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Decentralized Non-Fragile Event-Triggered H_∞ Filtering for Large-Scaled Power System Based on T-S Fuzzy Model

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ABSTRACT Considering the characteristics of the large-scaled power system and the communication networks, this paper studies the decentralized non-fragile H_∞ filtering for the large-scaled power system with the event-triggered strategy. The Takagi–Sugeno fuzzy model is used to approximate the system. Different from traditional time-triggered control, each subsystem transmits the output only when the defined error exceeds a given threshold in event-triggered control. Moreover, a non-fragile event-triggered H_∞ filtering with additive uncertainties is designed to solve the sensitive problem of the sensors. By applying a Lyapunov–Krasovskii functional and linear matrix inequalities, a method of decentralized event-triggered H_∞ filtering design is developed for the overall filtering error system concerned to be asymptotically stable with a given disturbance attenuation level. A simulation is finally given to show these effective methods in this paper.

INDEX TERMS Event-triggered strategy, non-fragile H_∞ filtering, large-scaled power system, linear matrix inequalities (LMIs), Takagi–Sugeno fuzzy system.

I. INTRODUCTION

The event-triggered control system has attracted growing attention recently [1]. Traditional control systems execute sampling and control commands periodically. We usually call this system time-triggered control system. However, the conventional time-triggered control scheme may have some negative effects on the control performance. For example, it wastes limited bandwidth and computing power, even shortens the lifetime of the whole system. So this paper adopts event-triggered filtering which is an important part of the control theory, only when the defined error reaches the specified threshold, the event is triggered and the data of the filtering is updated. therefore, the system achieves the balance between communication efficiency and control performance.

With the growth of the electricity industry, the centralized monitoring can not meet the needs anymore. The emergence of the communication networks has received much attention in the last decades which gives a new inspiration for the large-scaled systems, such as the power system and the multi-agent system [2], [3]. The communication networks, however, have many inherent disadvantages, such as the limited energy

resources and the bandwidth. Reference [4] proposed a centralized event-triggered control, in which the data is updated at the same time. Unlike centralized event-triggered control, each subsystem transmits its output based on information from itself in distributed event-triggered control [5]–[8]. Based on the previous study, we introduce the distributed event-triggered control scheme. Under the scheme introduced in [9], the subsystem transmits data over the networks only when its local specified threshold is exceeded. Consequently, it reduces the amount of data into the networks. Meanwhile, the energy consumption is reduced as well owing to the event-triggered releasing mechanism.

It is well-known that the power system itself is a strong coupling, non-linear system which is called the large-scaled system. In the large-scaled system, subsystems connect to each other and transmit the information for each other. These subsystems which have their own control inputs, measurement outputs as well as states. The T-S fuzzy model, an universal approximator for non-linear systems, has many studies [10], [11]. We use T-S fuzzy model to approximate the power system for stability analysis and filtering design. The term 'fuzzy' means it can deal with the concepts which

can't be accurate expressed as 'true' or 'false' but rather as 'partially true'. So, the fuzzy logic is better to describe non-linear systems. For example, T-S fuzzy model-based networked systems had become a popular issue [12]. Additionally, [13] investigated the H_∞ filtering for discrete-time non-linear systems based on T-S fuzzy model to consider multiple sensor faults. The above problems and realities also motivate this paper.

H_∞ filtering, which attracts much attention in the past decades [14]–[18], is a good tool for estimating the system states. The H_∞ filtering does not make any assumption about noise characteristics, only requires that the disturbance is energy limited and the energy gain from the disturbance to the estimation error is bounded by a certain level. In the actual power system, the filtering affected by software and hardware will cause some parameter errors because sensor nodes are distributed in the harsh environment usually. And the parameters of these sensors are very sensitive when there is a slight change in the surrounding environment, which is called 'fragility'. Thus, the filtering should be designed insensitive to some errors with respect to its gain. Inspired by this imperfect, theoretical results on non-fragile could be found in the following references. Reference [19] studied the non-fragile H_∞ filtering of non-linear system described by a T-S fuzzy model. Reference [20] discussed the non-fragile H_∞ filtering of non-linear delay systems. Reference [21] investigated the non-fragile controllers based on event-triggered control. In this paper, we study the non-fragile filtering for the large-scaled power system to guarantee the H_∞ performance.

Motivated by the above reasons, this paper concerns the non-fragile event-triggered H_∞ filtering for the large-scaled power system. The main contributions of this paper are summarized as follows:

- 1) We first propose non-fragile H_∞ filtering for large-scaled power system with decentralized event-triggered control (Fig. 1). The subsystem can be transmitted to the adjacent filtering only when its local state error exceeds the threshold value, which reduces the frequency of the update of the signal and saves the resources.
- 2) We adopt decentralized non-fragile filtering for the wireless sensor networks due to two common problems. (i) Sensor nodes, which are deployed in a harsh environment, are corrupted or destroyed easily. Thus the disturbance of the parameters called 'fragility'. (ii) With the growing expansion of power systems,

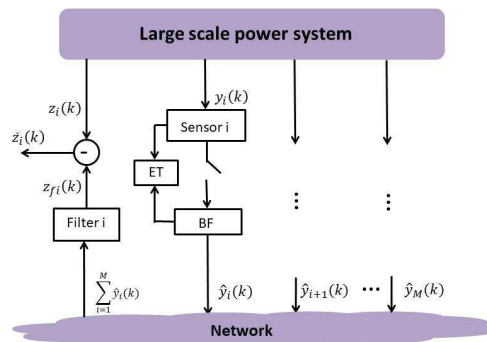


FIGURE 1. Event-triggered filtering of large-scaled power system.

the traditional estimation method for the overall state is no longer applicable. So we construct decentralized filtering to estimate the state of each subsystem. What's more, distributed filtering collect data from itself and its all neighbors.

- 3) The T-S fuzzy model has been proposed to cope with the event-triggered non-fragile filtering. Based on the Lyapunov-Krasovskii functional, a new sufficient condition is obtained so that the filtering error system is asymptotically stable and achieves a prescribed filtering performance. A sufficient condition for the event-triggered non-fragile filter is established by using linear matrix inequalities (LMIs).

II. PROBLEM FORMULATION

The event-triggered filtering system is illustrated in Fig. 1. In this section, we will construct the large-scaled power system with non-fragile T-S fuzzy model and the event-triggered condition mathematically.

Considering a M machine power subsystems with disturbance as [22]. First, we transform the large-scaled system toward a discrete-time T-S fuzzy system by the general linearisation and the discretisation with sampling time T (see (1), as shown at the bottom of this page).

where $x_{i1}(k)$, $x_{i2}(k)$ are the absolute rotor angle and angular velocity of the i th machine, respectively; M_i is the inertia coefficient and D_i is the damping coefficient; E_i is the internal voltage; Y_{mi} is the modulus of the transfer admittance between the m th and i th machines; θ_{mi} is the phase angle of the transfer admittance between the m th and i th machines; δ_{mi}^0 is the phase angle in steady state of the transfer admittance between the i th and m th machines.

$$\begin{cases} x_{i1}(k+1) = x_{i1}(k) + Tx_{i2}(k) \\ x_{i2}(k+1) = x_{i2}(k) \\ \quad + T\left\{\frac{D_i}{M_i}x_{i2}(k) + \sum_{m=1, m \neq i}^M \frac{E_i E_m Y_{mi}}{M_i} [\cos(\delta_{mi}^0 - \theta_{mi}) - \cos(x_{i1}(k) - x_{m1}(k) + \delta_{mi}^0 - \theta_{mi})] + w_i(k)\right\} \\ y_i(k) = x_i(k). \end{cases} \quad (1)$$

The above large-scaled power system can be described by the T-S model as follow, which represents the subsystem S_i by fuzzy IF-THEN rules:

Rule j : IF $\theta_{i1}(t)$ is F_{ij1} and...and $\theta_{ig}(t)$ is F_{ijg} THEN

$$S_i : \begin{cases} x_i(k+1) = \sum_{j=1}^{r_i} h_{ij}(\theta_i(k)) [A_{ij}x_i(k) + W_{ij}w_i(k) \\ \quad + \sum_{m=1, m \neq i}^M C_{mi}x_m(k)] \\ z_i(k) = \sum_{j=1}^{r_i} h_{ij}(\theta_i(k)) [H_{ij}x_i(k)]. \end{cases} \quad (2)$$

For the sensors, the measurement model is given as

$$y_i(k) = \sum_{j=1}^{r_i} h_{ij}(\theta_i(k)) D_{ij}x_i(k), \quad (3)$$

where $j = \{1, 2, \dots, r_i\}$ is the j th inference rule, and r_i denotes the number of inference rules; θ_i are premise variables; F_{ijg} denote fuzzy sets. $x_i(k) \in R^{n_{ix}}$ stands for the state vector, $y_i(k) \in R^{n_{iy}}$ is the output of the plant, $z_i(k) \in R^{n_{iz}}$ is the signal to be estimated, $w_i(k) \in R^{n_{iw}}$ is the disturbance. A_{ij} , W_{ij} , E_{ij} , H_{ij} and D_{ij} are constant known matrices. C_{mi} denotes the interconnected effect between the m th and i th subsystem, and $C_{mi} = 0$ for $m = i$.

Denote

$$h_{ij}(\theta_i(k)) = \frac{\mu_{ij}(\theta_i(k))}{\sum_{j=1}^{r_i} \mu_{ij}(\theta_i(k))},$$

which is assumed that $\mu_{ij}(\theta_i(k)) \geq 0$, $h_{ij}(\theta_i(k)) \geq 0$, $\sum_{j=1}^{r_i} h_{ij}(\theta_i(k)) = 1$ where $i = 1, 2, \dots, M$.

In this paper, we consider the following non-fragile filtering for the fuzzy subsystem S_{fi} :

$$S_{fi} : \begin{cases} x_{fi}(k+1) = \sum_{j=1}^{r_i} h_{ij}(\theta_i(k)) [(A_{fij} + \Delta A_{fij})x_{fi}(k) \\ \quad + B_{fij}y_i(t_k) + \sum_{m=1, m \neq i}^M B_{fmi}y_m(t_k)] \\ z_{fi}(k) = \sum_{j=1}^{r_i} h_{ij}(\theta_i(k)) C_{fij}x_{fi}(k), \end{cases} \quad (4)$$

with

$$\Delta A_{fij} = E_{ij}\Delta(k)F_{ij},$$

where $x_{fi}(k)$, $z_{fi}(k)$ are the state variable of the filtering and the estimation of the $z_i(k)$; A_{fi} , B_{fi} and C_{fi} are filtering parameter matrices to be determined. ΔA_{fij} is a gain variation. E_{ij} and F_{ij} are known real matrices and $\Delta(k)$ satisfies

$$\Delta^T(k)\Delta(k) \leq I.$$

In the large-scaled power system which is introduced in Fig. 1, in order to save communication resources, event-triggered control is introduced to determine whether or not

the sampled data should be transmitted. We compare the output error between the current data $y_i(k)$ and the last transmitted data $y_i(t_k)$ for each subsystem i . If the current data $y_i(k)$ satisfies a given condition, then it will be transmitted into the networks and become the new last released data, else, the current data will not be transmitted and the last released data will not be changed. Considering the following discrete-time event-triggered detector:

$$t_{k+1} = \inf \left\{ k \mid e_i^T(k) Q_i e_i(k) \geq \sigma_i^2 y_i^T(k) Q_i y_i(k) \right\}, \quad (5)$$

where $k \in \mathbb{N}$ and $Q_i = Q_i^T > 0$. $e_i(k) = y_i(k) - y_i(t_k)$ denotes the deviation from the last transmitted data to the current data. Considering (5), an event-triggered strategy is given by

$$y_i(t_{k+1}) = \begin{cases} y_i(k), & e_i^T(k) Q_i e_i(k) \geq \sigma_i^2 y_i^T(k) Q_i y_i(k) \\ y_i(t_k), & e_i^T(k) Q_i e_i(k) < \sigma_i^2 y_i^T(k) Q_i y_i(k). \end{cases} \quad (6)$$

Then, the fuzzy non-fragile filtering can be written as follow:

$$S_{fi} : \begin{cases} x_{fi}(k+1) = \sum_{j=1}^{r_i} h_{ij}(\theta_i(k)) [(A_{fij} + \Delta A_{fij})x_{fi}(k) \\ \quad + B_{fij}(e_i(k) + y_i(k)) \\ \quad + \sum_{m=1, m \neq i}^M B_{fmi}(e_m(k) + y_m(k))] \\ z_{fi}(k) = \sum_{j=1}^{r_i} h_{ij}(\theta_i(k)) C_{fij}x_{fi}(k). \end{cases} \quad (7)$$

By defining a new state vector $\xi_i(k) = [x_{fi}^T(k) \quad x_{fi}^T(k)]^T$ and $\tilde{z}_i(k) = z_i(k) - z_{fi}(k)$, we can obtain the filtering error subsystem i :

$$\Xi_i : \begin{cases} \xi_i(k+1) = \sum_{j=1}^{r_i} h_{ij}(\theta_i(k)) [\tilde{A}_{ij}\xi_i(k) + \tilde{W}_{ij}w_i(k) \\ \quad + \tilde{B}_{fij}e_i(k) + \sum_{m=1, m \neq i}^M \tilde{B}_{fmi}e_m(k) \\ \quad + \sum_{m=1, m \neq i}^M \tilde{C}_{mi}\xi_m(k)] \\ \tilde{z}_i(k) = \sum_{j=1}^{r_i} h_{ij}(\theta_i(k)) \tilde{H}_{fij}\xi_i(k), \end{cases} \quad (8)$$

where

$$\begin{aligned} \tilde{A}_{ij} &= \begin{bmatrix} A_{ij} & 0 \\ B_{fij}D_{ij} & A_{fij} + \Delta A_{fij} \end{bmatrix}, & \tilde{W}_{ij} &= \begin{bmatrix} W_{ij} \\ 0 \end{bmatrix}, \\ \tilde{B}_{fmi} &= \begin{bmatrix} 0 \\ B_{fmi} \end{bmatrix}, & \tilde{B}_{fij} &= \begin{bmatrix} 0 \\ B_{fij} \end{bmatrix}, \\ \tilde{C}_{mi} &= \begin{bmatrix} C_{mi} & 0 \\ B_{fmi}D_m & 0 \end{bmatrix}, & \tilde{H}_{ij} &= [H_{ij} \quad -C_{fij}], \\ \tilde{D}_{ij} &= [D_{ij} \quad 0], \\ \tilde{E}_{ij} &= \begin{bmatrix} 0 & 0 \\ 0 & E_{ij} \end{bmatrix}, & \tilde{F}_{ij} &= \begin{bmatrix} 0 & 0 \\ 0 & F_{ij} \end{bmatrix}. \end{aligned} \quad (9)$$

The non-fragile event-triggered filtering problem of this fuzzy system to be addressed in this paper can be formulated as follows. Given a prescribed level of noise attenuation $\gamma > 0$, $\tilde{z}(k) = [\tilde{z}_1^T(k) \tilde{z}_2^T(k), \dots, \tilde{z}_M^T(k)]^T$ and $w(k) = [w_1^T(k) w_2^T(k), \dots, w_M^T(k)]^T$. The filtering in the form of (4) for satisfying the following two requirements:

1) Asymptotically Stability: The error system composed of M filtering-error subsystems (8) with $w(t) = 0$ is asymptotically stable.

2) H_∞ Performance: Under zero initial conditions, the following overall H_∞ performance is satisfied

$$\|\tilde{z}(k)\|_2 < \gamma \|w(k)\|_2.$$

Remark 1: With the growing expansion of power systems, a large-scaled power system has become a regular model. The traditional method for a large-scaled power system, estimating the overall state, is no longer applicable. To obtain the overall state estimation in one site, the filtering has a heavy burden, and the calculation is very large, so monitoring each subsystem states is particularly important. In this paper, we will estimate the state of each subsystem $\tilde{z}_i(k) = [(z_1(k) - z_{f1}(k))^T \dots (z_J(k) - z_{fJ}(k))^T]^T$.

Remark 2: In the event-triggered control, only some of the data will be sent out to the filtering side. If we decrease σ , the rate of data transmission will increase accordingly. At one extreme, if the value of σ tends to zero, the event-triggered scheme is not considered and $t_{k+1} = t_k + 1$ from (5), we will have a traditional time-triggered filtering system.

Remark 3: We can consider $e_i(k) = y_i(k) - y_i(t_k)$ as a disturbance. If $\|y_i(k) - y_i(t_k)\|$ is sufficiently small, no transmissions between the sensor and filtering systems are needed.

The following, some lemmas are presented.

Lemma 1 [23]: For matrices \mathcal{H} , $\Delta(t)$ and \mathcal{W} with suitable dimensions and $\Delta^T(t) \Delta(t) \leq I$, the following inequality:

$$\mathcal{H} \Delta(t) \mathcal{W} + \mathcal{W}^T \Delta^T(t) \mathcal{H}^T \leq \kappa \mathcal{H} \mathcal{H}^T + \kappa^{-1} \mathcal{W}^T \mathcal{W}.$$

Lemma 2 (Schur Complement) [3]: For a given symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$, the matrix S_{11} is the $r \times r$ matrix. The following three conditions are equivalent :

- 1) $S < 0$,
- 2) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$,
- 3) $S_{22} < 0$, $S_{11} - S_{12}^T S_{22}^{-1} S_{12} < 0$.

III. MAIN RESULTS

A. STABILITY AND EVENT-TRIGGERED NON-FRAGILE H_∞ PERFORMANCE ANALYSIS

In this section, we will present a sufficient condition for the non-fragile event-triggered H_∞ filtering in the form of (6).

Theorem 1: For given scalars $\gamma, \sigma, \varepsilon_{i1}, \varepsilon_{i2}, \varepsilon_{i3}, \varepsilon_{i4}$, the non-fragile event-triggered filtering error system which is composed of M filtering-error subsystems as (8) is asymptotically stable. If there exist matrices $P_i > 0, Q_i > 0, \Phi_i > 0$ with

appropriate dimensions, satisfying

$$\begin{bmatrix} \Phi & \Pi_i^T \\ * & -I \end{bmatrix} < 0, \tag{10}$$

for all $i = 1, 2, \dots, M$, where

$$\Phi = \begin{bmatrix} \Xi & \mathbb{B}_{fmi} & \mathbb{C}_{mi} \\ * & -\mathbb{M}_i & 0 \\ * & * & -\mathbb{N}_i \end{bmatrix},$$

$$\Xi = \Xi_1 + \Xi_2,$$

$$\Xi_1 = \begin{bmatrix} P_i & 0 & 0 & 0 \\ 0 & \sigma_i^2 \tilde{D}_{ij}^T Q_i \tilde{D}_{ij} - P_i + \sum_{i=1}^M N_i & 0 & 0 \\ 0 & 0 & \sum_{i=1}^M M_i - Q_i & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Xi_2 = \Psi_i \begin{bmatrix} -I & \tilde{A}_{ij} & \tilde{B}_{fij} & \tilde{W}_{ij} \\ -I & \tilde{A}_{ij} & \tilde{B}_{fij} & \tilde{W}_{ij} \end{bmatrix}^T \Psi_i^T,$$

$$\mathbb{B}_{fmi} = [\Psi_i \tilde{B}_{f1i} \dots \Psi_i \tilde{B}_{fmi, m \neq i} \dots \Psi_i \tilde{B}_{fMi}],$$

$$\mathbb{C}_{mi} = [\Psi_i \tilde{C}_{1i} \dots \Psi_i \tilde{C}_{mi, m \neq i} \dots \Psi_i \tilde{C}_{Mi}],$$

$$\mathbb{M}_i = \text{diag}[M_1 \dots M_{m, m \neq i} \dots M_M],$$

$$\mathbb{N}_i = \text{diag}[N_1 \dots N_{m, m \neq i} \dots N_M],$$

$$\Pi_i = [\tilde{H}_{ij} \ 0 \ 0 \ 0].$$

Proof: Our main research is to guarantee the event-triggered non-fragile filtering error system that is composed of M filtering-error H_∞ performance under zero initial conditions. First, we construct a Lyapunov-function for the filtering error systems as:

$$V(k) = \sum_{m=1}^M V_i(k) = \sum_{m=1}^M \xi_i^T(k) P_i \xi_i(k) \tag{11}$$

where $P_i > 0$. Defining $\Delta V_i(k) = V_i(k+1) - V_i(k)$, we get

$$\Delta V(k) = \sum_{i=1}^M [\xi_i^T(k+1) P_i \xi_i(k+1) - \xi_i^T(k) P_i \xi_i(k)]. \tag{12}$$

In addition, we know the following:

$$\begin{aligned} 0 &= 2 \sum_{i=1}^M \chi_i^T(k) \{ \Psi_i [-\xi_i(k+1) + \tilde{A}_{ij} \xi_i(k) \\ &\quad + \tilde{W}_{ij} w_i(k) + \tilde{B}_{fij} e_i(k)] \\ &\quad + [\sum_{m=1, m \neq i}^M \tilde{B}_{fmi} e_m(k) + \sum_{m=1, m \neq i}^M \tilde{C}_{mi} \xi_m(k)] \}, \end{aligned} \tag{13}$$

where $\Psi_i = [R_i^T \ 0 \ 0 \ 0]^T$. Here, we introduce the slack matrix R_i^T . It is very useful for the derivation of linear matrix

inequalities (LMIs) to the H_∞ filtering design. Now (13) can be rewritten as:

$$\begin{aligned}
 0 &= 2 \sum_{i=1}^M \chi_i^T(k) \{ \Psi_i [-\xi_i(k+1) + \tilde{A}_{ij} \xi_i(k) \\
 &\quad + \tilde{W}_{ij} w_i(k) + \tilde{B}_{fij} e_i(k)] \\
 &\quad + [\sum_{m=1, m \neq i}^M \tilde{B}_{fmi} e_m(k) + \sum_{m=1, m \neq i}^M \tilde{C}_{mi} \xi_m(k)] \} \\
 &\equiv 2 \sum_{i=1}^M \chi_i^T(k) \Psi_i \\
 &\quad \times [-I \tilde{A}_{ij} \tilde{W}_{ij} \tilde{B}_{fij}] \chi_i(k) \\
 &\quad + \sum_{i=1}^M \chi_i^T(k) \Psi_i \sum_{m=1, m \neq i}^M \tilde{B}_{fmi} e_m(k) \\
 &\quad + \sum_{i=1}^M \chi_i^T(k) \Psi_i \sum_{m=1, m \neq i}^M \tilde{C}_{mi} \xi_m(k). \tag{14}
 \end{aligned}$$

Note that

$$2a^T b \leq a^T M^{-1} a + b^T M b, \tag{15}$$

where $M = M^T > 0$, and we have the following result:

$$\begin{aligned}
 &2 \sum_{i=1}^M \chi_i^T(k) \Psi_i \sum_{m=1, m \neq i}^M \tilde{B}_{fmi} e_m(k) \\
 &\leq \sum_{i=1}^M \sum_{m=1, m \neq i}^M \chi_i^T(k) \Psi_i \tilde{B}_{fmi} M_i^{-1} \tilde{B}_{fmi}^T \Psi_i^T \chi_i(k) \\
 &\quad + \sum_{i=1}^M \sum_{m=1, m \neq i}^M e_i^T(k) M_i e_i(k), \tag{16}
 \end{aligned}$$

and

$$\begin{aligned}
 &2 \sum_{i=1}^M \chi_i^T(k) \Psi_i \sum_{m=1, m \neq i}^M \tilde{C}_{mi} \xi_m(k) \\
 &\leq \sum_{i=1}^M \sum_{m=1, m \neq i}^M \chi_i^T(k) \Psi_i \tilde{C}_{mi} N_i^{-1} \tilde{C}_{mi}^T \Psi_i^T \chi_i(k) \\
 &\quad + \sum_{i=1}^M \sum_{m=1, m \neq i}^M \xi_i^T(k) N_i \xi_i(k). \tag{17}
 \end{aligned}$$

Defining $\chi_i(k) = [\xi_i^T(k+1) \ \xi_i^T(k) \ e_i^T(k) \ w_i^T(k)]^T$ and $\Delta V_i(k) = V_i(k+1) - V_i(k)$. And we can conclude from the event-triggered strategy (6) that

$$\sigma_i^2 y_i^T(k) Q_i y_i(k) - e_i^T(k) Q_i e_i(k) \geq 0. \tag{18}$$

Then we gets from (12)-(18) that

$$\begin{aligned}
 \Delta V(k) &= \sum_{i=1}^M \left[\xi_i^T(k+1) P_i \xi_i(k+1) - \xi_i^T(k) P_i \xi_i(k) \right] \\
 &\quad + 2 \sum_{i=1}^M \chi_i^T(k) \Psi_i [-\xi_i(k+1) + \tilde{A}_{ij}(k) \\
 &\quad + \tilde{W}_{ij} w_i(k) + \tilde{B}_{fij}(k)] \\
 &\quad + 2 \sum_{i=1}^M \chi_i^T(k) \Psi_i \sum_{m=1, m \neq i}^M \tilde{B}_{fmi} e_m(k) \\
 &\quad + 2 \sum_{i=1}^M \chi_i^T(k) \Psi_i \sum_{m=1, m \neq i}^M \tilde{C}_{mi} \xi_m(k) \\
 &\quad + \sum_{i=1}^M \left(\sigma_i^2 y_i^T(k) Q_i y_i(k) - e_i^T(k) Q_i e_i(k) \right) \\
 &= \sum_{i=1}^M \left[\xi_i^T(k+1) P_i \xi_i(k+1) - \xi_i^T(k) P_i \xi_i(k) \right] \\
 &\quad + 2 \sum_{i=1}^M \chi_i^T(k) \Psi_i [-\xi_i(k+1) + \tilde{A}_{ij} \xi_i(k) \\
 &\quad + W_{ij} w_i(k) + B_{fij} e_i(k)] \\
 &\quad + \sum_{i=1}^M \sum_{m=1, m \neq i}^M \xi_i^T(k) N_i^{-1} \xi_i(k) \\
 &\quad + \sum_{i=1}^M \sum_{m=1, m \neq i}^M e_i^T(k) M_i^{-1} e_i(k) \\
 &\quad + \sum_{i=1}^M \sum_{m=1, m \neq i}^M \chi_i^T(k) \Psi_i \tilde{C}_{mi} N_i^{-1} \tilde{C}_{mi}^T \Psi_i^T \chi_i(k) \\
 &\quad + \sum_{i=1}^M \sum_{m=1, m \neq i}^M \chi_i^T(k) \Psi_i \tilde{B}_{fmi} M_i^{-1} \tilde{B}_{fmi}^T \Psi_i^T \chi_i(k) \\
 &\quad + \sum_{i=1}^M \left(\sigma_i^2 y_i^T(k) Q_i y_i(k) - e_i^T(k) Q_i e_i(k) \right) \\
 &= \sum_{i=1}^M \left[\xi_i^T(k+1) P_i \xi_i(k+1) \right. \\
 &\quad \left. + \xi_i^T(k) \left(\sum_{m=1, m \neq i}^M N_i + \sigma_i^2 \tilde{D}_{ij}^T Q_i \tilde{D}_{ij} - P_i \right) \xi_i(k) \right. \\
 &\quad \left. + e_i^T(k) \left(\sum_{m=1, m \neq i}^M M_i - Q_i \right) e_i(k) \right]
 \end{aligned}$$

$$\begin{aligned}
 &+ 2 \sum_{i=1}^M \chi_i^T(k) \Psi_i [-I \quad \tilde{A}_{ij} \quad \tilde{B}_{fij} \quad \tilde{W}_{ij}] \chi_i \\
 &+ \sum_{i=1}^M \sum_{m=1, m \neq i}^M \chi_i^T(k) \Psi_i \tilde{C}_{mi} N_i^{-1} \tilde{C}_{mi}^T \Psi_i^T \chi_i(k) \\
 &+ \sum_{i=1}^M \sum_{m=1, m \neq i}^M \chi_i^T(k) \Psi_i \tilde{B}_{fmi} M_i^{-1} \tilde{B}_{fmi}^T \Psi_i^T \chi_i(k) \\
 &= \sum_{i=1}^M \chi_i^T(k) (\Phi_1 + \Phi_2) \chi_i. \tag{19}
 \end{aligned}$$

Then, by using the Schur complement, inequality (19) is equivalent to

$$\Phi + [\Pi_i \quad 0]^T [\Pi_i \quad 0] < 0, \tag{20}$$

which implies

$$\sum_{i=1}^M \chi_i^T(k) \Phi \chi_i(k) - \gamma^2 w^T(k) w(k) + \tilde{z}^T(k) \tilde{z}(k) < 0. \tag{21}$$

Under zero initial conditions, we have

$$\|\tilde{z}(k)\|_2 < \gamma \|w(k)\|_2. \tag{22}$$

Now, the asymptotic stability of the error system (8) is given under the condition $w(k) = 0$. Hence, the proof is complete.

Remark 4: For (18), If $k = t_k$, from the definition of $e_i(k)$ in (5), we have $e_i(k) = 0$, then from (18) that $\sigma_i y_i^T(k) Q_i y_i(k) \geq 0$. Else if $k \neq t_k$, there is no event occurring at the time $k \in (t_k, t_{k+1})$. According to (5), we obtain that $e_i^T(k) Q_i e_i(k) < \sigma_i y_i^T(k) Q_i y_i(k)$. Therefore, it is concluded that at any time, the inequality (18) can be concluded.

Remark 5: In order to prove the stability of system (8), we introduce the matrices M_i and N_i . If considering the matrices M_{ij} and N_{ij} , the obtained results may be less conservatism.

B. H_∞ FILTER DESIGN

In this section, the non-fragile event-triggered H_∞ filtering design problem for the large-scaled fuzzy system is addressed. The following Theorem provides a simple way to determine the filtering parameters A_{fij} , B_{fij} , C_{fij} .

Theorem 2 (Non-Fragile Event-Triggered H_∞ Filtering Design): The H_∞ filtering design problem is solved by given a constant $\gamma > 0$. If there are the positive-definite symmetric matrices \bar{P}_{ij} , \bar{M}_i , \bar{N}_i , G_i , $\bar{M}_i^0 \leq \bar{M}_i$, $\bar{N}_i^0 \leq \bar{N}_i$, $\bar{Q}_i = Q_i^{-1}$. We define $\bar{M}_i = M_i^{-1}$, $\bar{N}_i = N_i^{-1}$ and the positive constants σ_i and ε_i , which satisfy

$$\begin{bmatrix} \Phi & \Pi_i^T \\ * & -I \end{bmatrix} < 0, \tag{23}$$

where

$$\Phi = \begin{bmatrix} \Theta & \mathbb{B}_{fmi} & \mathbb{C}_{mi} & \tilde{Z}_i & \varepsilon_i \mathbb{E}_i & \mathbb{F}_i^T \\ * & \tilde{M}_i & 0 & 0 & 0 & 0 \\ * & * & \tilde{N}_i & 0 & 0 & 0 \\ * & * & * & -\mathbb{X}_i & 0 & 0 \\ * & * & * & * & -\varepsilon_i I & 0 \\ * & * & * & * & * & -\varepsilon_i I \end{bmatrix}, \tag{24}$$

$$\Theta = \begin{bmatrix} P_i - R_i - R_i^T & R_i \tilde{A}_{ij} & R_i \tilde{B}_{fij} & R_i \tilde{W}_{ij} \\ * & -P_i & 0 & 0 \\ * & * & \bar{Q}_i - G_i - G_i^T & 0 \\ * & * & * & -\gamma^2 \end{bmatrix}, \tag{25}$$

$$\mathbb{B}_{fmi} = [\Psi_i \tilde{B}_{f1i} \quad \dots \quad \Psi_i \tilde{B}_{fmi, m \neq i} \quad \dots \quad \Psi_i \tilde{B}_{fMi}], \tag{26}$$

$$\mathbb{C}_{mi} = [\Psi_i \tilde{C}_{1i} \quad \dots \quad \Psi_i \tilde{C}_{mi, m \neq i} \quad \dots \quad \Psi_i \tilde{C}_{Mi}], \tag{27}$$

$$\tilde{M}_i = \text{diag}_{M-1} \{ \bar{M}_i - G_i - G_i^T \} \tag{28}$$

$$\tilde{N}_i = \text{diag}_{M-1} \{ \bar{M}_i - R_i - R_i^T \} \tag{29}$$

$$\tilde{Z}_i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & R_i^T & 0 & \sigma_i R_i^T \tilde{D}_{ij}^T \\ 0 & 0 & R_i^T & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{30}$$

$$\mathbb{X}_i = \text{diag} \left[0 \frac{\bar{N}_i^0}{M-1} \quad \frac{\bar{M}_i^0}{M-1} \quad \bar{Q}_i \right], \tag{31}$$

$$\mathbb{E}_{ij} = \begin{bmatrix} \tilde{E}_{ij} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{32}$$

$$\mathbb{F}_{ij} = [0 \quad \tilde{F}_{ij} \quad 0 \quad 0]. \tag{33}$$

Furthermore, the parameter matrices of the H_∞ filtering are given as

$$A_{fij} = R_{2i}^{-1} \hat{A}_{fij}, B_{fij} = R_{2i}^{-1} \hat{B}_{fij}, C_{fij} = \hat{C}_{fij}. \tag{34}$$

Proof: Noticing that

$$-Y_i^T X_i^{-1} Y_i \leq X_i - Y_i - Y_i^T \tag{35}$$

where $X_i = X_i^T > 0$. Define $T = \text{diag} \{ I \ I \ G_i \ I \ G_i \ \mathbb{R}_i \ I \ I \ I \ I \ I \}$ and $\mathbb{G}_i = \text{diag} \{ G_1, \dots, G_{m, m \neq i}, \dots, G_M \}$, $\mathbb{R}_i = \text{diag} \{ R_1, \dots, R_{m, m \neq i}, \dots, R_M \}$. Let multiply (10) from the left and right by T , respectively, then one can obtain (23). The proof of necessity is completed.

Inequality (21) implies $P_i - R_i - R_i^T < 0$. Considering $P_i > 0$, we know that $R_i > 0$, which means R_i is a nonsingular matrix. Then, we have

$$\begin{aligned}
 R_i \tilde{A}_{ij} &= \begin{bmatrix} R_{1i} A_{ij} + R_{2i} B_{fij} D_{ij} & R_{2i} A_{fij} \\ R_{2i} A_{ij} + R_{2i} B_{fij} D_{ij} & R_{2i} A_{fij} \end{bmatrix}, \\
 R_i \tilde{B}_{fij} &= \begin{bmatrix} R_{2i} B_{fij} \\ R_{2i} B_{fij} \end{bmatrix}, \quad R_i \tilde{W}_{ij} = \begin{bmatrix} R_{1i} W_{ij} \\ R_{2i} W_{ij} \end{bmatrix}. \tag{36}
 \end{aligned}$$

By the introduction of the new matrices:

$$R_{2i} A_{fij} = \hat{A}_{fij}, R_{2i} B_{fij} = \hat{B}_{fij}, \tag{37}$$

the proof is completed.

IV. NUMERICAL EXAMPLE

In this section, a two-machine interconnected system is considered as follows [18]

$$\begin{cases} \dot{x}_{i1}(t) = x_{i2}(t) \\ \dot{x}_{i2}(t) = \frac{D_i}{M_i}x_{i2}(t) + \sum_{m=1, m \neq i}^2 \frac{E_i E_m Y_{mi}}{M_i} [\cos(\delta_{mi}^0 - \theta_{mi}) - \cos(x_{i1}(t) - x_{m1}(t) + \delta_{mi}^0 - \theta_{mi})] + w_i(t). \end{cases} \quad (38)$$

Linearizing the subsystems at these points, i.e., $x_{i1}(k) = \pm\pi/2, i = 1, 2$, one can obtain the following two-rule fuzzy model for the subsystem S_i .

Subsystem S_i :

Rule 1: IF $x_{i1}(k)$ is about $-\pi/2$, THEN

$$\begin{cases} x_i(k+1) = A_{i1}x_i(k) + B_{i1}w_i(k) + \sum_{m=1, m \neq i}^2 C_{mi}x_m(k) \\ y_i(k) = D_{i1}x_i(k), \end{cases}$$

Rule 2: IF $x_{i1}(k)$ is about $\pi/2$, THEN

$$\begin{cases} x_i(k+1) = A_{i2}x_i(k) + B_{i2}w_i(k) + \sum_{m=1, m \neq i}^2 C_{mi}x_m(k) \\ y_i(k) = D_{i2}x_i(k), \end{cases}$$

where

$$A_{11} = \begin{bmatrix} -0.15 & -0.325 \\ 0.01 & 0.4 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -0.1 & -0.325 \\ 0.01 & 0.4 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} -0.15 & -0.325 \\ 0.15 & 0.7 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -0.1 & -0.325 \\ 0.15 & 0.7 \end{bmatrix},$$

$$B_{11} = B_{12} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \quad B_{21} = B_{22} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix},$$

$$D_{11} = D_{12} = D_{21} = D_{22} = [1 \quad 0],$$

$$H_{11} = H_{12} = H_{21} = H_{22} = [0 \quad 0.5],$$

$$C_{12} = \begin{bmatrix} 0 & 0 \\ 0.025 & 0 \end{bmatrix}, \quad C_{21} = \begin{bmatrix} 0 & 0 \\ 0.06 & 0 \end{bmatrix}.$$

In order to design the non-fragile filtering, the parameters are given as following:

$$M_{111} = \begin{bmatrix} 0.1 & 0.05 \\ 0.01 & 0.03 \end{bmatrix}, \quad M_{112} = \begin{bmatrix} 0.15 & 0.01 \\ 0.1 & 0.1 \end{bmatrix},$$

$$M_{121} = \begin{bmatrix} 0.15 & 0.1 \\ 0.05 & 0.05 \end{bmatrix}, \quad M_{122} = \begin{bmatrix} 0.15 & 0.2 \\ 0.1 & 0.05 \end{bmatrix}.$$

$$M_{211} = [0.1 \quad 0.01], \quad M_{212} = [0.1 \quad 0.015],$$

$$M_{221} = [0.2 \quad 0.01], \quad M_{222} = [0.2 \quad 0.1].$$

$$N_{11} = \begin{bmatrix} 0.1 & 0 \\ 0.01 & 0.05 \end{bmatrix}, \quad N_{12} = \begin{bmatrix} 0.15 & 0 \\ 0.01 & 0.01 \end{bmatrix},$$

$$N_{21} = \begin{bmatrix} 0.15 & 0.1 \\ 0.2 & 0.05 \end{bmatrix}, \quad N_{22} = \begin{bmatrix} 0.15 & 0.2 \\ 0.1 & 0.05 \end{bmatrix}.$$

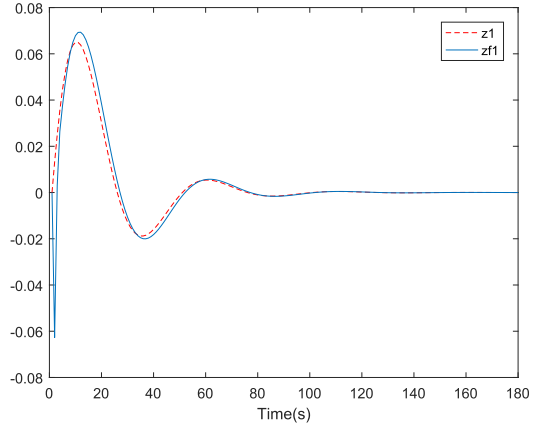


FIGURE 2. Trajectories of $z_1(k)$ and its estimation $z_{f1}(k)$.

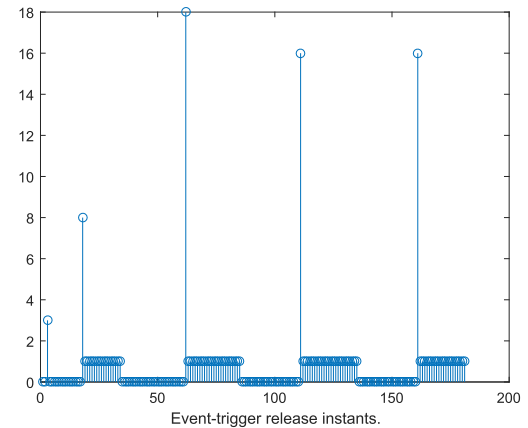


FIGURE 3. Event-triggered release instants of subsystem 1.

With the initial condition $x_1(k) = [0.3, 0]^T$ and $x_2(k) = [0.3, 0]^T$, the disturbance is set to be $w = \sin(0.04\pi k) * e^{-0.05k}$. We choose $\gamma = 1.2$ and $\sigma_1 = 0.4, \sigma_2 = 0.2$. Figs. 2 and 3 show the simulation results, and we obtain the event-triggered matrices $\Phi_1 = 2.8146$ and $\Phi_2 = 1.5323$, respectively. Then, we can get the filtering parameters as

$$A_{f11} = \begin{bmatrix} 0.789 & -0.192 \\ 0.104 & 0.067 \end{bmatrix}, \quad A_{f12} = \begin{bmatrix} 0.43 & -0.237 \\ -0.075 & 0.04 \end{bmatrix},$$

$$A_{f21} = \begin{bmatrix} 0.510 & -0.209 \\ -0.0035 & 0.0113 \end{bmatrix},$$

$$A_{f22} = \begin{bmatrix} -0.365 & -0.310 \\ -0.065 & 0.043 \end{bmatrix},$$

$$B_{f11} = \begin{bmatrix} 0.0992 \\ -0.0487 \end{bmatrix}, \quad B_{f12} = \begin{bmatrix} 0.0691 \\ -0.0355 \end{bmatrix},$$

$$B_{f21} = \begin{bmatrix} 0.0715 \\ -0.0706 \end{bmatrix}, \quad B_{f22} = \begin{bmatrix} 0.0584 \\ -0.0525 \end{bmatrix}.$$

$$C_{f11} = [0.0386 \quad 0.0301], \quad C_{f12} = [0.0471 \quad 0.0923],$$

$$C_{f21} = [0.0460 \quad 0.1227], \quad C_{f22} = [0.0620 \quad 0.298].$$

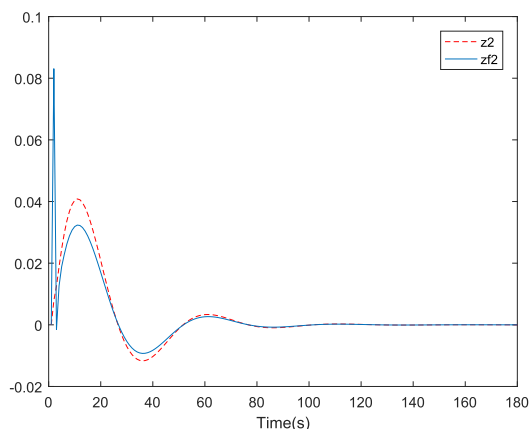


FIGURE 4. Trajectories of $z_2(k)$ and its estimation $z_{f2}(k)$.

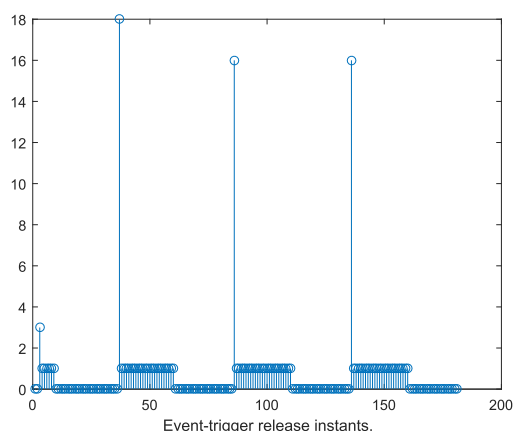


FIGURE 5. Event-triggered release instants of subsystem 2.

V. CONCLUSION

In this paper, The event triggered control is proposed in the large scale power systems, and the non-fragile H_∞ filtering performance problem has been investigated. The event-triggered means event-triggered control can determine whether the current data is necessary for transmission according to a triggered threshold. What's more, the filtering inevitably has gain uncertainty in real life. And thus we desire non-fragile event-triggered H_∞ filtering to guarantee the performance. By applying a Lyapunov-Krasovskii functional and linear matrix inequalities(LMIs), a method of event-triggered H_∞ distributed filtering design is developed for the filtering error system concerned to be asymptotically stable with a given disturbance attenuation level. A simulation is finally given to show the effective methods in this paper.

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