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Sampled-Data H_∞ Fuzzy Observer for Uncertain Oscillating Systems With Immeasurable Premise Variables

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ABSTRACT This paper proposes a sampled-data H_∞ fuzzy observer technique for nonlinear uncertain oscillating systems that are represented by the Takagi–Sugeno fuzzy model. The observers are designed for two cases: a measurable premise variable and an immeasurable premise variable. The error system between the nonlinear uncertain system and the sampled-data observer is constructed. The H_∞ performance function for the oscillating system is defined and is guaranteed in the Lyapunov sense. Sufficient conditions for the H_∞ sampled-data fuzzy observer are given in terms of linear matrix inequalities. Finally, the feasibility of the proposed technique is shown using two simulation examples.

INDEX TERMS H_∞ fuzzy observer, sampled-data system, uncertain oscillating system, Takagi–Sugeno (T–S) fuzzy model, linear matrix inequality (LMI).

I. INTRODUCTION

In recent years, sampled-data systems have gained much attention [1]–[3] because most engineering applications have both continuous plant and sampled-data computer-based implementations. Because continuous- and discrete-time states coexist in a sampled-data system, the conventional control techniques for continuous or discrete-time systems cannot be applied to the sampled-data system. To resolve this sampled-data problem, many methods have been proposed in [4]–[6]. Especially, when merged with the Takagi–Sugeno (T–S) fuzzy modeling, sampled-data control techniques have also been applied to nonlinear sampled-data systems because the T–S fuzzy model technique easily solves the nonlinearity problem using a convex combination method. These T–S fuzzy control methods for nonlinear sampled-data systems can be classified into two approaches: input-delay methods [7]–[9] and discretization methods [10]–[12]. In the first type of method, discrete-time states are converted to continuous-time states with a time-varying piecewise continuous delay, and using the time-delay system control technique, the stability of the sampled-data system is analyzed. Next, in the discretization method, the sampled-data system is mapped to the discrete-time system and analyzed in the discrete-time domain because the stability analysis of the discretized sampled-data system guarantees the stability of the original sampled-data system.

On the other hand, it is well known that state estimation is an important research issue for stable and reliable system operation and has been expanded by many methods such as filter design [13], [14] and observer design [15]–[21]. Previous filter design techniques were proposed for asymptotically stable systems. In addition, most conventional observers are designed by ensuring the convergence of the estimation error. Therefore, an observer is designed to be coupled with the controller [15]–[18] or based on an exactly known system model [19]–[21], and there have been few studies on state estimation techniques for uncertain oscillating systems. Very recently, in [22], the sampled-data observer design technique for uncertain oscillating system was proposed using a newly defined performance function. However, the performance function only guarantees the estimation error performance and does not consider the disturbance rejection performance. Furthermore, the state estimation technique for the immeasurable premise variable case is still insufficient.

State estimation for the immeasurable premise variable case is important for fuzzy estimators. This is because the measurable premise variable case is not realistic, as the premise variable is associated with the system states that should be estimated. To solve the immeasurable premise variable problem, many studies have been conducted in [8], [13], [16], [22], [23], and [25]. In [22] and [23], the membership functions of system and estimator are considered to be completely independent, but this approach usually leads to very conservative conditions. In [8], [13], and [16], the membership function of the estimator is assumed as an uncertain, but this method guarantees the performance only for the asymptotically stable system and the performance can be degraded when much of the system is assumed to be an uncertain. In [24], an interesting study on the membership function was conducted. Using this method, the membership function of the controller can be freely designed within certain constraints. Although this method was originally proposed for the purpose of giving the controller flexibility and robustness, it was also applied to solve other problems such as the nonlinear interconnections of large-scale system [14] and independence of the membership functions [25].

Motivated by the aforementioned studies, this paper proposes the sampled-data H_∞ fuzzy observer for nonlinear uncertain oscillating systems. The observers are designed for two cases, a measurable premise variable and an immeasurable premise variable. The T–S fuzzy model is used to construct the error system between the nonlinear uncertain system and sampled-data observer. To guarantee the performance in an oscillating system, an H_∞ performance function is defined. The H_∞ performance of the sampled-data observer is ensured in the Lyapunov sense. Sufficient conditions for measurable and immeasurable premise variable observer design techniques are given explicitly in terms of linear matrix inequalities (LMIs). Finally, simulation examples are provided to show the feasibility of the proposed techniques.

This paper is organized as follows: Section 2 formulates the sampled-data fuzzy observer with the measurable premise variable case. The fuzzy observer design technique for the immeasurable premise variable case is presented in Section 3. The procedure is validated via some simulation examples in Section 4. Finally, the conclusion is given in Section 5.

Notation: Subscripts i and j denote fuzzy rule indices. Notations $(\cdot)^T$ and $*$ are used for the transpose of the argument and the transposed element in the symmetric position, respectively. Further, $He\{A\}$ denotes $A + A^T$ and λ_A is the maximum eigenvalue of matrix A^TA. For a positive scalar r, $\mathcal{I}_r$ represents an integer set $\{1, 2, \ldots, r\}$. The observer gain L_i^{Thm1} means the observer gain L_i of Theorem 1.

II. SAMPLED-DATA FUZZY OBSERVER FOR THE MEASURABLE PREMISE VARIABLE CASE

Consider an uncertain nonlinear system which can be represented by following T–S fuzzy model:

$$R_i: \text{ IF } z_1(t) \text{ is } \Gamma_1^i \text{ and } \cdots \text{ and } z_q(t) \text{ is } \Gamma_q^i,$$
$$\text{THEN } \begin{cases} \dot{x}(t) = A_i x(t) + B_i w(t) + f(x(t)) \\ y(kT) = Cx(kT) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $w(t) \in \mathbb{L}_2^q$ is the disturbance, $y(kT) \in \mathbb{R}^m$ is the piecewise constant measurement output with sampling period $T \in \mathbb{R}_{>0}$ for $t \in [kT, kT + T)$, $k \in \mathbb{Z}_{\geq 0}$; Γ_p^i, $(i, p) \in \{\mathcal{I}_r := \{1, 2, \ldots, r\}\} \times \{\mathcal{I}_q := \{1, 2, \ldots, q\}\}$ is the fuzzy set for $z_p(t)$; $z_p(t)$ is the pth premise variable and $z(t)$ is the premise variable vector with $z_p(t)$, and A_i, B_i, and C are nominal system matrices with appropriate dimensions. In addition, $f(x(t))$ denotes unknown nonlinear state that is represented as a piece-wise continuous vector function with the following assumption:

Assumption 1: Vector function $f(x(t))$ is unknown but satisfies the following quadratic inequality:

$$f^T(x(t))f(x(t)) \leq \alpha^2 x^T(t)F^TFx(t), \tag{2}$$

where $\alpha \in \mathbb{R}_{>0}$ is a bound scalar constant of the uncertainty and F is a constant matrix with an appropriate dimension.

By fuzzy blending, the global dynamics of the uncertain fuzzy model (1) are inferred as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} h_i(z(t))\Big\{A_i x(t) + B_i w(t) + f(x(t))\Big\}, \\ y(kT) &= Cx(kT), \end{aligned} \tag{3}$$

where

$$h_i(z(t)) := \frac{\omega_i(z(t))}{\sum_{i=1}^{r}\omega_i(z(t))}, \quad \omega_i(z(t)) := \prod_{p=1}^{q}\Gamma_p^i(z_p(t)),$$

and $\Gamma_p^i(z_p(t)) : U_{z_p} \subset \mathbb{R} \to \mathbb{R}_{[0,1]}$ is the membership function of $z_p(t)$ on compact set U_{z_p}.

In addition, in this section, we need the following two assumptions to ensure the observability and measurability.

Assumption 2: All pairs (A_i, C) are observable for $i \in \mathcal{I}_r$.

Assumption 3: State variable $x(t)$ is not measurable, but premise variable $z(t)$ and output variable $y(kT)$ are measurable.

Based on the parallel distribute compensation (PDC) scheme, the sampled-data fuzzy observer can be constructed as follows:

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{i=1}^{r} h_i(z(t))\Big\{A_i\hat{x}(t) + L_i(y(kT) - \hat{y}(kT))\Big\}, \\ \hat{y}(kT) &= C\hat{x}(kT), \end{aligned} \tag{4}$$

where $\hat{x}(t) \in \mathbb{R}^n$, $\hat{y}(kT) \in \mathbb{R}^m$, and L_i are the state of the observer, observer output, and observer gain matrix to be designed, respectively.

Remark 1: The T–S fuzzy observer can be classified into two categories: a measurable premise variable observer and an immeasurable premise variable observer. Generally, the measurable observer is easier to design and allows a wider feasible region than the immeasurable one. However, it is not realistic in contrast to the immeasurable case, because the membership function of the observer is related to state $x(t)$. The work in this paper considers both cases. In this section, the measurable case is considered and the immeasurable case is discussed in the next section.

By defining estimation error $e(t) = x(t) - \hat{x}(t)$ and $e(kT) = x(kT) - \hat{x}(kT)$, the error system can be formulated as follows:

$$\dot{e}(t) = \sum_{i=1}^{r} h_i(z(t))\Big\{(A_i - L_iC)e(kT) + A_i\tilde{e}(t) + B_iw(t) + f(x(t))\Big\}, \quad (5)$$

where $\tilde{e}(t) = e(t) - e(kT)$.

Remark 2: The error system (5) is not fully represented in the T–S fuzzy model because of the unknown nonlinear function $f(x(t))$.

The purpose of this section is therefore to design a sampled-data observer in the form of (4) such that error system (5) with $w(t) = 0$ and $f(x(t)) = 0$ is asymptotically stable and there exists a given performance index $\gamma \geq 0$ satisfying the following H_∞ performance under the zero initial condition:

$$\int_0^\infty ||e(t)||^2 dt \leq \gamma^2 \int_0^\infty ||x(t)||^2 + ||w(t)||^2 dt. \quad (6)$$

Remark 3: Most previous fuzzy observers were designed only for exactly known or asymptotically stable systems, and there are few observer studies for uncertain oscillating systems. Using performance inequality (6), an improved fuzzy observer design technique is proposed that can be applied to not only to asymptotically stable systems but also to uncertain oscillating systems.

The following lemma is useful in the proof of our results.

Lemma 1 [26]: Given any function vectors $\varepsilon(t)$ and $\dot{x}(t)$, any matrix N_{ij}, and positive definite matrix Q, then the following inequality is satisfied:

$$-2\varepsilon^T(t)N_{ij}\int_{kT}^{t}\dot{x}(\tau)d\tau \leq T\varepsilon^T(t)N_{ij}Q^{-1}N_{ij}^T\varepsilon(t) + \int_{kT}^{t}\dot{x}^T(\tau)Q\dot{x}(\tau)d\tau. \quad (7)$$

Using the above H_∞ performance condition (6), the following sufficient condition for the sampled-data observer is proposed:

Theorem 1: Consider nonlinear uncertain system (3) and sampled-data fuzzy observer (4). For a prescribed real number γ, if there exist some symmetric and positive definite matrices P and Q, some matrices M, N_i, and S_i, and some given scalars α and λ_F such that the following LMIs are satisfied:

$$\begin{bmatrix} \Theta_i & * & * \\ (N_i)^T & -\frac{1}{T}Q & * \\ \tilde{M} & 0 & -\frac{\gamma^2}{3\alpha^2\lambda_F}I \end{bmatrix} \prec 0, \quad i \in \mathcal{I}_r, \quad (8)$$

where

$$\Theta_i = \begin{bmatrix} \Theta_i^{11} & * & * & * \\ \Theta_i^{21} & \Theta_i^{22} & * & * \\ \Theta_i^{31} & \Theta_i^{32} & -He\{M\} + TQ & * \\ B_i^TM & B_i^TM + N_{4i} & B_i^TM & -\gamma^2 I \end{bmatrix},$$

$$\begin{aligned}
\Theta_i^{11} &= He\{M^TA_i - S_iC\} + I \\
\Theta_i^{21} &= He\{M^TA_i\} - S_iC + N_{1i}^T + I, \\
\Theta_i^{22} &= He\{M^TA_i + N_{2i}\} + I \\
\Theta_i^{31} &= P - M + M^TA_i - S_iC, \\
\Theta_i^{32} &= P - M + M^TA_i + N_{3i}
\end{aligned}$$

$N_i^T = [N_{1i}^T \ N_{2i}^T \ N_{3i}^T \ N_{4i}^T]$, and $\tilde{M} = \mathrm{diag}\{M \ M \ M \ 0\}$, then error system (5) is asymptotically stable with $w(t) = 0$ and $f(x(t)) = 0$, and H_∞ performance (6) is guaranteed. Further, the matrix of the observer gain is given by $L_i = (M^T)^{-1}S_i$.

Proof: Choose the following Lyapunov function candidate $V(t)$ for (5):

$$V(t) = e^T(t)Pe(t) + \int_{kT}^{t}(T - t + \tau)\dot{e}^T(t)Q\dot{e}(t)d\tau, \quad (9)$$

where $P = P^T \succ 0$, $Q = Q^T \succ 0$, $t \in [kT, kT + T)$, and $k \in \mathbb{Z}_{\geq 0}$.

Then, the time derivative of (9) is as follows:

$$\begin{aligned}
\dot{V}(t) &= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) + T\dot{e}^T(t)Q\dot{e}(t) \\
&\quad - \int_{kT}^{t}\dot{e}^T(\tau)Q\dot{e}(\tau)d\tau \\
&= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) + T\dot{e}^T(t)Q\dot{e}(t) \\
&\quad - \int_{kT}^{t}\dot{e}^T(\tau)Q\dot{e}(\tau)d\tau \\
&\quad + 2\sum_{i=1}^{r}h_i(z(t))\varepsilon^T(t)N_i\Big[\tilde{e}(t) - \int_{kT}^{t}\dot{e}(\tau)d\tau\Big] \\
&\quad + 2\Big(Me(kT) + M\tilde{e}(t) + M\dot{e}(t)\Big)^T \\
&\quad \times\Big(- \dot{e}(t) + \sum_{i=1}^{r}h_i(z(t))\big\{(A_i - L_iC)e(kT) + A_i\tilde{e}(t) \\
&\qquad + B_iw(t) + f(x(t))\big\}\Big), \quad (10)
\end{aligned}$$

where $\varepsilon(t) = [e(kT) \quad \tilde{e}(t) \quad \dot{e}(t) \quad w(t)]$. Using Lemma 1 to (10), we have

$$\begin{aligned}
\dot{V}(t) &\leq \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) + T\dot{e}^T(t)Q\dot{e}(t) \\
&\quad - \int_{kT}^{t}\dot{e}^T(\tau)Q\dot{e}(\tau)d\tau + 2\sum_{i=1}^{r}h_i(z(t))\varepsilon^T(t)N_i\tilde{e}(t) \\
&\quad + T\varepsilon^T(t)N_i(Q)^{-1}N_i^T\varepsilon(t) + \int_{kT}^{t}\dot{e}^T(\tau)Q\dot{e}(\tau)d\tau \\
&\quad + \sum_{i=1}^{r}h_i(z(t))\Big\{ - 2e^T(kT)M^T\dot{e}(t) \\
&\quad + 2e^T(kT)M^T(A_i - L_iC)e(kT) + 2e^T(kT)M^TA_i\tilde{e}(t) \\
&\quad + 2e^T(kT)M^TB_iw(t) + 2e^T(kT)M^Tf(x(t)) \\
&\quad - 2\tilde{e}^T(t)M^T\dot{e}(t) + 2\tilde{e}^T(t)M^T(A_i - L_iC)e(kT) \\
&\quad + 2\tilde{e}^T(t)M^TA_i\tilde{e}(t) + 2\tilde{e}^T(t)M^TB_iw(t) \\
&\quad + 2\tilde{e}^T(t)M^Tf(x(t)) - 2\dot{e}^T(t)M^T\dot{e}(t) \\
&\quad + 2\dot{e}^T(t)M^T(A_i - L_iC)e(kT)
\end{aligned}$$

$$\begin{aligned}
&+2\dot{e}^T(t)M^TA_i\tilde{e}(t)+2\dot{e}^T(t)M^TB_iw(t)\\
&+2\dot{e}^T(t)M^Tf(x(t))\Big\}\\
=&\sum_{i=1}^{r}h_i(z(t))\Big\{\varepsilon^T(t)\hat{\Theta}_i\varepsilon(t)+2e^T(kT)M^Tf(x(t))\\
&+2\tilde{e}^T(t)M^Tf(x(t))+2\dot{e}^T(t)M^Tf(x(t))\\
&+T\varepsilon^T(t)N_i(Q)^{-1}N_i^T\varepsilon(t)\Big\}, \qquad (11)
\end{aligned}$$

where

$$\hat{\Theta}_i=\begin{bmatrix} He\{M^TA_i-S_iC\} & * & * & * \\ He\{M^TA_i\}-S_iC+N_{1i}^T & He\{M^TA_i+N_{2i}\} & * & * \\ P-M+M^TA_i-S_iC & P-M+M^TA_i+N_{3i} & -He\{M\}+TQ & * \\ B_i^TM & B_i^TM+N_{4i} & B_i^TM & 0 \end{bmatrix}.$$

From the well-known matrix inequality [31],

$$2X^TY\le\rho X^TX+\rho^{-1}Y^TY,$$

where X and Y are any matrices of appropriate dimensions and $\rho>0$, it is clear that $2e^T(kT)M^Tf(x(t))\le\rho e^T(kT)M^TMe(kT)+\rho^{-1}f^T(x(t))f(x(t))$. Then, (11) can be derived as

$$\begin{aligned}
\dot{V}(t)\le\sum_{i=1}^{r}h_i(z(t))\Big\{&\varepsilon^T(t)\{\hat{\Theta}_i+\rho\tilde{M}^T\tilde{M}+TN_i(Q)^{-1}N_i^T\}\varepsilon(t)\\
&+3\rho^{-1}f^T(x(t))f(x(t))\Big\}, \qquad (12)
\end{aligned}$$

where $\tilde{M}=\mathrm{diag}\{M\ M\ M\ 0\}$. Using Assumption 1 to (12), we can obtain

$$\begin{aligned}
\dot{V}(t)\le\sum_{i=1}^{r}h_i(z(t))\Big\{&\varepsilon^T(t)\{\hat{\Theta}_i+\rho\tilde{M}^T\tilde{M}+TN_i(Q)^{-1}N_i^T\}\varepsilon(t)\\
&+x^T(t)\{3\rho^{-1}\alpha^2F^TF\}x(t)\Big\}.
\end{aligned}$$

To guarantee H_∞ performance (6), the following inequality is needed:

$$\dot{V}(t)+e^T(t)e(t)-\gamma^2(x^T(t)x(t)+w^T(t)w(t))<0. \qquad (13)$$

Then, (13) can be induced as follows:

$$\begin{aligned}
&\dot{V}(t)+e^T(t)e(t)-\gamma^2(x^T(t)x(t)+w^T(t)w(t))\\
&\le\sum_{i=1}^{r}h_i(z(t))\Big\{\varepsilon^T(t)\{\Theta_i+\rho\tilde{M}^T\tilde{M}+TN_i(Q)^{-1}N_i^T\}\varepsilon(t)\\
&\quad+x^T(t)\{3\rho^{-1}\alpha^2F^TF-\gamma^2I\}x(t)\Big\}. \qquad (14)
\end{aligned}$$

Thus, the sufficient conditions for (14) can be constructed as

$$\hat{\Theta}_i+\rho\tilde{M}^T\tilde{M}+TN_i(Q)^{-1}N_i^T\prec 0, \qquad (15)$$
$$3\rho^{-1}\alpha^2\lambda_F-\gamma^2=0. \qquad (16)$$

Substituting (16) into (15) and applying the Schur complement to (15), we obtain the LMIs (8). Thus, (13) is satisfied if (8) holds.

Finally, to guarantee the H_∞ performance, we integrate (13) from 0 to ∞. Then,

$$V(\infty)-V(0)+\int_0^\infty ||e(t)||^2-\gamma^2(||x(t)||^2+||w(t)||^2)dt\le 0. \qquad (17)$$

Because $V(\infty)\ge 0$ and $V(0)=0$ under the zero initial condition, we can verify H_∞ performance (6). In addition, it is clear from (8) that $\dot{V}(t)<0$ is satisfied with $w(t)=0$ and $f(x(t))=0$, which indicates the asymptotic stability of error system (5) with $w(t)=0$ and $f(x(t))=0$. ■

Remark 4: Although Lyapunov function candidate (9) and its derivation process are similar to the input-delay converting method for the sampled-data system, there is no need to convert from the sampled-data system to the time-delay system, and any delay term is included in the stability analysis procedure. It has the advantage of reducing conservativeness of the stability condition.

Remark 5: Theorem 1 gives the sampled-data H_∞ fuzzy observer design technique for the nonlinear uncertain system. The main advantages of Theorem 1 is that it guarantees the H_∞ performance of oscillating systems and not fully modeled T–S fuzzy systems as well as asymptotically stable systems.

Theorem 1 is based on the measurable premise variable case. The sampled-data H_∞ fuzzy observer technique for the immeasurable premise variable case is discussed in the next section.

III. SAMPLED-DATA FUZZY OBSERVER FOR THE IMMEASURABLE PREMISE VARIABLE CASE

In this section, a sampled-data fuzzy observer for the immeasurable premise variable case is given. Instead of Assumption 3, we assume immeasurability of the premise variable $z(t)$.

Assumption 4: State variable $x(t)$ and premise variable $z(t)$ are not measurable, but the output variable $y(kT)$ is measurable.

The fuzzy observer IF-THEN rules for T–S fuzzy system (3) are constructed as follows:

$$R_i: \text{IF } \hat{z}_1(t) \text{ is } \Xi_1^i \text{ and } \cdots \text{ and } \hat{z}_o(t) \text{ is } \Xi_o^i$$
$$\text{THEN } \begin{cases}\dot{\hat{x}}(t)=A_i\hat{x}(t)+L_i(y(kT)-\hat{y}(kT))\\ \hat{y}(kT)=C\hat{x}(t)\end{cases} \qquad (18)$$

where Ξ_p^i, $(i,p)\in\mathcal{I}_r\times\mathcal{I}_o$ is the fuzzy set for $\hat{z}_p$.

Based on the fuzzy observer rules in (18), the sampled-data fuzzy observer is supposed in the following forms:

$$\begin{aligned}
\dot{\hat{x}}(t)&=\sum_{i=1}^{r}g_i(\hat{z}(t))\Big\{A_i\hat{x}(t)+L_i(y(kT)-\hat{y}(kT))\Big\},\\
\hat{y}(kT)&=C\hat{x}(kT), \qquad (19)
\end{aligned}$$

where

$$g_i(\hat{z}(t)) := \frac{\varphi_l^i(\hat{z}(t))}{\sum_{i=1}^r \varphi_i(\hat{z}(t))}, \quad \varphi_i(\hat{z}(t)) := \prod_{p=1}^{o} \Xi_p^i(\hat{z}_p(t)),$$

and $\Xi_p^i(\hat{z}_p(t)) : U_{\hat{z}_p} \subset \mathbb{R} \to \mathbb{R}_{[0,1]}$ is the membership function of $\hat{z}_p(t)$ on compact set $U_{\hat{z}_p}$.

Remark 6: Unlike measurable premise variable fuzzy observer (4), immeasurable fuzzy observer (19) is based on not only the states for observer $\hat{z}(t)$ but also imperfect premise matching condition $g_i(\hat{z}(t))$. Using this structure, an observer that is more realistic than the conventional PDC approach can be designed.

Defining the estimation error $e(t) = x(t) - \hat{x}(t)$, the error system can be formulated as follows:

$$\dot{e}(t) = \sum_{i=1}^r \sum_{i=1}^r h_i(z(t))g_j(\hat{z}(t))\Big\{(A_j - L_jC)e(kT) + A_j\tilde{e}(t) + (A_i - A_j)x(t) + B_iw(t) + f(x(t))\Big\}. \quad (20)$$

The purpose of this section is to design a sampled-data observer in the form of (19) such that error system (20) with $w(t) = 0$, $f(x(t)) = 0$, and $x(t) = 0$ is asymptotically stable and there exists a given performance index $\gamma \geq 0$ satisfying H_∞ performance inequality (6) under the zero initial condition.

The following Lemma is useful for solving the immeasurable premise variable problem and the proof of our main results.

Lemma 2: Inequality

$$\sum_{i=1}^r \sum_{j=1}^r h_i(z(t))g_j(\hat{z}(t))\varepsilon^T(t)\Psi_{ij}\varepsilon(t) < 0, (i,j) \in \mathcal{I}_r \times \mathcal{I}_r$$

holds for any symmetric matrix Ψ_{ij} and function vector $\varepsilon(t)$ if the inequality

$$g_i(\hat{z}(t)) - \mu_i h_i(z(t)) \geq 0 \quad (21)$$

is satisfied for any given scalar $0 < \mu_i < 1$ and there exist some symmetric matrix Λ_i and O_{ij} such that the following inequalities are satisfied:

$$\Psi_{ij} - \Lambda_i \prec 0, \quad (i,j) \in \mathcal{I}_r \times \mathcal{I}_r, \quad (22)$$

$$\Psi_{ii} - \Lambda_i - O_{ii} \prec 0, \quad i \in \mathcal{I}_r, \quad (23)$$

$$\mu_j(\Psi_{ij} - \Lambda_i - O_{ij}) + \mu_i(\Psi_{ji} - \Lambda_j - O_{ji}) \prec 0, \quad (i,j) \in \mathcal{I}_j \times \mathcal{I}_r, \quad (24)$$

$$\Lambda_i + \mu_i O_{ii} \prec 0, \quad i \in \mathcal{I}_r, \quad (25)$$

$$\Lambda_i + \mu_j O_{ij} + \Lambda_j + \mu_i O_{ji} \prec 0, \quad (i,j) \in \mathcal{I}_j \times \mathcal{I}_r. \quad (26)$$

Proof: The proof can be accomplished using a method to that in [24].

$$\begin{aligned}
&\sum_{i=1}^r \sum_{j=1}^r h_i(z(t))g_j(\hat{z}(t))\varepsilon^T(t)\Psi_{ij}\varepsilon(t) \\
&= \sum_{i=1}^r \sum_{j=1}^r h_i(z(t))\{g_j(\hat{z}(t)) + \mu_j h_j(z(t)) - \mu_j h_j(z(t))\} \\
&\quad \times \varepsilon^T(t)\Psi_{ij}\varepsilon(t) \\
&\quad + \sum_{i=1}^r \sum_{j=1}^r h_i(z(t))\{h_j(z(t)) - \mu_j h_j(z(t))\}\varepsilon^T(t)\Lambda_i\varepsilon(t) \\
&\quad - \sum_{i=1}^r \sum_{j=1}^r h_i(z(t))\{g_j(\hat{z}(t)) - \mu_j h_j(z(t))\}\varepsilon^T(t)\Lambda_i\varepsilon(t) \\
&\quad + \sum_{i=1}^r \sum_{j=1}^r h_i(z(t))h_j(z(t))\varepsilon^T(t)\mu_j O_{ij}\varepsilon(t) \\
&\quad - \sum_{i=1}^r \sum_{j=1}^r h_i(z(t))h_j(z(t))\varepsilon^T(t)\mu_j O_{ij}\varepsilon(t) \\
&= \sum_{i=1}^r \sum_{j=1}^r h_i(z(t))h_j(z(t))\mu_j\varepsilon^T(t)\{\Psi_{ij} - \Lambda_i - O_{ij}\}\varepsilon(t) \\
&\quad + \sum_{i=1}^r \sum_{j=1}^r h_i(z(t))\{g_j(\hat{z}(t)) - \mu_j h_j(z(t))\} \\
&\quad \times \varepsilon^T(t)\{\Psi_{ij} - \Lambda_i\}\varepsilon(t) \\
&\quad + \sum_{i=1}^r \sum_{j=1}^r h_i(z(t))h_j(z(t))\varepsilon^T(t)\{\Lambda_i + \mu_j O_{ij}\}\varepsilon(t). \quad (27)
\end{aligned}$$

Considering (21) and (22), then (27) can be derived as

$$\begin{aligned}
&\sum_{i=1}^r \sum_{j=1}^r h_i(z(t))g_j(\hat{z}(t))\varepsilon^T(t)\Psi_{ij}\varepsilon(t) \\
&\leq \sum_{i=1}^r \sum_{j=1}^r h_i(z(t))h_j(z(t))\mu_j\varepsilon^T(t)\{\Psi_{ij} - \Lambda_i - O_{ij}\}\varepsilon(t) \\
&\quad + \sum_{i=1}^r \sum_{j=1}^r h_i(z(t))h_j(z(t))\varepsilon^T(t)\{\Lambda_i + \mu_j O_{ij}\}\varepsilon(t) \\
&= \sum_{i=1}^r h_i^2(z(t))\mu_i\varepsilon^T(t)\{\Psi_{ii} - \Lambda_i - O_{ii}\}\varepsilon(t) \\
&\quad + \sum_{i=1}^r \sum_{j>i}^r h_i(z(t))h_j(z(t))\mu_j\varepsilon^T(t)\{\mu_j(\Psi_{ij} - \Lambda_i - O_{ij}) \\
&\quad + \mu_i(\Psi_{ji} - \Lambda_j - O_{ji})\}\varepsilon(t) \\
&\quad + \sum_{i=1}^r h_i^2(z(t))\varepsilon^T(t)\{\Lambda_i + \mu_i O_{ii}\}\varepsilon(t) \\
&\quad + \sum_{i=1}^r \sum_{j>1}^r h_i(z(t))h_j(z(t)) \\
&\quad \times \varepsilon^T(t)\{\Lambda_i + \mu_j O_{ij} + \Lambda_j + \mu_i O_{ji}\}\varepsilon(t).
\end{aligned}$$

Therefore, $\sum_{i=1}^r \sum_{j=1}^r h_i(z(t))g_j(\hat{z}(t))\varepsilon^T(t)\Psi_{ij}\varepsilon(t) < 0$ holds if (22)-(25) are satisfied. ■

Remark 7: If membership function constraint (21) is satisfied for all fuzzy rules, the membership function of observer $g_i(\hat{z}(t))$ can be freely designed. In addition, because inequality (21) is always satisfied if $g_i(\hat{z}(t)) = h_i(z(t))$, the PDC scheme is the special case of the imperfect premise matching condition.

Then, the sufficient condition for the sampled-data fuzzy observer with an immeasurable premise variable ban be summarized as the following Theorem:

Theorem 2: Consider nonlinear uncertain system (3) and sampled-data fuzzy observer (19). Let the membership functions of the T–S fuzzy system and sampled-data fuzzy observer satisfy $g_i(\hat{z}(t)) - \mu_i h_i(z(t)) \geq 0$ for any given scalar $0 < \mu_i < 1$. Further, there exist some symmetric and positive definite matrices P and Q, some matrices M, N_{ij}, and S_i, and some given scalars α, λ_F, and $\lambda_{\tilde{A}_{ij}}$ such that following LMIs are satisfied:

$$\Psi_{ij} - \Lambda_i \prec 0, \quad (i,j) \in \mathcal{I}_r \times \mathcal{I}_r, \tag{28}$$

$$\Psi_{ii} - \Lambda_i - O_{ii} \prec 0, \quad i \in \mathcal{I}_r, \tag{29}$$

$$\mu_j(\Psi_{ij} - \Lambda_i - O_{ij}) + \mu_i(\Psi_{ji} - \Lambda_j - O_{ji}) \prec 0, \quad (i,j) \in \mathcal{I}_j \times \mathcal{I}_r, \tag{30}$$

$$\Lambda_i + \mu_i O_{ii} \prec 0, \quad i \in \mathcal{I}_r, \tag{31}$$

$$\Lambda_i + \mu_j O_{ij} + \Lambda_j + \mu_i O_{ji} \prec 0, \quad (i,j) \in \mathcal{I}_j \times \mathcal{I}_r, \tag{32}$$

where

$$\Psi_{ij} = \begin{bmatrix} \Phi_{ij} & * & * \\ (N_{ij})^T & -\frac{1}{T}Q & * \\ \tilde{M} & 0 & -\frac{\gamma^2}{3(\alpha^2+1)(\lambda_F + \lambda_{\tilde{A}_{ij}})}I \end{bmatrix},$$

$$\Phi_{ij} = \begin{bmatrix} \Phi_{ij}^{11} & * & * & * \\ \Phi_{ij}^{21} & \Phi_{ij}^{22} & * & * \\ \Phi_{ij}^{31} & \Phi_{ij}^{32} & \Phi_{ij}^{33} & * \\ B_i^T M & B_i^T M + N_{4ij} & B_i^T M & -\gamma^2 I \end{bmatrix},$$

$$\begin{aligned} \Phi_{ij}^{11} &= He\{M^T A_j - S_j C\} + I, \\ \Phi_{ij}^{21} &= He\{M^T A_j\} - S_j C + N_{1ij}^T + I, \\ \Phi_{ij}^{22} &= He\{M^T A_j + N_{2ij}\} + I, \\ \Phi_{ij}^{31} &= P - M + M^T A_j - S_j C, \\ \Phi_{ij}^{32} &= P - M + M^T A_j + N_{3ij}, \\ \Phi_{ij}^{33} &= -He\{M\} + TQ. \end{aligned}$$

$N_{ij}^T = [N_{1ij}^T \; N_{2ij}^T \; N_{3ij}^T \; N_{4ij}^T]$, and $\tilde{M} = \mathrm{diag}\{M \;\; M \;\; M \;\; 0\}$. Then, error system (20) is asymptotically stable with $w(t) = 0$, $f(x(t)) = 0$, and $x(t) = 0$, and H_∞ performance (6) is guaranteed. Further, the matrix of the observer gain is given by $L_i = (M^T)^{-1} S_i$.

Proof: The proof can be derived in a similar way to that of Theorem 1. Then, we can obtain the following inequalities:

$$\begin{aligned} \dot{V}(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) g_j(\hat{z}(t)) \\ \times \Big\{ & \varepsilon^T(t)\hat{\Phi}_{ij}\varepsilon(t) + 2e^T(kT)M^T f(x(t)) \\ & + 2\tilde{e}^T(t)M^T f(x(t) \\ & + 2\dot{e}^T(t)M^T f(x(t)) + T\varepsilon^T(t)N_{ij}(Q)^{-1}N_{ij}^T\varepsilon(t) \\ & + 2e^T(kT)M^T(A_i - A_j)x(t) \\ & + 2\tilde{e}^T(t)M^T(A_i - A_j)x(t) \\ & + 2\dot{e}^T(t)M^T(A_i - A_j)x(t) \Big\}, \end{aligned} \tag{33}$$

where

$$\hat{\Phi}_{ij} = \begin{bmatrix} He\{M^T A_j - S_j C\} & * & * & * \\ He\{M^T A_j\} - S_j C + N_{1ij}^T & He\{M^T A_j + N_{2ij}\} & * & * \\ P - M + M^T A_j - S_j C & P - M + M^T A_j + N_{3ij} & -He\{M\} + TQ & * \\ B_i^T M & B_i^T M + N_{4ij} & B_i^T M & 0 \end{bmatrix}.$$

Let us define $\tilde{A}_{ij} = A_i - A_j$, then from the inequality in [31], it is clear that

$$\begin{aligned} & 2e^T(kT)M^T f(x(t)) \\ & \quad \leq \rho e^T(kT)M^T M e(kT) + \rho^{-1} f^T(x(t)) f(x(t)), \end{aligned} \tag{34}$$

$$\begin{aligned} & 2e^T(kT)M^T(A_i - A_j)x(t) \\ & \quad \leq \rho \frac{\lambda_{\tilde{A}_{ij}}}{\lambda_F} e^T(kT)M^T M e(kT) \\ & \qquad + \rho^{-1} \frac{\lambda_F}{\lambda_{\tilde{A}_{ij}}} x^T(t)(A_i - A_j)^T(A_i - A_j)x(t). \end{aligned} \tag{35}$$

By using (34), (35), and Assumption 1, the inequality (33) can be derived as

$$\begin{aligned} \dot{V}(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) g_j(\hat{z}(t)) \\ \times \Big\{ & \varepsilon^T(t)\{\hat{\Phi}_{ij} + \rho(1 + \frac{\lambda_{\tilde{A}_{ij}}}{\lambda_F})\tilde{M}^T\tilde{M} \\ & + TN_{ij}(Q)^{-1}N_{ij}^T\}\varepsilon(t) \\ & + x^T(t)\{3\rho^{-1}\alpha^2 F^T F \\ & + 3\rho^{-1}\frac{\lambda_F}{\lambda_{\tilde{A}_{ij}}}(A_i - A_j)^T(A_i - A_j)\}x(t) \Big\}. \end{aligned} \tag{36}$$

From the above inequality (36), the H_∞ performance condition (13) can be induced as follows:

$$\begin{aligned} & \dot{V}(t) + e^T(t)e(t) - \gamma^2(x^T(t)x(t) + w^T(t)w(t)) \\ & \quad \leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) g_j(\hat{z}(t)) \Big\{ \varepsilon^T(t)\{\Phi_{ij} \\ & \qquad + \rho(1 + \frac{\lambda_{\tilde{A}_{ij}}}{\lambda_F})\tilde{M}^T\tilde{M} + TN_{ij}(Q)^{-1}N_{ij}^T\}\varepsilon(t) \\ & \qquad + x^T(t)\{3\rho^{-1}\alpha^2 F^T F \\ & \qquad + 3\rho^{-1}\frac{\lambda_F}{\lambda_{\tilde{A}_{ij}}}(A_i - A_j)^T(A_i - A_j) - \gamma^2 I\}x(t) \Big\}. \end{aligned} \tag{37}$$

If $\rho = 3\lambda_F(\alpha^2 + 1)/\gamma^2$, (37) can be formulated as:

$$\begin{aligned} & \dot{V}(t) + e^T(t)e(t) - \gamma^2(x^T(t)x(t) + w^T(t)w(t)) \\ & \quad \leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) g_j(\hat{z}(t)) \varepsilon^T(t)\{\Phi_{ij} \end{aligned}$$

$$+\frac{3}{\gamma^2}(\lambda_F+\lambda_{\tilde{A}_{ij}})(\alpha^2+1)\tilde{M}^T\tilde{M}+TN_{ij}(Q)^{-1}N_{ij}^T\}\varepsilon(t) \tag{38}$$

Finally, by using the Schur complement and applying Lemma 22 to (38), it is obvious that the sufficient condition for (13) can be derived as LMIs (28)-(32). ■

Remark 8: Unlike Theorem 1, asymptotic stability is guaranteed when $x(t)=0$ as well as $f(x(t))=0$ and $w(t)=0$ because of the membership function mismatches between the system and observer.

Remark 9: This paper is mainly motivated by the approaches in [22]. However, there are following differences. First, this paper deals with the state estimation of the partially unknown or uncertain system, not the unknown interconnection of the interconnected system. Second, the disturbance attenuation performance is considered as well as the estimation of the oscillating system. Third, the imperfect premise matching condition is considered to solve the independence problem of the membership function [25]. Finally, since there are no discretization process in the development of the stability analysis, the proposed approaches give more feasible results than those of [22].

Remark 10: The main reasons of inherent conservatism in Theorem 1 and 2 are the construction of Lyapunov function (9) and the upper bound of Lemma 1 which is based on the Jensen inequality [27]. Thus, the conservatism problem can be reduced by using the different Lyapunov functions [28], [29] and the refined Jensen inequality [30].

Remark 11: The major contributions of this paper can be summarized as follows:

- The sampled-data H_∞ fuzzy observer is designed for a nonlinear uncertain system. Using the newly defined H_∞ performance function, the proposed design technique can be applied to an oscillating system and partially unknown system as well as an asymptotically stable system.
- Using the imperfect premise matching technique, the sampled-data fuzzy observers are designed for systems with an immeasurable premise variable as well as those with a measurable premise variable.

IV. SIMULATION EXAMPLES

In this section, two simulation examples are given to demonstrate the effectiveness of the proposed techniques. In Example 1, we show the effectiveness of the proposed techniques in an asymptotically stable system using a tunnel diode circuit system. In Example 2, using a mass-spring system, the simulation is performed in an oscillating system.

A. EXAMPLE 1

In order to demonstrate the performance of the proposed techniques for an asymptotically stable system, the following tunnel diode circuit system as shown in Fig. 1 is considered [32]:

$$\begin{aligned}
C\dot{v}_C(t) &= -0.002v_C(t)-0.01(v_C(t))^3+i_L(t)+f(v_C(t)),\\
L\dot{i}_L(t) &= -v_C(t)-Ri_L(t)+w(t),\\
y(kT) &= v_C(kT),
\end{aligned}$$

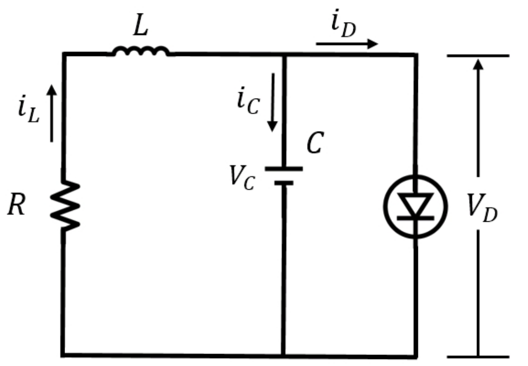

FIGURE 1. Tunnel diode circuit.

where $v_C(t)$ is the voltage of the capacitor, i_L is the current of the inductor, $f(v_C(t))$ is the uncertainty term, $w(t)$ is the disturbance, and $y(kT)$ is the output; the parameters of the capacitor, inductor, and resistance are set to $C=20\ mF$, $L=1000\ mH$, and $R=10\ \Omega$, respectively. We assume that $f(v_C(t))=0.05\sin(v_C(t))$ and by choosing $x(t)=[x_1^T\ x_2^T]^T=[v_C^T\ i_L^T]^T$, the membership functions are given by

$$h_1(z(t))=0.8-\frac{x_1^2(t)}{16},\quad h_2(z(t))=0.2+\frac{x_1^2(t)}{16},$$

and the T–S fuzzy system can be constructed as

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{r}h_i(z(t))\Big\{A_ix(t)+B_iw(t)+\alpha Fx(t)\Big\},\\
y(kT) &= Cx(kT),
\end{aligned}$$

where

$$A_1=\begin{bmatrix}0.8 & 50\\ -1 & -10\end{bmatrix},\quad A_2=\begin{bmatrix}-3.7 & 50\\ -1 & -10\end{bmatrix},$$
$$B_1=\begin{bmatrix}0\\1\end{bmatrix},\quad B_2=\begin{bmatrix}0\\1\end{bmatrix},$$
$$C=\begin{bmatrix}1 & 0\end{bmatrix},\quad F=\begin{bmatrix}0.5 & 0\\ 0 & 0\end{bmatrix},\quad \text{and}\quad \alpha=0.1.$$

The sampling time, disturbance, and initial values are set as $T=0.02$, $w(t)=e^{-0.7t}\sin(5t)$, and $x(0)=[0.1\ -0.05]^T$, $\hat{x}(0)=[0\ \ 0]^T$, respectively. In this subsection, we consider two cases: one with a measurable premise variable and one with an immeasurable premise variable.

Assuming a sampling period $T=0.02$ and solving the corresponding LMIs in Theorem 1 with $\gamma_{Thm1}=0.3$ and Theorem 2 with $\gamma_{Thm2}=0.7$ and $\mu_1=\mu_2=0.05$, the fuzzy observer gains can be obtained as follows:

$$L_1^{Thm1}=\begin{bmatrix}43.9405\\2.0981\end{bmatrix},\quad L_2^{Thm1}=\begin{bmatrix}39.5175\\2.1068\end{bmatrix},$$
$$L_1^{Thm2}=\begin{bmatrix}38.6216\\2.8336\end{bmatrix},\quad L_2^{Thm2}=\begin{bmatrix}34.2129\\2.8358\end{bmatrix}.$$

The time responses of the system and observer in the measurable premise variable case are shown in Figs. 2 and 3.

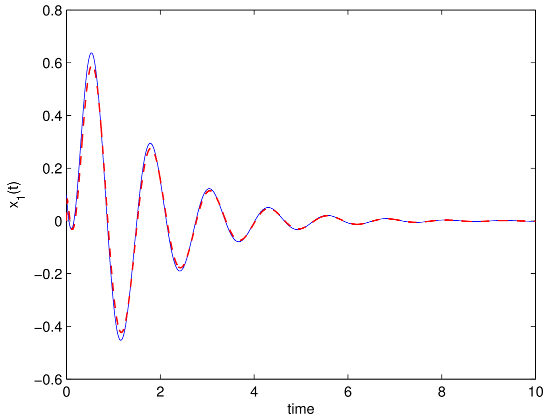

FIGURE 2. **State variable $x_1(t)$ and $\hat{x}_1(t)$ for the tunnel diode circuit system (measurable premise variable case): original (solid) and Theorem 1 (dashed).**

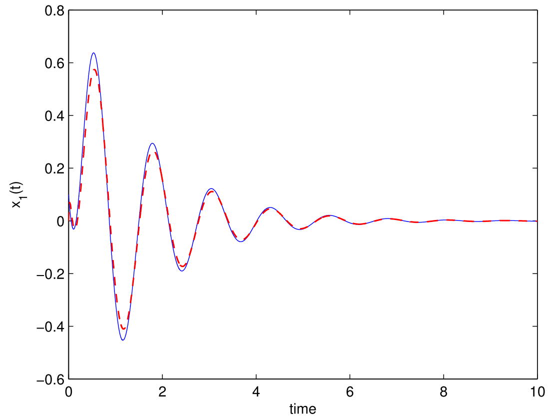

FIGURE 4. **State variable $x_1(t)$ and $\hat{x}_1(t)$ for the tunnel diode circuit system (immeasurable premise variable case): original (solid) and Theorem 2 (dashed).**

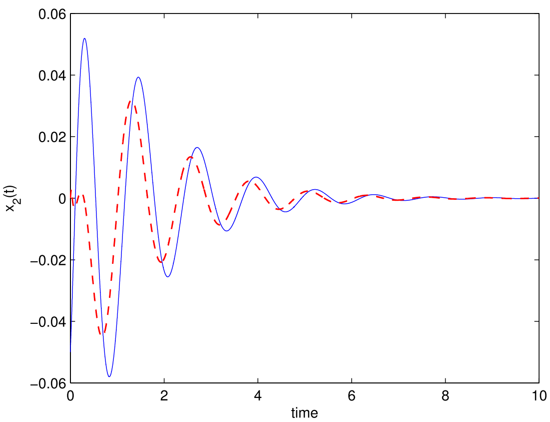

FIGURE 3. **State variable $x_2(t)$ and $\hat{x}_2(t)$ for the tunnel diode circuit system (measurable premise variable case): original (solid) and Theorem 1 (dashed).**

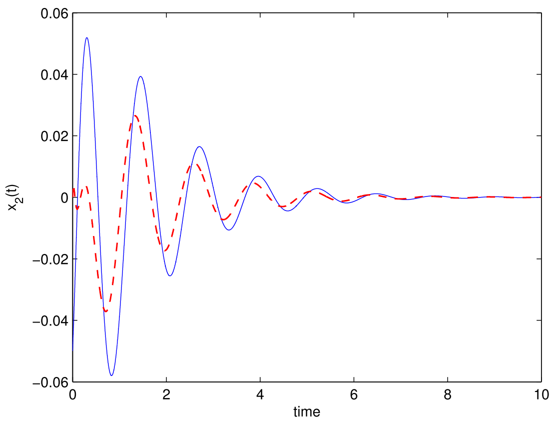

FIGURE 5. **State variable $x_2(t)$ and $\hat{x}_2(t)$ for the tunnel diode circuit system (immeasurable premise variable case): original (solid) and Theorem 2 (dashed).**

The states of the system $x_1(t)$ and $x_2(t)$ converge to zero when disturbance $w(t)$ and uncertain term $f(x(t))$ approach zero as $t \to \infty$ because the system is asymptotically stable. In addition, the error between the system and observer is bounded within some neighborhood of the origin. In order to conform the H_∞ performance of the proposed observer, the performance value is calculated under the zero initial condition $e(0) = [0\ \ 0]^T$ as follows:

$$\gamma^\star = \sqrt{\frac{\int_0^{10} e^T(\tau)e(\tau)d\tau}{\int_0^{10} w^T(\tau)w(\tau) + x^T(\tau)x(\tau)d\tau}} = 0.0820 < \gamma_{Thm1},$$

which implies that the H_∞ performance is well guaranteed.

Next, the membership functions of the observer in the immeasurable premise variable case are given by $g_1(\hat{z}(t)) = 0.5e^{\hat{x}_1^2(t)/3}$ and $g_2(\hat{z}(t)) = 1 - 0.5e^{\hat{x}_1^2(t)/3}$. The time responses of the system and observer are shown in Figs. 4 and 5. As in the measurable premise variable case, the states of the system and observer converge to zero as $t \to \infty$, and the error between the system and observer is bounded within some neighborhood of the origin. In addition, under the zero initial condition, the performance value is calculated as

$$\gamma^\star = \sqrt{\frac{\int_0^{10} e^T(\tau)e(\tau)d\tau}{\int_0^{10} w^T(\tau)w(\tau) + x^T(\tau)x(\tau)d\tau}} = 0.0909 < \gamma_{Thm2},$$

which implies that, although there is some performance degradation compared to the measurable premise variable case, the H_∞ performance is also well guaranteed in the immeasurable premise variable case.

B. EXAMPLE 2

In order to show the performance of the proposed techniques for an oscillating system, we consider the following mass-spring systems [22], [23]:

$$\ddot{x}(t) = -\vartheta(x(t)) - f(x(t)),$$
$$y(kT) = x(kT).$$

where $\vartheta(x(t)) = 0.01x(t) + 0.67x^3(t)$ and $f(x(t))$ is uncertain function that is assumed to be $f(x(t)) = 0.01\sin(x(t))$. Then, the membership functions are given by $h_1(z(t)) = 1 - x^2(t)$ and $h_2(z(t)) = x^2(t)$ and the mass-spring system can be constructed as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\Big\{A_i x(t) + B_i w(t) + \alpha F x(t)\Big\},$$
$$y(kT) = Cx(kT),$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ -0.01 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ -0.68 & 0 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 0 & 0 \\ -0.1 & 0 \end{bmatrix}, \quad \text{and} \quad \alpha = 0.1.$$

The sampling time, disturbance, and initial values are set as $T = 0.05$, $w(t) = 0.1e^{-0.5t}\sin(3t)$, and $x(0) = [0.5 \;\; 0.3]^T$, $\hat{x}(0) = [0 \;\; 0]^T$, respectively. In this section, we consider two cases: one with a measurable premise variable and one with an immeasurable premise variable.

First, to demonstrate the performance in the measurable premise variable case, using Theorem 1 with $\gamma_{Thm1} = 0.12$, and solving the corresponding LMIs, the sampled-data fuzzy observer gains are

$$L_1^{Thm1} = \begin{bmatrix} 6.8052 \\ 2.3806 \end{bmatrix}, \quad L_2^{Thm1} = \begin{bmatrix} 6.8113 \\ 1.7338 \end{bmatrix}.$$

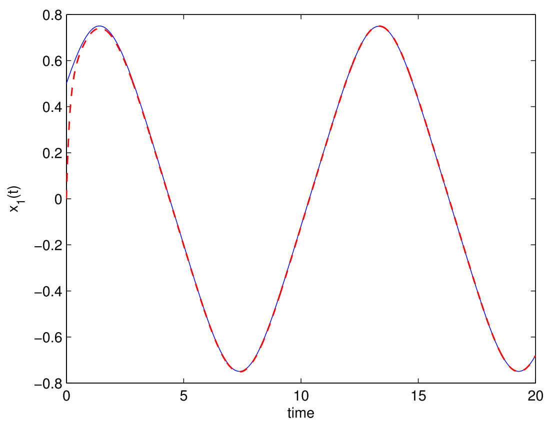

FIGURE 6. State variables $x_1(t)$ and $\hat{x}_1(t)$ for the mass-spring system (measurable premise variable case): original (solid) and Theorem 1 (dashed).

The time responses of the system and observer are shown in Figs. 6 and 7. As shown in figures, the error between the system and observer does not converge to zero although disturbance $w(t)$ approaches zero as $t \to \infty$ because uncertain term $f(x(t))$ is not zero. However, the error is bounded within some neighborhood of the origin. In addition, under the zero initial condition $e(0) = [0 \;\; 0]^T$, the performance scalar value is calculated as

$$\gamma^\star = \sqrt{\frac{\int_0^{20} e^T(\tau)e(\tau)d\tau}{\int_0^{20} w^T(\tau)w(\tau) + x^T(\tau)x(\tau)d\tau}} = 0.0147$$
$$< \gamma_{Thm1},$$

which implies that the H_∞ performance is well guaranteed.

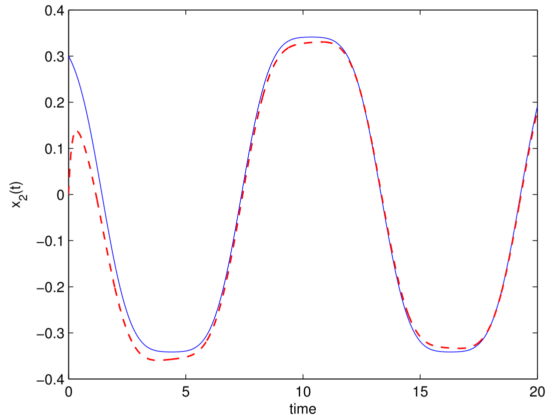

FIGURE 7. State variables $x_2(t)$ and $\hat{x}_2(t)$ for the mass-spring system (measurable premise variable case): original (solid) and Theorem 1 (dashed).

TABLE 1. Performance comparison for the mass-spring system with [8] and [22].

Sampling time T	0.05	0.10	0.15	0.20
Theorem 4.2 in [8]	Marginal infeasibility			
Corollary 2 in [22]	0.4626	0.3980	0.3335	0.2699
Corollary 3 in [22]	0.0666	0.1095	0.1318	0.1607
Theorem 2	0.0636	0.0748	0.0868	0.1036

Next, the immeasurable premise variable case is considered. Using Theorem 2 with $\mu_1 = \mu_2 = 0.01$ and $\gamma = 0.25$ and solving the corresponding LMIs, the sampled-data fuzzy observer gains are

$$L_1^{Thm2} = \begin{bmatrix} 9.8832 \\ 13.1487 \end{bmatrix}, \quad L_2^{Thm2} = \begin{bmatrix} 9.8864 \\ 12.5037 \end{bmatrix}.$$

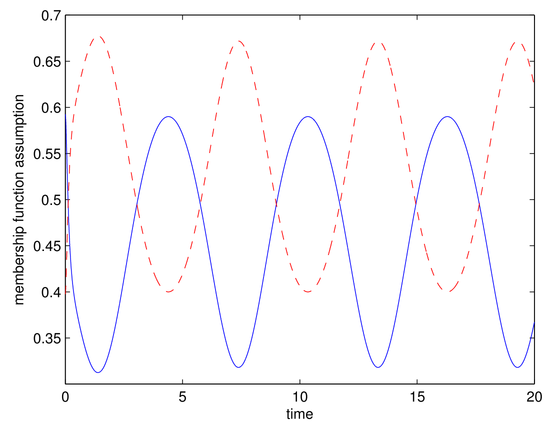

FIGURE 8. Membership function assumption: $g_1(\hat{z}(t)) - 0.05h_1(z(t))$ (solid) and $g_2(\hat{z}(t)) - 0.05h_2(z(t))$ (dashed).

The membership functions of the observer are given by $g_1(\hat{z}(t)) = 0.6 - 0.5\hat{x}_1^2(t)$ and $g_2(\hat{z}(t)) = 0.4 + 0.5\hat{x}_1^2(t)$. In Fig. 8, the time response of the membership function assumption $g_i(\hat{z}(t)) - 0.01h_i(z(t))$ is shown where the membership function inequality (21) is satisfied for all fuzzy rules.

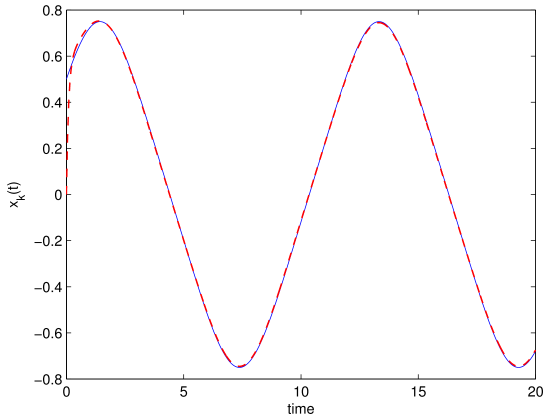

FIGURE 9. State variables $x_1(t)$ and $\hat{x}_1(t)$ for the mass-spring system (immeasurable premise variable case): original (solid) and Theorem 2 (dashed).

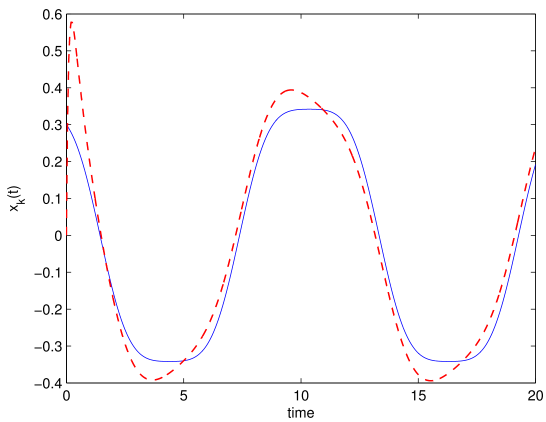

FIGURE 10. State variables $x_2(t)$ and $\hat{x}_2(t)$ for the mass-spring system (immeasurable premise variable case): original (solid) and Theorem 2 (dashed).

The time responses of the system and observer are shown in Figs. 9 and 10. Although there is a performance degradation compared to that of the measurable case because of the inconsistency of the membership function of the system and observer, the error is bounded within some neighborhood of the origin. In addition, under the zero initial condition $e(0) = [0 \;\; 0]^T$, the performance scalar value is calculated as

$$\gamma^\star = \sqrt{\frac{\int_0^{20} e^T(\tau)e(\tau)d\tau}{\int_0^{20} w^T(\tau)w(\tau) + x^T(\tau)x(\tau)d\tau}} = 0.1184 < \gamma_{Thm2},$$

which implies that the H_∞ performance is also well guaranteed in the immeasurable premise variable case of an oscillating system.

V. CONCLUSION

This paper established the sampled-data H_∞ fuzzy observer for nonlinear uncertain oscillating systems. To obtain a sampled-data fuzzy observer for the measurable premise variable case, the uncertain nonlinear system and the sampled-data observer were represented using the T–S fuzzy model. The estimation error system was constructed, and the H_∞ performance function for the oscillating system was defined. The H_∞ performance was guaranteed in the Lyapunov sense, and its sufficient conditions were derived in terms of the LMIs. The fuzzy observer in the immeasurable premise variable case was designed using the same process used in the measurable case, and the sufficient conditions were also expressed in LMI format. Finally, the feasibility of the proposed technique was confirmed via two simulation examples.

Future works

To improve the performance of observer design techniques, the sampled-data observer will be designed by using the fuzzy Lyapunov method [33] and refined Jensen inequality [30]. In addition, the proposed approach will be applied to the time-delay systems [34], [35] and observer-based control techniques [36].

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