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Multicell Downlink Beamforming With Limited Backhaul Signaling

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ABSTRACT A downlink beamforming design is proposed in multicell multi-input single-output downlink networks with limited backhaul signaling. In the proposed scheme, the beamforming vector at each base station (BS) is designed to minimize the sum of its *weighted* generating interference with local channel state information and the aid of information exchange between the BSs. The generating-interference weight coefficients are designed in pursuit of increasing the sum rate. Simulation results show that the proposed scheme outperforms the existing scheme in the mid-to-high signal-to-noise ratio regime even with much reduced amount of information exchange via backhaul.

INDEX TERMS Multicell downlink beamforming, interference channel, multi-input single-output (MISO), limited backhaul signaling, local channel state information (CSI).

I. INTRODUCTION

In dense cellular networks, e.g., small cells, a recent study has shown that each user's signal-to-interference-plus-noise ratio (SINR) converges to a constant or diverges depending on the path loss coefficient [1], which in turn shows the impact of the intercell interference. Multiple antennas employed at the transmitter provide significant spectral efficiency gain, mitigating or completely removing intercell or inter-user interference via transmit beamforming [2]–[4].

Full cooperation between the base stations (BSs) in designing the beamforming vectors offers optimal multiplexing gain [2] or a Pareto rate boundary [3]. However, these schemes require the global channel state information (CSI) at the transmitters, which limits the feasibility in practical applications with limited backhaul capacity [5].

Several studies have proposed cooperative beamforming methods with limited backhaul signaling or feedback [5]–[8]. In [5] and [8], separate and joint vector quantization for desired and interference channels is proposed to maximize the achievable rate. In [6], coordinated regularized multiuser multi-input single-output (MISO) precoding is proposed based on vector-quantized global CSI assuming massive antennas at the BSs. However, with the vector quantization, the number of quantization bits increases linearly with respect to the number of antennas to achieve the same achievable rate regardless of the number of transmit antennas.

Efforts have been made to obtain the optimal multiplexing gain only with local CSI [9], designing the beamforming transmit and receive vectors iteratively. Fully distributed non-iterative downlink beamforming designs with local CSI were proposed in [10] and [11], however their applications are limited due to strict conditions on the number of cells and channel coefficients [11], or on the number of users per cell [10]. A general framework maximizing signal-to-leakage-interference-plus-noise ratio (SLNR)[1] only with local CSI has been proposed [12], which is applicable to arbitrarily configured multicell networks and achieves the Pareto's optimal rate bound. Castanheira *et al.* [13] considered a robust max-SLNR beamforming design under CSI uncertainties for coordinated multi-point (CoMP) transmission with joint data transmission and with only coordinated beamforming.

Iterative beamforming design approaches, in which the BSs update their beamforming vectors iteratively exchanging interference pricing measures with other BSs or users, have

[1]The terminology is also known as signal-to-generating-interference-plus-noise ratio or virtual SINR [12]

been proposed in pursuit of maximizing the sum-rate of the two-user multi-input multi-output interference channel [14] and minimizing transmit power of the multicell MISO channel [15]–[17] with the use of limited backhaul signaling. In the scheme proposed in [18], beamforming vectors, receive equalizers and weight coefficients are designed iteratively between the transmitters and receivers. However, it requires excessive amount of backhaul signaling due to the vector information exchange about the beamforming vectors.

In this paper, we propose a non-iterative cooperative downlink beamforming scheme in multi-cell MISO networks, each cell of which consists of a BS with multiple antennas and a user with a single antenna, based on local CSI with limited backhaul signaling of a few scalar values. A novel distributed beamforming design with limited backhaul signaling among the BSs was proposed in [19] for multicell multiuser MISO networks, where the power allocation problem is solved in a distributed manner only with scalar information exchange among the BSs. However, in the scenario assumed in [19], all the BSs serve each user collaboratively, yielding the CoMP transmission with joint data transmission.

In the proposed scheme, the BSs collaborate with one another only to jointly design their beamforming vectors, and each BS serves a single user. In such a case, the backhaul is used only for the beamforming design, not for the data sharing, and thus the backhaul usage can be minimized. Specifically, the beamforming vector at each BS in the proposed scheme is designed to minimize the sum of weighted generating-interference (WGI) based on local CSI with *quantized scalar* information exchange among the BSs. In previous max-SLNR beamforming designs, the weight coefficients in the SLNR formulation are all identical to 1. In the proposed beamforming design, a weight of each generating interference term is optimized to improve a lower bound of the sum-rate with scalar information which is quantized and exchanged among the BSs via limited-capacity backhaul. Simulation results show that the proposed scheme outperforms the distributed iterative beamforming design [18] even with much reduced backhaul signaling in the mid to high signal-to-noise ratio (SNR).

The remainder of this paper is organized as follows. Section II presents the system model. The proposed beamforming design is provided in Section III. In Section IV, numerical evaluation of the proposed scheme is discussed, and Section V concludes the paper.

II. SYSTEM MODEL

It is assumed that each cell is composed of a single BS and user assuming frequency-, code-, or time-division multiuser orthogonal multiplexing as shown in Fig. 1. Each BS is assumed to have N_T antennas, whereas each user has a single antenna. The number of cells considered is denoted by N_C, and it is assumed that $N_C > N_T$. The channel vector from the m-th BS to the user in the n-th cell is denoted by $\mathbf{h}_{mn} \in \mathbb{C}^{N_T \times 1}$. Block fading and time-division duplexing with channel reciprocity are assumed. Resorting to channel reciprocity, each BS is assumed to have local CSI at the transmitter [12], i.e., the m-th BS has the information of $\mathbf{h}_{mn}$, $n \in \{1, \ldots, N_C\} \triangleq \mathcal{N}_C$.

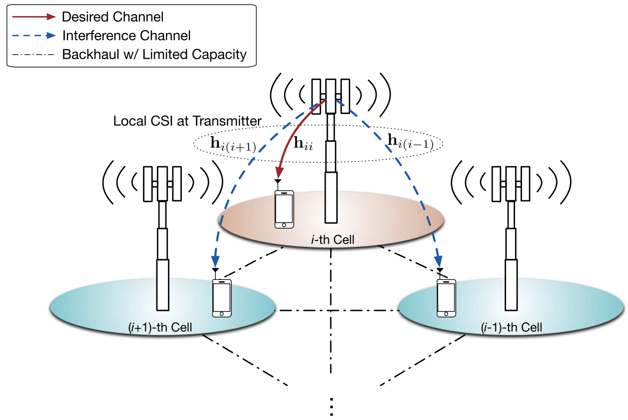

FIGURE 1. System model.

The beamforming vector at the m-th BS is denoted by $\mathbf{w}_m \in \mathbb{C}^{N_T \times 1}$, where $\|\mathbf{w}_m\|^2 = 1$. The received signal at the user in the m-th cell is written by

$$y_m = \underbrace{\mathbf{h}_{mm}^H \mathbf{w}_m x_m}_{\text{desired signal}} + \underbrace{\sum_{n=1, n\neq m}^{N_C} \mathbf{h}_{nm}^H \mathbf{w}_n x_n}_{\text{intercell interference}} + z_m, \quad (1)$$

where x_l is the unit-variance transmit symbol at the l-th BS, $l \in \mathcal{N}_C$, and z_m is the additive white Gaussian noise at the user in the m-th cell with zero-mean and variance of N_0. Thus, the corresponding SINR is expressed by

$$\rho_m = \frac{\left|\mathbf{h}_{mm}^H \mathbf{w}_m\right|^2}{\sum_{n=1, n\neq m}^{N_C} \left|\mathbf{h}_{nm}^H \mathbf{w}_n\right|^2 + N_0}, \quad (2)$$

and the achievable sum-rate is given by

$$R = \sum_{m=1}^{N_C} \log(1 + \rho_m). \quad (3)$$

III. PROPOSED BEAMFORMING DESIGN

A. PROBLEM FORMULATION

Let us consider the beamforming design of the i-th BS. The WGI at the i-th BS is defined as

$$\Omega_i(\mathbf{w}_i, \boldsymbol{\alpha}_i) = \sum_{j=1, j\neq i}^{N_C} \alpha_{ij} \left|\mathbf{h}_{ij}^H \mathbf{w}_i\right|^2, \quad (4)$$

where $\alpha_{ij} \geq 0$, $\sum_{j=1, j\neq i}^{N_C} \alpha_{ij} = 1$, and $\boldsymbol{\alpha}_i \triangleq \left[\alpha_{i1}, \ldots, \alpha_{i(i-1)}, \alpha_{i(i+1)}, \ldots, \alpha_{iN_C}\right]$. Here, the interference weight α_{ij} accounts for the relative emphasis on each interference channel. When minimizing (4), large α_{ij} leads to generating-interference (GI) from the i-th BS to the user in the j-th cell being more reduced compared to the other GI from the i-th BS. To minimize the WGI, the beamforming vector at the i-th BS is designed such that

$$\mathbf{w}_i = \arg_{\mathbf{w}} \min \Omega_i(\mathbf{w}, \boldsymbol{\alpha}_i), \quad \text{s.t.} \ \|\mathbf{w}\|^2 = 1. \tag{5}$$

Note that Ω_i can be expressed as

$$\Omega_i = \|\mathbf{G}_i \mathbf{w}_i\|^2, \tag{6}$$

where

$$\mathbf{G}_i(\boldsymbol{\alpha}_i) \triangleq \left[\sqrt{\alpha_{i1}}\mathbf{h}_{i1}, \ldots, \sqrt{\alpha_{i(i-1)}}\mathbf{h}_{i(i-1)}, \sqrt{\alpha_{i(i+1)}}\mathbf{h}_{i(i+1)}, \ldots, \sqrt{\alpha_{iN_C}}\mathbf{h}_{iN_C}\right]^H. \tag{7}$$

Let us denote the singular value decomposition of the matrix $\mathbf{G}_i \in \mathbb{C}^{(N_C-1)\times N_T}$ as $\mathbf{G}_i = \mathbf{U}_i\boldsymbol{\Sigma}_i\mathbf{V}_i^H$, where $\mathbf{U}_i \in \mathbb{C}^{(N_C-1)\times(N_C-1)}$ and $\mathbf{V}_i \in \mathbb{C}^{N_T\times N_T}$ consist of orthogonal columns, and $\boldsymbol{\Sigma}_i \in \mathbb{C}^{(N_C-1)\times N_T}$ is a diagonal matrix composed of the singular values of $\mathbf{G}_i$. The solution for the problem (5) is given by $\mathbf{w}_i = \mathbf{v}_i^{[N_T]}$, where $\mathbf{v}_i^{[N_T]}$ is the N_T-th column of $\mathbf{V}_i$, associated with the minimum singular value of $\mathbf{G}_i$. The solution of (5), however, is not unique, since $c\mathbf{v}_i^{[N_T]}$ also could be the solution for the problem (5), where c is an arbitrary complex number and $|c|^2 = 1$. However, all the solutions $c\mathbf{v}_i^{[N_T]}$ have the same WGI since $|c|^2 = 1$, and hence, the unique solution can be obtained without loss of generality as follows:

$$\mathbf{w}_i^* = \frac{\bar{v}_{i,1}^{[N_T]}}{\left|v_{i,1}^{[N_T]}\right|}\mathbf{v}_i^{[N_T]}, \tag{8}$$

where $v_{i,k}^{[N_T]}$ is the k-th element of $\mathbf{v}_i^{[N_T]}$, and $\bar{v}_{i,k}^{[N_T]}$ is the complex conjugate of $v_{i,k}^{[N_T]}$. Let us denote the set $\mathcal{A}$ by $\mathcal{A} \triangleq \{\mathbf{a} : \sum_{m=1}^{N_C-1} a_m = 1, \mathbf{a} = [a_1, \ldots, a_{N_C-1}] \in \mathbb{R}_{\geq 0}{}^{N_C-1}\}$. Then, since for any given $\boldsymbol{\alpha}_i \in \mathcal{A}$, a unique $\mathbf{G}_i$ is defined from (7), and thus a unique $\mathbf{w}_i^*$ is obtained from (8). Therefore, we can define the function $\boldsymbol{\omega}_i : \mathcal{A} \to \mathbb{C}^{N_T}$ by

$$\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i) = \mathbf{w}_i^*. \tag{9}$$

Remark 1: In fact, for any given $\alpha_{ij}, j = 1, \ldots, i-1, i+1, \ldots, N_C$, inserting $r \cdot \alpha_{ij}$ with any positive real value r into (7) leads to the same right singular matrix $\mathbf{V}_i$ and the same beamforming vector $\mathbf{w}_i^*$. Thus, the constraint $\sum_{j=1,j\neq i}^{N_C} \alpha_{ij} = 1$ is not necessarily needed. Nevertheless, posing this constraint does not lose any generality of the problem, while providing us with mathematical convenience in deriving the lower-bound of the sum-rate as shown later in Lemma 1.

In what follows, we propose a protocol to design $\boldsymbol{\alpha}_i$ with local CSI and limited backhaul signaling in pursuit of maximizing the sum-rate.

B. STEP 1: INFORMATION EXCHANGE BETWEEN BS'S

As an initialization step, with only local CSI, the j-th BS calculates its preparatory beamforming vector $\hat{\mathbf{w}}_i$ from

$$\hat{\mathbf{w}}_i = \arg_{\mathbf{w}} \min \sum_{j=1,j\neq i}^{N_C} \left|\mathbf{h}_{ij}^H \mathbf{w}\right|^2. \tag{10}$$

That is, $\hat{\mathbf{w}}_i$ is the beamforming vector merely minimizing the sum of GI [10]. Note that (10) includes only the outgoing channels from the i-th BS, and thus can be solved at the i-th BS only with local CSI. Then, the i-th BS shares the scalars $\left|\mathbf{h}_{il}^H \hat{\mathbf{w}}_i\right|^2, l \in \mathcal{N}_C$, with all other BSs via backhaul. Therefore, the amount of the information to be exchanged via backhaul does not increase with respect to the number of antennas. To consider limited backhaul capacity, scalar quantization shall be considered in Section III-D.

C. STEP 2: DESIGN OF α_i MAXIMIZING A LOWER BOUND OF THE SUM-RATE

Now, let us rewrite the sum-rate maximization problem as

$$\max_{\mathbf{w}_1,\ldots,\mathbf{w}_{N_C}} \sum_{l=1}^{N_C} \log(1+\rho_l) = \max_{\mathbf{w}_i} R_i(\mathbf{w}_i), \tag{11}$$

where

$$R_i(\mathbf{w}_i) = \max_{\mathbf{w}_1,\ldots,\mathbf{w}_{i-1},\mathbf{w}_{i+1},\ldots,\mathbf{w}_{N_C}} \sum_{l=1}^{N_C} \log(1+\rho_l). \tag{12}$$

Since $\mathbf{w}_i$ is a function of $\boldsymbol{\alpha}_i$ as in (9), we pose the following modified formulation:

$$\max_{\boldsymbol{\alpha}_i} R_i(\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)). \tag{13}$$

Since ρ_i in (2) is a function of all the beamforming vectors, i.e., function of $\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_{N_C}$, global CSI is required to compute $R_i(\mathbf{w}_i)$ in (12) and solve (13).

To circumvent the requirement of the global CSI acquisition, we propose to maximize a lower-bound on the sum-rate, which obviously results in an improved sum-rate. Specifically, we convert the sum-rate-maximizing problem into the maximization of a lower-bound on the sum-rate to separate the design of $\boldsymbol{\alpha}_i$ from the design of $\boldsymbol{\alpha}_j, j \neq i$. To design $\boldsymbol{\alpha}_i$, let us denote the modified SINRs of the users in the i-th cell and j-th cell, $j \neq i$, where $\left|\mathbf{h}_{ki}^H \mathbf{w}_k\right|^2$ and $\left|\mathbf{h}_{kj}^H \mathbf{w}_k\right|^2$, $k \neq i$, are substituted by $\left|\mathbf{h}_{ki}^H \hat{\mathbf{w}}_k\right|^2$ and $\left|\mathbf{h}_{kj}^H \hat{\mathbf{w}}_k\right|^2$, by $\rho_i^{[i]}$ and $\rho_j^{[i]}$, respectively. Note that $\left|\mathbf{h}_{ki}^H \hat{\mathbf{w}}_k\right|^2$ and $\left|\mathbf{h}_{kj}^H \hat{\mathbf{w}}_k\right|^2$ are calculated and shared in step 1.

Then, $\rho_i^{[i]}$ and $\rho_j^{[i]}$ for $j \neq i$ is given by

$$\rho_i^{[i]}(\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)) = \frac{\left|\mathbf{h}_{ii}^H \boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)\right|^2}{\sum_{j=1,j\neq i}^{N_C} \left|\mathbf{h}_{ji}^H \hat{\mathbf{w}}_j\right|^2 + N_0}, \tag{14}$$

$$\rho_j^{[i]}(\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)) = \frac{\left|\mathbf{h}_{jj}^H \hat{\mathbf{w}}_j\right|^2}{\left|\mathbf{h}_{ij}^H \boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)\right|^2 + \sum_{k=1,k\neq i,j}^{N_C} \left|\mathbf{h}_{kj}^H \hat{\mathbf{w}}_k\right|^2 + N_0}. \tag{15}$$

In other words, $\rho_i^{[i]}$ and $\rho_j^{[i]}$ can be viewed as the estimates of the SINRs for the users in the i-th and j-th cells, respectively, in the i-th BS's perspective. We have for all $\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)$

$$R_i(\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)) \geq \sum_{l=1}^{N_C} \log\left(1+\rho_l^{[i]}\left(\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)\right)\right), \tag{16}$$

because for given $\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)$, $R_i(\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i))$ is maximized with respect to $\mathbf{w}_1, \ldots, \mathbf{w}_{i-1}, \mathbf{w}_{i+1}, \ldots, \mathbf{w}_{N_C}$ as in (12), whereas fixed $\mathbf{w}_k = \hat{\mathbf{w}}_k$, $k \neq i$, is applied to calculate $\sum_{l=1}^{N_C} \log\left(1+\rho_l^{[i]}\left(\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)\right)\right)$.

Now, to bound $R_i(\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i))$ further, we establish the following lemma.

Lemma 1: Let us denote

$$\hat{\mathbf{G}}_i \triangleq \left[\mathbf{h}_{i1}, \ldots, \mathbf{h}_{i(i-1)}, \mathbf{h}_{i(i+1)}, \ldots, \mathbf{h}_{iN_C}\right]^H. \tag{17}$$

Then, the WGI in the proposed scheme at the i-th BS, $\|\mathbf{G}_i\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)\|^2$, is bounded by the GI in the min-GI scheme as

$$\|\mathbf{G}_i\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)\|^2 \leq \left\|\hat{\mathbf{G}}_i\hat{\mathbf{w}}_i\right\|^2. \tag{18}$$

Proof: See Appendix A. ■

In addition, using Lemma 1, the following theorem is established to derive a lower-bound of the sum-rate.

Theorem 1: For given $\boldsymbol{\alpha}_i$, We have

$$R_i(\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)) \geq \sum_{j=1, j\neq i}^{N_C} \log(1+\alpha_{ij}C_{ij}), \tag{19}$$

where

$$C_{ij} = \frac{\left|\mathbf{h}_{jj}^H\hat{\mathbf{w}}_j\right|^2}{\left\|\hat{\mathbf{G}}_i\hat{\mathbf{w}}_i\right\|^2 + \sum_{k=1,k\neq i,j}^{N_C}\left|\mathbf{h}_{kj}^H\hat{\mathbf{w}}_k\right|^2 + N_0}. \tag{20}$$

Proof: See Appendix B. ■

Note that the i-th BS can compute C_{ij} in (20) using the exchanged information. From (19), α_{ij} which maximizes the lower bound of the sum-rate can be obtained as

$$\boldsymbol{\alpha}_i^* = \arg\max_{\boldsymbol{\alpha}_i} \sum_{j=1, j\neq i}^{N_C} \log(1+\alpha_{ij}C_{ij}) \longrightarrow \alpha_{ij}^* = \left[\gamma - \frac{1}{C_{ij}}\right]^+, \tag{21}$$

where $[\chi]^+ = \max(\chi, 0)$ and γ can be calculated from $\sum_{j=1,j\neq i}^{N_C}\alpha_{ij}^* = 1$. Note that the numerator of C_{ij} defined in (20) is the desired signal of the j-th cell with the min-GI beamforming design, and the denominator includes the interference signal to the j-th cell with the min-GI beamforming design. Thus, α_{ij} is designed by (21) considering both of the desired signal and the intercell interference signal which are related to the SINR, thus improving the lower-bound of the sum-rate.

Finally, inserting $\boldsymbol{\alpha}_i^*$ obtained from (21) into (4), the beamforming vector at the i-th BS can be obtained from

$$\mathbf{w}_i = \boldsymbol{\omega}_i(\boldsymbol{\alpha}_i^*). \tag{22}$$

Note that $\hat{\mathbf{w}}_i$ in (10) can be obtained by the i-th BS with only local CSI. In addition, C_{ij} in (21) can also be calculated with exchanged information and only local CSI by definition of (20). For given $\boldsymbol{\alpha}_i^*$ obtained from solving (21), the solution of $\mathbf{w}_i$ in (22) can be immediately computed by establishing $\mathbf{G}_i$ for given $\boldsymbol{\alpha}_i^*$ as in (7) and computing $\mathbf{w}_i^*$ from (8), which requires only local CSI. Therefore, though the proposed scheme obtains a sub-optimal solution that improves the lower bound of the sum-rate, it provides a nice compromise between the amount of backhaul signaling and the sum-rate performance, which shall be shown by numerical results in Section IV.

D. SCALAR VALUE QUANTIZATION

To minimize the amount of the use of backhaul for exchanging $\left|\mathbf{h}_{ij}^H\hat{\mathbf{w}}_i\right|^2$, $i, j \in \mathcal{N}_C$, an M-level non-uniform quantization is considered. Let us denote the quantization points by y_j, $j = 1, \ldots, M$, and the boundaries by b_k, $k = 1, \ldots, M-1$. The following theorem establishes the necessary condition for a non-uniform quantization minimizing the mean-square quantization error (MSQE).

Theorem 2: A necessary condition of minimizing the MSQE is given by

$$y_j = \frac{[tF(t)]_{b_{j-1}}^{b_j} - \int_{b_{j-1}}^{b_j} F(t)\mathrm{d}t}{[F(t)]_{b_{j-1}}^{b_j}}, \tag{23}$$

where $[g(x)]_{\delta_2}^{\delta_1} \triangleq g(\delta_1) - g(\delta_2)$ and $F(t)$ is the cumulative density function (CDF) of $\left|\mathbf{h}_{ij}^H\hat{\mathbf{w}}_i\right|^2$ given by

$$F(t) = \int_0^1 \left(f_1(x)\cdot f_2\left(\frac{t}{x}\right)\right)\mathrm{d}x. \tag{24}$$

Here,

$$f_1(x) = \frac{1}{\beta(1, N_C-2)}(1-x)^{N_C-3}, \quad \text{for } 0 \leq x \leq 1, \tag{25}$$

$$f_2(x) = 1 - \mathrm{etr}(-x\Sigma^{-1}) \sum_{k=0}^{N_T(N_C-N_T-1)} \widehat{\sum_\kappa} \frac{C_\kappa(x\Sigma^{-1})}{k!}, \tag{26}$$

where $\beta(\cdot)$ is the beta function, $\kappa = (k_1, \ldots, k_{N_T})$ denotes the partition of integer k with $k_1 \geq \cdots \geq k_{N_T}$ and $k = k_1 + \cdots + k_{N_T}$, $\mathrm{etr}(\cdot)$ is $\exp(\mathrm{tr}(\cdot))$, $\sum_\kappa$ denotes summation over the partitions $\kappa = (k_1, \ldots, k_{N_T})$ of k with $k_1 \leq N_C - N_T - 1$, and $C_\kappa(\cdot)$ is the complex zonal polynomials (a.k.a. Schur polynomials). For $N_T = 2$ and $N_C = 3$, $F(t)$ is simplified as

$$F(t) = 1 - e^{-2t} + 2t\cdot\Gamma(0, 2t), \tag{27}$$

where $\Gamma(s, x) = \int_x^\infty t^{s-1}e^{-t}\mathrm{d}t$ is the upper incomplete gamma function.

Proof: See Appendix C. ■

Proposition 1: Through integration by parts, the MSQE defined by $Q = \sum_{j=1}^M \int_{b_{j-1}}^{b_j}\left((t-y_j)^2\cdot f(t)\right)\mathrm{d}t$, where $f(t)$ is the probability density function (PDF) of $\left|\mathbf{h}_{ij}^H\hat{\mathbf{w}}_i\right|^2$, can be written as a function of the CDF $F(t)$ as

$$Q = \sum_{j=1}^{M} \int_{b_{j-1}}^{b_j} 2(t - y_j)(1 - F(t))\mathrm{d}t + \sum_{j=1}^{M} \left[-(t - y_j)^2 (1 - F(t)) \right]_{b_{j-1}}^{b_j}. \quad (28)$$

From Theorem 2 and Proposition 1, the Lloyd-Max algorithm minimizing the MSQE is obtained as Algorithm 1. Let n_f be the number of bits required to represent each quantized scalar information. Then, for given N_T and n_f, we define the codebook by $\mathcal{K}_{N_T,n_f}$ which consists of 2^{n_f} elements. The codebooks designed by Algorithm 1 are shown in Appendix V in cases where (N_T, n_f) are (2, 1), (2, 2), (2, 3), (4, 1), (4, 2), and (4, 3). For given codebook $\mathcal{K}_{N_T,n_f}$, the scalar value $|\mathbf{h}_{ij}^H \hat{\mathbf{w}}_i|^2$, $i, j \in \mathcal{N}_C$, to be shared by the BSs is quantized to $\psi_{ij}^{[N_T,n_f]}$ as follows:

$$\psi_{ij}^{[N_T,n_f]} = \arg \min_{\tau \in \mathcal{K}_{N_T,n_f}} \left| \left| \mathbf{h}_{ij}^H \hat{\mathbf{w}}_i \right|^2 - \tau \right|. \quad (29)$$

Algorithm 1 Iterative Lloyd-Max Quantization Algorithm

1) Set initial representative levels y_j for $j = 1, \ldots, M$.
2) Calculate decision thresholds $b_k = \frac{1}{2}(y_k + y_{k+1})$, for $k = 1, \ldots, M-1$.
3) Calculate new representative levels y_j, $j = 1, \ldots, M$, satisfying the necessary condition from (23).
4) Repeat **2)** and **3)** until no further reduction in the MSQE Q given by (28).

IV. NUMERICAL SIMULATIONS

A. BAKCHAUL SIGNALING COMPARISON

In this subsection, the amount of backhaul signaling required in the proposed scheme is compared to those of some existing schemes. The weighted minimizing mean-square error (WMMSE) scheme [18] maximizing the sum-rate is considered, where each beamforming vector is designed iteratively between the transmitters and receivers. It is known that the 'WMMSE' scheme is the most efficient scheme that achieves the optimal sum-rate bound iteratively but in a distributed manner. In the 'WMMSE' scheme, the vector information about beamforming vectors and the scalar information about receive equalizers and weight coefficients need to be exchanged between the transmitters and receivers. The number of iteration in the 'WMMSE' scheme is denoted by π. The 'Global' scheme where all the beamforming vectors are jointly optimized in pursuit of maximizing the sum-rate [5] with the quantized channel vectors is also considered. For the optimal quantization of the channel vectors for the 'Global' scheme, the Grassmannian codebook is applied. Recall that the number of bits required for the quantization of each scalar value or vector by n_f. Table 1 summarizes the amount of required backhaul signaling in bits for the considered schemes. As shown in Table 1, the amount of the required backhaul signaling of the propose scheme is less than or equal to that of the 'WMMSE' scheme. Moreover, the required backhaul signaling of the 'WMMSE' scheme increases in proportion to the number of iteration π, which is shown by comparing the case of $N_C = 3$ and $\pi = 1$ and the case of $N_C = 3$ and $\pi = 2$.

TABLE 1. Amount of required backhaul signaling.

Scheme			Proposed	WMMSE	Global
Amount of backhaul sinaling (in bits)	General		$n_f N_C^2 (N_C - 1)$	$3\pi n_f N_C^2$	$n_f N_C^2 (N_C - 1)$
	$N_C = 3$, $\pi = 1$	$n_f = 1$	18	27	18
		$n_f = 2$	36	54	36
		$n_f = 3$	54	81	54
	$N_C = 3$, $\pi = 2$	$n_f = 1$	18	54	18
		$n_f = 2$	36	108	36
		$n_f = 3$	54	162	54
	$N_C = 7$, $\pi = 2$	$n_f = 1$	294	294	294
		$n_f = 2$	588	588	588
		$n_f = 3$	882	882	882

B. SPECTRAL EFFICIENCY COMPARISON

Figures 2 and 3 demonstrate the average rates per cell versus SNR for the case of $N_T = 2$ and $N_C = 3$ and the case of and $N_T = 4$ and $N_C = 7$, respectively. The achievable sum-rate of the proposed scheme is evaluated under Rayleigh fading environment compared with other existing schemes. For the 'WMMSE' scheme, π is assumed to be 1 for $N_C = 3$ and 2 for $N_C = 7$ for fair comparison of the amount of the backhaul signaling. In addition, three schemes requiring only local CSI without information exchange are also considered as follows to show the impact of the exchanged information of the proposed scheme. First, the 'Max-SNR' scheme is considered, where all the beamforming vectors are designed only to maximize the desired signals. Second, the 'Min-GI' scheme [10] is considered, where all the beamforming vectors are determined only to minimize GI. Third, in the 'Max-SLNR' scheme [12], all the beamforming vectors are constructed maximizing SLNR. For the proposed scheme, the scalar quantization discussed in Section III-D with the codebook in Appendix D is used, whereas in the 'WMMSE' scheme, the Grassmannian codebook is applied for the beamforming vector quantization, and respective optimal scalar quantization is applied for the quantization of the weight coefficients and receive equalizers. For the 'Global' scheme, the Grassmannian codebook is applied for the channel vector quantization and one bit is used for magnitude, whereas two bits are used for angle in cases of $n_f = 3$. As a baseline, 'Random' is considered, where each beamforming vector is randomly determined. For comparison, unquantized versions of the 'WMMSE' scheme and the proposed scheme are evaluated.

As seen in Figs. 2a and 3a, the 'Max-SLNR' scheme shows relatively high performance even without backhaul signaling. However, as the SNR increases, the proposed scheme shows notable rate gain compared with all the other schemes,

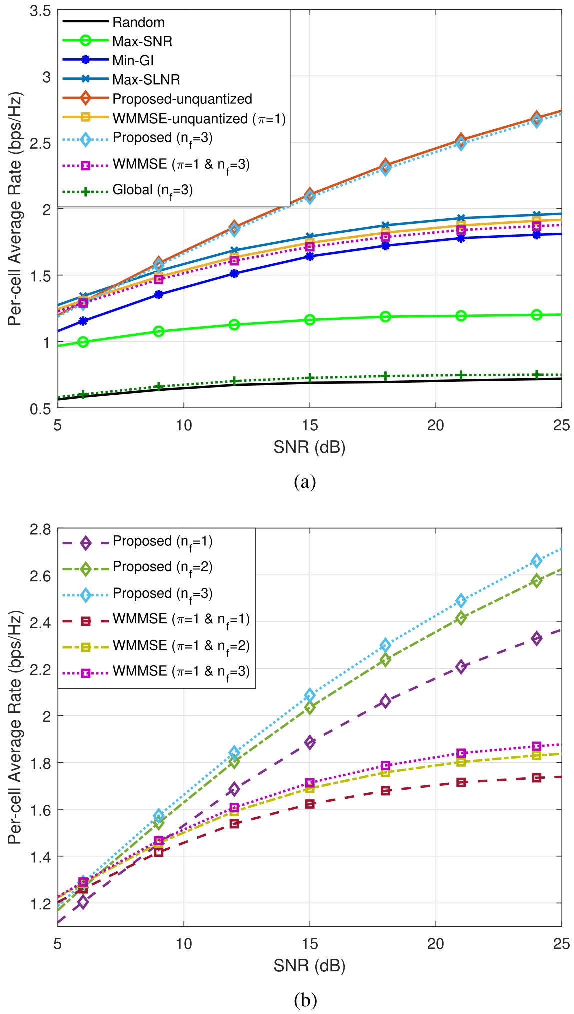

FIGURE 2. Per-cell average rate versus SNR for $N_T = 2$ and $N_C = 3$. (a) Per-cell average rate versus SNR of the unquantized version of the proposed scheme compared to other existing scheme. (b) Per-cell average rate versus SNR of the proposed scheme and the 'WMMSE' scheme for $n_f = 1, 2, 3$ and $\pi = 1$.

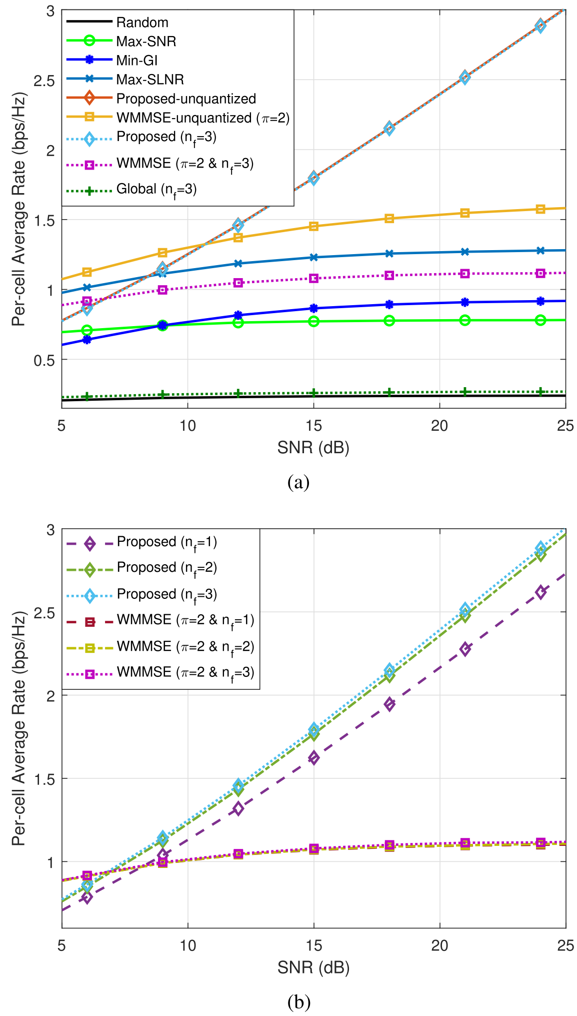

FIGURE 3. Per-cell average rate versus SNR for $N_T = 4$ and $N_C = 7$. (a) Per-cell average rate versus SNR of the unquantized version of the proposed scheme compared to other existing scheme. (b) Per-cell average rate versus SNR of the proposed scheme and the 'WMMSE' scheme for $n_f = 1, 2, 3$ and $\pi = 2$.

in which each BS minimizes its GI signals with different weights designed improving the lower bound of the sum-rate. On the other hand, the sum of GI is minimized maximizing the desired channel gain in the 'Max-SLNR' scheme, which in general does not directly relate to the sum-rate for $N_C > 2$. With only $n_f = 3$, the proposed scheme already shows achievable rates close to its performance upper-bound, i.e., the achievable rate of the unquantized version of the proposed scheme. On the other hand, $n_f = 3$ are too small to quantize channel vectors, and hence the 'Global' scheme shows almost the same performance as the 'Random' scheme due to significant quantization error in the vector quantization. According to Table 1 and Figs. 2b and 3b, the rates of the proposed scheme with 18 bits ($N_C = 3$ and $n_f = 1$) of backhaul signaling and 294 bits ($N_C = 7$ and $n_f = 1$) of backhaul signaling are even higher than the rates of the 'WMMSE' scheme with 81 bits ($N_C = 3$ and $n_f = 3$) of backhaul signaling and 882 bits ($N_C = 7$ and $n_f = 3$) of backhaul signaling, respectively, in the SNR regime higher than 7dB for $N_C = 3$ and 9dB for $N_C = 7$, thereby improving the spectral efficiency and reducing the backhaul signaling at the same time.

In Figs. 4 and 5, the average rate per cell of the proposed scheme versus SNR are compared with those of the existing schemes assuming antenna correlation between the transmit antennas. The Kronecker antenna correlation model [20] is used, and the antenna correlation matrices used for Figs. 4 and 5 are

$$\mathbf{R}_2 = \begin{bmatrix} 1 & 0.3 \\ 0.3 & 1 \end{bmatrix}, \tag{30}$$

$$\mathbf{R}_4 = \begin{bmatrix} 1 & 0.3 & 0.3^2 & 0.3^3 \\ 0.3 & 1 & 0.3 & 0.3^2 \\ 0.3^2 & 0.3 & 1 & 0.3 \\ 0.3^3 & 0.3^2 & 0.3 & 1 \end{bmatrix}, \tag{31}$$

respectively. As shown in Figs. 4 and 5, the overall per-cell average rates of all the considered schemes are relatively degraded compared to those of Figs. 2a and 3a, respectively,

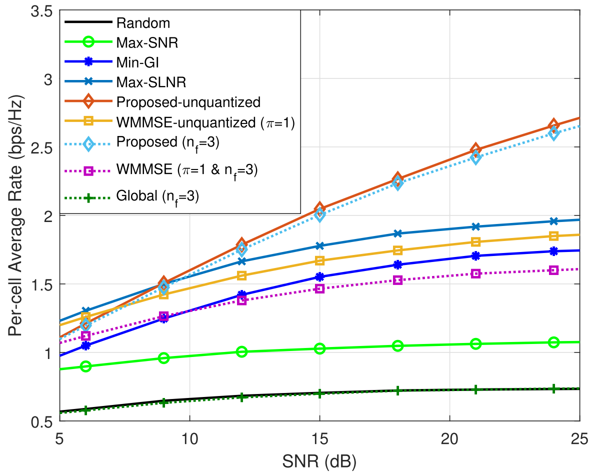

FIGURE 4. Per-cell average rate versus SNR for $N_T = 2$, $N_C = 3$, and the antenna correlation matrix $\mathbf{R}_2$.

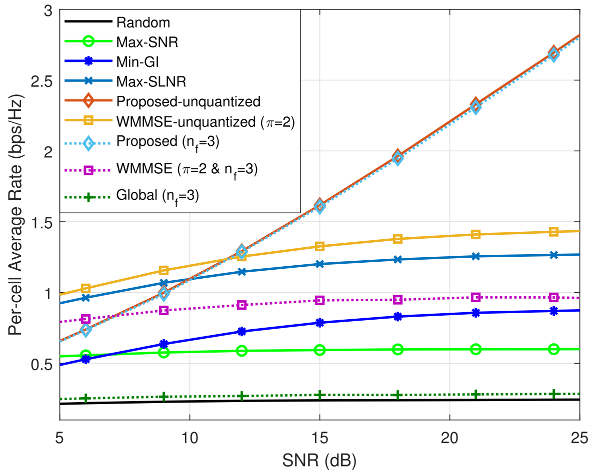

FIGURE 5. Per-cell average rate versus SNR for $N_T = 4$, $N_C = 7$, and the antenna correlation matrix $\mathbf{R}_4$.

due to the effect of the antenna correlation. However, the proposed scheme shows the maximum per-cell average rate in the SNR regime higher than 9dB and 12dB for the case of $N_T = 2$ and $N_C = 3$ and the case of $N_T = 4$ and $N_C = 7$, respectively. Moreover, the proposed scheme with $n_f = 3$ achieves the per-cell average rate which is close to that of the proposed scheme with unquantized scalar information, labeled by 'Proposed-unquantized,' with only 3 bits of backhaul signaling for each scalar value.

V. CONCLUSION

We have proposed a beamforming scheme to improve the total sum-rate of the MISO interference channel. The proposed beamforming design only requires local CSI and limited backhaul signaling between the BSs. Though there have been several schemes in which the beamforming vector is designed with limited backhaul signaling between the BSs, for given amount of backhaul signaling, the proposed scheme significantly outperforms the existing schemes in the mid to high SNR regime. Unlike a few previous schemes, the proposed scheme requires no iterative design between the transmitters and receivers, and the amount of information exchange does not increase as the number of antennas grows. These aforementioned benefits make the proposed scheme suitable in practical MISO interference channels.

APPENDIX A PROOF OF LEMMA 1

Let $\mathbf{A}$ and $\mathbf{B}$ be complex $m \times n$ matrices and let $q = \min\{m, n\}$. Then, the multiplicative Schur-Horn inequality (also known as Weyl inequality) [21] gives us

$$\lambda_{j+k-1}(\mathbf{A}\mathbf{B}^H) \le \lambda_j(\mathbf{A})\lambda_k(\mathbf{B}), \tag{32}$$

where $\lambda_i(\cdot)$ denotes the i-th singular value of a matrix, $1 \le j, k \le q$, and $j+k-1 \le q$. Since $\hat{\mathbf{G}}_i$ is a tall or square matrix when $N_C > N_T$, let us define $\mathbf{\Pi}_i$ by

$$\mathbf{\Pi}_i \triangleq \left[\mathbf{G}_i, \mathbf{0}_{(N_C-1)\times(N_C-1-N_T)}\right] \in \mathbb{C}^{(N_C-1)\times(N_C-1)}. \tag{33}$$

Then, $\hat{\mathbf{\Pi}}_i$ can be obtained by substituting $\mathbf{G}_i$ in (33) as $\hat{\mathbf{G}}_i$. Defining

$$\mathbf{\Xi}_i = \operatorname{diag}\left(\sqrt{\alpha_{i1}}, \ldots, \sqrt{\alpha_{i(i-1)}}, \sqrt{\alpha_{i(i+1)}}, \ldots \sqrt{\alpha_{iN_C}}\right), \tag{34}$$

we have $\mathbf{\Pi}_i = \mathbf{\Xi}_i\hat{\mathbf{\Pi}}_i$. Inserting $\mathbf{A} = \mathbf{\Xi}_i$ and $\mathbf{B} = \hat{\mathbf{\Pi}}_i^H$ into (32) gives us

$$\lambda_{j+k-1}\left(\mathbf{\Xi}_i\hat{\mathbf{\Pi}}_i\right) \le \lambda_j\left(\mathbf{\Xi}_i\right)\lambda_k\left(\hat{\mathbf{\Pi}}_i^H\right), \tag{35}$$

for $1 \le j,\ k \le N_T,\ j+k-1 \le N_T$. Thus, for $j = 1$, we have

$$\lambda_k\left(\mathbf{\Pi}_i\right) \le \lambda_1\left(\mathbf{\Xi}_i\right)\lambda_k\left(\hat{\mathbf{\Pi}}_i^H\right) \tag{36}$$

$$\leftrightarrow \lambda_k\left(\mathbf{G}_i\right) \le \lambda_1\left(\mathbf{\Xi}_i\right)\lambda_k\left(\hat{\mathbf{G}}_i^H\right), \quad \text{for } 1 \le k \le N_T, \tag{37}$$

where (37) follows from the fact that $\lambda_{N_T}\left(\hat{\mathbf{G}}_i^H\right) = \lambda_{N_T}\left(\hat{\mathbf{G}}_i\right)$ and that the largest singular value of $\mathbf{\Xi}_i$ is bounded by $\lambda_1\left(\mathbf{\Xi}_i\right) \le 1$, since $\sqrt{\alpha_{ij}} \le 1$. Therefore, we can get $\lambda_k\left(\mathbf{G}_i\right) \le \lambda_k\left(\hat{\mathbf{G}}_i\right)$, and inserting $k = N_T$ yields

$$\lambda_{N_T}\left(\mathbf{G}_i\right) = \|\mathbf{G}_i\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)\| \le \lambda_{N_T}\left(\hat{\mathbf{G}}_i\right) = \left\|\hat{\mathbf{G}}_i\hat{\mathbf{w}}_i\right\|, \tag{38}$$

which proves the lemma.

APPENDIX B PROOF OF THEOREM 1

Since $\sum_{j=1,j\neq i}^{N_C}\alpha_{ij}\left|\mathbf{h}_{ij}^H\mathbf{w}_i\right|^2 = \|\mathbf{G}_i\mathbf{w}_i\|^2$ by (4) and (6), from Lemma 1, we have

$$\left\|\hat{\mathbf{G}}_i\hat{\mathbf{w}}_i\right\|^2 \ge \|\mathbf{G}_i\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)\|^2 \ge \alpha_{ij}\left|\mathbf{h}_{ij}^H\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)\right|^2. \tag{39}$$

Then, for given $\boldsymbol{\alpha}_i$, we can obtain a lower bound on $\rho_j^{[i]}$ as followings:

$$\rho_j^{[i]}\left(\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)\right) = \frac{\left|\mathbf{h}_{jj}^H\hat{\mathbf{w}}_j\right|^2}{\left|\mathbf{h}_{ij}^H\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)\right|^2 + \sum_{k=1,k\neq i,j}^{N_C}\left|\mathbf{h}_{kj}^H\hat{\mathbf{w}}_k\right|^2 + N_0}$$

$$\ge \frac{\left|\mathbf{h}_{jj}^H\hat{\mathbf{w}}_j\right|^2}{\frac{\left\|\hat{\mathbf{G}}_i\hat{\mathbf{w}}_i\right\|^2}{\alpha_{ij}} + \sum_{k=1,k\neq i,j}^{N_C}\left|\mathbf{h}_{kj}^H\hat{\mathbf{w}}_k\right|^2 + N_0}, \tag{40}$$

$$\ge \alpha_{ij}C_{ij}, \tag{41}$$

where (40) follows from (39), and (41) follows from $0 \leq \alpha_{ij} \leq 1$. Here, C_{ij} is defined in (20). Therefore, we further have

$$R(\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i)) \geq \sum_{j=1, j\neq i}^{N_C} \log\left(1+\rho_j^{[i]}(\boldsymbol{\omega}_i(\boldsymbol{\alpha}_i))\right) \quad (42)$$

$$\geq \sum_{j=1, j\neq i}^{N_C} \log\left(1+\alpha_{ij}C_{ij}\right), \quad (43)$$

where (42) follows from (16), and (43) follows from (41).

APPENDIX C PROOF OF THEOREM 2

If we denote the PDF and CDF of $\left|\mathbf{h}_{ij}^H \hat{\mathbf{w}}_i\right|^2$, $i, j \in \mathcal{N}_C$, by $f(t)$ and $F(t)$, respectively, the necessary condition for minimizing the MSQE is known as $y_j = \frac{\int_{b_{j-1}}^{b_j} t \cdot f(t) \mathrm{d}t}{\int_{b_{j-1}}^{b_j} f(t) \mathrm{d}t}$ [22], which can be modified as following via the integration by parts:

$$y_j = \frac{\left[-t\left(1-F(t)\right)\right]_{b_{j-1}}^{b_j} + \int_{b_{j-1}}^{b_j}(1-F(t))\mathrm{d}t}{\left[F(t)\right]_{b_{j-1}}^{b_j}}. \quad (44)$$

Thus, (44) can be further simplified as (23).

The aim here is to derive the CDF $F(t)$. Since $\hat{\mathbf{w}}_i$ is obtained from (5) independently of $\mathbf{h}_{ii}$, $\forall i \in \mathcal{N}_C$, $\left|\mathbf{h}_{ii}^H \hat{\mathbf{w}}_i\right|^2$ is a chi-square random variable with degrees of freedom of 2. In case of $\left|\mathbf{h}_{ij}^H \hat{\mathbf{w}}_i\right|^2$, $i \neq j$, since $\hat{\mathbf{w}}_i$ is obtained as $\hat{\mathbf{w}}_i = \hat{\mathbf{v}}_i^{[N_T]}$, $\left|\mathbf{h}_{ij}^H \hat{\mathbf{w}}_i\right|^2 = 0$ if $(N_C - 1) < N_T$ and $\left|\mathbf{h}_{ij}^H \hat{\mathbf{w}}_i\right|^2 = \left(\hat{\sigma}_i^{[N_T]}\right)^2 \left|\hat{u}_i^{[j', N_T]}\right|^2$ otherwise, where $j' = j - 1$ if $i < j$ and $j' = j$ if $i > j$. Here, $\hat{\mathbf{v}}_i^{[N_T]}$ is the N_T-th column of the right singular matrix of $\hat{\mathbf{G}}_i$, $\hat{\sigma}_i^{[N_T]}$ is the N_T-th singular value of $\hat{\mathbf{G}}_i$, and $\hat{u}_i^{[j', N_T]}$ is the (j', N_T)-th element of the left singular matrix of $\hat{\mathbf{G}}_i$. If we define $\nu_i^{[N_T]}$ by $\nu_i^{[N_T]} = \left(\hat{\sigma}_i^{[N_T]}\right)^2$, the CDF of $\nu_i^{[N_T]}$ can be represented as f_2 in (26) [23]. If we define ξ_j by $\xi_j = \left|\hat{u}_i^{[j', N_T]}\right|^2$, the PDF of ξ_j is known as f_1 in (25) [24]. Since $\left(\hat{\sigma}_i^{[N_T]}\right)^2$ and $\left|\hat{u}_i^{[j', N_T]}\right|^2$ are independent, the CDF of $\left|\mathbf{h}_{ij}^H \hat{\mathbf{w}}_i\right|^2$, the product of $\left(\hat{\sigma}_i^{[N_T]}\right)^2$ and $\left|\hat{u}_i^{[j', N_T]}\right|^2$, can be represented as (24).

If $N_C - 1 = N_T$, the CDF of $\nu_i^{[N_T]}$ is simplified as [25]

$$f_2(x) = 1 - e^{-N_T x}. \quad (45)$$

Inserting (45) for $N_T = 2$ and $N_C = 3$ into (24) gives us (27).

APPENDIX D CODEBOOKS FOR THE PROPOSED SCHEME

The codebooks which are used for the quantization of the proposed scheme are

$$\mathcal{K}_{2,1} = \{1.1936, 5.1831\}, \quad (46)$$

$$\mathcal{K}_{2,2} = \{0.6693, 2.3922, 4.8012, 8.8195\}, \quad (47)$$

$$\mathcal{K}_{2,3} = \{0.3559, 1.1559, 2.0747, 3.1561, 4.4735, 6.1395, 8.5331, 12.6185\}, \quad (48)$$

$$\mathcal{K}_{4,1} = \{1.1779, 5.1301\}, \quad (49)$$

$$\mathcal{K}_{4,2} = \{0.6540, 2.3447, 4.7196, 8.7581\}, \quad (50)$$

$$\mathcal{K}_{4,3} = \{0.3471, 1.1306, 2.0333, 3.0925, 4.3799, 6.0385, 8.3822, 12.3984\}. \quad (51)$$

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