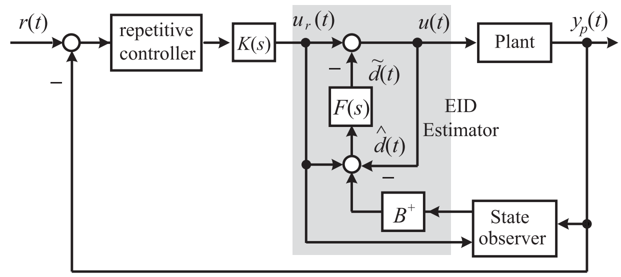

FIGURE 7. Configuration of EID-based MRCS.

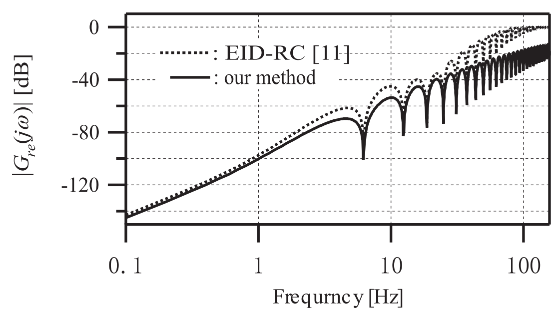

FIGURE 8. Bode magnitude plots of $G_{re}(s)$ without nonlinear part.

The related poles of the EID estimator are $p_{eid} = \begin{bmatrix} -10 & -10 \end{bmatrix}^T$.

From (25), (37), and (38) yields

$$\begin{cases} G_1(s) = 1 - B^+ L C_p [sI - (A_p - LC_p)]^{-1} B_p \\ \qquad = \dfrac{200s^2 + 400s + 7400}{s^3 + 220s^2 + 4100s + 20000} \\ P(s) = C_p(sI - A_p)^{-1} B_p = \dfrac{1}{s^2 + 3s + 1} \\ G(s) = P(s)K(s) = \dfrac{1000s^2 + 9.05 \times 10^4 s + 9 \times 10^4}{s^4 + 4s^3 + 4s^2 + s}. \end{cases} \tag{39}$$

Since $\|q(s)[1 + G(s)]^{-1}\| = 1.0647 > 1$ and two poles of $[1 + G(s)]^{-1} G(s)$ are located in right half plane, the EID-based MRCS (FIG. 7) is not stable. Therefore, the EID-RC method in [11] is not available for system (25). So, the proposed GESO-based repetitive-control method broadens the practical scope.

Moreover, the Bode magnitude of the transfer function from the reference input $r(t)$ to the tracking error, $G_{re}(s)$, (i.e., the sensitivity function) (Fig. 8) interprets the simulation results. Clearly, $G_{re}(j\omega)$ is higher for the EID-based MRCS in [11] than for the proposed GESO-based MRCS over almost the whole frequency band. Therefore, the GESO-based MRCS exhibits better disturbance-rejection performance than the EID-based MRCS in [11] and thus broadens the applications of repetitive control.

In addition, for SISO plant (37), since $C_p B_p = 0$, $P(s) = C_p(sI - A_p)^{-1} B_p$ has no zeros and the resulting EID-based MRCS are difficult to be stabilized using the methods in [23]–[25]. So, the proposed GESO-based repetitive-control method provides a better solution for this problem.

V. CONCLUSION

In this paper, a GESO-based repetitive-control method is first presented for a class of MIMO systems with mismatched disturbances and nonintegral-chain form. It involves real-time estimation and active compensation for the lumped disturbance. This method has significant advantages over other repetitive control methods.

a) By appropriately choosing a disturbance compensation gain, the incorporating of the disturbance estimate through GESO into the repetitive control law enables the active rejection of any type of disturbance/uncertainty and provides better control performance than a conventional MRCS does.
b) The GESO and the repetitive control law can be designed independently and a well-designed GESO can be plugged into a well-posed MRCS.
c) A slight correction for the delay of the repetitive controller enhances the control performance for the periodic signals.
d) The simplicity of the integrated control law makes the system easy to be implemented.

Guidelines for the selection of the controller parameters are given. Based on the examinations of simulation results, the roles of the GESO and the integrated control law are explained. In the future, we will explore the mechanism of disturbance/uncertainty estimation in a general nonlinear MRCS and the design of such a system.

APPENDIX

A. DETAIL INTERPRETATION FOR THE CHOICE OF T [34]

Let

$$e^{-Ts}q(s) = e^{-T_r s}q_1(s) \tag{40}$$

where $q_1(j\omega)$ should be as close as possible to 1 for $|\ \omega\ | \leq \ \omega_r$ so as to get the poles of $C_{MR}(s)$ as close as possible to $ik\omega_0$, $k = 1, 2, 3, \cdots$. From (40) yields

$$q_1(s) = e^{-(T-T_r)s}q(s) = \frac{e^{-(T-T_r)s}}{1/\omega_c s + 1}$$
$$= 1 + (T_r - T - 1/\omega_c)s + o(|\ s\ |^2).$$

Thus, a good way of getting $C_{MR}(s) \approx \dfrac{1}{1 - e^{-T_r s}}$ is to make

$$T_r - T - 1/\omega_c = 0. \tag{41}$$

We choose the cutoff frequency ω_c in (8) satisfying $\omega_0 \ll \omega_r \ll \omega_c$. Eq. (11) can be understood as a slight correction $1/\omega_c$ subtracted from the value T_r used in conventional repetitive controller, which is the same as the period of the periodic signals.

REFERENCES

[1] S. Liuzzo and P. Tomei, "A global adaptive learning control for robotic manipulators," *Automatica*, vol. 44, no. 5, pp. 1379–1384, 2008.
[2] T.-Y. Doh, J. R. Ryoo, and M. J. Chung, "Design of a repetitive controller: An application to the track-following servo system of optical disk drives," *IEE Proc.-Control Theory Appl.*, vol. 153, no. 3, pp. 323–330, May 2006.
[3] G. A. Ramos, J. Cortés-Romero, and H. Coral-Enriquez, "Spatial observer-based repetitive controller: An active disturbance rejection approach," *Control Eng. Pract.*, vol. 42, pp. 1–11, Sep. 2015.
[4] B. A. Francis and W. M. Wonham, "The internal model principle for linear multivariable regulators," *Appl. Math. Optim.*, vol. 2, no. 2, pp. 170–194, Jun. 1975.
[5] T. Inoue, M. Nakano, and S. Iwai, "High accuracy control of servomechanism for repeated contouring," in *Proc. 10th Annu. Symp. Incremental Motion Control Syst. Devices*, 1981, pp. 285–292.
[6] S. Hara, Y. Yamamoto, T. Omata, and M. Nakano, "Repetitive control system: A new type servo system for periodic exogenous signals," *IEEE Trans. Autom. Control*, vol. 33, no. 7, pp. 659–668, Jul. 1988.
[7] N. O. Pérez-Arancibia, T.-C. Tsao, and J. S. Gibson, "A new method for synthesizing multiple-period adaptive–repetitive controllers and its application to the control of hard disk drives," *Automatica*, vol. 46, no. 7, pp. 1186–1195, 2010.
[8] Y. Shan and K. K. Leang, "Accounting for hysteresis in repetitive control design: Nanopositioning example," *Automatica*, vol. 48, no. 8, pp. 1751–1758, 2012.
[9] M. Wu, L. Zhou, and J. She, "Design of observer-based H_∞ robust repetitive-control system," *IEEE Trans. Autom. Control*, vol. 56, no. 6, pp. 1452–1457, Jun. 2011.
[10] L. Zhou, J. She, M. Wu, and Y. He, "Design of a robust observer-based modified repetitive-control system," *ISA Trans.*, vol. 52, no. 3, pp. 375–382, 2013.
[11] M. Wu, B. Xu, W. Cao, and J. She, "Aperiodic disturbance rejection in repetitive-control systems," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 3, pp. 1044–1051, May 2014.
[12] C. Luo, J. Yao, F. Chen, L. Li, and Q. Xu, "Adaptive repetitive control of hydraulic load simulator with RISE feedback," *IEEE Access*, vol. 5, pp. 23901–23911, 2017.
[13] X.-D. Li, T. W. S. Chow, and J. K. L. Ho, "Quasi-sliding mode based repetitive control for nonlinear continuous-time systems with rejection of periodic disturbances," *Automatica*, vol. 45, no. 1, pp. 103–108, 2009.
[14] G. Weiss, Q.-C. Zhong, T. C. Green, and J. Liang, "H_∞ repetitive control of DC-AC converters in microgrids," *IEEE Trans. Power Electron.*, vol. 19, no. 1, pp. 219–230, Jan. 2004.
[15] J. Back and H. Shim, "Adding robustness to nominal output-feedback controllers for uncertain nonlinear systems: A nonlinear version of disturbance observer," *Automatica*, vol. 44, no. 10, pp. 2528–2537, 2008.
[16] J. Back and H. Shim, "An inner-loop controller guaranteeing robust transient performance for uncertain MIMO nonlinear systems," *IEEE Trans. Autom. Control*, vol. 54, no. 7, pp. 1601–1607, Jul. 2009.
[17] J. Yang, W.-H. Chen, and S. H. Li, "Non-linear disturbance observer-based robust control for systems with mismatched disturbances/uncertainties," *IET Control Theory Appl.*, vol. 5, no. 18, pp. 2053–2062, 2011.
[18] X. Zhao, P. Shi, and X. Zheng, "Fuzzy adaptive control design and discretization for a class of nonlinear uncertain systems," *IEEE Trans. Cybern.*, vol. 46, no. 6, pp. 1476–1483, Jun. 2016.
[19] H. Wang, P. X. Liu, and P. Shi, "Observer-based fuzzy adaptive output-feedback control of stochastic nonlinear multiple time-delay systems," *IEEE Trans. Cybern.*, vol. 47, no. 9, pp. 2568–2578, Sep. 2017.
[20] H. Wang, W. Sun, and P. X. Liu, "Adaptive intelligent control of nonaffine nonlinear time-delay systems with dynamic uncertainties," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 7, pp. 1474–1485, Jul. 2017.
[21] W.-H. Chen, J. Yang, L. Guo, and S. Li, "Disturbance-observer-based control and related methods—An overview," *IEEE Trans. Ind. Electron.*, vol. 63, no. 2, pp. 1083–1095, Feb. 2016.
[22] C. Smith and M. Tomizuka, "Shock rejection for repetitive control using a disturbance observer," in *Proc. 35th Conf. Decision Control*, Kobe, Japan, 1996, pp. 2503–2504.
[23] R.-J. Liu, G.-P. Liu, M. Wu, J. She, and Z.-Y. Nie, "Robust disturbance rejection in modified repetitive control system," *Syst. Control Lett.*, vol. 70, pp. 100–108, Aug. 2014.
[24] P. Yu, M. Wu, J. She, K.-Z. Liu, and Y. Nakanishi, "An improved equivalent-input-disturbance approach for repetitive control system with state delay and disturbance," *IEEE Trans. Ind. Electron.*, vol. 65, no. 1, pp. 521–531, Jan. 2018.

[25] L. Zhou, J. She, M. Wu, Y. He, and S. Zhou, "Estimation and rejection of aperiodic disturbance in a modified repetitive-control system," *IET Control Theory Appl.*, vol. 8, no. 10, pp. 882–889, 2014.

[26] A. H. M. Sayem, Z. W. Cao, and Z. H. Man, "Performance enhancement of ADRC using RC for load frequency control of power system," in *Proc. IEEE 8th Conf. Ind. Electron. Appl.*, 2013, pp. 433–438.

[27] C. Yang, K. Huang, H. Cheng, Y. Li, and C.-Y. Su, "Haptic identification by ELM-controlled uncertain manipulator," *IEEE Trans. Syst., Man, Cybern. A, Syst.*, vol. 47, no. 8, pp. 2398–2409, Aug. 2017.

[28] X. Liu, C. Yang, Z. Chen, M. Wang, and C.-Y. Su, "Neuro-adaptive observer based control of flexible joint robot," *Neurocomputing*, vol. 275, pp. 73–82, Jan. 2018.

[29] S. Li, J. Yang, W.-C. Chen, and X. Chen, "Generalized extended state observer based control for systems with mismatched uncertainties," *IEEE Trans. Ind. Electron.*, vol. 59, no. 12, pp. 4792–4802, Dec. 2012.

[30] B. Barmish and G. Leitmann, "On ultimate boundness control of uncertain systems in the absence of matching assumption," *IEEE Trans. Autom. Control*, vol. 27, no. 1, pp. 153–158, Feb. 1982.

[31] C. Edwards and S. Spurgeon, *Sliding Mode Control: Theory and Applications*. London, U.K.: Taylor & Francis, 1998.

[32] Z. Gao, "Active disturbance rejection control: A paradigm shift in feedback control system design," in *Proc. Amer. Control Conf.*, 2006, pp. 2399–2405.

[33] A. Isidori, *Nonlinear Control Systems: An Introduction*, 3rd ed. New York, NY, USA: Springer-Verlag, 1995.

[34] G. Weiss and M. HäKfele, "Repetitive control of MIMO systems using H^{∞} design," *Automatica*, vol. 35, no. 7, pp. 1185–1199, 1999.

[35] K. Ogata, *Modern Control Engineering*, 3rd ed. Englewood Cliffs, NJ, USA: Prentice-Hall, 1997.

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