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Finite-Time H-Infinity Control of a Fractional-Order Hydraulic Turbine Governing System

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ABSTRACT The finite-time H_∞ control of a fractional-order hydraulic turbine governing system (HTGS) is studied. First, a fractional-order HTGS as well as its generalized Takagi–Sugeno fuzzy model is presented. Then, based on the fractional-order stability theorem, the robust H_∞ state feedback control is designed to guarantee that the HTGS is asymptotically stable with prescribed H_∞ performance. Furthermore, the H_∞ control is integrated with finite-time control theory, a finite-time H_∞ control is proposed for the fractional-order HTGS, and the stability condition is given in terms of linear matrix inequalities. Finally, simulation results verify the validity and superiority of the proposed control method.

INDEX TERMS Fractional-order stability, hydraulic turbine governing system, finite-time control, H_∞ control, linear matrix inequality.

I. INTRODUCTION

With the continuous expansion of the scale of the power system, the hydropower station plays a more and more important role in the task of peak regulation and frequency modulation in the power system [1]–[4]. Therefore, there is an urgent need of better regulating performance for the HTGS to meet the stable operation of power system. However, the HTGS is a complex nonlinear coupling system involving hydraulic, mechanical and electrical system [5]–[8]. Many factors make the control of HTGS very difficult such as the inertia and fluctuation of hydraulic parameters in the pressure diversion system, the appearance and attenuation of water hammer phenomenon, the nonlinear characteristics of mechanical and electrical coupling of water-turbine generator set, and the load disturbance of power system [9]–[13]. The increase of hydropower stations with high head and large capacity requires desperately better control of the HTGS.

There have been many results on the modeling and dynamic analysis for HTGS [14]–[19]. The integer-order calculus is always adopted for HTGS modeling. Recently, It has been found that fractional calculus has more advantages in describing soft, memory, strong dependence and viscoelastic attributes of numerous processes and materials [20]–[22]. Many projects could be better described by fractional calculus, such as brushless DC motors [23], wind turbine generators [24], electromechanical gyrostat systems [25], and memristor [26]. Therefore, according to the memory characteristics and historical dependence of the hydraulic-servo system, a more practical fractional-order HTGS is considered in the study.

At present, the control of the HTGS is mainly focused on PID control, fuzzy control, sliding model control and predictive control [27]–[30]. However, the above control methods are based on the asymptotic stability. Theoretically, the time that the asymptotic control system tends to be stable is infinite. From the point of view of the time optimization, The optimal control method should guarantee the HTGS stable in a finite time [31]–[33]. Finite-time control can improve the transition time of the HTGS. Besides, H_∞ control has a certain effect on enhancing the dynamic performance and the anti-interference ability [34]–[37]. Clearly, both finite-time control and H_∞ control have a specific advantage on HTGS performance. Can the combination of the two methods improve the transition process of the HTGS? No report has been found.

In light of the above analysis, some advantages are concluded from this study. Based on the generalized Takagi-Sugeno (T-S) fuzzy model, the fuzzy modeling of a HTGS is given. Considering the HTGS with uncertainty and external disturbance, based on the fractional-order stability theorem, by combining finite-time control and H_∞ control theory, a finite-time H_∞ control method for the HTGS is proposed. The stability condition with prescribed H_∞ performance is given in terms of the linear matrix inequality (LMI). Finally, simulation results are in agreement with the theoretical analysis.

II. PRELIMINARIES

A. FRACTIONAL CALCULUS DEFINITION

Definition 1 [38]: The Caputo definition of fractional derivative is defined as

$$D^\alpha f(t) = \frac{d^\alpha f(t)}{dt^\alpha} = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^n(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau, \quad \times (n-1<\alpha<n)$$

where α is the fractional order and the gamma function $\Gamma(\cdot)$ is defined as $\Gamma(\tau) = \int_0^\infty t^{\tau-1} e^{-t} dt$.

B. GENERALIZED T-S FUZZY MODEL

The T-S fuzzy model is described by the IF-THEN fuzzy rules. Local dynamics in different state space regions are represented over a linear realization. Then, the combination of the linear model is used to represent the nonlinear system [39], [40]. The generalized T-S fuzzy model is the generalization of integer-order fuzzy model, which is given as:

Rule R^i: IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_n(t)$ is M_{in}

$$\text{THEN } \frac{d^\alpha x(t)}{dt^\alpha} = A_i x(t) + B_i u(t) + B_\omega \omega(t), \quad (i = 1, 2, \ldots, r).$$

where $z(t) = \left[z_1(t)\ z(t) \cdots z_n(t) \right]$ is the premise variable, $M_{ij}(j = 1, 2, \cdots, n)$ is the fuzzy set, r is the fuzzy rule number, $u(t)$ is the control input,$\omega(t)$ is the external disturbance, $x(t) \in R^n$ is the state variable, $A_i \in R^{n\times n}$, $B_i \in R^{n\times n}$, $B_\omega \in R^{n\times n}$.

III. HTGS MODEL

The mathematical model of the HTGS is presented as [5]:

$$\begin{cases} \frac{d\delta}{dt} = \omega_0 \omega \\ \frac{d\omega}{dt} = \frac{1}{T_{ab}} \left[m_t - D\omega - \frac{E_q V_s}{x_{d\Sigma}} \sin\delta - \frac{V_s^2}{2} \frac{x_{d\Sigma} - x_{q\Sigma}}{x_{d\Sigma} x_{q\Sigma}} \sin 2\delta \right] \\ \frac{dm_t}{dt} = \frac{1}{e_{qh} T_w} \left[-m_t + e_y y + \frac{e e_y T_w}{T_y} y \right] \\ \frac{dy}{dt} = -\frac{1}{T_y} y, \end{cases} \tag{1}$$

where δ, ω, m_t, and y are the generator rotor angle deviation, the rotational speed relative deviation of the generator, the hydro-turbine output incremental torque deviation and the incremental deviation of the guide vane opening, respectively; ω_0 is the rated speed; T_{ab} is the inertia time constant of the rotating part; D is the damping coefficient of the generator; E_q is the quadrature-axis transient electromotive force; V_s is the bus voltage; $x_{d\Sigma}$ is the direct-axis transients reactance of total system; $x_{q\Sigma}$ is the quadrature-axis reactance of total system; e_{qh} is the transfer coefficient of water head; T_w is the inertia time constant of water flow in pressure diversion system; e_y is the transfer coefficient of the turbine torque with respect to the main relay stroke; e is the transfer coefficient; T_y is the major relay connector response time.

Considering the significant historical reliance of the hydraulic servo system, the following fractional-order hydraulic servo system is adopted [41]:

$$\frac{d^\alpha y}{dt^\alpha} = -\frac{1}{T_y} y, \tag{2}$$

where α is the fractional order, and T_y is the major relay connector response time.

For convenience, the x_1, x_2, x_3 and x_4 is used to replace the δ, ω, m_t and y, respectively. Considering the randomness of the load, and according to (1) and (2), the factional-order HTGS is represented as:

$$\begin{cases} \frac{dx_1}{dt} = \omega_0 x_2 + 0.1\text{rand}(1) \\ \frac{dx_2}{dt} = \frac{1}{T_{ab}} \left[x_3 - Dx_2 - \frac{E_q V_s}{x_{d\Sigma}} \sin x_1 - \frac{V_s^2}{2} \frac{x_{d\Sigma} - x_{q\Sigma}}{x_{d\Sigma} x_{q\Sigma}} \sin 2x_1 \right] + 0.1\text{rand}(1) \\ \frac{dx_3}{dt} = \frac{1}{e_{qh} T_w} \left[-x_3 + e_y x_4 + \frac{e e_y T_w}{T_y} x_4 \right] + 0.1\text{rand}(1) \\ \frac{d^\alpha x_4}{dt^\alpha} = -\frac{1}{T_y} x_4 + 0.1\text{rand}(1). \end{cases} \tag{3}$$

For system (3), the parameters are selected as follows: $\omega_0 = 314$, $T_{ab} = 9.0$s, $D = 2.0$, $E_q' = 1.35$, $x_{d\Sigma}' = 1.15$, $x_{q\Sigma} = 1.474$, $T_w = 0.8$s, $T_y = 0.1$s, $V_s = 1.0$, $e_{gh} = 0.5$, $e_y = 1.0$, $e = 0.7$, $\alpha = 0.98$. The state trajectories of

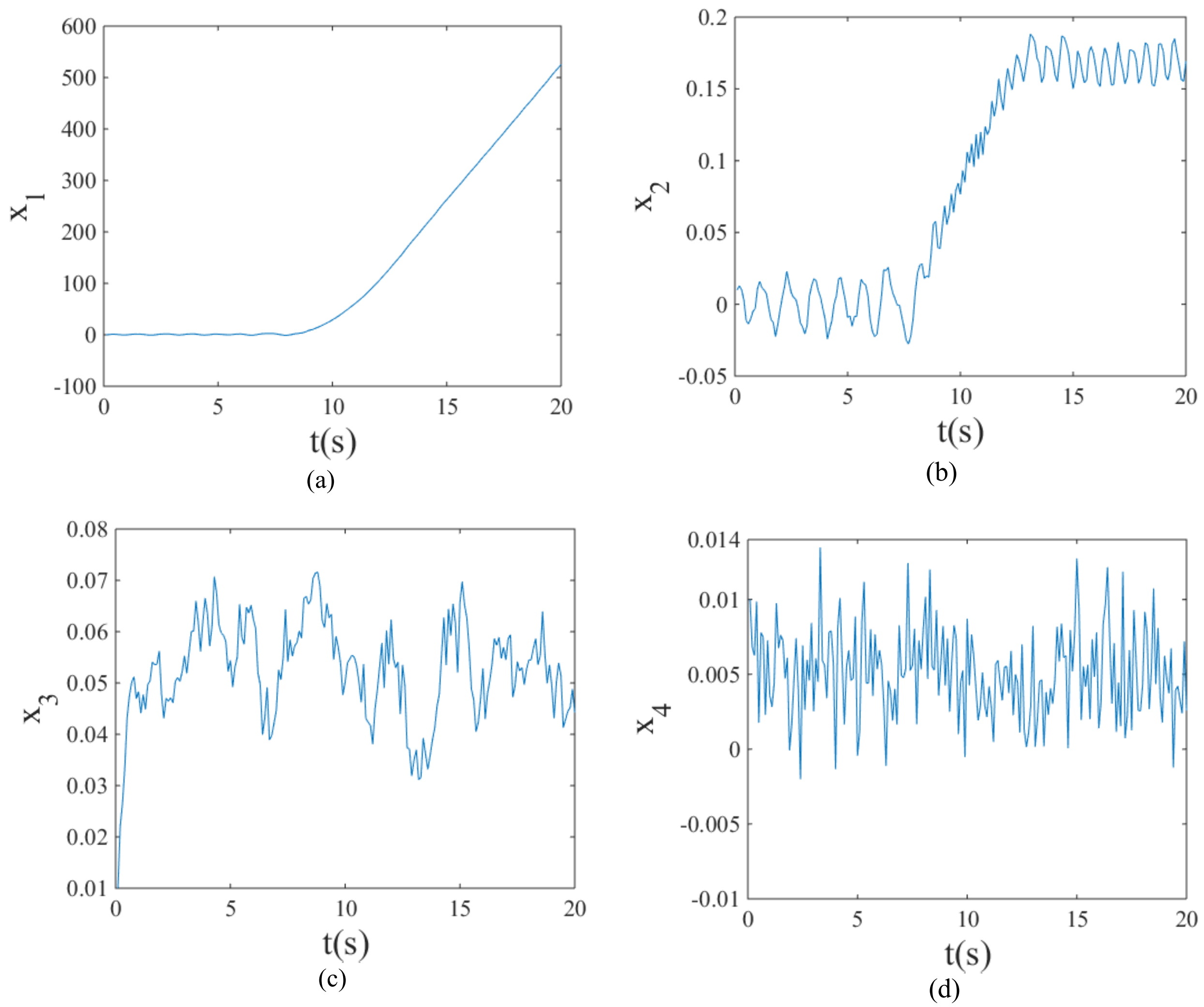

FIGURE 1. State trajectories of fractional-order HTGS (3). (a) $x_1 - t$. (b) $x_2 - t$. (c) $x_3 - t$. (d) $x_4 - t$.

fractional-order HTGS (3) are shown in Figure (1). It is clear the system is in irregular and unstable vibrations. So it is necessary to design a controller.

IV. CONTROLLER DESIGN

Considering the boundedness, select $x_1 \in [-d, d]$, $d = 4$. The following fuzzy rules of fractional-order HTGS (3) can be obtained.

R^1 : IF x_1 is $M_1(x_1(t))$ (near 0),

$$\text{THEN } \frac{d^\alpha x(t)}{dt^\alpha} = A_1 x(t) + B_1 u(t) + B_\omega \omega(t).$$

R^2 : IF x_1 is $M_2(x_1(t))$ (near $\pm d$),

$$\text{THEN } \frac{d^\alpha x(t)}{dt^\alpha} = A_2 x(t) + B_2 u(t) + B_\omega \omega(t).$$

The membership function can be selected as

$$M_1(x_1(t)) = \frac{1}{2}\left(1 + \frac{x_1(t)}{d}\right),$$
$$M_2(x_1(t)) = \frac{1}{2}\left(1 - \frac{x_1(t)}{d}\right).$$

According to fuzzy theory, the fuzzy model of fractional-order HTGS (3) can be got as

$$\frac{d^\alpha x(t)}{dt^\alpha} = \sum_{i=1}^{2} h_i(z(t))\left(A_i x(t) + B_i u(t) + B_\omega \omega(t)\right). \quad (4)$$

where

$$\begin{cases} h_i(z(t)) = \dfrac{\prod_{j=1}^{n} M_{ij}(z_j(t))}{\sum_{i=1}^{r}\prod_{j=1}^{n} M_{ij}(z_j(t))} \geq 0, \\ \sum_{i=1}^{r} h_i(z(t)) = 1. \end{cases}$$

Taking the uncertainty into consideration, and for the fractional-order HTGS (4), x_1, x_2 and x_3 is chosen as subsystem S_1, and x_4 as subsystem S_2. There is,

$$\frac{d^\alpha x_i(t)}{dt^\alpha} = (A_{ii} + \Delta A_{ii}) x_i(t) + (B_i + \Delta B_i) u_i(t) + \sum_{j=1, j\neq i}^{2} \left(A_{ij} + \Delta A_{ij}\right) x_j(t) + B_{\omega i}\omega_i(t), \quad (5)$$

$$z_i(t) = C_i x(t) + D_i u(t), \quad (6)$$

where $i = 1, 2$, $x_i(t)$, $u_i(t)$, $\omega_i(t)$ and $z_i(t)$ are the state, control input, disturbance input and performance output of the ith subsystems, respectively. The real matrices $A_{ii} \in R^{2\times 2}$, $B_i \in R^{2\times 2}$, $A_{ij} \in R^{2\times 2}$, $B_{\omega i} \in R^{2\times 2}$, $C_i \in R^{2\times 2}$, $D_i \in R^{2\times 2}$ are constant. A_{ij} and ΔA_{ij} are the interconnection matrices from subsystem j to subsystem i. The real matrices $\Delta A_{ii} \in R^{2\times 2}$, $\Delta B_i \in R^{2\times 2}$ and $\Delta A_{ij} \in R^{2\times 2}$ denote time-invariant uncertainties in the system, control input and interconnection matrices, respectively. The uncertain matrices ΔA_{ij} and ΔB_i are assumed to be of the following form:

$$\Delta A_{ij} = M_{aij}F_{aij}E_{aij},$$
$$\Delta B_i = M_{bi}F_{bi}E_{bi},$$

where F_{aij} and F_{bi} are real uncertain matrices of appropriate dimensions satisfying:

$$F_{aij}^T F_{aij} \le I, F_{bi}^T F_{bi} \le I.$$

M_{aij}, E_{aij}, M_{bi} and E_{bi} are known real constant matrices of appropriate dimensions, which specify how the elements of the system nominal matrices A_{ij} and B_i are affected by the uncertain parameters in F_{aij} and F_{bi}.

Further, the HTGS (5) can be represented as

$$\frac{d^\alpha x(t)}{dt^\alpha} = (A + \Delta A)\, x(t) + (B + \Delta B)\, \mathrm{u}(t) + B_\omega \omega(t). \quad (7)$$

Define $A = A + \Delta A$, $B = B + \Delta B$ and a state feedback controller is adopted as

$$u_i(\mathrm{t}) = K_i x_i(t),$$

where the gain matrix $K_i \in R^{2\times 2}$ is the fixed gain that will be designed later.

Therefore, the HTGS (7) is rewritten as

$$\frac{d^\alpha x(t)}{dt^\alpha} = (A + BK)\, x(t) + B_\omega \omega(t),$$
$$z(t) = Cx(t) + Du(t) = (C + DK)\, x(t), \quad (8)$$

where

$$\omega = \left(\omega_1^T, \omega_2^T, \omega_3^T, \omega_4^T\right) \epsilon R^4,$$
$$K = \mathrm{diag}(K_1, K_2) \in R^{4\times 4},$$
$$B_\omega = \begin{bmatrix} B_{\omega 11} & B_{\omega 12} \\ B_{\omega 21} & B_{\omega 22} \end{bmatrix} \in R^{4\times 4}.$$

For convenience, the HTGS (7) is simplified as

$$\frac{d^\alpha x(t)}{dt^\alpha} = A_{cl} x(t) + B_\omega \omega(t),$$
$$z(t) = C_{cl} x(t), \quad (9)$$

where $A_{cl} = A + BK$, $C_{cl} = C + DK$,

$$A_{cl} = \begin{bmatrix} A_{cl11} & A_{cl12} \\ A_{cl21} & A_{cl22} \end{bmatrix} \in R^{4\times 4}.$$

Then the transfer function from the disturbance to the output is obtained as:

$$T_{z\omega}(s) = C_{cl}\left(s^\alpha I - A_{cl}\right)^{-1} B_\omega. \quad (10)$$

The objective is to get the state feedback gain matrix K_i such that the fractional-order HTGS (9) is finite-time stable and satisfies the H_∞ performance. The following Definition and lemmas are given.

Definition 2 [42]: An n-by-n Hermitian matrix A is said to be negative definite if for all non-zero $x \in C^n$,

$$x^* A x < 0. \quad (11)$$

Definition 3 [43]: The time-varying linear system

$$\dot{x}(t) = A_{cl} x(t), \quad t \in [0, T],$$

is said to be finite-time stable (FTS) with respect to (c_1, c_2, T, R), with $c_2 > c_1$ and $R > 0$ if $x^T(0)\, Rx(0) \le c_1 \Rightarrow x^T(t)\, Rx(t) < c_2, \forall t \in [0, T]$.

Assumption 1: As to the following system

$$\frac{d^\alpha x(t)}{dt^\alpha} = A_{cl} x(t) + B_\omega \omega(t),$$

for the given positive constants c_1, T and a positive definite matrix R, suppose that $x_0^T R x_0 \le c_1\ (\forall t \in (0, T))$. Here, $\forall \omega(t)^T \omega(t) \le m$, m is the upper bound of the product of $\omega(t)^T$ and $\omega(t)$.

Lemma 1 [44]: for any matrices X and Y with appropriate dimensions and any $\varepsilon > 0$, the following inequality holds:

$$X^* Y + Y^* X \le \varepsilon X^* X + \varepsilon^{-1} Y Y^*. \quad (12)$$

Lemma 2 [45]: if there exists $\gamma > 0$ and $X = X^*$ which satisfy the following inequality, then the fractional-order HTGS (9) is asymptotically stable and satisfies$||T_{z\omega}(s)||_\infty \le \gamma$.

$$\begin{bmatrix} sym\left\{A_{cl}\left(e^{\theta i}X + e^{-\theta i}\overline{X}\right)\right\} & * & * \\ C_{cl}\left(e^{\theta i}X + e^{-\theta i}\overline{X}\right) & -\gamma I & * \\ B_\omega^T & 0 & -\gamma I \end{bmatrix} < 0, \quad (13)$$

where $\theta = (1 - \alpha)\frac{\pi}{2}$.

Lemma 3 [46]: If there is a positive definite matrix P which satisfies $J = x^T P \frac{d^\alpha x}{dt^\alpha} \le 0$($x^T P \frac{d^\alpha x}{dt^\alpha}$ is named as J function), then the fractional-order HTGS (9) is globally asymptotically stable and satisfies $||T_{z\omega}(s)||_\infty \le \gamma$, $J = x^T P \frac{d^\alpha x}{dt^\alpha} \le 0$ is equivalent to

$$J_0 = x^T P \frac{d^\alpha x}{dt^\alpha} + \frac{d^\alpha x}{dt^\alpha}^T P x \le 0. \quad (14)$$

Theorem 1: The fractional-order HTGS (9) with state feedback controller is finite-time stable and satisfies $||T_{z\omega}(s)||_\infty \le \gamma$ if there exists a positive definite Hermitian matrix $X_i = X_i^* \in \mathrm{C}^{n_i \times n_i}$ and $Y_i \in R^{m_i \times n_i}$, scalars $\beta_i > 0$, $\varepsilon_{ii} > 0$, $\delta_{ji} > 0$, $\mu_{ji} > 0$ $(i, j = 1, 2 \ldots, N, i \ne j)$, $\sigma_i \le 0$, positive definite matrices $P_i > 0$, $Q_i > 0$

such that the inequalities (15)-(17), [(15), as shown at the bottom of this page.] holds.

$$\begin{bmatrix} P_iA_{clii} + A_{clii}^TP_i - \sigma_iP_i & P_iB_{\omega ii} \\ B_{\omega ii}^TP_i & -\sigma_iQ_i \end{bmatrix} < 0, \tag{16}$$

$$c_1\lambda_{max}\left(\tilde{P}_i\right) + \mathrm{d}_i\lambda_{max}(Q_i) < c_2e^{-\sigma_iT}\lambda_{min}\left(\tilde{P}_i\right), \tag{17}$$

where $i, j = 1, 2, \ldots N, i \neq j$.

$$\begin{aligned} \Gamma_i^{11} &= Sym\left\{A_{ii}\left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right) + B_iY_i\right\} + \gamma^{-1}B_{\omega i}B_{\omega i}^T \\ &\quad + \beta_iM_{bi}M_{bi}^T + \varepsilon_{aii}M_{aii}M_{aii}^T \\ &\quad + \sum\nolimits_{j=1,j\neq i}^{N}\left\{\delta_{ij}A_{ij}A_{ij}^T + \mu_{ij}M_{aij}M_{aij}^T\right\}. \\ \tilde{P}_i &= R^{1\frac{1}{2}}P_iR^{-\frac{1}{2}} \end{aligned}$$

The K_i is obtained as

$$K_i = Y_i\left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right)^{-1}, \quad i = 1, 2, \ldots, N. \tag{18}$$

Proof: first, according to Schur complement, Lemma 2 is equivalent to

$$\begin{aligned} \Omega &= sym\left\{A_{cl}\left(e^{\theta i}X + e^{-\theta i}\overline{X}\right)\right\} + \gamma^{-1}B_\omega B_\omega^T \\ &\quad + \gamma^{-1}\left[C_{cl}\left(e^{\theta i}X + e^{-\theta i}\overline{X}\right)\right]^T \\ &\quad \times \left[C_{cl}\left(e^{\theta i}X + e^{-\theta i}\overline{X}\right)\right] < 0. \end{aligned} \tag{19}$$

From Definition 2, there is

$$\begin{aligned} &\xi^*\Omega\xi \\ &= \xi^*\{sym\left[A_{cl}\left(e^{\theta i}X + e^{-\theta i}\overline{X}\right)\right] + \gamma^{-1}B_\omega B_\omega^T \\ &\quad + \gamma^{-1}\left[C_{cl}\left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right)\right]^T C_{cl}\left(e^{\theta i}X + e^{-\theta i}\overline{X}\right)\}\xi \\ &= \sum_{i=1}^{N}\xi_i^*\{sym\{(A_{ii} + \Delta A_{ii} + (B_i + \Delta B_i)K_i) \\ &\quad \times \left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right)\} \\ &\quad + \gamma^{-1}\left[(C_i + D_iK_i)\left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right)\right]^T \\ &\quad \times \left[(C_i + D_iK_i)\left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right)\right] + \gamma^{-1}B_\omega B_\omega^T\}\xi_i \\ &\quad + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\xi_i^*sym\{(A_{ii} + \Delta A_{ii}) \\ &\quad \times \left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right)\}\xi_i < 0. \end{aligned} \tag{20}$$

From Lemma 1, one can be obtained

$$\begin{aligned} &\mathrm{Sym}\left\{\Delta B_iK_i\left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right)\right\} \\ &\quad = Sym\{M_{bi}F_{bi}E_{bi}Y_i\} \\ &\quad \leq \beta_iM_{bi}M_{bi}^T + \beta_i^{-1}[E_{bi}Y_i]^T[E_{bi}Y_i]. \end{aligned} \tag{21}$$

Similarly, one gets

$$\begin{aligned} &\mathrm{Sym}\left\{\Delta A_{ii}\left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right)\right\} \\ &\quad = Sym\left\{M_{aij}F_{aij}E_{aij}\left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right)\right\} \\ &\quad \leq \varepsilon_{ii}M_{aii}M_{aii}^T + \varepsilon_{ii}^{-1}\left[E_{aii}\left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right)\right]^T \\ &\quad\quad \times \left[E_{aii}\left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right)\right]. \end{aligned} \tag{22}$$

$$\begin{aligned} &\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\xi_i^*\mathrm{Sym}\{A_{ij}\left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right)\}\xi_j \\ &\quad \leq \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\{\delta_{ij}\xi_i^*A_{ij}A_{ij}^T\xi_i \\ &\quad\quad + \delta_{ij}^{-1}\xi_j^*\left(e^{\theta i}X_j + e^{-\theta i}\overline{X_j}\right)^T \times \left(e^{\theta i}X_j + e^{-\theta i}\overline{X_j}\right)\xi_j\} \\ &\quad = \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\xi_i^*\{\delta_{ij}A_{ij}A_{ij}^T + \delta_{ij}^{-1}\left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right)^T \\ &\quad\quad \times \left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right)\}\xi_i. \end{aligned} \tag{23}$$

$$\begin{aligned} &\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\xi_i^*Sym\{\Delta A_{ij}\left(e^{\theta i}X_j + e^{-\theta i}\overline{X_j}\right)\}\xi_j \\ &\quad = \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\xi_i^*Sym\{M_{aij}F_{aij}E_{aij}\left(e^{\theta i}X_j + e^{-\theta i}\overline{X_j}\right)\}\xi_j\} \end{aligned}$$

$$\Gamma_i = \begin{bmatrix} \Gamma_i^{11} & * & * & \cdots & * & \cdots & * & \cdots & * \\ C_i\left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right) + D_iY_i & -\gamma I & * & \cdots & * & \cdots & * & \cdots & * \\ E_{aii}\left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right) & 0 & -\varepsilon_{ii}I & \cdots & * & \cdots & * & \cdots & * \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right) & 0 & 0 & \cdots & -\delta_{ji}I & \cdots & * & \cdots & * \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ E_{aji}\left(e^{\theta i}X_i + e^{-\theta i}\overline{X_i}\right) & 0 & 0 & \cdots & 0 & \cdots & -\mu_{ji}I & \cdots & * \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ E_{bi}Y_i & 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & -\beta_iI \end{bmatrix} < 0, \tag{15}$$

$$\leq \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\{\mu_{ij}\xi_i^{*}M_{aij}M_{aij}^{T}\xi_i + \mu_{ij}^{-1}\xi_j^{*}\left[E_{aij}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\right]^{T} \times E_{aij}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\xi_j\}$$
$$= \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\xi_i^{*}\{\mu_{ij}M_{aij}M_{aij}^{T} + \mu_{ij}^{-1}\left[E_{aji}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\right]^{T} \times E_{aji}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\}\xi_i. \quad (24)$$

Substituting (21)-(24) into (19), one has

$$\xi^{*}\Omega\xi \leq \sum_{i=1}^{N}\xi_i^{*}\{Sym\{A_{ii}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)+B_iY_i\} + \gamma^{-1}B_{\omega}B_{\omega}^{T}+\gamma^{-1}\left[(C_i+D_iK_i)\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\right]^{T} \times\left[(C_i+D_iK_i)\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\right]+\beta_iM_{bi}M_{bi}^{T} + \beta_i^{-1}\left[E_{bi}Y_i\right]^{T}\left[E_{bi}Y_i\right]+\varepsilon_{ii}M_{aii}M_{aii}^{T} + \varepsilon_{ii}^{-1}\left[E_{aii}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\right]^{T} \times\left[E_{aii}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\right]+\sum_{j=1,j\neq i}^{N}\{\delta_{ij}A_{ij}A_{ij}^{T} + \delta_{ji}^{-1}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)^{T}\times\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\} + \sum_{j=1,j\neq i}^{N}\{\mu_{ij}M_{aij}M_{aij}^{T}+\mu_{ji}^{-1}\left[E_{aji}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\right]^{T} \times E_{aji}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\}\}\xi_i. \quad (25)$$

Then,

$$Sym\{A_{ii}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)+B_iY_i\}+\gamma^{-1}B_{\omega i}B_{\omega i}^{T} +\gamma^{-1}\left[(C_i+D_iK_i)\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\right]^{T} \times\left[(C_i+D_iK_i)\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\right] +\beta_iM_{bi}M_{bi}^{T}+\beta_i^{-1}\left[E_{bi}Y_i\right]^{T}\left[E_{bi}Y_i\right]+\varepsilon_{ii}M_{aii}M_{aii}^{T} +\varepsilon_{ii}^{-1}\left[E_{aii}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\right]^{T} \times\left[E_{aii}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\right]+\sum_{j=1,j\neq i}^{N}\{\delta_{ij}A_{ij}A_{ij}^{T} +\delta_{ji}^{-1}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)^{T}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\} +\sum_{j=1,j\neq i}^{N}\{\mu_{ij}M_{aij}M_{aij}^{T}+\mu_{ji}^{-1}\left[E_{aji}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\right]^{T} \times E_{aji}\left(e^{\theta i}X_i+e^{-\theta i}\overline{X_i}\right)\}<0, \quad i=1,2. \quad (26)$$

According to the Schur complement, (26) is equivalent to (15).

Next, it is proved that if the inequalities (16) and (17) hold, the HTGS (9) will be finite-time stable.

Select J function as

$$J\left(x\left(t\right),\omega\left(t\right)\right)=x^{T}P_ix+\omega^{T}Q_i\omega.$$

Assume that

$$\left(\frac{d^{\alpha}x}{dt^{\alpha}}\right)^{T}P_ix+x^{T}P_i\left(\frac{d^{\alpha}x}{dt^{\alpha}}\right)<\sigma_iJ\left(x\left(t\right),\omega\left(t\right)\right). \quad (27)$$

Since $J\left(x\left(t\right),\omega\left(t\right)\right)>0$, $\sigma_i<0$, according to (27), there is

$$\left(\frac{d^{\alpha}x}{dt^{\alpha}}\right)^{T}P_ix+x^{T}P_i\left(\frac{d^{\alpha}x}{dt^{\alpha}}\right)<\sigma_iJ\left(x\left(t\right),\omega\left(t\right)\right)<0. \quad (28)$$

Make further treatment of (27) to obtain the more practical condition.

$$\begin{aligned}&\left(\frac{d^{\alpha}x}{dt^{\alpha}}\right)^{T}P_ix+x^{T}P_i\left(\frac{d^{\alpha}x}{dt^{\alpha}}\right)-\sigma_iJ\left(x\left(t\right),\omega\left(t\right)\right)\\&=\left(\frac{d^{\alpha}x}{dt^{\alpha}}\right)^{T}P_ix+x^{T}P_i\left(\frac{d^{\alpha}x}{dt^{\alpha}}\right)-\sigma_ix^{T}P_ix-\sigma_i\omega^{T}Q_i\omega\\&=\left(A_{clii}x+B_{\omega ii}\omega\right)^{T}P_ix+x^{T}P_i\left(A_{clii}x+B_{\omega ii}\omega\right)\\&\quad-\sigma_ix^{T}P_ix-\sigma_i\omega^{T}P_i\omega\\&=x^{T}\left(A_{clii}^{T}P_i+P_iA_{clii}-\sigma_iP_i\right)x\\&\quad+\omega^{T}B_{\omega ii}^{T}P_ix+x^{T}P_iB_{\omega ii}\omega-\sigma_i\omega^{T}Q_i\omega\\&=\begin{bmatrix}x\\\omega\end{bmatrix}^{T}\begin{bmatrix}P_iA_{clii}+A_{clii}^{T}P_i-\sigma_iP_i & P_iB_{\omega ii}\\B_{\omega ii}^{T}P_i & -\sigma Q_i\end{bmatrix}\begin{bmatrix}x\\\omega\end{bmatrix}<0.\end{aligned} \quad (29)$$

According to Definition 2, that is to say

$$\begin{bmatrix}P_iA_{clii}+A_{clii}^{T}P_i-\sigma_iP_i & P_iB_{\omega ii}\\B_{\omega ii}^{T}P_i & -\sigma_iQ_i\end{bmatrix}<0. \quad (30)$$

Therefore, (27) is equivalent to (16).

$$\begin{aligned}&J\left(x\left(t\right),\omega\left(t\right)\right)\\&\quad=x\left(t\right)^{T}P_ix\left(t\right)+\omega\left(t\right)^{T}Q_i\omega\left(t\right)\\&\quad=x\left(t\right)^{T}R^{1/2}P\tilde{P}_iR^{1/2}x(t)+\omega(t)^{T}Q_i\omega(t)\\&\quad\geq\lambda_{min}\left(\tilde{P}_i\right)x(t)^{T}Rx(t)\end{aligned} \quad (31)$$

$$\begin{aligned}&J\left(x\left(0\right),\omega\left(0\right)\right)e^{\sigma_it}\\&\quad=(x\left(0\right)^{T}Px\left(0\right)+\omega\left(0\right)^{T}Q_i\omega\left(0\right))e^{\sigma_it}\\&\quad=x\left(0\right)^{T}R^{1/2}\tilde{P}R^{1/2}x(0)+\omega\left(0\right)Q_i\omega(0)e^{\sigma_it}\\&\quad\leq(\lambda_{max}\left(\tilde{P}_i\right)x(0)^{T}Rx(0)+\lambda_{max}(Q_i)\omega(0)^{T}\omega(0))e^{\sigma_it}\\&\quad=(\lambda_{max}(\tilde{P}_i)c_1+\lambda_{max}(Q_i)d)e^{\sigma_it}.\end{aligned} \quad (32)$$

According to (27), (31) and (32), one obtains

$$\begin{aligned}&\lambda_{min}(\tilde{P}_i)x(t)^{T}Rx(t)\\&\quad\leq J(x(t),\omega(t))<J(x(t),\omega(t))e^{\sigma_tt}\\&\quad\leq(\lambda_{max}(\tilde{P}_i)c_1+\lambda_{max}(Q_i)d)e^{\sigma_tt}\end{aligned}$$

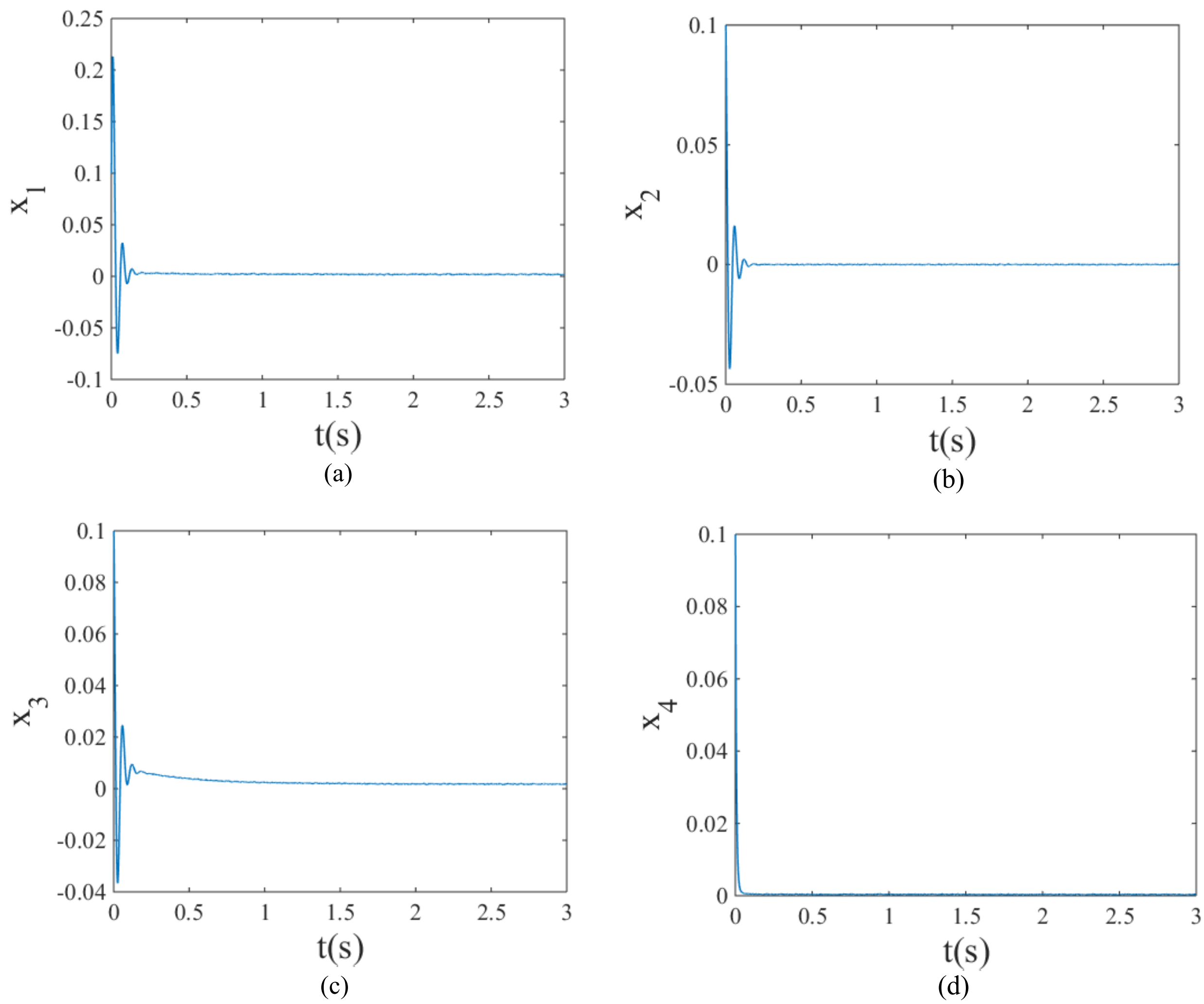

FIGURE 2. State trajectories of fractional-order HTGS (9) with the proposed finite-time H_∞ control. (a) $x_1 - t$. (b) $x_2 - t$. (c) $x_3 - t$. (d) $x_4 - t$.

$$\begin{aligned}
&\Rightarrow \lambda_{min}(\tilde{P}_i)x(\tilde{P}_i)x(t)^T Rx(t)\\
&\leq (\lambda_{max}(\tilde{P}_i)c_i + \lambda_{max}(Q_i)d)e^{\sigma_i t}\\
&\Rightarrow x(t)^T Rx(t)\\
&\leq \frac{\left(\lambda_{max}(\tilde{P}_i)c_i + \lambda_{max}(Q_i)d\right)e^{\sigma_i t}}{\lambda_{min}}. \qquad (33)
\end{aligned}$$

Here, let

$$c_2 = \frac{\left(\lambda_{max}\left(\tilde{P}_i\right)c_{1i} + \lambda_{max}\left(Q_i\right)d\right)e^{\sigma_i t}}{\lambda_{min}(\tilde{P}_i)} \qquad (34)$$

So (33) can be written as

$$x\left(t\right)^T Rx\left(t\right) \leq c_2, \quad \forall t \in [0, T]. \qquad (35)$$

The proof is completed. ■

The controller K_i can be easily got by solving the inequalities (15)-(17) via Matlab's LMI toolbox.

V. NUMERICAL SIMULATION

For the fractional-order HTGS (9), the coefficient matrix is obtained as follows:

$$\begin{aligned}
A_1 &= \begin{bmatrix} 0 & 314 & 0 & 0 \\ 1.02 & -0.22 & 0.11 & 0 \\ 0 & 0 & -2.5 & 16.5 \\ 0 & 0 & 0 & -10 \end{bmatrix},\\
A_2 &= \begin{bmatrix} 0 & 314 & 0 & 0 \\ 0.09 & -0.22 & 0.11 & 0 \\ 0 & 0 & -2.5 & 16.5 \\ 0 & 0 & 0 & -10 \end{bmatrix},\\
B_{\omega 1} &= B_{\omega 2} = I_{4\times 4}.
\end{aligned}$$

The parameters are selected as:

$$\Delta A_{11} = M_{a11}F_{a11}E_{a11} = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.1 \end{bmatrix} F_{a11}\left[0.6\ 0.4\ 0.4\right],$$

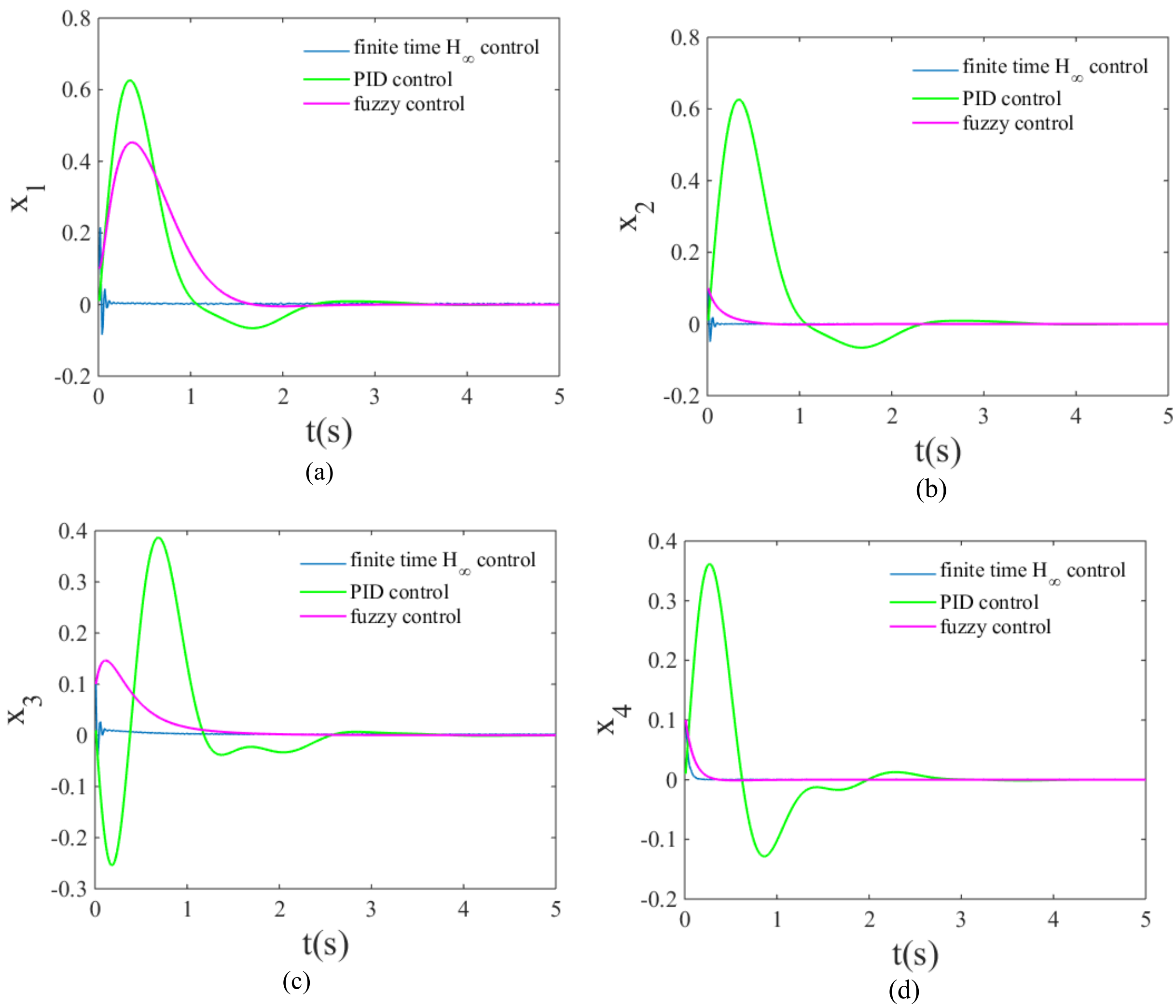

FIGURE 3. State trajectories of fractional-order HTGS (9) under different control methods. (a) $x_1 - t$. (b) $x_2 - t$. (c) $x_3 - t$. (d) $x_4 - t$.

$$\Delta A_{12} = M_{a12}F_{a12}E_{a12} = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.1 \end{bmatrix} F_{a12}\,[0.6]\,,$$
$$\Delta A_{22} = M_{a22}F_{a22}E_{a22} = [0.3]\,F_{a22}\,[0.2]\,,$$
$$\Delta A_{21} = M_{a21}F_{a21}E_{a21} = [0.3]\,F_{a11}\begin{bmatrix} 0.2 & 0.4 & 0.4 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^{\mathrm{T}},$$
$$\Delta B_1 = M_{b1}F_{b1}E_{b1} = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.1 \end{bmatrix} F_{b1}0.6,$$
$$C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$
$$B_2 = [1]\,,\ B_{\omega 2} = \begin{bmatrix} 1\ 0\ 0 \\ 0\ 1\ 0 \\ 0\ 0\ 1 \end{bmatrix},$$
$$\Delta B_2 = M_{b2}F_{b2}E_{b2} = [0.3]\,F_{b2}0.1,$$
$$C_2 = [1]\,,\ D_2 = [0]\,,\ c_1 = 1,\ c_2 = 2,$$
$$\mathrm{T} = 1,\ d = 1,\ R = \mathrm{I},\ \gamma = 1.$$

When the coefficient matrix is A_1,
For S_1: $\alpha = 1$,

$$A_{11} = \begin{bmatrix} 0 & 314 & 0 \\ 1.02 & -0.22 & 0.11 \\ 0 & 0 & -2.5 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 \\ 0 \\ 16.5 \end{bmatrix}.$$

For S_2: $\alpha = 0.98$,

$$A_{22} = [-10]\,, \quad A_{21} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}.$$

The state feedback control gain K_i is got.

$$K_1 = \begin{bmatrix} -47.0988 & -47.6155 & 16.5130 \end{bmatrix},$$
$$K_2 = [-189.4218]\,,$$
$$A_{cl} = \begin{bmatrix} \text{-52.6307 260.6991 18.5746 0.1200} \\ \text{-48.8483 -50.7034 17.6549 0.0600} \\ \text{-49.8648 -50.4812 15.0438 16.5600} \\ \text{0.0600 0.1200 0.1200 -205.0444} \end{bmatrix}.$$

When the coefficient matrix is A_2,

For S_1: $\alpha = 1$,

$$A_{11} = \begin{bmatrix} 0 & 314 & 0 \\ 0.09 & -0.22 & 0.11 \\ 0 & 0 & -2.5 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 \\ 0 \\ 16.5 \end{bmatrix}.$$

For S_2: $\alpha = 0.98$,

$$A_{22} = [-10], \quad A_{21} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}.$$

The state feedback control gain K_i is obtained

$$K_1 = [-19.12 - 17.32.37], \quad K_2 = [-74.69],$$
$$A_{cl} = \begin{bmatrix} -21.2911 & 294.6836 & 0.49110.1200 & \\ -20.1111 & -18.5395 & 0.5402 & 0.0600 \\ -20.2041 & -18.3173 & -2.0709 & 16.5600 \\ 0.0600 & 0.1200 & 0.1200 & -86.8660 \end{bmatrix}.$$

Corresponding simulation results are given in Figure 2. It is obvious that the system state are stable about the equilibrium point rapidly, which shows the effectiveness of the proposed scheme. To compare the performance of the proposed finite-time H_∞ control method, the pure fuzzy control method and PID control are applied to fractional-order HTGS (9). Figure 3 shows the simulation results with different control method. Compared with the fuzzy control and PID control, the overshoot is lower and the control time is shorter. There is a significant improvement in the transition process, which demonstrates the robustness and superiority of the proposed approach.

VI. CONCLUSION

This paper studied the finite-time H_∞ control of a fractional-order HTGS. Based on the generalized T-S fuzzy model, the fractional-order fuzzy model of a HTGS was presented. By combining finite-time control and H_∞ control theory, a finite-time H_∞ state feedback control was proposed for the HTGS. The control is based on the fractional-order stability theorem. Compared with the pure fuzzy control and PID control, the proposed finite-time H_∞ control showed the advantages of smaller overshoot, less stable time and fewer oscillations. It demonstrated the validity and advantage of the proposed method.

In the future, we will consider and extend the application of the approach in the stability control for linear and nonlinear switched systems [47]–[49].

VII. CONFLICT OF INTERESTS

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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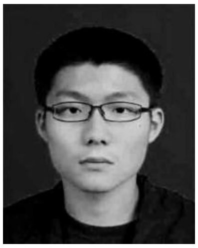

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