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Self-Tuning Distributed Fusion Filter for Multi-Sensor Systems Subject to Unknown Model Parameters and Missing Measurement Rates

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ABSTRACT A self-tuning fusion estimation problem is addressed for multi-sensor (MS) linear discrete-time stochastic systems subject to unknown model parameters (UMPs) and missing measurement rates. The phenomena of missing measurements for different sensors are described by random variable sequences obeying Bernoulli distributions. The UMPs and missing measurement rates are identified online by the recursive extended least squares (RELS) algorithms and correlation functions, respectively. A distributed fusion identifier for UMPs is presented by using matrix-weighted fusion estimation (MWFE) algorithm in the linear unbiased minimum variance sense. Furthermore, the corresponding self-tuning state estimation algorithms are obtained by substituting the identified model parameters and missing measurement rates into the local optimal filters, cross-covariance matrices (CCMs), and distribution optimal fusion filter. Finally, the convergence of the presented algorithms is analyzed. A numerical example shows the effectiveness of the presented algorithms.

INDEX TERMS RELS, correlation function, multi-sensor system, unknown model parameter, unknown missing measurement rate, self-tuning fusion filter.

I. INTRODUCTION

Recently, the research on MS information fusion filter has achieved much attention owing to its wide applications in practical systems, such as industrial monitor, navigation and guidance, object identification, and so on. The classical Kalman filtering theory [1] requires the known model parameters and noise statistical characteristics, which restricts its application fields. This pushes the development of the adaptive or self-tuning estimation theory.

For MS systems subject to unknown variances of noises, the self-tuning distributed fusion state predictors are obtained in [2]–[4] where unknown variances of noises are identified by correlation functions and weighted average methods. Using the same identification methods as [2]–[4], a self-tuning full-order weighted measurement fusion (WMF) Kalman filter is proposed in [5] for MS descriptor systems subject to unknown variances of noises. For systems subject to UMPs, the self-tuning fusion estimation problems are discussed in [6]–[8]. The weighted average method is adopted to obtain the fused UMPs, and then a self-tuning fusion filter is proposed in [6]. Then, the corresponding WMF self-tuning fusion filter is also proposed for multi-channel signals in [7]. The RELS algorithm is adopted to identify the UMPs, and then the self-tuning WMF Kalman filter is presented for MS discrete systems with correlated measurement noises in [8]. In the aforementioned literature, the self-tuning fusion estimation algorithms proposed are all by the complete measurement data. However, the sensor data may be incomplete or disturbed in sensor networks or networked systems because of the aging, fault, saturation, and bias of sensors, or fading measurements, packet droppings and random transmission delays induced by unreliable networks, and so on [9]–[13].

For discrete linear stochastic systems subject to multiple random transmission delays and packet droppings, optimal linear estimators have been designed in [14] and [15].

For nonlinear systems subject to missing measurements, the event-based filtering algorithms [16], [17], the receding horizon filtering algorithm [18], and the variance-constrained approach [19] have been designed, respectively. However, multiple sensors are not considered in [14]–[19]. For MS systems subject to random transmission delays, centralized fusion (CF) estimators are proposed in [20]. However, CF has the bad reliability, which means that a faulty sensor may bring the divergence of CF estimators. Distributed fusion (DF) filters have good reliability because of the parallel structures. Therefore, DF filter obtains much attention in the recent years. DF estimators are presented for sensor networks subject to packet droppings in [21] and [22] and for networked systems subject to transmission delays and packet droppings in [23] and [24]. The estimation algorithms proposed in the above literature have assumed the known missing measurements rates, delay rates and packet dropping rates.

Thus far, the research on self-tuning DF estimation problem for systems subject to unknown missing measurement rates has been rarely reported, particularly, as well as UMPs together. In this paper, the RELS algorithm is adopted to identify the UMPs and the correlation function is adopted to identify the unknown missing measurement rates of different sensors. A self-tuning DF state filter is presented for MS systems subject to UMPs and unknown missing measurement rates. The main contributions include: 1) the studied systems are subjected to both UMPs and unknown missing measurement rates; 2) the correlation functions are utilized to identify the unknown missing measurement rates; 3) a DF identifier for model parameters is presented by using the MWFE algorithm in the LUMV sense in [25]; 4) a self-tuning DF state filter is obtained based on the identified model parameters and missing measurement rates; and 5) the convergence of the presented algorithms is analyzed by using the dynamic error system analysis (DESA) method and dynamic variance error system analysis (DVESA) method in [26].

II. PROBLEM FORMULATION

Consider the following MS discrete-time stochastic system with missing measurements

$$x(k+1) = \Phi x(k) + \Gamma w(k) \tag{1}$$

$$y_i(k) = \mu_i(k) H_i x(k) + v_i(k), \quad i = 1, \cdots, L \tag{2}$$

where $x(k) \in R^n$ is a state, $y_i(k) \in R$ is a scalar measurement. The subscript i means the ith sensor. L is the number of sensors. $\{\mu_i(k)\}$ is a random variable sequence obeying the Bernoulli distribution with the probabilities $\text{Prob}\{\mu_i(k) = 1\} = \alpha_i$ and $\text{Prob}\{\mu_i(k) = 0\} = 1 - \alpha_i$. $\mu_i(k) = 1$ means the perfect signal delivery and $\mu_i(k) = 0$ means the missing measurement. α_i is a receiving measurement rate. $1 - \alpha_i$ is a missing measurement rate. $\mu_i(k)$ is independent of other stochastic variables. Φ, Γ and H_i are time-invariant matrices of suitable dimensions.

Assumption 1: $w(k)$ and $v_i(k)$ are uncorrelated white noises of zero mean and variances Q_w and Q_{v_i}.

Assumption 2: The initial value $x(0)$ is uncorrelated with $w(k)$ and $v_i(k)$, and satisfies

$$\text{E}\{x(0)\} = u_0, \quad \text{E}\left\{[x(0) - u_0][x(0) - u_0]^{\text{T}}\right\} = P_0$$

where E is the mathematical expectation, the superscript T denotes the transpose.

Assumption 3: Φ is a stable matrix.

Assumption 4: $y_i(k)$, $i = 1, \cdots, L$ are bounded.

Assumption 5: α_i and part parameters of Φ are unknown.

Assumptions 1 and 2 are general in estimation problem. Assumptions 3 and 4 will be used in the convergence proof of the proposed algorithms in later text. Assumption 5 shows the problem to be solved.

Our objective is to design the self-tuning DF filter $\hat{x}_s(k|k)$ for the state $x(k)$ based on the measurement data $(y_i(1), \ldots y_i(k))$, $i = 1, \ldots, L$ by identifying the unknown parameters in Assumption 5 and then substituting them into the optimal DF filter.

III. OPTIMAL FUSION FILTERING ALGORITHM

In this section, one first need drive the distributed optimal fusion filter under the condition of the known model parameters and missing measurement rates. Next, one will present it under Assumptions 1-3.

Eq. (2) can be rewritten as

$$y_i(k) = \alpha_i H_i x(k) + V_i(k) \tag{3}$$

where $V_i(k) = (\mu_i(k) - \alpha_i) H_i x(k) + v_i(k)$ is a fictitious white noise with zero mean and the variance matrix $Q_{V_i}(k) = \alpha_i(1 - \alpha_i) H_i X(k) H_i^{\text{T}} + Q_{v_i}$. The state second-order moment $X(k) = \text{E}[x(k)x^{\text{T}}(k)]$ is recursively calculated by

$$X(k+1) = \Phi X(k) \Phi^{\text{T}} + \Gamma Q_w \Gamma^{\text{T}} \tag{4}$$

under the initial value $X(0) = u_0 u_0^{\text{T}} + P_0$. From Assumption 3, it follows that $X(k)$ is bounded.

The following Lemma 1 introduces the algorithm of local optimal filter. Lemma 2 presents the formula of computing the CCMs. The distributed optimal fusion filter by using MWFE algorithm is given in Lemma 3.

Lemma 1 [25]: Under Assumptions 1-3, the local optimal filtering algorithm based on the measurement data of each sensor subsystem for systems (1) and (3) is given as

$$\hat{x}_i(k+1|k) = \Phi \hat{x}_i(k|k) \tag{5}$$

$$C_i(k+1) = y_i(k+1) - F_i \hat{x}_i(k+1|k) \tag{6}$$

$$\Sigma_i(k+1|k) = \Phi P_i(k|k) \Phi^{\text{T}} + \Gamma Q_w \Gamma^{\text{T}} \tag{7}$$

$$Q_{C_i}(k+1) = F_i \Sigma_i(k+1|k) F_i^{\text{T}} + Q_{V_i}(k+1) \tag{8}$$

$$K_i(k+1) = \Sigma_i(k+1|k) F_i^{\text{T}} Q_{C_i}^{-1}(k+1) \tag{9}$$

$$P_i(k+1|k+1) = [I_n - K_i(k+1) F_i] \Sigma_i(k+1|k) \tag{10}$$

$$\hat{x}_i(k+1|k+1) = \Psi_{fi}(k+1) \hat{x}_i(k|k) + K_i(k+1) y_i(k+1) \tag{11}$$

where $F_i = \alpha_i H_i$, $\Psi_{fi}(k+1) = [I_n - K_i(k+1) F_i] \Phi$. $\hat{x}_i(k|k)$ and $\hat{x}_i(k+1|k)$ are the filter and predictor with corresponding variance matrices $P_i(k|k)$ and $\Sigma_i(k+1|k)$. $K_i(k)$ is the filtering gain matrix. The initial values are $\hat{x}_i(0|0) = u_0$ and $P_i(0|0) = P_0$.

Lemma 2 [25]: Defining the filtering error CCM between arbitrary two local filters as $P_{ij}(k|k) = \mathrm{E}[\tilde{x}_i(k|k)\tilde{x}_j^T(k|k)]$, $i, j = 1, \cdots, L$, $i \neq j$ with $\tilde{x}_i(k|k) = x(k) - \hat{x}_i(k|k)$, one has the recursive computational formula as

$$\begin{aligned} &P_{ij}(k+1|k+1) \\ &\quad = [I_n - K_i(k+1)F_i][\Phi P_{ij}(k|k)\Phi^{\mathrm{T}} + \Gamma Q_w \Gamma^{\mathrm{T}}] \\ &\qquad \times [I_n - K_j(k+1)F_j]^{\mathrm{T}} \end{aligned} \tag{12}$$

under the initial value $P_{ij}(0|0) = P_0$.

Lemma 3 [25]: Under Lemma 1 and Lemma 2, the MWFE filter in the LUMV sense is calculated as

$$\hat{x}_o(k|k) = \sum_{i=1}^{L} W_i(k)\hat{x}_i(k|k) \tag{13}$$

where the optimal weighting coefficient matrices are calculated as

$$[W_1(k), \ldots, W_L(k)] = (e^{\mathrm{T}}P^{-1}(k|k)e)^{-1}e^{\mathrm{T}}P^{-1}(k|k) \tag{14}$$

where $e = [I_n, \ldots, I_n]^T \in R^{nL\times n}$ and $P(k|k) = \left[P_{ij}(k|k)\right] \in R^{nL\times nL}$. The variance matrix of the DF filter is calculated as

$$P_o(k|k) = (e^{\mathrm{T}}P^{-1}(k|k)e)^{-1} \tag{15}$$

Also, the relation $P_o(k|k) \leq P_i(k|k)$, $i = 1, \cdots, L$ holds.

IV. FUSION IDENTIFIER OF UNKNOWN MODEL PARAMETERS

When Φ contains unknown parameters, one identifies the UMPs based on the RELS algorithm in this section.

From (1), it follows that

$$x(k) = (I_n - q^{-1}\Phi)^{-1}q^{-1}\Gamma w(k) \tag{16}$$

where q^{-1} is the backward shift operator, i.e., $q^{-1}x(k) = x(k-1)$. Substituting (16) into (3) leads to

$$y_i(k) = \alpha_i H_i(I_n - q^{-1}\Phi)^{-1}q^{-1}\Gamma w(k) + V_i(k) \tag{17}$$

From (17), one can obtain a new measurement model

$$A(q^{-1})y_i(k) = \alpha_i B_i(q^{-1})w(k) + A(q^{-1})V_i(k) \tag{18}$$

with

$$A(q^{-1}) = \det(I_n - q^{-1}\Phi) \tag{19}$$

$$B_i(q^{-1}) = H_i \mathrm{adj}(I_n - q^{-1}\Phi)q^{-1}\Gamma \tag{20}$$

where the 'det' denotes the matrix determinant and the 'adj' denotes the adjoint matrix. $A(q^{-1})$ and $B_i(q^{-1})$ are polynomials with forms as

$$\begin{aligned} A(q^{-1}) &= 1 + a_1q^{-1} + \cdots + a_{n_a}q^{-n_a}, \\ B_i(q^{-1}) &= B_{i1}q^{-1} + \cdots + B_{in_{B_i}}q^{-n_{B_i}} \end{aligned}$$

a_t, $t = 1, \ldots, n_a$ and B_{it}, $t = 1, \ldots, n_{B_i}$ are the coefficients of $A(q^{-1})$ and $B(q^{-1})$, and n_a and n_{B_i} are their orders, respectively.

From [26], two moving average (MA) processes in the right hand side of (18) are equivalent to a stable MA process $D_i(q^{-1})\varepsilon_i(k)$, where $\varepsilon_i(k)$ is the white noise with unknown variance $\sigma_{\varepsilon_i}^2$, and $D_i(q^{-1})$ is the polynomial with the form as

$$D_i(q^{-1}) = 1 + d_{i1}q^{-1} + \cdots + d_{in_{D_i}}q^{-n_{D_i}}$$

d_{it}, $t = 1, \ldots, n_{D_i}$ are the coefficients of $D_i(q^{-1})$ and n_{D_i} is the order of $D_i(q^{-1})$.

One can rewrite (18) as

$$A(q^{-1})y_i(k) = D_i(q^{-1})\varepsilon_i(k) \tag{21}$$

Setting

$$\begin{aligned} \varphi_i^{\mathrm{T}}(k) &= [-y_i(k-1), \cdots, -y_i(k-n_a), \\ &\qquad \hat{\varepsilon}_i(k-1), \cdots, \hat{\varepsilon}_i(k-n_{in_{D_i}})], \\ \vartheta_i &= [a_1, \cdots, a_{n_a}, d_{i1}, \cdots d_{in_{D_i}}]^{\mathrm{T}} \end{aligned}$$

one has

$$y_i(k) = \varphi_i^{\mathrm{T}}(k)\vartheta_i + \varepsilon_i(k) \tag{22}$$

Then, one has the parameter estimator based on the RELS algorithm as follows [26]:

$$\hat{\vartheta}_i(k+1) = \hat{\vartheta}_i(k) + M_i(k+1)\hat{\varepsilon}_i(k+1) \tag{23}$$

$$\hat{\varepsilon}_i(k+1) = y_i(k+1) - \varphi_i^{\mathrm{T}}(k+1)\hat{\vartheta}_i(k) \tag{24}$$

$$M_i(k+1) = \frac{Z_i(k)\varphi_i(k+1)}{1 + \varphi_i^{\mathrm{T}}(k+1)Z_i(k)\varphi_i(k+1)} \tag{25}$$

$$Z_i(k+1) = [I_{n_a+n_{D_i}} - M_i(k+1)\varphi_i^{\mathrm{T}}(k+1)]Z_i(k) \tag{26}$$

under the initial values $\hat{\vartheta}_i(0) = 0$ and $Z_i(0) = \beta_i I$, with a sufficiently large positive number β_i, $\hat{\varepsilon}_i(k) = y_i(k) = 0$, $(k \leq 0)$. From [26], one has that the parameter estimator based on the RELS algorithm is consistent, i.e., the estimates converge to the true values with probability 1, $\hat{\vartheta}_i(k) \to \vartheta_i$ as $k \to \infty$.

Based on the measurements of single sensor, one can obtain the local estimators $\hat{\vartheta}_i(k)$ at time k of unknown parameters ϑ_i by above algorithms. From the definition of ϑ_i, one knows that the parameters a_t, $t = 1, \ldots, n_a$ have been estimated L times based on L sensors, so one can obtain the fusion estimates of parameters a_t by applying MWFE algorithm in [26]. One needs to calculate the estimation error variances of the local parameter estimators and the CCMs between any two local parameter estimators. Next, one will solve them.

From (23), the local estimation error $\tilde{\vartheta}_i(k) = \hat{\vartheta}_i(k) - \vartheta_i$ satisfies the equation

$$\begin{aligned} \tilde{\vartheta}_i(k+1) = [I_{n_A+n_{D_i}} &- M_i(k+1)\varphi_i^{\mathrm{T}}(k+1)]\tilde{\vartheta}_i(k) \\ &+ M_i(k+1)\varepsilon_i(k+1) \end{aligned} \tag{27}$$

The CCM $P_{\vartheta_{ij}}(k) = \mathrm{E}[\tilde{\vartheta}_i(k)\tilde{\vartheta}_j^{\mathrm{T}}(k)]$ between any two local parameter estimation errors can be solved by

$$\begin{aligned} P_{\vartheta_{ij}}(k+1) = [I_{n_A+n_{D_i}} &- M_i(k+1)\varphi_i^{\mathrm{T}}(k+1)]P_{\vartheta_{ij}}(k) \\ &\times [I_{n_A+n_{D_j}} - M_j(k+1)\varphi_j^{\mathrm{T}}(k+1)]^{\mathrm{T}} \\ &+ M_i(k+1)\hat{\sigma}_{\varepsilon_{ij}}^2(k+1)M_j^{\mathrm{T}}(k+1) \end{aligned} \tag{28}$$

where the cross-covariance $\sigma^2_{\varepsilon_{ij}}$ between $\varepsilon_i(k)$ and $\varepsilon_j(k)$ can be approximately computed by the following recursion

$$\begin{aligned}\hat{\sigma}^2_{\varepsilon_{ij}}(k) &= \frac{1}{k}\sum_{l=1}^{k}\hat{\varepsilon}_i(l)\hat{\varepsilon}_j(l)\\ &= \hat{\sigma}^2_{\varepsilon_{ij}}(k-1) + \frac{1}{k}[\hat{\varepsilon}_i(k)\hat{\varepsilon}_j(k) - \hat{\sigma}^2_{\varepsilon_{ij}}(k-1)] \quad (29)\end{aligned}$$

The initial value is $\hat{\sigma}^2_{\varepsilon_{ij}}(0) = y_i(0)y_j(0)$. $P_{\vartheta_{ii}}(k)$ is the local parameter estimation error variance $P_{\vartheta_i}(k)$ when $i = j$.

Setting $\vartheta_A = [a_1, \cdots, a_{n_a}]^{\mathrm{T}}$, then $\vartheta_A = [I_{n_a}, 0]\vartheta_i$. One has local parameter estimators and estimation error CCMs of ϑ_A as

$$\hat{\vartheta}_{Ai}(k) = [I_{n_a}, 0]\hat{\vartheta}_i(k), P_{\vartheta_{Aij}}(k) = [I_{n_a}, 0]P_{\vartheta_{ij}}(k)[I_{n_a}, 0]^{\mathrm{T}} \quad (30)$$

where $P_{\vartheta_{Aii}}(k)$ is the local parameter estimation error variance $P_{\vartheta_{Ai}}(k)$ when $i = j$.

Based on the local parameter estimators $\hat{\vartheta}_{Ai}(k)$ and estimation error CCMs $P_{\vartheta_{Aij}}(k)$, applying the MWFE algorithm in the LUMV sense [25], one can obtain the DF parameter estimator.

Theorem 1: The matrix-weighted DF parameter estimator in the LUMV sense is calculated as

$$\hat{\vartheta}_{Ao}(k) = \sum_{i=1}^{L} W_i^{\vartheta_A}(k)\hat{\vartheta}_{Ai}(k) \quad (31)$$

The parameter fusion weighted matrices are calculated as

$$[W_1^{\vartheta_A}(k), \ldots, W_L^{\vartheta_A}(k)] = (e_\vartheta^{\mathrm{T}} P_{\vartheta_A}^{-1}(k)e_\vartheta)^{-1}e_\vartheta^{\mathrm{T}} P_{\vartheta_A}^{-1}(k) \quad (32)$$

where $e_\vartheta = \begin{bmatrix} I_{n_a} & \cdots & I_{n_a}\end{bmatrix}^T_{n_aL\times n_a}$, $P_{\vartheta_A}(k) = \begin{bmatrix}P_{\vartheta_{Aij}}(k)\end{bmatrix}_{n_aL\times n_aL}$, $i, j = 1, \cdots, L$, is an $n_aL \times n_aL$ matrix.

The parameter fusion estimation error variance is given as

$$P_{\vartheta_{Ao}}(k) = (e_\vartheta^{\mathrm{T}} P_{\vartheta_A}^{-1}(k)e_\vartheta)^{-1} \quad (33)$$

Also, the relation $P_{\vartheta_{Ao}}(k|k) \le P_{\vartheta_{Ai}}(k|k)$, $i = 1, \ldots, L$ holds.

From (19), one knows that the parameter $\vartheta_A = [a_1, \cdots, a_{n_a}]^{\mathrm{T}}$ is a function of the unknown parameters of Φ. If the unknown parameters of Φ can be uniquely identified by ϑ_A, one can obtain the unknown parameter fusion estimator of Φ based on the identified fusion parameter estimator $\hat{\vartheta}_{Ao}(k)$. Further, one gets the estimator $\hat{\Phi}(k)$ of Φ. From the above analysis, one can see that $\hat{\Phi}(k)$ converges to Φ with probability 1, i.e., $\hat{\Phi}(k) \to \Phi$ as $k \to \infty$.

V. IDENTIFICATION OF THE UNKNOWN RECEIVING MEASUREMENT RATES

Substituting $\hat{\Phi}(k)$ into (4), one obtains

$$\hat{X}(k+1) = \hat{\Phi}(k)\hat{X}(k)\hat{\Phi}^{\mathrm{T}}(k) + \Gamma Q_w \Gamma^T \quad (34)$$

One introduces the zero-step correlation function $R_i(k) = \mathrm{E}[y_i^2(k)]$, from (2) it is computed as

$$R_i(k) = \alpha_i H_i \hat{X}(k) H_i^{\mathrm{T}} + Q_{v_i} \quad (35)$$

The correlation function can be approximated by the sampling correlation function which can be recursively calculated as [26]:

$$\hat{R}_i(k) = \frac{1}{k}\sum_{l=1}^{k} y_i^2(l) = \hat{R}_i(k-1) + \frac{1}{k}[y_i^2(k) - \hat{R}_i(k-1)] \quad (36)$$

under the initial value $\hat{R}_i(0) = 0$.

From (35) one can obtain the receiving measurement rate of the ith sensor at each time as

$$\hat{\alpha}_i(k) = \frac{\hat{R}_i(k) - Q_{v_i}}{H_i\hat{X}(k)H_i^{\mathrm{T}}} \quad (37)$$

From Section 3, $\hat{\Phi}(k) \to \Phi$ as $k \to \infty$. From the stability of Φ and (34), $\hat{X}(k) \to X(k)$ as $k \to \infty$. From the ergodicity of stochastic process [26], $\hat{R}_i(k) \to R_i(k)$ as $k \to \infty$. One can obtain that the estimate $\hat{\alpha}_i(k)$ converges to the true value α_i, i.e., $\hat{\alpha}_i(k) \to \alpha_i$ as $k \to \infty$. Further, one has

$$\begin{aligned}\hat{Q}_{V_i}(k) &= \hat{\alpha}_i(k)(1-\hat{\alpha}_i(k))H_i\hat{X}(k)H_i^T + Q_{v_i}\\ &\to \alpha_i(1-\alpha_i)H_iX(k)H_i^T + Q_{v_i} = Q_{V_i}(k), \quad k\to\infty\end{aligned}$$

Substituting the estimates $\hat{\Phi}(k)$ and $\hat{\alpha}_i(k)$ into the local optimal filter in Lemma 1, CCMs in Lemma 2 and distributed MWFE algorithm in Lemma 3, respectively. One can obtain the corresponding self-tuning filtering algorithms. To save space, the detail is omitted. Denote the corresponding self-tuning local filter, prediction error variance, filtering error CCM, and fusion filter by $\hat{x}_{si}(k|k)$, $\hat{\Sigma}_{s_i}(k|k-1)$, $\hat{P}_{sij}(k|k)$, and $\hat{x}_s(k|k)$, respectively.

VI. CONVERGENCE ANALYSIS OF SELF-TUNING FUSION FILTER

A. PRELIMINARY LEMMAS

The following Lemma 4 and Lemma 5 present a stability criterion in terms of Lyapunov equations, which constitutes the DVESA method and DESA method. Further, utilizing the two methods, the convergence of the proposed self-tuning fusion filter is proven.

Lemma 4 [26]: Consider a time-varying Lyapunov equation

$$J(k) = T_1(k)J(k-1)T_2^{\mathrm{T}}(k) + U(k)$$

where $J(k) \in R^{n\times n}$ and $U(k) \in R^{n\times n}$. The matrices $T_1(k) \in R^{n\times n}$ and $T_2(k) \in R^{n\times n}$ are uniformly asymptotically stable, i.e., there are constants $c_i > 0$ and $0 < \rho_i < 1$, such that

$$\|T_i(k,t)\| \le c_i\rho_i^{k-t}, \quad \forall k \ge t \ge 0,\ i = 1, 2$$

where $T_i(k,k) = I_n$, $T_i(k,t) = T_i(k)T_i(k-1)\cdots T_i(t+1)$, $k > t$. Then, $J(k)$ is bounded if $U(k)$ is bounded, further $J(k) \to 0$ as $k \to \infty$ if $U(k) \to 0$ as $k \to \infty$.

Lemma 5 [26]: Consider the dynamic error system

$$\delta(k) = T(k)\delta(k-1) + u(k)$$

where $\delta(k) \in R^n$ and $u(k) \in R^n$. The matrix $T(k) \in R^{n\times n}$ is uniformly asymptotically stable. Then, $\delta(k)$ is bounded if $u(k)$ is bounded, further $\delta(k) \to 0$ as $k \to \infty$ if $u(k) \to 0$ as $k \to \infty$.

B. CONVERGENCE OF THE SELF-TUNING LOCAL FILTERING AND PREDICTION ERROR VARIANCE MATRICES

Denote the systems (1) and (3) with known model parameters and receiving measurement rates by the notation $(\Phi, \Gamma, F_i, Q_w, Q_{V_i}(k))$, and the systems (1) and (3) with UMPs and unknown receiving measurement rates by the notation $(\hat{\Phi}(k), \Gamma, \hat{F}_i(k), Q_w, \hat{Q}_{V_i}(k))$.

Lemma 6 [26]: For system $(\Phi, \Gamma, F_i, Q_w, Q_{V_i}(k))$ under Assumptions 1-5, the state transition matrix $\Psi_{fi}(k) = [I_n - K_i(k)F_i]\Phi$ of local optimal filter and the state transition matrix $\Psi_{pi}(k) = \Phi[I_n - K_i(k)F_i]$ of local optimal predictor are both uniformly asymptotically stable. Both $\Sigma_i(k|k-1)$ and $K_i(k)$ are bounded. For system $(\hat{\Phi}(k), \Gamma, \hat{F}_i(k), Q_w, \hat{Q}_{V_i}(k))$, the state transition matrix $\hat{\Psi}_{sfi}(k) = [I_n - \hat{K}_{si}(k)\hat{F}_i(k)]\hat{\Phi}(k)$ of local self-tuning filter and the state transition matrix $\hat{\Psi}_{spi}(k) = \hat{\Phi}(k)[I_n - \hat{K}_{si}(k)\hat{F}_i(k)]$ of local self-tuning predictor are uniformly asymptotically stable. Both $\hat{\Sigma}_{si}(k|k-1)$ and $\hat{K}_{si}(k)$ are bounded.

Theorem 2: Under Assumptions 1-5, the self-tuning prediction error variance matrix $\hat{\Sigma}_{si}(k+1|k)$ with the consistent estimates $\hat{\Phi}(k)$ and $\hat{\alpha}_i(k)$ converges to the optimal prediction error variance matrix $\Sigma_i(k+1|k)$ with the true values Φ and α_i with probability 1. The local self-tuning filtering error variance $\hat{P}_{si}(k|k)$ converges to the optimal filtering error variance $P_i(k|k)$ with probability 1, i.e.,

$$[\hat{\Sigma}_{si}(k+1|k) - \Sigma_i(k+1|k)] \to 0, \quad k \to \infty \tag{38}$$
$$[\hat{P}_{si}(k|k) - P_i(k|k)] \to 0, \quad k \to \infty \tag{39}$$

Further, one has $[\hat{K}_{si}(k) - K_i(k)] \to 0$, $k \to \infty$.

Proof: From Lemma 1, $\hat{\Sigma}_{si}(k+1|k)$ and $\Sigma_i(k+1|k)$ satisfy the Riccati equation

$$\hat{\Sigma}_{si}(k+1|k) = \hat{\Phi}(k)[\hat{\Sigma}_{si}(k|k-1) - \hat{K}_{si}(k)\hat{F}_i(k) \times \hat{\Sigma}_{si}(k|k-1)]\hat{\Phi}^{\mathrm{T}}(k) + \Gamma Q_w \Gamma^{\mathrm{T}} \tag{40}$$
$$\Sigma_i(k+1|k) = \Phi[\Sigma_i(k+1|k) - K_i(k)F_i\Sigma_i(k+1|k)]\Phi^{\mathrm{T}} + \Gamma Q_w \Gamma^{\mathrm{T}} \tag{41}$$

Let $\Delta\hat{\Phi}(k) = \hat{\Phi}(k) - \Phi$. Substituting $\hat{\Phi}(k) = \Phi + \Delta\hat{\Phi}(k)$ into (40) and subtracting (41) from (40) yield

$$\begin{aligned}\hat{\Sigma}_{si}(k+1|k) &- \Sigma_i(k+1|k)\\ &= \hat{\Phi}(k)[\hat{\Sigma}_{si}(k|k-1) - \Sigma_i(k|k-1)\\ &\quad -\hat{K}_{si}(k)\hat{F}_i(k)\hat{\Sigma}_{si}(k|k-1)\\ &\quad +K_i(k)F_i\Sigma_i(k|k-1)]\Phi^{\mathrm{T}} + U_{i1}(k)\end{aligned} \tag{42}$$
$$\begin{aligned}U_{i1}(k) &= \hat{\Phi}(k)[I_n - \hat{K}_{si}(k)\hat{F}_i(k)]\hat{\Sigma}_{si}(k|k-1)\Delta\hat{\Phi}^{\mathrm{T}}(k)\\ &\quad + \Delta\hat{\Phi}(k)[I_n - K_i(k)F_i]\Sigma_i(k|k-1)\Phi^{\mathrm{T}}\end{aligned} \tag{43}$$

From the definitions of $\hat{\Psi}_{spi}(k)$ and $\Psi_{pi}(k)$ in Lemma 6, it follows that

$$\begin{aligned}\hat{\Psi}_{spi}(k)&(\hat{\Sigma}_{si}(k|k-1) - \Sigma_i(k|k-1))\Psi_{pi}^{\mathrm{T}}(k)\\ &= \hat{\Phi}(k)[\hat{\Sigma}_{si}(k|k-1) - \Sigma_i(k|k-1)\\ &\quad - \hat{K}_{si}(k)\hat{F}_i(k)\hat{\Sigma}_{si}(k|k-1)\\ &\quad + K_i(k)F_i\Sigma_i(k|k-1)]\Phi^{\mathrm{T}} + U_{i2}(k)\end{aligned} \tag{44}$$
$$\begin{aligned}U_{i2}(k) &= \hat{\Phi}(k)[\hat{K}_{si}(k)\hat{F}_i(k)\Sigma_i(k|k-1)\\ &\quad + \hat{K}_{si}(k)\hat{F}_i(k)\hat{\Sigma}_{si}(k|k-1)F_i^{\mathrm{T}}K_i^{\mathrm{T}}(k)\\ &\quad - \hat{\Sigma}_{si}(k|k-1)F_i^{\mathrm{T}}K_i^{\mathrm{T}}(k)\\ &\quad - \hat{K}_{si}(k)\hat{F}_i(k)\Sigma_i(k|k-1)F_i^{\mathrm{T}}K_i^{\mathrm{T}}(k)]\Phi^{\mathrm{T}}\end{aligned} \tag{45}$$

Setting $\hat{F}_i(k) = F_i + \Delta\hat{F}_i(k)$, one has

$$\begin{aligned}\hat{K}_{si}(k)&\hat{F}_i(k)\hat{\Sigma}_{si}(k|k-1)F_i^{\mathrm{T}}\\ &= \hat{K}_{si}(k)\hat{F}_i(k)\hat{\Sigma}_{si}(k|k-1)\hat{F}_i^{\mathrm{T}}(k)\\ &\quad -\hat{K}_{si}(k)\hat{F}_i(k)\hat{\Sigma}_{si}(k|k-1)\Delta\hat{F}_i^{\mathrm{T}}(k)\\ &= \hat{\Sigma}_{si}(k|k-1)\hat{F}_i^{\mathrm{T}}(k) - \hat{K}_{si}(k)\hat{Q}_{V_i}(k)\\ &\quad -\hat{K}_{si}(k)\hat{F}_i(k)\hat{\Sigma}_{si}(k|k-1)\Delta\hat{F}_i^{\mathrm{T}}(k)\end{aligned} \tag{46}$$
$$\begin{aligned}\hat{F}_i(k)&\Sigma_i(k|k-1)F_i^{\mathrm{T}}K_i^{\mathrm{T}}(k)\\ &= F_i\Sigma_i(k|k-1)F_i^{\mathrm{T}}K_i^{\mathrm{T}}(k)\\ &\quad + \Delta\hat{F}_i(k)\Sigma_i(k|k-1)F_i^{\mathrm{T}}K_i^{\mathrm{T}}(k)\\ &= F_i\Sigma_i(k|k-1) - Q_{V_i}(k)K_i^{\mathrm{T}}(k)\\ &\quad + \Delta\hat{F}_i(k)\Sigma_i(k|k-1)F_i^{\mathrm{T}}K_i^{\mathrm{T}}(k)\end{aligned} \tag{47}$$

Substituting (46) and (47) into (45) yields

$$\begin{aligned}&U_{i2}(k)\\ &= \hat{\Phi}(k)[\hat{K}_{si}(k)\Delta\hat{F}_i(k)\Sigma_i(k|k-1)\\ &\quad + \hat{\Sigma}_{si}(k|k-1)\Delta\hat{F}_i^{\mathrm{T}}(k)K_i^{\mathrm{T}}(k) - \hat{K}_{si}(k)(\hat{Q}_{V_i}(k)\\ &\quad - Q_{V_i}(k))K_i^{\mathrm{T}}(k) - \hat{K}_{si}(k)\hat{F}_i(k)\hat{\Sigma}_{si}(k|k-1)\Delta F_i^{\mathrm{T}}(k)K_i^{\mathrm{T}}(k)\\ &\quad - \hat{K}_{si}(k)\Delta\hat{F}_i(k)\Sigma_i(k|k-1)F_i^{\mathrm{T}}K_i^{\mathrm{T}}(k)]\Phi^{\mathrm{T}}\end{aligned} \tag{48}$$

From (42) and (44), and defining the variance error $\Upsilon_i(k) = \hat{\Sigma}_{si}(k|k-1) - \Sigma_i(k|k-1)$, one has the dynamic variance error system as

$$\Upsilon_i(k+1) = \hat{\Psi}_{spi}(k)\Upsilon_i(k)\Psi_{pi}^{\mathrm{T}}(k) + U_i(k) \tag{49}$$
$$U_i(k) = U_{i1}(k) - U_{i2}(k) \tag{50}$$

To prove (38), one only needs to prove $\Upsilon_i(k) \to 0$ as $k \to \infty$. Considering the dynamic variance error systems (49) and (50), according to Lemma 6 one knows that $\hat{\Sigma}_{si}(k+1|k)$, $\hat{K}_{si}(k)$, $\Sigma_i(k+1|k)$ and $K_i(k)$ are bounded as $k \to \infty$, then from $\Delta\hat{\Phi}(k) \to 0$, $\Delta\hat{F}_i(k) \to 0$ and $\hat{Q}_{V_i}(k) \to Q_{V_i}(k)$ as $k \to \infty$, it holds that

$$U_{i1}(k) \to 0, \quad U_{i2}(k) \to 0 \tag{51}$$

Then, $U_i(k) \to 0$ as $k \to \infty$. From (49), according to the uniformly asymptotic stability of $\hat{\Psi}_{spi}(k)$ and $\Psi_{pi}(k)$, based on Lemma 4, one has $\Upsilon_i(k) \to 0$ as $k \to \infty$, i.e., (38) is true. Similarly, (39) is also true. Further, using $\hat{K}_i(k) = \hat{\Sigma}_{si}(k|k-1)\hat{F}_i^{\mathrm{T}}(k)\hat{Q}_{C_i}^{-1}(k)$, $K_i(k) = \Sigma_i(k|k-1)F_i^{\mathrm{T}}Q_{C_i}^{-1}(k)$, (38), and $\hat{X}(k) \to X(k)$, then $\hat{K}_{si}(k) \to K_i(k)$ as $k \to \infty$ is true. The proof is completed.

C. CONVERGENCE OF SELF-TUNING FILTERING ERROR CCM

Substituting the estimates $\hat{\Phi}(k)$ and $\hat{\alpha}_i(k)$ into (12), one can obtain the self-tuning cross-covariance matrix as

$$\begin{aligned}\hat{P}_{sij}(k+1|k+1) = {} & [I_n - \hat{K}_{si}(k+1)\hat{F}_i(k+1)] \\ & \times [\hat{\Phi}(k)\hat{P}_{sij}(k|k)\hat{\Phi}^{\mathrm{T}}(k) + \Gamma Q_w \Gamma^{\mathrm{T}}][I_n \\ & - \hat{K}_{sj}(k+1)\hat{F}_j(k+1)]^{\mathrm{T}} \end{aligned} \quad (52)$$

Theorem 3: Under Assumptions 1-5, the solution of self-tuning Lyapunov equation (52) converges to the solution of optimal Lyapunov equation (12), i.e.,

$$[\hat{P}_{sij}(k|k) - P_{ij}(k|k)] \to 0, k \to \infty \quad (53)$$

Proof: Let $\Delta_{ij}(k) = \hat{P}_{sij}(k|k) - P_{ij}(k|k)$, $\hat{K}_{si}(k) = K_i(k) + \Delta\hat{K}_i(k)$, $\hat{\Psi}_{sfi}(k) = \Psi_{fi}(k) + \Delta\hat{\Psi}_{fi}(k)$. Then, $\Delta\hat{K}_i(k) \to 0$, $\Delta\hat{\Psi}_{fi}(k) \to 0$ as $k \to \infty$.

By subtracting (12) from (52), the dynamic variance error satisfies the following Lyapunov equation as

$$\Delta_{ij}(k) = \Psi_{fi}(k)\Delta_{ij}(k-1)\Psi_{fj}^{\mathrm{T}}(k) + U_{ij}(k) \quad (54)$$

$$\begin{aligned} U_{ij}(k) = {} & \Delta\hat{\Psi}_{fi}(k)\hat{P}_{sij}(k-1|k-1)\Psi_{fj}^{\mathrm{T}}(k) \\ & + \Delta\hat{\Psi}_{fi}(k)\hat{P}_{sij}(k-1|k-1)\Delta\hat{\Psi}_{fj}^{\mathrm{T}}(k) \\ & + \Psi_{fi}(k)\hat{P}_{sij}(k-1|k-1)\Delta\hat{\Psi}_{fj}^{\mathrm{T}}(k) \\ & + [I_n - \hat{K}_{si}(k)\hat{F}_i(k)]\Gamma Q_w \Gamma^{\mathrm{T}}[I_n - \hat{K}_{sj}(k)\hat{F}_j(k)]^{\mathrm{T}} \\ & - [I_n - K_i(k)F_i]\Gamma Q_w \Gamma^{\mathrm{T}}[I_n - K_j(k)F_j]^{\mathrm{T}} \end{aligned} \quad (55)$$

Using the boundness of $\hat{K}_{si}(k)$, $\hat{K}_{sj}(k)$, $\hat{F}_i(k)$ and $\hat{F}_j(k)$, one has $[I_n - \hat{K}_{si}(k)\hat{F}_i(k)]\Gamma Q_w \Gamma^{\mathrm{T}}[I_n - \hat{K}_{sj}(k)\hat{F}_j(k)]^{\mathrm{T}} - [I_n - K_i(k)F_i]\Gamma Q_w \Gamma^{\mathrm{T}}[I_n - K_j(k)F_j]^{\mathrm{T}} \to 0$ as $k \to \infty$.

Using Lemma 4, Lemma 6 and the uniformly asymptotic stability of $\hat{\Psi}_{sfi}(k)$ and $\Psi_{fi}(k)$, one can obtain that $\hat{P}_{sij}(k|k)$ is bounded. Hence, from $\hat{\alpha}_i(k) \to \alpha_i$ and $\Delta\hat{K}_i(k) \to 0$, $\Delta\hat{\Psi}_{fi}(k) \to 0$ as $k \to \infty$, one has $U_{ij}(k) \to 0$. Using (54) and Lemma 4, one has that (53) is true. The proof is completed.

D. CONVERGENCE OF LOCAL AND FUSION SELF-TUNING FILTERS

Theorem 4: Under Assumptions 1-5, the local self-tuning filter $\hat{x}_{si}(k|k)$ converges to the local optimal filter $\hat{x}_i(k|k)$, i.e.,

$$[\hat{x}_{si}(k|k) - \hat{x}_i(k|k)] \to 0, \quad k \to \infty \quad (56)$$

Proof: From (11), the local self-tuning filter is given as

$$\hat{x}_{si}(k|k) = \hat{\Psi}_{sfi}(k)\hat{x}_{si}(k-1|k-1) + \hat{K}_{si}(k)y_i(k) \quad (57)$$

Because $\hat{K}_{si}(k)$ and $y_i(k)$ are bounded and $\hat{\Psi}_{sfi}(k)$ is uniformly asymptotically stable, $\hat{x}_{si}(k|k)$ is bounded from Lemma 5. Let $\delta_i(k) = \hat{x}_{si}(k|k) - \hat{x}_i(k|k)$. Subtracting (11) from (57), one has the dynamic error system

$$\delta_i(k) = \Psi_{fi}(k)\delta_i(k-1) + u_i(k) \quad (58)$$

where $u_i(k) = \Delta\hat{\Psi}_{fi}(k)\hat{x}_{si}(k-1|k-1) - \Delta\hat{K}_i(k)y_i(k)$. From the boundedness of $\hat{x}_{si}(k|k)$ and $y_i(k)$, and $\Delta\hat{K}_i(k) \to 0$ and $\Delta\hat{\Psi}_{fi}(k) \to 0$, one obtains $u_i(k) \to 0$. Applying Lemma 5 to (58) leads to $\delta_i(k) \to 0$, as $k \to \infty$, i.e., (56) is true. The proof is completed.

Theorem 5: Under Assumptions 1-5, the self-tuning DF filter $\hat{x}_s(k|k)$ converges to the optimal DF filter $\hat{x}_o(k|k)$, i.e.,

$$[\hat{x}_s(k|k) - \hat{x}_o(k|k)] \to 0, \quad k \to \infty \quad (59)$$

Proof: From (53), one has $[\hat{W}_{si}(k) - W_i(k)] \to 0$. Let $\hat{W}_{si}(k) = W_i(k) + \Delta\hat{W}_i(k)$. Then, $\Delta\hat{W}_i(k) \to 0$. Using (56) and the boundedness of $\hat{x}_{si}(k|k)$, one obtains

$$\begin{aligned}\hat{x}_s(k|k) - \hat{x}_o(k|k) = {} & \sum_{i=1}^{L} W_i(k)[\hat{x}_{si}(k|k) - \hat{x}_i(k|k)] \\ & + \sum_{i=1}^{L} \Delta\hat{W}_i(k)\hat{x}_{si}(k|k) \to 0 \end{aligned} \quad (60)$$

i.e., (59) is true. The proof is completed.

Remark 1: From the preceding sections, one can find that the main difficulties and keys of the work in this paper include: 1) introduction of the fictitious white noise $V_i(k)$ in (3) and construction of ARMA model (21); 2) the computation of cross-covariance matrices (28) for parameters; 3) Convergence analysis (Theorem 2-Theorem 5) of the proposed algorithms.

VII. SIMULATION EXAMPLE

Considering a numerical example as (1) and (2) with 3 sensors. In the simulation, the parameters are taken as $\Phi = \begin{bmatrix} a_{11} & a_{12} \\ 0.3 & -0.5 \end{bmatrix}$, $\Gamma = \begin{bmatrix} 0.5 \\ 0.6 \end{bmatrix}$, $H_1 = \begin{bmatrix} 1 & 1.2 \end{bmatrix}$, $H_2 = \begin{bmatrix} 1.2 & 1.5 \end{bmatrix}$, $H_3 = \begin{bmatrix} 0.4 & 1 \end{bmatrix}$, $Q_w = 2$, $Q_{v_1} = 1.21$, $Q_{v_2} = 0.81$, $Q_{v_3} = 0.3$. The UMPs are set as $a_{11} = 0.8$, $a_{12} = -0.2$ and the unknown receiving measurement rates are set as $\alpha_1 = 0.3$, $\alpha_2 = 0.7$, $\alpha_3 = 0.9$. The initial values are $\hat{x}_i(0|0) = 0$, $P_i(0|0) = 0.1I_2$, and $P_{ij}(0|0) = 0.1I_2$, $i, j = 1, 2, 3$.

Our aim is to identify the unknown receiving measurement rates α_i and UMPs a_{11} and a_{12}, and solve the self-tuning fusion filter $\hat{x}_s(k|k)$.

Applying the algorithm proposed in Section 5, the identification results of unknown receiving measurement rates for 3 sensors by the correlation function method are given in Fig 1. As the time increases, one sees that the identified results of receiving measurement rates converge to the true values of receiving measurement rates.

Applying the algorithm proposed in Section 4, the fusion identification results of the UMPs are shown in Fig 2. As the time increases, one sees that the identification results of UMPs converge to the true values of model parameters.

To show the advantage of our MWFE parameter identifier, one does the comparison with weighted average (WA) fusion estimation algorithm in [6] and [26]. So, the weighted average fusion estimation algorithm is also given in the simulation. The local estimation error variances, weighted average estimation error variances and MWFE error variances for UMPs a_{11} and a_{12} can be computed below.

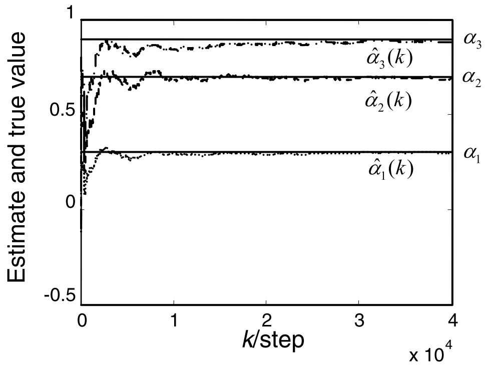


FIGURE 1. Identification of receiving measurement rates.

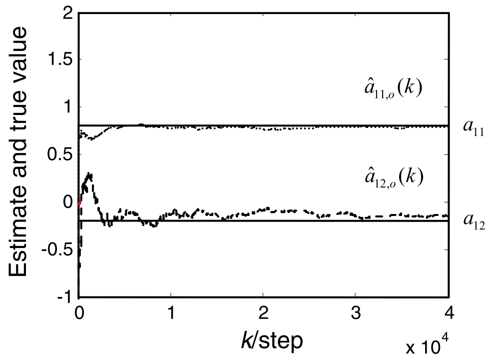


FIGURE 2. Identification of parameters of Φ.

A. LOCAL ESTIMATION ERROR VARIANCE

From (19), one obtains the relationship between a_t, $t=1,2$ and the model parameters of Φ as:

$$a_{11} = -(a_1 + a_{22}) \tag{61}$$

$$a_{12} = \frac{-1}{a_{21}}(a_2 - a_{22}a_{11}) \tag{62}$$

From (30), one can obtain local estimates $\hat{a}_{t,i}(k)$, $t=1,2$, $i=1,2,3$ for a_t. Then, local estimates of a_{11} and a_{12} are obtained as

$$\hat{a}_{11,i}(k) = -(\hat{a}_{1,i}(k) + a_{22}),$$
$$\hat{a}_{12,i}(k) = \frac{-1}{a_{21}}(\hat{a}_{2,i}(k) - a_{22}\hat{a}_{11,i}(k)),$$

Defining the local estimation error variances as $P_{a_{11,i}}(k) = \mathrm{E}[\tilde{a}^2_{11,i}(k)]$ and $P_{a_{12,i}}(k) = \mathrm{E}[\tilde{a}_{12,i}(k)\tilde{a}^{\mathrm{T}}_{12,i}(k)]$ where $\tilde{a}_{11,i}(k) = \hat{a}_{11,i}(k) - a_{11} = \tilde{a}_{1,i}(k)$ and $\tilde{a}_{12,i}(k) = \hat{a}_{12,i}(k) - a_{12} = \frac{1}{a_{21}}(\tilde{a}_{2,i}(k) + a_{22}\tilde{a}_{1,i}(k))$. Then, one has the local estimation error variances of a_{11} and a_{12} as

$$P_{a_{11,i}}(k) = P^{(1,1)}_{\vartheta_{Ai}}(k),$$
$$P_{a_{12,i}}(k) = \frac{1}{a^2_{21}}(P^{(2,2)}_{\vartheta_{Ai}}(k) + 2a_{22}P^{(1,2)}_{\vartheta_{Ai}}(k) + a^2_{22}P^{(1,1)}_{\vartheta_{Ai}}(k)),$$

B. WEIGHTED AVERAGE ESTIMATION ERROR VARIANCE

Defining the WA estimate of a_t as

$$\hat{\bar{a}}_t(k) = \frac{1}{3}\sum_{i=1}^{3}\hat{a}_{t,i}(k),$$

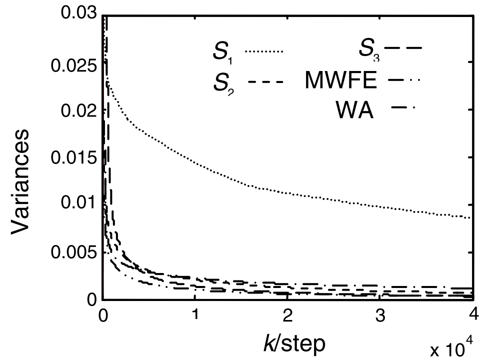


FIGURE 3. Estimation error variance of a_{11}.

From (61) and (62), the WA estimates of a_{11} and a_{12} are given as

$$\hat{\bar{a}}_{11}(k) = -(\hat{\bar{a}}_1(k) + a_{22}),$$
$$\hat{\bar{a}}_{12}(k) = \frac{-1}{a_{21}}(\hat{\bar{a}}_2(k) - a_{22}\hat{\bar{a}}_{11}(k)),$$

Defining the WA estimation error variances as $P_{\tilde{\bar{a}}_{11}}(k) = \mathrm{E}[\tilde{\bar{a}}^2_{11}(k)]$ and $P_{\tilde{\bar{a}}_{12}}(k) = \mathrm{E}[\tilde{\bar{a}}^2_{12}(k)]$, where $\tilde{\bar{a}}_{11}(k) = \hat{\bar{a}}_{11}(k) - a_{11} = \frac{1}{3}(\sum_{i=1}^{3}\tilde{a}_{1,i}(k))$ and $\tilde{\bar{a}}_{12}(k) = \hat{\bar{a}}_{12}(k) - a_{12} = \frac{1}{3a_{21}}\sum_{i=1}^{3}(\tilde{a}_{2,i}(k) + a_{22}\tilde{a}_{1,i}(k))$. One has the WA fusion estimation error variances of a_{11} and a_{12} as

$$\begin{aligned}
P_{\tilde{\bar{a}}_{11}}(k) = {} & \frac{1}{9}(P^{(1,1)}_{\vartheta_{A1}}(k) + P^{(1,1)}_{\vartheta_{A2}}(k) + P^{(1,1)}_{\vartheta_{A3}}(k) \\
& + 2P^{(1,1)}_{\vartheta_{A12}}(k) + 2P^{(1,1)}_{\vartheta_{A13}}(k) + 2P^{(1,1)}_{\vartheta_{A23}}(k)),\\
P_{\tilde{\bar{a}}_{12}}(k) = {} & \frac{1}{9a^2_{21}}(P^{(2,2)}_{\vartheta_{A1}}(k) + P^{(2,2)}_{\vartheta_{A2}}(k) + P^{(2,2)}_{\vartheta_{A3}}(k) \\
& + a^2_{22}(P^{(1,1)}_{\vartheta_{A1}}(k) + P^{(1,1)}_{\vartheta_{A2}}(k) + P^{(1,1)}_{\vartheta_{A3}}(k)) \\
& + 2(P^{(2,2)}_{\vartheta_{A12}}(k) + P^{(2,2)}_{\vartheta_{A13}}(k) + a_{22}P^{(1,2)}_{\vartheta_{A1}}(k) \\
& + a_{22}P^{(2,1)}_{\vartheta_{A12}}(k) + a_{22}P^{(2,1)}_{\vartheta_{A13}}(k) + P^{(2,2)}_{\vartheta_{A23}}(k) \\
& + a_{22}P^{(2,1)}_{\vartheta_{A21}}(k) + a_{22}P^{(1,2)}_{\vartheta_{A2}}(k) \\
& + a_{22}P^{(2,1)}_{\vartheta_{A23}}(k) + a_{22}P^{(2,1)}_{\vartheta_{A31}}(k) + a_{22}P^{(2,1)}_{\vartheta_{A32}}(k) \\
& + a_{22}P^{(1,2)}_{\vartheta_{A3}}(k) + a^2_{22}P^{(1,1)}_{\vartheta_{A12}}(k) + a^2_{22}P^{(1,1)}_{\vartheta_{A13}}(k) \\
& + a^2_{22}P^{(1,1)}_{\vartheta_{A23}}(k))),
\end{aligned}$$

C. MWFE ERROR VARIANCE

From (31), one can obtain the fusion estimates $\hat{a}_{1,o}(k)$ and $\hat{a}_{2,o}(k)$ of parameters a_1 and a_2, respectively. Then, from (61) and (62) one has the fusion estimates of a_{11} and a_{12} as

$$\hat{a}_{11,o}(k) = -(\hat{a}_{1,o}(k) + a_{22}),$$
$$\hat{a}_{12,o}(k) = \frac{-1}{a_{21}}(\hat{a}_{2,o}(k) - a_{22}\hat{a}_{11,o}(k)),$$

Defining the fusion estimation error variances as $P_{a_{11,o}}(k) = \mathrm{E}[\tilde{a}^2_{11,o}(k)]$ and $P_{a_{12,o}}(k) = \mathrm{E}[\tilde{a}^2_{12,o}(k)]$ where

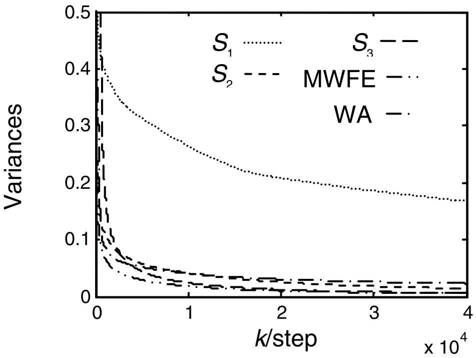
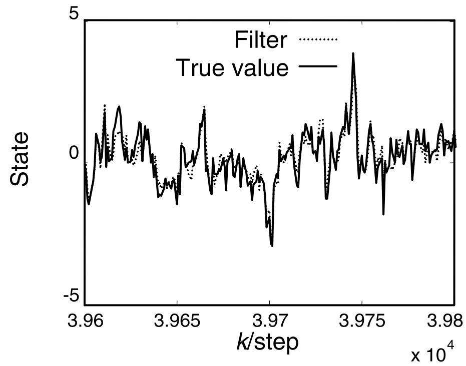
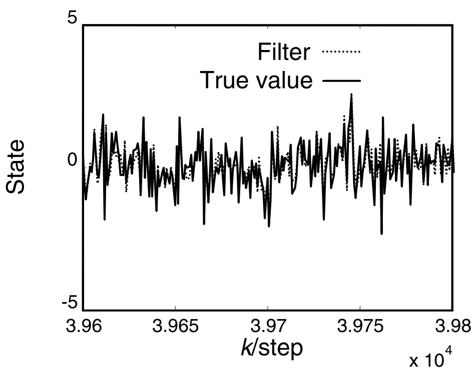


FIGURE 4. Estimation error variance of a_{12}.



(a)



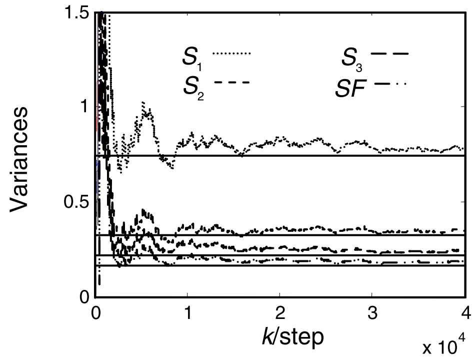
(b)

FIGURE 5. Self-tuning DF filter. (a) The first state component. (b) The second state component.

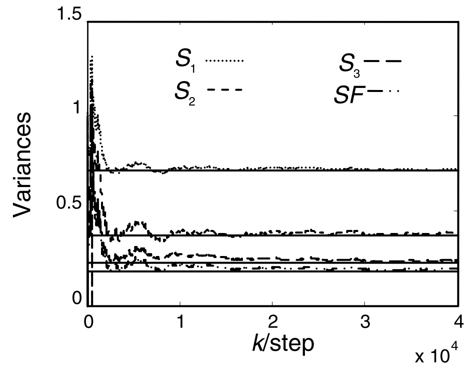
$\tilde{a}_{11,o}(k) = \hat{a}_{11,o}(k) - a_{11} = \tilde{a}_{1,o}(k)$ and $\tilde{a}_{12,o}(k) = \hat{a}_{12,o}(k) - a_{12} = \frac{1}{a_{21}}(\tilde{a}_{2,o}(k) + a_{22}\tilde{a}_{1,o}(k))$. Then, one has the MWFE error variances of a_{11} and a_{12} as

$$P_{a_{11,o}}(k) = P_{\vartheta_o}^{(1,1)}(k),$$
$$P_{a_{12,o}}(k) = \frac{1}{a_{21}^2}(P_{\vartheta_o}^{(2,2)}(k) + 2a_{22}P_{\vartheta_o}^{(1,2)}(k) + a_{22}^2 P_{\vartheta_o}^{(1,1)}(k)),$$

The local estimation error variances, WA fusion estimation error variances and MWFE error variances of UMPs a_{11} and a_{12} are shown in Fig 3 and Fig 4, respectively, where S_i, $i = 1, 2, 3$ denote the local estimation error variances. It can be seen that our MWFE variances are lower than local variances and WA variances.



(a)

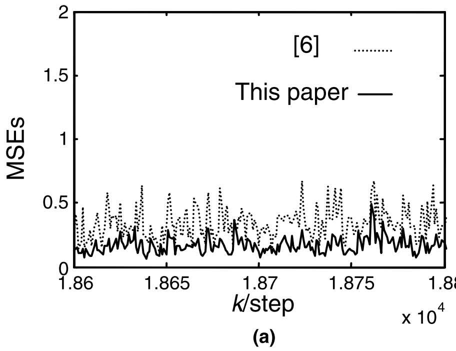


(b)

FIGURE 6. Variances of local, fusion optimal and self-tuning filters. (a) Variance of the first state component. (b) Variance of the second state component.

The self-tuning DF filter is given in Fig 5. One sees that the self-tuning DF filter is effective. The local and fusion error variances of optimal and self-tuning state filters are given in Fig 6 where S_i, i=1,2,3 denote the self-tuning local estimation error variances, the SF denotes the self-tuning fusion variance, and the lines denote the corresponding optimal variances. From Fig 6, one sees that the variances of self-tuning local filters converge to those of optimal local filters, and the variance of self-tuning fusion filter converges to that of optimal fusion filter. So, they have the asymptotic optimality. Moreover, the self-tuning DF filter has better accuracy than any self-tuning local filter.

Under the case of missing measurements, the self-tuning algorithms in the existing literature [6] that do not consider missing measurements will lose the asymptotic optimality. Fig.7 shows the comparison of the mean square errors (MSEs) by 30 times Monte Carlo runs for the self-tuning fusion filter without considering the missing measurements in [6] and our self-tuning fusion filter with considering the missing measurements. It can be seen that our self-tuning fusion filter with considering the missing measurements has better accuracy under the case of missing measurements.



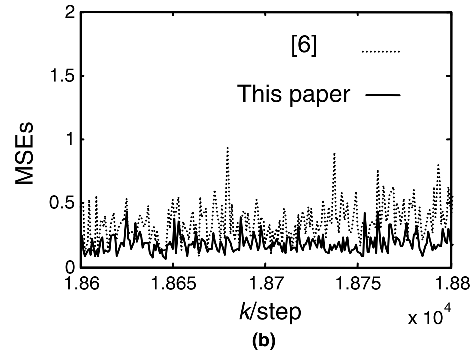


FIGURE 7. **MSEs of the self-tuning fusion flters with/without considering missing measurements. (a) MSEs of the first state component. (b) MSEs of the second component.**

VIII. CONCLUSION

A self-tuning DF filter has been proposed for MS systems subject to UMPs and unknown missing measurement rates. The model parameters and missing measurement rates are identified online based on the RELS algorithm and correlation function, respectively. A DF identifier for UMPs is proposed by using MWFE algorithm in the LUMV sense. The corresponding self-tuning state filtering algorithms are obtained by substituting the identified model parameters and missing measurement rates into the optimal local and fusion filtering algorithms. By utilizing the DVESA method, it has been proven that the local self-tuning filtering error variance converges to the optimal filtering error variance, and the solution of self-tuning filtering error CCM converges to that of optimal filtering error CCM. By utilizing DESA method, it has been proven that the self-tuning local filter converges to the optimal local filter, and the self-tuning DF filter converges to the optimal DF filter.

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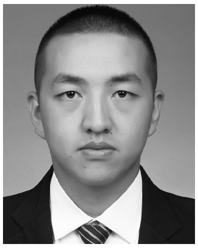

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