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Distributed Decisions on TV Spectrum Allocation Considering Spatial and Temporal Variation

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ABSTRACT TV spectrum has lower path loss, longer transmission range, and higher penetration capability, resulting in a wide range of potential important applications. However, unlike Wi-Fi bands, TV spectrum is subjected to high spatial and temporal variations due to the random arrivals and departures of primary users (PUs), which results in new technical challenges in TV spectrum utilization. One important issue is how to allocate TV spectrum to secondary users (SUs) by taking the spatial and temporal variations into consideration. This has been largely ignored in previous studies. In this paper, we first formulate the TV spectrum allocation problem as a 0-1 integer optimization problem, and then we approximate our optimal objective via Log-Sum-Exp function. Thereafter, we solve this problem by implementing a Markov chain in a distributed manner. Furthermore, we extend the static problem setting to a dynamic environment where the number of vacant TV channels varies with time due to the arrivals and departures of PUs. Simulation results show that our proposed distributed algorithm can converge very fast to the optimal solution, and can achieve a close-to optimal performance with a guaranteed loss bound.

INDEX TERMS TV spectrum allocation, distributed algorithm, time-reversible Markov chains, spatial variation, temporal variation.

I. INTRODUCTION

According to the report by Cisco Visual Networking Index (CVNI), worldwide data traffic grew 63% in 2016, and is predicted to continue its rapid growth for several years [1]. This increasing demand for high data-rate wireless services has led to an impending spectrum crisis. After the transition from analog to digital television broadcast, a substantial amount of TV spectrum that was previously used by analog transmission will become available due to the higher spectrum efficiency of digital TV. These newly freed up spectra are referred to as TV white spaces (TVWS), which have drawn much attention as they could provide promising means to mitigate the spectrum scarcity problem as well as offer enormous opportunities for new applications [2]–[4]. In 2008, Federal Communications Commission (FCC) permitted the use of TVWS on an unlicensed basis [5] so long as requirements, such as minimizing the interference to the licensed users are met. Furthermore, in 2010, FCC released the final rule to approve TVWS for unlicensed operation [6].

Previous literatures have been mainly focused on how to detect and quantify the availability of TVWS [7], [8]. Recently, researchers and regulators have begun to investigate how to allocate TVWS for wireless services. The spectrum allocation problem has been extensively studied in wireless networks [9]–[12]. However, due to the unique nature of TVWS, spectrum allocation is very different from that in WiFi network. First, in order not to interfere with the licensed users, only the vacant TV spectrum can be allocated to the secondary users (SUs). Second, according to the measurement of available TV spectrum at Harvard University [13], TVWS displays significant spatial variation; therefore different SUs at different locations may have different sets of available TV spectrum for exploitation. Third, since the operations of licensed users are highly unpredictable and thus they can become active at any time. Hence temporal variation of TVWS is also expected. In this case, the TV spectrum allocated to SUs may vary with time [14]. Thus, spectrum allocation in TVWS faces great challenges, and is very much different from that in WiFi network.

Although some papers have studied TVWS allocation issues [15]–[20], they did not take the spatial and temporal variations of TVWS into consideration. To fill this gap, in this paper, we propose a distributed algorithm for TV spectrum allocation which can provide performance guarantee without the propagation of global information. The basic idea of this distributed algorithm is as follows: 1) we first approximate the optimal objective via log-sum-exp function, and then design a Markov Chain with steady-state distribution specific to this problem. We show that the Markov chain can be implemented in a distributed manner, which directly yields a distributed algorithm for our TV spectrum allocation problem. 2) We further extend the static spectrum problem setting to a dynamic environment where the licensed user can join or leave the system randomly.

The main contributions of this paper are as follows:

1) We first formulate the TV spectrum allocation problem as a 0-1 integer optimization problem, and then we use log-sum-exp function to approximate the optimal objective. Thereafter, we construct a Markov Chain to approach the optimal solution in a distributed manner.
2) We further extend our distributed algorithm to handle TV spectrum spatial and temporal variations by considering a more realistic case where the PU randomly departs or arrives at the system, which brings a greater challenge in the algorithm implementation. To the best of the authors' knowledge, this case has not been fully investigated by previous works.
3) Both the theoretical analysis and the simulation results show that the approximation gap and the convergence of our proposed distributed algorithm are guaranteed.

The rest of this paper is organized as follows. Some related works are briefly reviewed in Section II. The system model and problem formulation are given in Section III. The proposed distributed algorithm is designed in Section IV. A dynamic version of the TV spectrum allocation problem is investigated in Section V. Simulation results and evaluations are given in Section VI. Finally, Section VII concludes the paper.

II. RELATED WORKS

Research on spectrum allocation has attracted a lot of attention in the WiFi scenario. For example, in [9], a sensing and allocation strategy with one SU and multiple channels is proposed, and the optimal allocation strategy is obtained via linear programming. In [10], the spectrum allocation problem is formulated as an oligopoly market, in which several system parameters such as spectrum substitutability and channel quality on the Nash equilibrium are studied. In [11], a weighted semi-matching algorithm is proposed for resources allocation in CRNs with the network performance being improved by combining with removal algorithms and power control. In [12], the spectrum allocation problem is further investigated under a more practical scenario where the heterogeneous characteristics of both secondary sender-destination and primary channel are taken into consideration. In [21], a spectrum auction mechanism for heterogeneous secondary wireless service provisioning in CRNs is proposed, where time-dependent buyer valuation information is taken into consideration. By joint consideration of both flexible spectrum demands and the satisfaction of SUs' QoS expectations, a multi-unit spectrum auction in CR networks with power-constrained is further studied in [22]. In [23], energy-efficient maximizing resource allocation in downlink of heterogeneous networks is investigated, where the problem is formulated as a mixed-integer nonlinear fractional programing.

Different from typical WiFi bands, TVWS are subject to high spatial variation and temporal variation, hence spectrum allocation is significantly harder than that in regular WiFi network. Recently, spectrum allocation in TVWS has received extensive attention. For example, in [13], an adaptive spectrum allocation algorithm that periodically reevaluates the allocation based on TVWS availability is proposed. In [15], the spectrum allocation problem is formulated as an optimization problem with the goal of distributing spectrum fairly among all APs. In [16], the TV spectrum selection and client assignment are considered as coupled problems, which is significantly more complex than traditional methods. In [17], a business model is presented by jointly taking the pricing and admission control into consideration. In [18], an oligopoly TVWS market has been investigated, in which several SUs compete with one another to serve the end users. With the consideration of fixed and variable pricing services for end users, a TVWS database architecture is developed in [19]. Saifullah *et al.* [20] design a complete MAC protocol which features a location-aware spectrum allocation for mitigating hidden terminal effects. In [24], a throughput-efficient channel allocation framework for multi-channel cognitive vehicular networks is proposed with the objective of maximizing vehicular short-term utility. In [25], the TVWS sharing problem is modeled as a multi-objective optimization problem, and an evolutionary algorithm that shares the TVWS among coexisting networks taking care of channel occupancy requirements is designed. Huang *et al.* [26] propose a low complexity spectrum policy for the spectrum manager which assigns fragmented white spaces for heterogeneous bandwidth requests in a centralized network. In [27], a hybrid model of fuzzy rule based technique and genetic algorithm for optimal TVWS allocation is proposed.

Our work differs from previous studies in three aspects.

- First, in this paper, we mainly focus on the TV spectrum allocation problem by taking the spatial variation and temporal variation into account, which has been largely ignored in [15]–[20].
- Second, compared with [24], [26], our proposed algorithm can be implemented in a distributed

way based on the Markov approximation technique. We prove that the constructed Markov chain is time-reversible and can converge very fast to the optimal solution.

- We further extend our distributed algorithm to a dynamic environment where the PU can randomly depart or arrive at the system.

III. SYSTEM MODEL AND PROBLEM FORMULATION

In this paper, we attempt to investigate the TV spectrum allocation problem by taking the spatial and temporal variation into account. In this case, different SUs may have different available TV channels at their locations, and the available channels may change with time due to the sudden appearance of PU. Thus how to allocate TV channel to SUs based on the current TVWS availability is one of the most important problems in TV networks.

A. SYSTEM MODEL

We consider a TV network with two types of users operating in the same licensed TV channel: the PU (e.g. TV receiver) has the right to access the licensed TV channel at any time, while the SU is permitted to access the TV channel on an unlicensed basis. The operation of PU should be protected whilst the SU operates on the vacant TV channel by querying a geo-location database in advance. We assume there are N SUs and M TV channels. We use $C = \{ch_1, ch_2, \ldots, ch_M\}$ to denote the set of TV channels, and $U = \{u_1, u_2, \ldots, u_N\}$ to represent the set of SUs. Then we have $|C| = M$ and $|U| = N$. As stated in [13], TV spectrum availability varies from one location to another. Therefore, different SUs at different locations may have different sets of vacant TV channels. Due to this spatial variation, the spectrum allocated to SU should be limited to the vacant TV spectrum at its location. Let C_{u_i} be the set of available TV channels at SU u_i, with its cardinality $|C_{u_i}| = m_i$. As the temporal variation of TVWS is also expected, TVWS availability will vary with time at a given location. Thus, the value of m_i will change as the PU is turned on or turned off at a given location.

We model the topology of TV network as a general bipartite graph $G(C \cup U, \ell)$. Vertex set C corresponds to the TV channels in the network, and set U contains the SUs. An edge exists between $(ch, u) \in \ell$, $ch \in C$ and $u \in U$, if and only if the TV channel ch is available for SU u at its location. As shown in Fig. 1(a), the sets of available channels for users $u_i, 1 \leq i \leq 4$ are $C_{u_1} = \{ch_1, ch_3, ch_5\}$, $C_{u_2} = \{ch_2, ch_4\}$, $C_{u_3} = \{ch_1, ch_2, ch_4\}$, $C_{u_4} = \{ch_3, ch_5\}$. In order to control channel congestion, we assume that each TV channel can only serve content up to B_c number of users simultaneously. Therefore, we consider the constraint that each TV channel ch_j has a degree bound B_j and we allow each channel to have a different degree bound. Figs. 1(b)-(d) show a network configuration with 4 users and 5 channels under a degree bound of 2 for each TV channel.

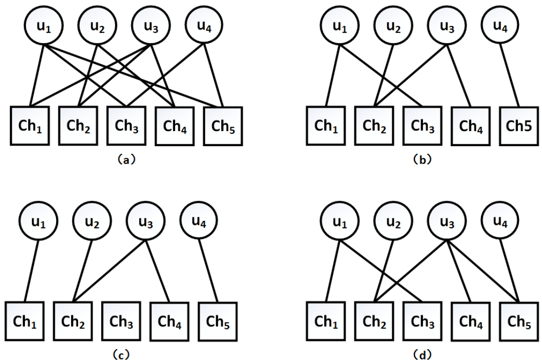

FIGURE 1. (a)Network architecture for 4 SUs and 5 TV channels in Fig.1, (b)-(d) three feasible TV spectrum allocation under degree bound 2 for each channel.

B. PROBLEM FORMULATION

Let $[\mathbb{V}]_{N\times M}$ denote the TV spectrum allocation matrix with its element v_{ij} defined as,

$$v_{ij} = \begin{cases} 1 & \text{if TV channel } ch_j \text{ is allocated to user } u_i \\ 0 & \text{Otherwise} \end{cases}$$

Since the SUs share the same TV spectrum resources in both the time and frequency domains, collision may occur between two SUs if they are within the range of each other and trying to access the same channel. To represent this collision, we define a matrix $\mathbb{A}_{N\times N\times M}$ as follows:

$$\mathcal{A}_{i_1,i_2,j} = \begin{cases} 1 & \text{if SUs } u_{i_1} \text{ and } u_{i_2} \text{ collide on channel } ch_j \\ 0 & \text{Otherwise} \end{cases}$$

The meaning of $\mathcal{A}_{i_1,i_2,j} = 1$ is that SU u_{i_1} and SU u_{i_2} collide with each other on TV channel ch_j. Thus channel ch_j cannot be allocated to SU u_{i_1} and SU u_{i_2} simultaneously for communication. Therefore, the TV spectrum allocation should satisfy the multi-user channel access condition, that is

$$\mathcal{A}_{i_1,i_2,j} = 1 \Rightarrow v_{i_1j}v_{i_2j} = 0, \quad 1 \leq i_1,\ i_2 \leq N,\ 1 \leq j \leq M \tag{1}$$

In addition, a channel accessed by a massive number of users may cause heavy congestion. Therefore, the number of SUs can be served by a channel simultaneously is limited. This constraint can be represented as

$$\sum_i v_{ij} \leq B_j, \quad 1 \leq j \leq M \tag{2}$$

Further, we assume that once a TV channel is allocated to a SU, a transmission rate can be achieved. Let x_{ij} denote the transmission rate of user u_i transmitting on TV channel ch_j. Finally, with the objective of maximizing the achievable transmission rate, the TV spectrum allocation problem can be formulated as the following optimization problem:

$$CUP-I: \max \sum_i \sum_j v_{ij} x_{ij} \quad (3)$$

$$s.t. \sum_i v_{ij} \leq B_j, 1 \leq j \leq M \quad (4)$$

$$v_{i_1 j} v_{i_2 j} = 0, \quad if\, \mathcal{A}_{i_1,i_2,j} = 1,$$

$$1 \leq i_1, \quad i_2 \leq N, \ 1 \leq j \leq M \quad (5)$$

$$v_{ij} \in \{0,1\}, \quad 1 \leq i \leq N, \ 1 \leq j \leq M \quad (6)$$

Let $\mathscr{F}$ be the set of all feasible configurations that satisfy all the constraints for the TV spectrum allocation problem. For a configuration $f \in \mathscr{F}$, we use x_f to denote the objective function under configuration f. Then, the **CUP-I** problem can be formulated as follows:

$$\max_{f \in \mathscr{F}} x_f \quad (7)$$

It is obviously that the formulated problem is a combinatorial optimization problem and the complexity to find the optimal solution will grow exponentially as the number of SUs and TV channels increases. In the next section, we propose a distributed algorithm to obtain a close-to-optimal solution.

IV. THE DISTRIBUTED ALGORITHM

Due to the combinatorial feature, it seems very challenging to solve this problem in polynomial time. In [28], a Markov approximation technique in designing distributed algorithms for solving combinatorial problems approximately is proposed. Benefiting from this framework, in this paper, we can leverage on the Markov approximation to construct a distributed algorithm for solving our combinatorial problem defined in CUP-I approximately. In the following, we describe in detail the two steps in the design of the distributed algorithm: log-sum-exp approximation and Markov Chain implementation.

A. LOG-SUM-EXP APPROXIMATION

We first use log-sum-exp function to approximate the maximum objective function in (7), that is

$$\max_{f \in \mathscr{F}} x_f \approx \frac{1}{\xi} \log\Big(\sum_{f \in \mathscr{F}} \exp(\xi x_f)\Big) \quad (8)$$

where ξ is a positive constant. Let $|\mathscr{F}|$ be the size of $\mathscr{F}$, then the approximation gap is upper-bounded by $\frac{1}{\xi} \log |\mathscr{F}|$ [28]. Thus, as $\xi \to \infty$, the approximation gap goes to zero.

To provide a better understanding, we associate each configuration f with a probability p_f, resulting in the following equivalent problem

$$CUP-II: \max_{\boldsymbol{P} \geq 0} \sum_{f \in \mathscr{F}} p_f x_f \quad (9)$$

$$s.t. \sum_{f \in \mathscr{F}} p_f = 1 \quad (10)$$

where $\boldsymbol{P} = [p_f]_{f \in \mathscr{F}}$ is the associated probability for all configurations. We can easily observe that the optimal value of CUP-II is the same as (7) and can be obtained by setting the probability of the optimal configuration to be 1, and the other configurations to be 0.

According to Theorem 1 in [28], we can rewrite **CUP-II** problem in the following approximated form, that is

$$CUP-III: \max_{\boldsymbol{P} \geq 0} \sum_{f \in \mathscr{F}} p_f x_f - \frac{1}{\xi} \sum_{f \in \mathscr{F}} p_f \log(p_f) \quad (11)$$

$$s.t. \sum_{f \in \mathscr{F}} p_f = 1 \quad (12)$$

By solving the Karush-Kuhn-Tucker(KKT) conditions on the **CUP-III** problem [29], we can come to the following conclusion in Theorem 1, that is

Theorem 1: The optimal solution and optimal value of the ***CUP-III*** *problem are given by*

$$p_f^*(\boldsymbol{x}) = \frac{\exp(\xi x_f)}{\sum_{f' \in \mathscr{F}} \exp(\xi x_{f'})}, \quad \forall f \in \mathscr{F} \quad (13)$$

and

$$\frac{1}{\xi} \log\Big[\sum_{f \in \mathscr{F}} \exp(\xi x_f)\Big] \approx \max_{f \in \mathscr{F}} x_f \quad (14)$$

Since the optimal value of CUP-III is $\frac{1}{\xi} \log(\sum_{f \in \mathscr{F}} \exp(\xi x_f))$, then the original problem in (7) is implicitly solved by computing an approximated version of the problem CUP-II, off by an entropy term $\frac{1}{\xi} \sum_{f \in \mathscr{F}} p_f \log(p_f)$ with approximation gap bounded by $\frac{1}{\xi} \log |\mathscr{F}|$. Therefore, if we can construct a Markov Chain with steady-state distribution according to (13) in a distributed manner, the system will time share among all the feasible configurations, when this Markov Chain converges.

B. MARKOV CHAIN ALGORITHM DESIGN

We use C_u to denote the set of vacant channels for user $u \in U$. Let C_u^f be the set of in-use TV channels for user u under configuration f. The set of not-in-used channels is given by C_u/C_u^f. Further, we denote $\mathscr{C}_f = \{\{u, ch\}, \forall u \in U, ch \in C_u^f\}$ as the set of all user-channel association under the configuration f. An example is shown in Fig. 1(b), $\mathscr{C}_f = \{\{u_1, ch_1\}, \{u_1, ch_3\}, \{u_2, ch_2\}, \{u_3, ch_2\}, \{u_3, ch_4\}, \{u_4, ch_5\}\}$. In order to construct our Markov Chain, we only allow direct transition between two adjacent configurations f and f' if such transition corresponds to a user u selecting a channel from C_u/C_u^f to use or removing a channel from C_u^f, which means that the transition rate $q_{f,f'}$ is set to zero unless $|\mathscr{C}_f/\mathscr{C}_{f'}| = 1$ or $|\mathscr{C}_{f'}/\mathscr{C}_f| = 1$. Under such assumption, the transition between configuration f in Fig. 1(b) to configuration f' in Fig. 1(c) is feasible, while the direct transition from the configuration in Fig. 1(c) to that in Fig. 1(d) is not permitted.

In order to make the designed Markov Chain time-reversible, the choice of transition rate has to satisfy the following detailed balance equation

$$p_f^*(\boldsymbol{x}) q_{f,f'} = p_{f'}^*(\boldsymbol{x}) q_{f',f}, \quad \forall f, f' \in \mathscr{F} \quad (15)$$

In our paper, we design the transition rates $q_{f,f'}$ and $q_{f',f}$ as

$$q_{f,f'} = \frac{1}{2}\frac{1}{\exp(\tau)}\frac{\exp(\xi x_{f'})}{\exp(\xi x_f)+\exp(\xi x_{f'})} \quad (16)$$

$$q_{f',f} = \frac{1}{2}\frac{1}{\exp(\tau)}\frac{\exp(\xi x_{f})}{\exp(\xi x_f)+\exp(\xi x_{f'})} \quad (17)$$

where τ is a constant. We can easily verify that the detailed balance equation holds. It can be noted that with such design, the system seems to favor configuration with a larger transmission rate.

Next, we design the distributed implementation of Markov Chain as follows:

- **Initially:** Each user $u \in U$ randomly selects TV channels from C_u under the degree bound and multi-user channel access condition, and uses these channels for transmission.
- **Step 1:** Each user $u \in U$ generates a timer which is independent and exponential with mean $\frac{2\exp(\tau)}{|C_u|}$ and starts to count down.
- **Step 2:** When the timer expires, user u measures its received transmission rate to estimate x_f. User u will go to Step 3a with probability $\frac{|C_u^f|}{|C_u|}$ and go to Step 3b with probability $\frac{|C_u|-|C_u^f|}{|C_u|}$.
  - **Step 3a:** User u randomly selects a in-use channel from C_u^f to remove, and then estimates rate x_f'.
  - **Step 3b:** User u randomly selects a channel from not-in-use C_u/C_u^f to transmit, and then estimates rate x_f'.
- **Step 4:** With the estimates of x_f and x_f', user u switches to the new configuration f' with probability $\frac{\exp(\xi x_{f'})}{\exp(\xi x_f)+\exp(\xi x_{f'})}$, and back to configuration f with probability $1-\frac{\exp(\xi x_{f'})}{\exp(\xi x_f)+\exp(\xi x_{f'})}$. Then repeat Step 1.

The proposed implemented algorithm for the Markov Chain is further illustrated in detail in Algorithm 1.

From the above algorithm, we can note that the generation of count-down timer does not require the global information of the system. We use the user's received transmission rate to estimate $x_{f'}$. Thus the algorithm can be implemented in a distributed manner and runs on each individual user independently.

Theorem 2: The proposed Algorithm 1 realizes a time-reversible Markov Chain and its stationary distribution $p_f^*(\boldsymbol{x})$ is given by (13), $\forall f \in \mathscr{F}$.

The proof of Theorem 2 is relegated to Appendix-A.

Algorithm 1 Implemented Algorithm for Markov Chain

Require: U : User set;
C_{u_i} : Available TV channels set for user u_i, $\forall u_i \in U$.
Initialization: Each user $u_i \in U$ randomly selects channels from C_{u_i} under the degree bound and multi-user channel access condition for use.

Procedure Selection (u_i)
  Each user u_i generates a timer which follows an exponential distribution with a mean of $\frac{2\exp(\tau)}{|C_{u_i}|}$;
End Procedure
while the timer expires **do**
  With probability $\frac{|C_{u_i}^f|}{|C_{u_i}|}$,
    User u_i randomly removes a in-use TV channel
    from $C_{u_i}^f$, and estimates $x_{f'}$ for configuration f';
    $f \to f'$ with probability $\frac{\exp(\xi x_{f'})}{\exp(\xi x_f)+\exp(\xi x_{f'})}$;
  With probability $1-\frac{|C_{u_i}^f|}{|C_{u_i}|}$,
    User u_i randomly chooses a channel from
    $C_{u_i}/C_{u_i}^f$, and estimates transmission rate $x_{f'}$;
    $f \to f'$ with probability $\frac{\exp(\xi x_{f'})}{\exp(\xi x_f)+\exp(\xi x_{f'})}$;
  refresh the timer and begin counting down.
end while
return *rules*

C. CONVERGENCE ANALYSIS

In the following subsection, we will analyze the convergence of our designed algorithm. In our Markov Chain, the estimate of x_f may be inaccurate, which will lead to the algorithm not converging to the stationary distribution $p_f^*(\boldsymbol{x})$.

For each feasible configuration $f \in \mathscr{F}$, let ϕ_f denote the inaccuracy bound, then $[-\phi_f, \phi_f]$ is the bounded region of the inaccurate rate [30]. Furthermore, we quantify the observed rate to $2k_f+1$ discrete values:

$$[x_f-\phi_f, \ldots, x_f-\frac{1}{k_f}\phi_f, x_f, x_f+\frac{1}{k_f}\phi_f, \ldots, x_f+\phi_f]$$

Let π_{f_j} be the probability that the observed rate takes value $x_f+\frac{j}{k_f}\phi_f$, $\forall j \in \{-k_f, \ldots, k_f\}$ and $\sum_{j=-k_f}^{k_f}\pi_{f_j}=1$. Given a configuration $f \in \mathscr{F}$ and its estimated rate x_f in the original Markov Chain, the system transits to a new configuration f' with probability $\frac{\exp(\xi x_{f'})}{\exp(\xi x_f)+\exp(\xi x_{f'})}$, and stays in configuration f with probability $1-\frac{\exp(\xi x_{f'})}{\exp(\xi x_f)+\exp(\xi x_{f'})}$. However, due to the inaccurate observed rate, there are extra $2k_f+1$ states for configuration f: $(f, x_f+\frac{j}{k_f}\phi_f), j \in \{-k_f, \ldots, k_f\}$. In this case, the system in state $(f, x_f+\frac{j}{k_f}\phi_f)$ will switch to new a configuration f' with probability

$$\frac{\exp(\xi(x_{f'}+\frac{j}{k_{f'}}\phi_{f'}))}{\exp(\xi(x_f+\frac{j}{k_f}\phi_f))+\exp(\xi(x_{f'}+\frac{j}{k_{f'}}\phi_{f'}))}$$

and stay in configuration f with probability

$$1-\frac{\exp(\xi(x_{f'}+\frac{j}{k_{f'}}\phi_{f'}))}{\exp(\xi(x_f+\frac{j}{k_f}\phi_f))+\exp(\xi(x_{f'}+\frac{j}{k_{f'}}\phi_{f'}))}$$

For ease of expression, in the following, we use f_j and f_j' to represent the states $(f, x_f+\frac{j}{k_f}\phi_f)$ and $(f', x_{f'}+\frac{j}{k_{f'}}\phi_{f'})$, for all $f, f' \in \mathscr{F}, j \in \{-k_f, \ldots, k_f\}$. Similar to (16), the transition rate from state $(f, x_f + \frac{j}{k_f}\phi_f)$ to state $(f', x_{f'} + \frac{j}{k_{f'}}\phi_{f'})$ is given by

$$q^*_{f_j f'_j} = \frac{\exp(\xi(x_{f'} + \frac{j}{k_{f'}})\phi_{f'})}{\exp(\xi(x_{f'} + \frac{j}{k_{f'}})\phi_{f'}) + \exp(\xi(x_f + \frac{j}{k_f})\phi_f)} \cdot \frac{\pi_{f'_j}}{2\exp(\tau)} \quad (18)$$

and we also have

$$q^*_{f'_j f_j} = \frac{\exp(\xi(x_f + \frac{j}{k_f})\phi_f)}{\exp(\xi(x_{f'} + \frac{j}{k_{f'}})\phi_{f'}) + \exp(\xi(x_f + \frac{j}{k_f})\phi_f)} \cdot \frac{\pi_{f_j}}{2\exp(\tau)} \quad (19)$$

where $\sum_{j=-k_f}^{k_f} \pi_{f_j} = 1$ and $\sum_{j=-k_{f'}}^{k_{f'}} \pi_{f'_j} = 1$. We refer to such new Markov Chain as an extended version of the original one proposed in the previous subsection. The extended Markov Chain has a finite number of states, and any two states are reachable from each other. Therefore, this extended Markov Chain is irreducible and a unique stationary distribution exists.

Let $\bar{p}_{f, x_f + \frac{j}{k_f}\phi_f}$ denote the stationary distribution under the configuration $f \in \mathscr{F}$ with observed rate $x_f + \frac{j}{k_f}\phi_f$, then we have

$$\bar{P} = [\bar{p}_{f, x_f + \frac{j}{k_f}\phi_f}, j \in \{-k_f, \ldots, k_f\}, f \in \mathscr{F}] \quad (20)$$

As there are $2k_f + 1$ states under configuration f in the new Markov Chain, the stationary distribution of configuration f is given by

$$\tilde{p}_f(\mathbf{x}) = \sum_{j=-k_f}^{k_f} \bar{p}_{f, x_f + \frac{j}{k_f}}, \quad \forall f \in \mathscr{F} \quad (21)$$

Thus, the stationary distribution of the new Markov Chain is denoted by $\tilde{P} = [\tilde{p}_f(\mathbf{x}), f \in \mathscr{F}]$.

After the extended Markov Chain has been designed, we can study the impact of the inaccurate transmission rate. To provide a convergence analysis, the total variance distance is used to quantify the difference between P^* and $\tilde{P}$ [31], which is defined as

$$d_{TV}(P^*, \tilde{P}) = \frac{\sum_{f \in \mathscr{F}} |p^* - \tilde{p}|}{2} \quad (22)$$

where $P^* = [p^*_f(\mathbf{x}), \forall f \in \mathscr{F}]$ is the stationary distribution of the original Markov Chain. Then we come to the conclusion in following theorem 3.

Theorem 3: The bound of $d_{TV}(P^*, \tilde{P})$ is given by :

$$0 \le d_{TV}(P^*, \tilde{P}) \le 1 - \exp(-2\xi\phi_{max}) \quad (23)$$

where $\phi_{\max} = \max_{f \in \mathscr{F}} \phi_f$. Moreover, let $x_{\max} = \max_{f \in \mathscr{F}} x_f$, then the optimality gap in transmission rate $|P^*\mathbf{x}^T - \tilde{P}\mathbf{x}^T|$ is bounded by

$$|P^*\mathbf{x}^T - \tilde{P}\mathbf{x}^T| \le 2x_{\max}(1 - \exp(-2\xi\phi_{\max})) \quad (24)$$

The proof of Theorem 3 is relegated to Appendix-B.

V. DYNAMIC MARKOV CHAIN

In this section, we extend the above static problem setting to a dynamic environment, which will bring about new significant challenges. As observed in [13], the behaviors of PUs are highly unpredictable as they can become active at any time without any warning. In this case, the SU needs to promptly vacate the occupied TV channel in order not to interfere with the PUs. Due to this temporal variation of TVWS, when a PU arrives, the dedicated channel must be vacated to serve the PU. When a PU departs, the channel will be released and reused by the SUs. Therefore the number of vacant TV channels for each SU varies with time.

To model such temporal variation, we assume that the PUs arrive in the system according to a Poisson process with parameter λ, and stay for a time that is exponentially distributed with parameter μ. Thus, the TV channel becomes vacant at a rate of μ and unavailable at a rate of λ. Under this setting, the number of TV channels in the system will obey a $M/M/m$ queue [33]. Let $\varphi = \frac{\mu}{\lambda}$, the stationary distribution is given by:

$$\Psi_m = \frac{\varphi^m e^{-\varphi}}{m!} \quad (25)$$

Furthermore, we can get the following steady-state equation

$$\Psi_m \mu = \Psi_{m+1}(m+1)\lambda \quad (26)$$

Let $\mathscr{F}_m$ denote the set of all feasible configurations where there are m TV channels available, and $x^m_{f_m}$ be the system objective function for a given $f_m \in \mathscr{F}_m$. Then we have the optimal performance for m channels as follows

$$x^m_{\max} = \max_{f_m \in \mathscr{F}_m} x^m_{f_m} \quad (27)$$

Meanwhile, the long-term averaged system performance can be denoted as:

$$\Upsilon^* = \sum_{m=0}^{\infty} \Psi_m x^m_{\max} \quad (28)$$

A. LOG-SUM-EXP APPROXIMATION

Likewise,we have the following function by using the Log-Sum-Exp approximation:

$$x^m_{\max} \approx \frac{1}{\xi} \log[\sum_{f_m \in \mathscr{F}_m} \exp(\xi x^m_{f_m})] \quad (29)$$

Combining (28) and(29), the long-term averaged system performance can be rewritten as

$$\Upsilon^* \approx \sum_{m=0}^{\infty} \Psi_m \frac{1}{\xi} \log[\sum_{f_m \in \mathscr{F}_m} \exp(\xi x^m_{f_m})] \quad (30)$$

Next, we use p_{f_m} to denote the percentage of time associated with configuration $f_m \in \mathscr{F}_m$. Using similar argument as in the previous section, we get the following theorem

Theorem 4: The optimal value of the following optimization problem

$$\max_{\boldsymbol{P}\geq 0} \sum_{m=0}^{\infty}\sum_{f_m\in\mathscr{F}_m} p_{f_m}x_{f_m}^{m} - \frac{1}{\xi}\sum_{m=0}^{\infty}\sum_{f_m\in\mathscr{F}_m} p_{f_m}\log(p_{f_m}) \tag{31}$$

$$\text{s.t.} \sum_{f_m\in\mathscr{F}_m} p_{f_m} = \Psi_m, \quad \forall f_m \in \mathscr{F}_m,\ m = 0, 1, 2, \ldots \tag{32}$$

is (30), and the optimal solution is given by

$$p^*_{f_m} = \Psi_m \frac{\exp(\xi x^m_{f_m})}{\sum_{f'_m\in\mathscr{F}_m}\exp(\xi x^m_{f'_m})}, \quad \forall f_m \in \mathscr{F}_m,\ m=0, 1, 2, \ldots \tag{33}$$

B. MARKOV CHAIN ALGORITHM DESIGN

Now, we design a Markov Chain with stationary distribution $p^*_{f_m}$. We only allow transitions between two states when a PU arrives or departs. Suppose there are m TV channels in the system. When a PU departs, a TV channel occupied by the PU will be released and the system will leave state f_m and enter into state f_{m+1}. When a PU arrives, one of the vacant TV channels will be removed from the system and the state will transit from f_{m+1} to f_m. We use $q_{f_m\to f_{m+1}}$ to denote the transition rate from state f_m to f_{m+1}, which is defined as follows

$$q_{f_m\to f_{m+1}} = \mu\frac{\exp(\xi x^{m+1}_{f_{m+1}})}{\sum_{s'\in\mathscr{S}, f_{m+1}=f_m\cup s'}\exp(\xi x^{m+1}_{f_{m+1}})} \tag{34}$$

where $\mathscr{S}$ is the set of local configurations that are available for the new channel, and $f_{m+1} = f_m \cup \{s\}$.

When a PU arrives, one of TV channels will be vacated, and the system will transit from state f_{m+1} to state f_m with rate $q_{f_{m+1}\to f_m}$ given by

$$q_{f_{m+1}\to f_m} = \lambda(m+1)\frac{\sum_{f_{m+1}\in\mathscr{F}_{m+1}}\exp(\xi x^{m+1}_{f_{m+1}})}{\sum_{f_m\in\mathscr{F}_m}\exp(\xi x^m_{f_m})} \times \frac{\exp(\xi x^m_{f_m})}{\sum_{s'\in\mathscr{S}, f'_{m+1}=f_m\cup s'}\exp(\xi x^{m+1}_{f'_{m+1}})} \tag{35}$$

Next, we illustrate the implemented algorithm for the Markov Chain, which should be time-reversible, and satisfy the following detailed balance function

$$p^*_{f_m} q_{f_m\to f_{m+1}} = p^*_{f_{m+1}} q_{f_{m+1}\to f_m} \tag{36}$$

- **Step 1:** Let f_m be the current configuration, when a PU departs, the system will leave state f_m and transit to state $f_{m+1} = f_m \cup s$.
- **Step 2:** The new channel will be associated to local configuration $s \in \mathscr{S}$ with a probability of

$$\frac{\exp(\xi x^{m+1}_{f_{m+1}})}{\sum_{s'\in S, f_{m+1}=f_m\cup s'}\exp(\xi x^{m+1}_{f_{m+1}})}$$

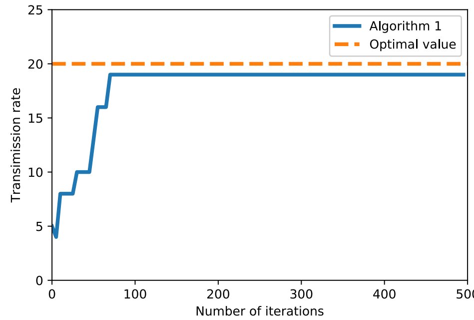

FIGURE 2. The achieved transmission rate with $N = 5$ and $M = 5$.

- **Step 3:** When a PU arrives, one of the TV channels will be removed from use with a probability

$$\frac{\sum_{f_{m+1}\in\mathscr{F}_{m+1}}\exp(\xi x^{m+1}_{f_{m+1}})}{\sum_{f_m\in\mathscr{F}_m}\exp(\xi x^m_{f_m})}\cdot\frac{\exp(\xi x^m_{f_m})}{\sum_{s'\in\mathscr{S}, f'_{m+1}=f_m\cup s'}\exp(\xi x^{m+1}_{f'_{m+1}})}$$

  and the system will enter state f_m.

Theorem 5: The implementation of Algorithm 2 realizes a time-reversible Markov Chain with stationary distribution $p^*_{f_m}$.

The proof of Theorem 5 is relegated to Appendix-C.

VI. SIMULATION RESULTS

In this section, we use simulations to evaluate our proposed distributed algorithms. We set $\xi = 2$ and $\tau = 6$. In order to model the random characteristic of SUs, the transmission rates x_{ij} are randomly generated between 1 and 4. We let the number of SUs and TV channels vary from 5 to 25. We run the simulations 1000 times to obtain the average system performance.

A. EVALUATION OF ALGORITHM 1

In this subsection, we evaluate our proposed distributed algorithm by checking the following two aspects:

1) Does the algorithm converge to the optimal solution as expected from the theoretical analysis?
2) How fast does it converge?

We randomly choose a feasible configuration and run our algorithm over it. The results are displayed in Figs. 2-4. From these three figures, we can see that the transmission rate increases with the iterations, which means that our algorithm is likely to make the system transit to a configuration with better performance until it converges. This can be achieved within 80 iterations for a small network with $N = 5$ and $M = 5$ in Fig. 2, 149 iterations for a medium network with $N = 10$ and $M = 15$ in Fig. 3, and 420 iterations for a relatively large network with $N = 25$ and $M = 25$ in Fig. 4. These results illustrate that the number of iterations needed for convergence increases as the network size increases. Furthermore, we can observe that the transmission rate achieved is very close to the optimal value obtained by using the exhaustive method. The approximation gap is 1 for small and medium networks as shown in Figs. 2-3, and 3 for a large

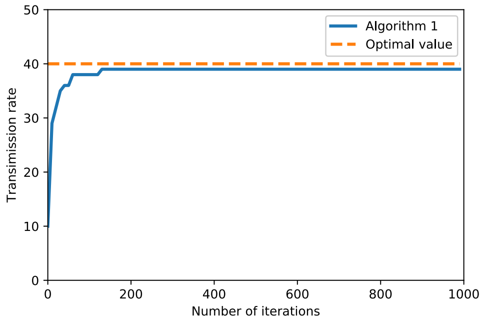

FIGURE 3. **The achieved transmission rate with $N = 10$ and $M = 15$.**

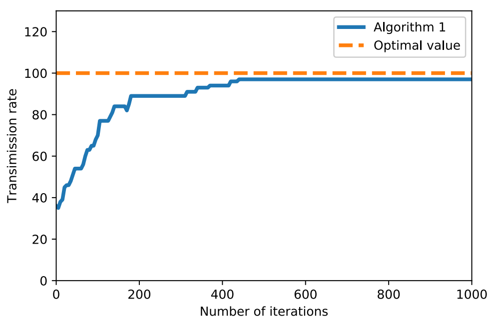

FIGURE 4. **The achieved transmission rate with $N = 25$ and $M = 25$.**

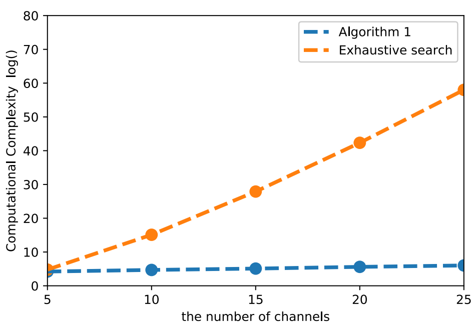

FIGURE 5. **Computational complexity for original NP hard problem and our proposed algorithm.**

network as shown in Fig. 4. This is consistent with the computed theoretical result of $\frac{1}{\xi}\log|\mathscr{F}| = 2.39$. Therefore, from Figs. 2-4, we can verify that our algorithm converges very fast to the optimal solution, and can achieve a close-to optimal performance.

Next, we will study the performance of complexity over the number of channels. As illustrated in Fig.5, our proposed algorithm is much simpler than the original NP hard problem where the optimal value is obtained using exhaustive search, especially for a large number of channels. Thus the designed distributed algorithm can achieve a close-to-optimal solution with less complexity.

B. PERFORMANCE COMPARISON

Next, we compare our distributed algorithm with the following two methods when the number of channels M varies from 5 to 25.

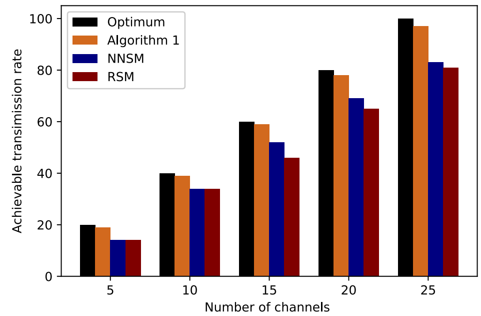

FIGURE 6. **Comparison between our algorithm, Optimum, NNSM and RSM.**

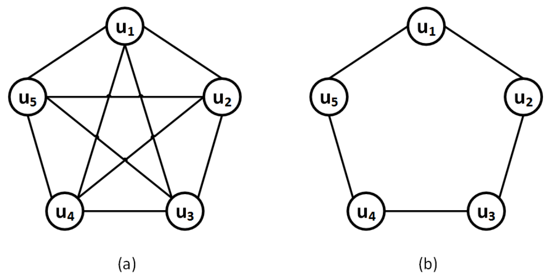

FIGURE 7. **Collision graph settings for simulation.**

1) Nearest Neighbor Selection Method (NNSM): Each user transmits on the channel that is closest to it.
2) Random Selection Method (RSM): Each user randomly chooses one vacant channel to use.

The simulation result is illustrated in Fig. 6. As a comparison, the transmission rate achieved by our distributed algorithm is also shown. From Fig. 6, we can see that Algorithm 1 outperforms RSM and NNSM in transmission rate, and is closest to the optimal value. Furthermore, this improvement becomes larger and larger as the number of TV channels increases. The reason is that our proposed distributed algorithm can make the system transit to a better configuration, and stay on the best configuration for most of time, while for NNSM and RSM, each user only considers its own preference as the system is unable to reach the global optimum. From Fig. 6, it can also be noted that the transmission rate increases as the number of TV channels increases.

C. EFFECT OF COLLISION GRAPH

In order to better understand how our proposed algorithm performs, we run our algorithm under the following three collision graph cases:

- Case I: As shown in Fig. 7(a), all the SUs collide with one another, which means that any two users cannot be allocated with the same TV channel.
- Case II: As shown in Fig. 7(b), any SU only collides with two adjacent users, thus TV channels re-utilization is possible in some cases.
- Case III: The collision graph is randomly generated.

From Fig. 8, we can see that the achieved transmission rate increases as the number of TV channels increases.

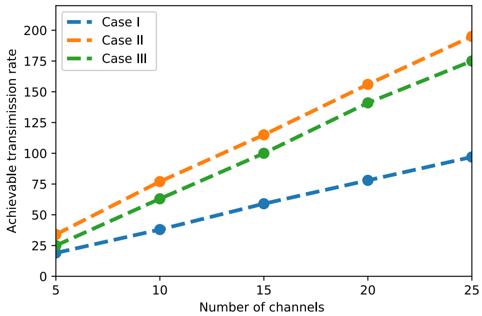

FIGURE 8. Transmission rates for different collision graph settings.

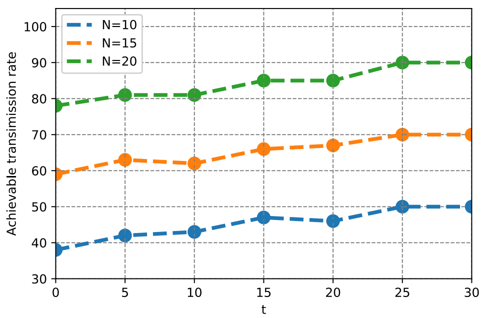

FIGURE 9. Transmission rate for $\mu = 6, \lambda = 3$.

Furthermore, we note that Case II can provide optimal performance in terms of transmission rate. The reason is obvious since for Case II, the same TV channel can be allocated to different non-conflicting SUs, which increases the achievable transmission rate. While for Case I, all the SUs conflict with one another, thus no TV channel can be re-utilized.

D. EVALUATION OF THE ALGORITHM IN DYNAMIC SCENARIO

In this subsection, we evaluate our algorithm in a dynamic scenario where the PUs randomly arrive and depart. The unit time is set as 30, and is divided into 30 slots of equal length. We then evaluate our algorithm for the following two settings.

- Setting I: $\mu = 6$, $\lambda = 3$, which means that there are 3 PUs arriving and 6 PUs departing per unit time.
- Setting II: $\mu = 3$, $\lambda = 5$, which means that there are 5 PUs arriving and 3 PUs departing per unit time.

Figs. 9 and 10 depict the achievable transmission rate as a function of the number of TV channels for different values of $N = \{10, 15, 20\}$ under setting I and setting II. From Fig. 9, we can observe that the achievable transmission rate increases at time point $t = \{5, 15, 25\}$ due to the departure of PU, and keeps invariant at time point $t = \{10, 20, 30\}$ because the arrival and departure of PUs both happen at these points. From Fig. 10, we can see that the transmission rate decreases at time point $t = \{6, 12, 18, 24\}$. The reason is that there are PU arrivals and but no PU departure at these point, which requires the PUs to occupy the TV channel being used by SUs. Therefore, for the case of $\mu = 3$ and $\lambda = 5$, the achievable transmission rate decreases as time goes on because that more PUs join the system.

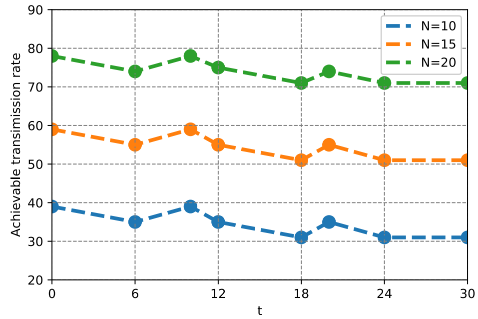

FIGURE 10. Transmission rate for $\mu = 10, \lambda = 3$.

VII. CONCLUSION

In this paper, we focus on the TV spectrum allocation problem by taking the spatial and temporal variations of TVWS into consideration. This has always been ignored in most of the literatures. With the objective to maximize the transmission rate, we formulate the TV spectrum allocation problem as a 0-1 integer optimization problem, which is a combinatorial optimization problem. We propose a distributed algorithm to approach the optimal solution by Markov approximation method. Furthermore, we extend our distributed algorithm to a more realistic case where the PU randomly departs or arrives at the system. Evaluation results demonstrate that our proposed distributed algorithm can converge very fast to the optimal solution.

APPENDIX

A. PROOF OF THEOREM 2

Proof: We only allow direct transitions between two configurations f and f' if $|\mathscr{C}_f/\mathscr{C}_{f'}| = 1$ or $|\mathscr{C}_{f'}/\mathscr{C}_f| = 1$ in our Markov Chain design. In other words, we allow a user to remove a channel from C_u^f or add a new channel from C_u/C_u^f. Let $Pr\{f \rightarrow f'\}$ denote the probability that the process will enter state f' while leaving state f when the timer expires, which contains the following two cases:

Case I: $|\mathscr{C}_f/\mathscr{C}_{f'}| = 1$, which means that user u removes a channel from C_u^f, the probability is

$$\begin{aligned} Pr(f \rightarrow f') &= \frac{|C_u^f|}{|C_u|} \cdot \frac{1}{|C_u^f|} \cdot \frac{\exp(\xi x_{f'})}{\exp(\xi x_f) + \exp(\xi x_{f'})} \\ &\quad \cdot \frac{\frac{|C_u|}{2\exp(\tau)}}{\sum_{v \in U} \frac{|C_v|}{2\exp(\tau)}} \\ &= \frac{1}{\sum_{u \in U} |C_u|} \cdot \frac{\exp(\xi x_{f'})}{\exp(\xi x_f) + \exp(\xi x_{f'})} \end{aligned} \quad (37)$$

Case II: $|\mathscr{C}_{f'}/\mathscr{C}_f| = 1$, which means that user u adds a new channel from C_u/C_u^f, the probability is

$$Pr(f \to f') = \frac{|C_u| - |C_u^f|}{|C_u|} \cdot \frac{1}{|C_u| - |C_u^f|} \cdot \frac{\exp(\xi x_{f'})}{\exp(\xi x_f) + \exp(\xi x_{f'})} \cdot \frac{\frac{|C_u|}{2\exp(\tau)}}{\sum_{v \in U} \frac{|C_v|}{2\exp(\tau)}} = \frac{1}{\sum_{u \in U} |C_u|} \cdot \frac{\exp(\xi x_{f'})}{\exp(\xi x_f) + \exp(\xi x_{f'})} \tag{38}$$

Each user u counts down at a rate given by

$$\frac{|C_u|}{2} \exp^{-1}(\tau) \tag{39}$$

Therefore, the process leaves state f at a rate of

$$\sum_{u \in U} \frac{|C_u|}{2} \exp^{-1}(\tau) \tag{40}$$

By combining (37), (38) and (40), we can calculate the transition rate from f to f' as:

$$q_{f,f'} = \frac{1}{\sum_{u \in U} |C_u|} \cdot \frac{\exp(\xi x_{f'})}{\exp(\xi x_f) + \exp(\xi x_{f'})} \cdot \sum_{u \in U} \frac{|C_u|}{2} \exp^{-1}(\tau) = \frac{1}{2\exp(\tau)} \cdot \frac{\exp(\xi x_{f'})}{\exp(\xi x_f) + \exp(\xi x_{f'})} \tag{41}$$

Together with (13), we can easily verify that the balance equation satisfies between any two adjacent states f and f'. Thus the constructed Markov Chain is time-reversible and its stationary distribution is (13) according to Theorem 1.3 and Theorem 1.4 in [32].

This completes the proof. □

B. PROOF OF THEOREM 3

Proof: Let $\mathscr{M}$ be the original Markov Chain, and $\mathscr{M}'$ be the extended Markov Chain. Suppose there exist direct transitions between configurations f and f' in the original Markov Chain $\mathscr{M}$, then there are direct transitions between states f_j and f'_l in the extended Markov chain $\mathscr{M}'$. The transition rates are given by

$$q_{f_j f'_l} = \frac{\exp(\xi(x_{f'} + \frac{l}{k_{f'}}\phi_{f'}))}{\exp(\xi(x_f + \frac{j}{k_f}\phi_f)) + \exp(\xi(x_{f'} + \frac{l}{k_{f'}}\phi_{f'}))} \cdot \frac{\pi_{f'_l}}{2\exp(\tau)} \tag{42}$$

and

$$q_{f'_l f_j} = \frac{\exp(\xi(x_f + \frac{j}{k_f}\phi_f))}{\exp(\xi(x_f + \frac{j}{k_f}\phi_f)) + \exp(\xi(x_{f'} + \frac{l}{k_{f'}}\phi_{f'}))} \cdot \frac{\pi_{f_j}}{2\exp(\tau)} \tag{43}$$

where $\sum_{j=-k_f}^{k_f} \pi_{f_j} = 1$ and $\sum_{l=-k_{f'}}^{k_{f'}} \pi_{f'_l} = 1$

According to the detailed balance equation $p_{f_j} q_{f_j, f'_l} = p_{f'_l} q_{f'_l, f_j}$, $\forall j \in \{-k_f, \ldots, k_f\}$, $l \in \{-k_{f'}, \ldots, k_{f'}\}$, we have

$$\frac{p_{f_j}}{\pi_{f_j} \exp(\xi(x_f + \frac{j}{k_f}\phi_f))} = \frac{p_{f'_l}}{\pi_{f'_l} \exp \xi(x_{f'} + \frac{l}{k_{f'}}\phi_{f'})} \tag{44}$$

When $j = l = 0$, we have

$$\frac{p_{f_0}}{\pi_{f_0} \cdot \exp(\xi x_f)} = \frac{p_{f'_0}}{\pi_{f'_0} \exp(\xi x_{f'})} \tag{45}$$

Combining (44) and (45), we have

$$\frac{p_{f'_l}}{p_{f'_0}} = \frac{\pi_{f'_l}}{\pi_{f'_0}} \cdot \exp(\xi \frac{l}{k_{f'}}\phi_{f'}), \quad \forall l \in \{-k_{f'}, \ldots, k_{f'}\} \tag{46}$$

For an arbitrary state $\hat{f}_0 \in \mathscr{F}$ in $\mathscr{M}'$, and $\hat{f}_0 \neq f, f'$, since the extended Markov Chain $\mathscr{M}'$ is irreducible, state $\hat{f}_0$ is able to reach state f_0 through a series of adjacent states $\bar{f}(1)_0, \ldots, \bar{f}(H)_0$. Then we have

$$\frac{p_{\hat{f}_0}}{p_{f_0}} = \prod_{h=1}^{H-1} \frac{p_{\bar{f}(h+1)_0}}{p_{\bar{f}(h)_0}} \tag{47}$$

Using (45), we have

$$\frac{p_{\bar{f}(h+1)_0}}{\pi_{\bar{f}(h+1)_0} \cdot \exp(\xi x_{\bar{f}(h+1)})} = \frac{p_{\bar{f}(h)_0}}{\pi_{\bar{f}(h)_0} \exp(\xi x_{\bar{f}(h)})} \tag{48}$$

and

$$\frac{p_{\hat{f}_0}}{\pi_{\hat{f}_0} \cdot \exp(\xi x_{\hat{f}})} = \frac{p_{f_0}}{\pi_{f_0} \cdot \exp(\xi x_f)} \tag{49}$$

Combining (43) and (46), we have

$$\frac{p_{f_j}}{p_{f_0}} = \frac{\pi_{f_j}}{\pi_{f_0}} \cdot \exp(\xi \frac{j}{k_f}\phi_f), \quad \forall j \in \{-k_f, \ldots, k_f\} \tag{50}$$

and

$$\frac{p_{f_0}}{\pi_{f_0} \cdot \exp(\xi x_f)} \textit{ is a constant}, \quad \forall f \in \mathscr{F} \tag{51}$$

Recall that for all $f \in \mathscr{F}$, and $j \in \{-k_f, \ldots, k_f\}$, we have

$$\sum_{f \in \mathscr{F}} \sum_{j=-k_f}^{k_f} p_{f_j} = 1 \tag{52}$$

Now, the combination of (50), (51) and (52) yields the stationary distribution of the extended Markov Chain $\mathscr{M}'$, that is

$$\bar{p}_{f_j} = \frac{\pi_{f_j} \cdot \exp(\xi(x_f + \frac{j}{k_f}\phi_f))}{\sum_{f' \in \mathscr{F}} \sum_{l=-k_{f'}}^{k_{f'}} \pi_{f'_l} \cdot \exp(\xi(x_{f'} + \frac{l}{k_{f'}}\phi_{f'}))} \tag{53}$$

for all $f \in \mathscr{F}$, and $j \in \{-k_f, \ldots, k_f\}$.

Since there are $2k_f + 1$ states for configuration f in the extended Markov Chain $\mathscr{M}'$, the stationary distribution is given by

$$\tilde{p}_f = \sum_{j=-k_f}^{k_f} \bar{p}_{f_j}, \quad \forall f \in \mathscr{F} \tag{54}$$

Then we have

$$\tilde{p}_f = \frac{\varrho_f \exp(\xi x_f)}{\sum_{f\in\mathscr{F}} \varrho_{f'} \exp(\xi x_{f'})}, \quad \forall f \in \mathscr{F} \tag{55}$$

where

$$\varrho_f = \sum_{j=-k_f}^{k_f} \pi_{f_j} \cdot \exp(\xi \frac{j}{k_f}\phi_f), \forall f \in \mathscr{F} \tag{56}$$

Recall that the stationary distribution of the original Markov Chain $\mathscr{M}$ is given by

$$p_f^* = \frac{\exp(\xi x_f)}{\sum_{f'\in\mathscr{F}} \exp(\xi x_{f'})} \tag{57}$$

By (55), (56) and (57), we can get

$$\frac{\tilde{p}_f}{p_f^*} = \frac{\varrho^*}{\varrho_f} \tag{58}$$

where

$$\varrho^* = \frac{\sum_{f'\in\mathscr{F}} \varrho_{f'} \exp(\xi x_{f'})}{\sum_{f'\in\mathscr{F}} \exp(\xi x_{f'})} \tag{59}$$

Let $\Omega = \{f \in \mathscr{F} : p_f^* \geq \tilde{p}_f\}$, then the total variation distance is given by

$$\begin{aligned} d_{TV}(P^*, \tilde{P}) &= \frac{\sum_{f\in\mathscr{F}} |p_f^* - \tilde{p}_f|}{2} \\ &= \sum_{f\in\Omega} (p_f^* - \tilde{p}_f) \end{aligned} \tag{60}$$

Furthermore, we can get

$$\begin{aligned} p_f^* - \tilde{p}_f &= \frac{\exp(\xi x_f)}{\sum_{f'\in\mathscr{F}} \exp(\xi x_{f'})} - \frac{\varrho_f \exp(\xi x_f)}{\sum_{f'\in\mathscr{F}} \varrho_{f'} \exp(\xi x_{f'})} \\ &= \frac{\exp(\xi x_f)}{\sum_{f'\in\mathscr{F}} \exp(\xi x_{f'})}[1 - \frac{\varrho_f}{\varrho^*}] \end{aligned} \tag{61}$$

Next, we give an upper bound of $1 - \frac{\varrho_f}{\varrho^*}$. For all $f \in \mathscr{F}$, and $j \in \{-k_f, \ldots, k_f\}$, we have

$$\exp(\xi \frac{j}{k_f}\phi_f) \geq \exp(-\xi\phi_f) \geq \exp(-\xi\phi_{\max}) \tag{62}$$

and

$$\exp(\xi \frac{j}{k_f}\phi_f) \leq \exp(\xi\phi_f) \leq \exp(\xi\phi_{\max}) \tag{63}$$

where $\phi_{\max} = \max_{f\in\mathscr{F}} \phi_f$. Thus we get that

$$\begin{aligned} \varrho_f &= \sum_{j=-k_f}^{k_f} \pi_{f_j} \cdot \exp(\xi \frac{j}{k_f}\phi_f) \\ &\geq \sum_{j=-k_f}^{k_f} \pi_{f_j} \cdot \exp(-\xi\phi_{\max}) \\ &= \exp(-\xi\phi_{\max}) \end{aligned} \tag{64}$$

and

$$\begin{aligned} \varrho_f &= \sum_{j=-k_f}^{k_f} \pi_{f_j} \cdot \exp(\xi \frac{j}{k_f}\phi_f) \\ &\leq \sum_{j=-k_f}^{k_f} \pi_{f_j} \cdot \exp(\xi\phi_{\max}) \\ &= \exp(\xi\phi_{\max}) \end{aligned} \tag{65}$$

By (59) and (65), we can also have

$$\begin{aligned} \varrho_f^* &= \frac{\sum_{f'\in\mathscr{F}} \varrho_{f'} \exp(\xi x_{f'})}{\sum_{f'\in\mathscr{F}} \exp(\xi x_{f'})} \\ &\leq \frac{\sum_{f'\in\mathscr{F}} \exp(\xi\phi_{\max}) \exp(\xi x_{f'})}{\sum_{f'\in\mathscr{F}} \exp(\xi x_{f'})} \\ &= \exp(\xi\phi_{\max}) \end{aligned} \tag{66}$$

Combining (64), (65) and (66), $\forall f \in \Omega \in \mathscr{F}$, we have the upper bound of $1 - \frac{\varrho_f}{\varrho^*}$

$$1 - \frac{\varrho_f}{\varrho^*} \leq 1 - \frac{\exp(-\xi\phi_{\max})}{\exp(\xi\phi_{\max})} = 1 - \exp(-2\xi\phi_{\max}) \tag{67}$$

Therefore, we have

$$p_f^* - \tilde{p}_f \leq \frac{\exp(\xi x_f)}{\sum_{f'\in\mathscr{F}} \exp(\xi x_{f'})}[1 - \exp(-2\xi\phi_{\max})] \tag{68}$$

So, by (68), we can get

$$\begin{aligned} d_{TV}(P^*, \tilde{P}) &= \sum_{f\in\Omega} (p_f - p_f^*) \\ &\leq \sum_{f\in\Omega} \frac{\exp(\xi x_f)}{\sum_{f'\in\mathscr{F}} \exp(\xi x_{f'})}[1 - \exp(-2\xi\phi_{\max})] \\ &\leq \sum_{f\in\mathscr{F}} \frac{\exp(\xi x_f)}{\sum_{f'\in\mathscr{F}} \exp(\xi x_{f'})}[1 - \exp(-2\xi\phi_{\max})] \\ &= [1 - \exp(-2\xi\phi_{\max})] \end{aligned} \tag{69}$$

Finally, the optimality gap in transmission rate is bounded by

$$\begin{aligned} |P^*\mathbf{x}^T - \tilde{P}\mathbf{x}^T| &= |\sum_{f\in\mathscr{F}} (p_f^* - \tilde{p}_f)x_f| \\ &\leq x_{\max}|(p_f - p_f^*)| \\ &= 2x_{\max} d_{TV}(P^*, \tilde{P}) \\ &\leq 2x_{\max}(1 - \exp(-2\xi\phi_{\max})) \end{aligned} \tag{70}$$

This completes the proof. □

C. PROOF OF THEOREM 5

Proof: We denote $Pr(f_m \to f_{m+1})$ as the probability that the system enters state f_{m+1} after leaving state f_m when the PU departs, then we have

$$Pr(f_m \to f_{m+1}) = \frac{\exp(\xi x_{f_{m+1}}^{m+1})}{\sum_{s'\in\mathscr{S}, f_{m+1}=f_m\cup s'} \exp(\xi x_{f_{m+1}}^{m+1})} \tag{71}$$

Furthermore, the PU departs at a rate of μ, so the system leaves state f_m at rate μ. Thus the transition rate $q_{f_m \to f_{m+1}}$ can be calculated as

$$q_{f_m \to f_{m+1}} = \mu \frac{\exp(\xi x^{m+1}_{f_{m+1}})}{\sum_{s' \in \mathscr{S}, f_{m+1} = f_m \cup s'} \exp(\xi x^{m+1}_{f_{m+1}})} \tag{72}$$

On the other hand, the PU arrives at a rate of λ. Thus the TV channel becomes unavailable at a rate of λ. As there is a total of $m+1$ TV channels in the system, the system leaves statef_{m+1} at a rate of $(m+1)\lambda$. Thus we can calculate the transition rate $q_{f_{m+1}} \to f_m$ as

$$q_{f_{m+1} \to f_m} = \lambda(m+1) \frac{\sum_{f_{m+1} \in \mathscr{F}_{m+1}} \exp(\xi x^{m+1}_{f_{m+1}})}{\sum_{f_m \in \mathscr{F}_m} \exp(\xi x^{m}_{f_m})} \times \frac{\exp(\xi x^{m}_{f_m})}{\sum_{s' \in \mathscr{S}, f'_{m+1} = f_m \cup s'} \exp(\xi x^{m+1}_{f'_{m+1}})} \tag{73}$$

Together with (33), we can see that the balance equation holds. Thus the constructed Markov Chain is time-reversible and its stationary distribution is (33).

This completes the proof. □

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