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# Sparse Bayesian Learning for Channel Estimation in Time-Varying Underwater Acoustic OFDM Communication

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**ABSTRACT** In this paper, we study the sparse Bayesian learning (SBL) framework for channel estimation in underwater acoustic (UWA) orthogonal frequency-division multiplexing (OFDM) communication systems, which provides a desirable property of preventing structural error with fewer convergence errors for sparse signal reconstruction compared with the compress sensing (CS)-based methods. First, we design a SBL-based channel estimator for block-by-block processing using the channel sparse structure independently in each block. Then, we propose a joint channel model after Doppler compensation for multi-block joint processing, where the delays of the channels for several consecutive blocks are similar and the path gains exhibit temporal correlation, and we denote a temporal correlation coefficient for path gains to evaluate the strength of the correlation. Furthermore, we propose the temporal multiple SBL (TMSBL)-based channel estimator to jointly estimate the channels by taking advantage of the channel coherence between consecutive OFDM blocks. Results of numerical simulation and sea trial demonstrate the effectiveness of the SBL and TMSBL channel estimator algorithms in time-varying UWA channel, which achieve better channel estimation performance and lower bit error rate compared with the existing CS-based methods, such as orthogonal matching pursuit (OMP) and simultaneous OMP, especially the TMSBL estimator achieves the best performance in strong temporal correlated channels and maintains robustness in weak temporal correlated channels.

**INDEX TERMS** Time-varying UWA channels, sparse channel estimation, sparse Bayesian learning, orthogonal frequency division multiplexing.

## I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a multicarrier transmission technology, attracting much attention in both radio and UWA communications due to its high spectrum efficiency and resistance to frequency selective fading [1]–[5]. The UWA channel is one of the most challenging wireless communication channels with large delay spread and significant Doppler effects, and it is a time-varying channel. So accurate channel estimation and equalization at the receiver are important for the performance of OFDM systems, and this paper we will address the methods of the UWA channel estimation.

In recent years the UWA channels are exploited to be naturally sparse, meaning that most channel energy is concentrated in a few paths. Based on the sparsity of the UWA channels, many sparse channel estimation methods has been extensively studied. Compressed Sensing (CS) based sparse channel estimation methods are widely adopted. In CS based algorithms, greedy algorithms such as Matching Pursuit (MP) and OMP have been proposed in UWA channel estimations as a solution to the $l_0$ norm problem of channel estimation. Li and Preisig [6] adopted these two algorithms to estimate the UWA channel impulse response (CIR) and the channel delay-Doppler-spread function, finally improved the estimation performance. Since solving the $l_0$ norm problem is known to be NP-hard, convex optimization based techniques transform the $l_0$ norm to a convex $l_1$ norm question. In the convex optimization algorithms, such as Basis Pursuit (BP) and Basis Pursuit denoising (BPDN) algorithms are proposed to be the solutions to sparse channel estimation. Berger *et al.* [7] used CS techniques, specifically OMP and BP algorithms to achieve sparse time-varying channel estimation, which had estimated the path delays and path Doppler scales at the same time and achieved better performance than conventional methods. Yin *et al.* [8] applied the BPDN method to estimate the UWA CIR and achieved outstanding performance over time-invariant channels. Although the BP algorithm is marked by demonstrable successes, it's also hampered by a significant shortcoming that we can achieve the global minimum of the cost function in BP which does not necessarily coincide with the sparsest solutions, we refer to this misalignment as structural error. Except these there is also a $l_p$ (i.e., $p$ strictly less than one) norm solution to the sparse channel estimation question, which is called Focal Underdetermined System Solver (FOCUSS) algorithm. This algorithm has many local minimums, so frequently converges to suboptimal local minimum termed convergence errors [9]. As there are the shortcomings of these algorithms, people seek for alternative methods to estimate the sparse channel. In [10], a SOMP based method was utilized by exploiting joint sparsity among adjacent OFDM blocks, the authors assumed a joint sparsity model framework and verified the superiority of the channel estimation method comparing conventional CS based methods by the experimental performance under field test. However this method merely focused on exploiting sparse path delays for multi-block, ignoring temporal correlation for gains which is more common in UWA channels.

Recently, sparse Bayesian learning (SBL) based channel estimation methods have been presented in radio OFDM systems [11], [12]. The main feature is that the Bayesian techniques evaluate the posterior distribution of channel impulse response conditioned on received data, unlike the CS based methods that provide point estimates of the sparse vector. Meanwhile the SBL algorithm has robustness with highly structured dictionaries [13] in which case the performance breaks down in most CS based algorithms. And it turns out that CS based approaches such as BP can also be viewed as a problem in the Bayesian framework, where the goal is to obtain a maximum a posteriori estimate by using a fixed sparsity inducing prior distribution. Furthermore it turns out the SBL algorithm to be an iterative reweighted $l_1$ minimization, which is more possible to reach the real sparse solution. All in all, comparing with the CS based methods, the SBL based methods provide a desirable property of preventing structural error with fewer convergence errors [9]. In paper [11], the EM-SBL algorithm was applied to jointly estimate the sparse channel, unknown data symbols and the second order statistics of the channel, to further improve the performance of the EM-SBL algorithm, a threshold-based pruning of the estimated second order statistics that are input to the algorithm was also proposed. Prasad *et al.* [12] employed the SBL algorithm for channel estimation and proposed a joint SBL (JSBL) and a low-complexity recursive JSBL algorithm for joint channel estimation and data detection in a quasi-static, block-fading scenario. And in a time-varying scenario, the authors used a first-order autoregressive model for the wireless channel and proposed recursive and low-complexity Kalman filtering-based SBL (KSBL) algorithm and joint KSBL (JKSBL) algorithm for channel estimation.

In the SBL research field, there are also some algorithms for solving the signals recovery problem with multiple measurement vectors (MMV) [14]–[16]. The authors proposed multiple sparse Bayesian learning (MSBL) algorithm to simultaneously estimate sparse signals in [14], finding that it often outperformed the multiple response extensions of CS based algorithms. Further considering the temporal correlation in multiple measurement vectors, in paper [15], the authors presented a block sparse Bayesian learning (bSBL) framework and derived the temporal SBL (TSBL) and temporal MSBL (TMSBL) algorithms to the estimation problems in the presence of temporal correlation. Extensive experiments had shown that the proposed algorithms had superior performance to many state-of-the-art algorithms and theoretical analysis also had shown that the proposed algorithms had desirable global and local minimum properties to the MMV problem.

As we know the UWA channel is a typical time-varying sparse channel with a limited number of paths, where the delays are similar and the gains exhibit strong temporal correlation over a time scale less than the channel coherence time. Such as in [17] the channel coherence time had been studied using experimental data in calm sea, the results showed that several sparse paths arrivaled with relatively stable delays. In addition the Doppler shifting caused by the motion of the transmitter/receiver or any reflection/scattering points in the channel leads to time-varying path delays on a small time scale, which can be considered to change from one time interval to another and can be estimated and compensated accordingly. As such, both the channel sparse structure and temporal correlation can be utilized to improve channel estimation performance.

In this paper, we introduce the sparse Bayesian learning framework in UWA OFDM communication systems. Firstly we propose the SBL based channel estimator for block-by-block processing after Doppler compensation and we study the performance of the SBL algorithm in UWA channel estimation problem. Then based on the sparsity structure in time-varying UWA channel we propose a joint channel model and denote a temporal correlation coefficient to describe the temporal correlation for gains. Further we propose the TMSBL based channel estimator for multi-block joint processing, where we jointly estimate the channels of several consecutive blocks by exploiting the temporal correlation. Through simulation and experimental data results, we confirm the effectiveness of the SBL based channel estimators, which achieve better channel estimation performance and lower BER compared with the OMP and SOMP channel estimators, especially the TMSBL estimator achieves the best performance in strong temporal correlated channels and maintains robustness in weak temporal correlated channels.

*Notations:* upper (lower) bold letter denotes matrices (column vectors); $\mathbf{A}^H$ denote the Hermitian of $\mathbf{A}$; $\cdot$ denotes the element-wise product between two vectors.

## II. SYSTEM MODEL

### A. CP-OFDM SYSTEM

We consider a cyclic prefix (CP) OFDM system here. Assume there are $K$ subcarriers in one OFDM block, and the transmitted symbol on the $k$th subcarrier is $X_k$. Let $T$ denote one OFDM symbol duration and $T_{\text{cp}}$ denote the cyclic prefix length. The $k$th subcarrier is at frequency

$$f_k = f_c + k/T, \quad k = -K/2, \dots, K/2-1 \tag{1}$$

where $f_c$ is the carrier frequency. $K_d$ data subcarriers $\mathbf{X_D}$, $K_p$ pilot subcarriers $\mathbf{X_P}$, $K_n$ null subcarriers $\mathbf{X_N}$ satisfy $K = K_d + K_p + K_n$. The transmitted signal is given by

$$\tilde{x}(t) = 2Re\left\{\sum_{k=-K/2}^{K/2-1} X_k e^{j2\pi f_k t} q(t)\right\}, \quad t \in [-T_{\text{cp}}, T] \tag{2}$$

where $q(t)$ is the pulse shaping filter

$$q(t) = \begin{cases} 1, & t \in [-T_{\text{cp}}, T] \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

### B. BASIC CHANNEL MODEL

Fig. 1 shows a 12 frames history of the channel impulse response with 8 consecutive OFDM symbols in one frame, which is obtained from experimental data collected in South China Sea in May 2014. In the experiment, the source-receiver range was approximately 5km, the source depth is 27m and the receiver depth is 30m. We estimate the CIR after Doppler compensation. From the channel of experimental data, we notice that the channel are extremely sparse with several significant paths. The total delay spread is around 10ms. Besides the delays of major paths with strong energy remain stable across the whole OFDM signal, and these path gains vary with time in a small range.

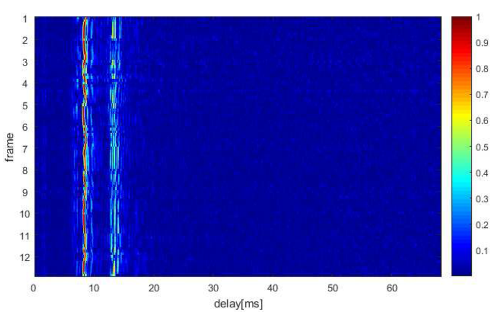


**FIGURE 1.** Channel impulse response estimate.

We adopt the time-varying channel model commonly considered in UWA communications as

$$h(\tau, t) = \sum_{l=1}^{L} A_l(t)\delta(\tau - (\tau_l - at)) \tag{4}$$

where we assume that the channel is a multipath channel with $L$ paths. $A_l$ and $\tau_l$ denote the path gain and path delay of the $l$th path, and all the paths have the equal Doppler scale factor $a$ (e.g. horizontal shallow water transmission with range much greater than depth [18]). Furthermore, based on that the gains and delays of propagation paths exhibit large-scale variation on a larger time scale [19], we assume that path delays remain stable across several consecutive OFDM blocks and the path gains and Doppler scale factors are constant during one OFDM block but vary from block to block.

### C. RECEIVER PROCESSING

The received passband signal is

$$\tilde{y}(t) = \sum_{l=1}^{L} A_l \tilde{x}([1+a]t - \tau_l) + \tilde{w}(t), \tag{5}$$

where the $\tilde{w}(t)$ denotes the additive noise.

Then we adopt the method using self-correlators for CP in [20] to synchronize and estimate the Doppler scales block by block. After resampling the received data with estimated Doppler factor $\hat{a}$ and CP-OFDM demodulation, the dominating Doppler shiftings are considered to be compensated and the residual are considered as additive noise. We can model the $K \times 1$ received signal $\mathbf{Y}$ for each block as

$$\mathbf{Y} = \mathbf{XFh} + \mathbf{W} \tag{6}$$

where $\mathbf{F}$ is the $K \times L$ Discrete Fourier Transform (DFT) matrix, $\mathbf{X}$ is an $K \times K$ diagonal matrix consisting of the $K$ transmitted symbols and $K$ is the DFT size, $\mathbf{W}$ is additive Gaussian noise. The overall channel is represented as $\mathbf{h} = [h_1, h_2, \cdots, h_L]^T$. One frame consists of $M$ consecutive OFDM blocks.

The system model considering only $P$ pilot subcarriers can be written as

$$\mathbf{Y_p} = \mathbf{X_p F_p h} + \mathbf{W_p} \tag{7}$$

where $\mathbf{Y_p}$ is a $P \times 1$ vector containing received $P$ pilot subcarriers in $\mathbf{Y}$, $\mathbf{X_p}$ is the $P \times P$ diagonal matrix with the known pilots along its diagonal, $\mathbf{F_p}$ is the $P \times L$ DFT matrix and $\mathbf{W_p}$ contains the elements of $\mathbf{W}$ at pilot locations.

## III. UWA SPARSE CHANNEL ESTIMATION

### A. SBL BASED BLOCK-BY-BLOCK PROCESSING

We rewrite (7) in a basic mathematical model as

$$\mathbf{Y_p} = \Phi_\mathbf{p} \mathbf{h} + \mathbf{W_p} \tag{8}$$

where $\Phi_\mathbf{p} = \mathbf{X_p F_p}$ is a known dictionary matrix, and the task is to estimate the vector $\mathbf{h}$ in which most elements are zeros.

In a SBL framework, we model the channel as $\mathbf{h} \sim \mathcal{CN}(0, \boldsymbol{\Gamma})$, where $\boldsymbol{\Gamma}$ is a diagonal matrix with $\boldsymbol{\gamma}$ for

$\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \cdots, \gamma_L]^T$. Note that for $i \in [1, L]$ if $\gamma_i \to 0$, then the corresponding $h_i \to 0$. Sparse Bayesian learning relies on a parameterized prior to obtain sparse solutions in regression. The parametric form of SBL prior can be written as

$$p(\mathbf{h}; \boldsymbol{\Gamma}) = \prod_{i=1}^{L} (\pi\gamma_i)^{-1} \exp\left(-\frac{|h_i|^2}{\gamma_i}\right), \tag{9}$$

where $\boldsymbol{\Gamma}$ constitutes the hyperparameters, which control the variance of each of the channel coefficients. These hyperparameters can be estimated using the type-II maximum-likelihood (ML) procedure, i.e., by maximizing the marginalized probability distribution function (pdf) $p(\mathbf{Y_p}; \boldsymbol{\Gamma})$

$$\gamma_{i,ML} = \arg\max_{\gamma_i} p(\mathbf{Y_p}; \boldsymbol{\Gamma}). \tag{10}$$

The above problem cannot be solved in closed form. Expectation maximization (EM) algorithm is used to obtain the hyperparameters in an iterative way, and it can offer the advantage of convergence at the same time. For unknown values of hyperparameters governing the prior density, $\mathbf{h}$ is considered as nuisance variable and $\boldsymbol{\Gamma}$ is eatimated. The $E$ and the $M$ steps of the algorithm in the $r$th iteration can be given as

$$E-step: Q(\boldsymbol{\Gamma}/\boldsymbol{\Gamma}^{(r)}) = E_{\mathbf{h}/\mathbf{Y_p},\boldsymbol{\Gamma}^{(r)}}[\log p(\mathbf{Y_p}, \mathbf{h}; \boldsymbol{\Gamma})] \tag{11}$$

$$M-step: \gamma_i^{(r+1)} = \arg\max_{\gamma_i>0} Q(\boldsymbol{\Gamma}/\boldsymbol{\Gamma}^{(r)}). \tag{12}$$

The $E$-step above requires the posterior density of the sparse vector with the hyperparameter $\boldsymbol{\Gamma} = \boldsymbol{\Gamma}^{(r)}$, which can be computed as

$$p(\mathbf{h}/\mathbf{Y_p}; \boldsymbol{\Gamma}^{(r)}) = \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma}), \tag{13}$$

where $\boldsymbol{\mu} = \sigma^{-2}\Sigma\Phi_{\mathbf{p}}^H\mathbf{Y_p}$ and $\boldsymbol{\Sigma} = (\sigma^{-2}\Phi_{\mathbf{p}}^H\Phi_{\mathbf{p}} + \boldsymbol{\Gamma}^{(r)^{-1}})^{-1}$. By using the posterior density, we can obtain the maximum a posteriori (MAP) estimate of the sparse channel vector at the end of the EM iterations, i.e., $\hat{\mathbf{h}} = \boldsymbol{\mu}$. The $M$-step can be simplified to obtain

$$\gamma_i^{(r+1)} = \arg\max_{\gamma_i>0} E_{\mathbf{h}/\mathbf{Y_p},\boldsymbol{\Gamma}^{(r)}}[\log p(\mathbf{Y_p}, \mathbf{h}; \boldsymbol{\Gamma})] \tag{14}$$

$$= \arg\max_{\gamma_i>0} E_{\mathbf{h}/\mathbf{Y_p},\boldsymbol{\Gamma}^{(r)}}[\log p(\mathbf{h}; \boldsymbol{\Gamma})] \tag{15}$$

$$= E_{\mathbf{h}/\mathbf{Y_p},\boldsymbol{\Gamma}^{(r)}}[|h_i|^2] \tag{16}$$

$$= \Sigma_{(i,i)} + |\mu_i|^2, \tag{17}$$

where $\Sigma_{(i,i)}$ is the $i$th diagonal component of $\boldsymbol{\Sigma}$ and $\mu_i$ is the $i$th component of $\boldsymbol{\mu}$. In (14), the term $E_{\mathbf{h}/\mathbf{Y_p},\boldsymbol{\Gamma}^{(r)}}[\log p(\mathbf{Y_p}|\mathbf{h}; \boldsymbol{\Gamma})]$ has been dropped, as it is not a function of $\gamma_i$.

We then give the steps of the SBL algorithm in the following TABLE as in Algorithm 1.

Note that the noise variance $\sigma^2$ is obtained by the null subcarriers as

$$\sigma^2 = E[|\mathbf{Y_n}|^2]. \tag{18}$$

**Algorithm 1** The SBL-Based Channel Estimation Method

1. Input: the received vector $\mathbf{Y_p}$, the dictionary matrix $\Phi_{\mathbf{p}}$, the maximum iteration number $r_{\max}$, the threshold $e$, the noise variance $\sigma^2$.
2. Initialize: the hyperparameters matrix $\boldsymbol{\Gamma}^{(0)} = \mathbf{I}_L$, the iteration counter $r = 0$.
3. $E$-step:
$$\boldsymbol{\mu} = \sigma^{-2}\Sigma\Phi_{\mathbf{p}}^H\mathbf{Y_p},$$
$$\boldsymbol{\Sigma} = (\sigma^{-2}\Phi_{\mathbf{p}}^H\Phi_{\mathbf{p}} + \boldsymbol{\Gamma}^{(r)^{-1}})^{-1}.$$
4. $M$-step:
$$\gamma_i^{(r+1)} = \Sigma_{(i,i)} + |\mu_i|^2, \text{ for } i = 1, 2, \cdots, L.$$
5. Increase $r$ and return to Step3 if $r < r_{\max}$, or $\| \boldsymbol{\gamma}^{(r+1)} - \boldsymbol{\gamma}^{(r)} \|_2^2 \leq e$ end the iteration.
6. Output: the estimated sparse channel vector $\hat{\mathbf{h}} = \boldsymbol{\mu}$, the estimated hyperparameters vector $\boldsymbol{\gamma}$.

### B. TMSBL BASED MULTI-BLOCK JOINT PROCESSING

#### 1) THE JOINT CHANNEL MODEL

Although we can adopt the SBL algorithm above for block-by-block channel estimation, in fact for most of the cases the UWA channel exhibits strong temporal correlation across multiple blocks. As we know the UWA channel is a typical time-varying sparse channel with a limited number of non-zero paths, where the delays are similar and the gains exhibit strong temporal correlation over a time scale less than the channel coherence time. Besides the Doppler shifting caused by the motion of the transmitter/receiver or any reflection/scattering points in the channel leads to time-varying path delays on a small time scale, which can be considered as changing from block to block. These small variation of delays due to relative motion can be solved mostly by Doppler compensation.

So after compensating the Doppler shifting block-by-block, we can propose the joint channel model for $M$ consecutive OFDM blocks as

$$\bar{\mathbf{h}} \triangleq [\mathbf{h}^1, \cdots, \mathbf{h}^m, \cdots, \mathbf{h}^M] \tag{19}$$

where $\mathbf{h}^m$ ($m \in [1, M]$) is the channel vector for the $m$th block. For every $\mathbf{h}^m$ the positions of non-zero delays are similar and the corresponding gains have temporal correlation.

#### 2) TEMPORAL CORRELATION COEFFICIENT

To describe the correlation for path gains overall, we denote the temporal correlation coefficient as

$$\eta(m, n) = \frac{E\{|\mathbf{h}^m \cdot \mathbf{h}^n|\}}{\sqrt{E\{|\mathbf{h}^m|^2\}E\{|\mathbf{h}^n|^2\}}}, \quad m, n \in [1, M] \tag{20}$$

where the coefficient $\eta(m, n)$ describes the strength of temporal correlation between the path gains of the $m$th and $n$th block. When the delays and gains of paths for each $\mathbf{h}$ are exactly the same, the channel is linear time-invariant and the temporal correlation is the strongest with $\eta = 1$. For the time-varying channel, the coefficient declines by the variation of delays or gains. For the case that the location of the transmitter and receiver is relatively fixed without drastic changes, the several strongest paths appearing in each $\mathbf{h}$ with the similar delays usually dominate the strength of the correlation, while the weak paths affect little for the strength of the correlation. Later we will analyze the channel estimation performance for different $\eta$ in the following section.

In the way the basic model in (8) has been extended as

$$\overline{\mathbf{Y}}_{\mathbf{p}} = \Phi_{\mathbf{p}}\overline{\mathbf{h}} + \overline{\mathbf{W}}_{\mathbf{p}} \tag{21}$$

where $\overline{\mathbf{Y}}_{\mathbf{p}} \triangleq [\mathbf{Y}_{\mathbf{p}}^1, \cdots, \mathbf{Y}_{\mathbf{p}}^M]$ is an available measurement matrix consisting of $M$ consecutive OFDM received vectors, $\mathbf{Y}_{\mathbf{p}}^m$ is the received signal in $m$th block. Note that the $M$ here depends on channel coherence time.

### 3) TMSBL BASED CHANNEL ESTIMATION METHOD

For the estimation problem in (21) we adopt the TMSBL algorithm to jointly estimate the $\overline{\mathbf{h}}$ by exploiting the temporal correlation.

Firstly we model the parametric form of prior of each $\overline{\mathbf{h}}_i$ as

$$p(\overline{\mathbf{h}}_i; \gamma_i, \mathbf{B}_i) \sim \mathcal{N}(0, \gamma_i \mathbf{B}_i), \quad i = 1, \cdots, L \tag{22}$$

where $\overline{\mathbf{h}}_i$ means the $i$th row of the $\overline{\mathbf{h}}$, and $\gamma_i$ is a nonnegative hyperparameter controlling the row sparsity of $\overline{\mathbf{h}}$ as in the SBL algorithm. The $\mathbf{\Gamma}$ is also a diagonal matrix with $\boldsymbol{\gamma}$ for $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \cdots, \gamma_L]^T$, the associated all elements in $\overline{\mathbf{h}}_i$ become zeros if $\gamma_i \to 0$. $\mathbf{B}_i$ is a positive definite matrix that captures the correlation structure of $\overline{\mathbf{h}}_i$ (the correlation for multiple blocks) and can be estimated. By combining each of these row priors, we can get the full weight prior

$$p(\overline{\mathbf{h}}; \Gamma, \mathbf{B}) = \prod_{i=1}^{L} p(\overline{\mathbf{h}}_i; \gamma_i, \mathbf{B}_i). \tag{23}$$

And we directly list the posterior density of each column of $\overline{\mathbf{h}}$ as

$$p(\mathbf{h}^m | \mathbf{Y}_{\mathbf{p}}^m; \Gamma) = \mathcal{N}(\boldsymbol{\mu}^m, \mathbf{\Sigma}), \quad m = 1, \cdots, M \tag{24}$$

with covariance and mean given by

$$\Sigma = (\sigma^{-2}\Phi_{\mathbf{p}}^H \Phi_{\mathbf{p}} + \mathbf{\Gamma}^{(r)^{-1}})^{-1}, \tag{25}$$
$$\mathcal{M} = [\boldsymbol{\mu}^1, \cdots, \boldsymbol{\mu}^M] = \sigma^{-2}\Sigma\Phi_{\mathbf{p}}^H \overline{\mathbf{Y}}_{\mathbf{p}}. \tag{26}$$

The $\boldsymbol{\mu}^m$ and $\mathcal{M}$ are the estimated $\hat{\mathbf{h}}^m$ and $\hat{\overline{\mathbf{h}}}$ respectively, and the $\mathbf{\Gamma}^{(r)}$ means the update $\mathbf{\Gamma}$ matrix in the $r$ iteration. To estimate the hyperparameters we can also use EM algorithm. For the E-step, this requires computation of the posterior parameters using in (25-26), while the M-step is expressed via the update rule

$$\gamma_i = \frac{1}{L}\mathcal{M}_i \mathbf{B}^{-1} \mathcal{M}_i^H + \mathbf{\Sigma}_{(i,i)} \tag{27}$$

where $\mathcal{M}_i$ means the $i$th row of the $\mathcal{M}$ matrix and we use one positive definite matrix $\mathbf{B}$ instead of $\mathbf{B}_i$ in (27) to describe correlation structure of all the paths, by this we can prevent overfitting due to limited data and too many parameters [21], [22], more importantly this strategy does not destroy the global minimum property and brings acceptable performance results.

Furthermore we adopt the learning rule for $\mathbf{B}$ in [15] as

$$\hat{\mathbf{B}} \leftarrow \sum_{i=1}^{L} \frac{\mathcal{M}_i^H \mathcal{M}_i}{\gamma_i} + \eta \mathbf{I} \tag{28}$$

where $\eta$ is a positive scalar, this regularized form ensures that the estimated $\hat{\mathbf{B}}$ is positive definite to increase the robustness.

We then give the steps of the TMSBL algorithm in the following TABLE as in Algorithm 2.

**Algorithm 2** The TMSBL-Based Channel Estimation Method

1 Input: the received vector $\overline{\mathbf{Y}}_{\mathbf{p}}$, the dictionary matrix $\Phi_{\mathbf{p}}$, the maximum iteration number $r_{\max}$, the threshold $e$, the noise variance $\sigma^2$.

2 Initialize: the hyperparameters matrix $\mathbf{\Gamma}^{(0)} = \mathbf{I}_L$, the iteration counter $r = 0$, $\mathbf{B} = \mathbf{I}_M$.

3 $E$-step:
$$\mathcal{M} = \sigma^{-2}\Sigma\Phi_{\mathbf{p}}^H \overline{\mathbf{Y}}_{\mathbf{p}},$$
$$\mathbf{\Sigma} = (\sigma^{-2}\Phi_{\mathbf{p}}^H \Phi_{\mathbf{p}} + \mathbf{\Gamma}^{(r)^{-1}})^{-1}.$$

4 $M$-step:
$\gamma_i^{(r+1)} = \frac{1}{L}\mathcal{M}_i \mathbf{B}^{-1} \mathcal{M}_i^H + \mathbf{\Sigma}_{(i,i)}$, for $i = 1, 2, \cdots, L$.

5 Update the $\mathbf{B}$ matrix:
$$\hat{\mathbf{B}} = \sum_{i=1}^{L} \frac{\mathcal{M}_i^H \mathcal{M}_i}{\gamma_i^{(r+1)}} + \eta \mathbf{I}_M.$$

6 Increase $r$ and return to Step3 if $r < r_{\max}$, or $\| \boldsymbol{\gamma}^{(r+1)} - \boldsymbol{\gamma}^{(r)} \|_2^2 \leq e$ end the iteration.

7 Output: the estimated sparse channel vector $\hat{\overline{\mathbf{h}}} = \mathcal{M}$, the estimated hyperparameters vector $\boldsymbol{\gamma}$, the estimated $\hat{\mathbf{B}}$ matrix.

The noise variance $\sigma^2$ is obtained by the null subcarriers as

$$\sigma^2 = E[|\overline{\mathbf{Y}}_{\mathbf{n}}|^2] \tag{29}$$

and we set the positive scalar $\eta = 2$ in (28) to guarantee the $\mathbf{B}$ matrix is positive.

## IV. SIMULATION RESULTS

For purpose of numerical simulations, we adopt the UWA CP-OFDM system settings in TABLE 1.

**TABLE 1.** UWA CP-OFDM settings.

| | | |
|---|---|---|
| Bandwidth | $B$ | 1.5kHz |
| Carrier frequency | $f_c$ | 2.25kHz |
| Sampling frequency | $f_s$ | 12kHz |
| No.subcarriers | $K$ | 256 |
| No.data subcarriers | $K_d$ | 200 |
| No.pilot subcarriers | $K_p$ | 32 |
| No.null subcarriers | $K_n$ | 24 |
| Symbol duration | $T$ | 171ms |
| Cyclic-prefix length | $T_{cp}$ | 10ms |
| Blocks in one frame | $N_b$ | 4 |

And we assume that the channel has 10 randomly generated paths, where the inter-arrival times are distributed exponentially with mean 0.5ms, and we assume the delays remain fixed within one OFDM frame, but the Doppler scales vary from block to block which are randomly chosen in $[-v_p/c, v_p/c]$ with $v_p = 1.5$m/s and $c = 1500$m/s. The amplitudes of paths are Rayleigh distributed with the average power decreasing exponentially with delay, and we set different temporal correlation coefficients for analysis. Further the data subcarriers are encoded using 1/2 nonbinary low density parity check (LDPC) code with quadrature phase shift keying (QPSK) modulation.

Here we adopt least square(LS), OMP and SBL algorithms to estimate the channel block-by-block, then we use SOMP and TMSBL algorithms for joint estimation across 4 blocks in each frame. The performance of the simulations is measured in terms of the frequency domain channel mean square error (MSE) and BER. We also include a cruve based on perfect channel state information (CSI) as a benchmark in BER performance. The performance of several algorithms is verified over 1000 simulations, computing the MSE and BER at each value of the signal-to-noise ratio (SNR).

In Fig. 2 and Fig. 3 we study the performance in strong temporal correlated channels, so we set the temporal correlation coefficients from 0.7 to 0.99 for different blocks, where most delays of strong paths are similar and corresponding gains vary in a small scale for the 4 blocks. And there are also some random paths appearing in different blocks with random delays, which are with less power.

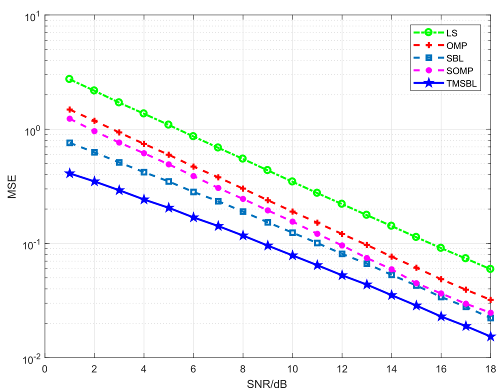


**FIGURE 2.** **The comparison of MSE performance in strong temporal correlated channels.**

Fig. 2 shows the MSE performance of the above mentioned methods. It is observed that the MSE performance of LS method is the worst, and SBL method outperforms the OMP method about 2 dB. Based on the joint estimation the SOMP method achieves better performance than the OMP method, but less than the SBL method. By considering the temporal correlation, the TMSBL gains the best MSE performance of all, with about 2 dB higher than the SBL algorithm.

Fig. 3 shows the decoded BER performance of the above mentioned methods. We can find that the BER performance of LS method is still the worst. The BER performance of the

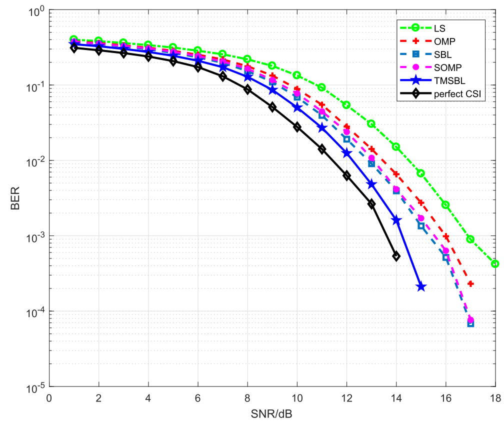


**FIGURE 3.** **The comparison of decoded BER performance in strong temporal correlated channels.**

OMP method is about 0.5 dB and 1.5 dB lower than SBL and TMSBL method, and the SOMP method is between OMP and SBL method. Meanwhile the performance of TMSBL method is the one most close to the perfect CSI cruve.

In Fig. 4 and Fig. 5 we study the performance in weak temporal correlated channels, so we set the temporal correlation coefficients from 0.1 to 0.3 for different blocks, where only several delays of strong power paths are similar and corresponding gains vary mildly for the 4 blocks. And most paths appear in different blocks with random delays, which are with random power.

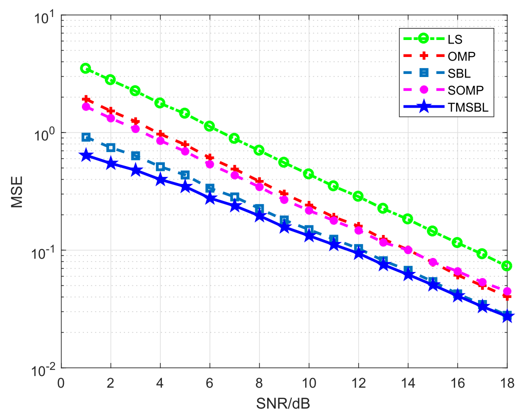


**FIGURE 4.** **The comparison of MSE performance in weak temporal correlated channels.**

Fig. 4 shows the MSE performance of the above mentioned methods. We can find that the MSE performance of LS method is the worst, and the performance of the OMP method and the SOMP method are very close. Meanwhile the performance of the SBL method and the TMSBL method are also close and about 2dB better than the OMP and SOMP methods.

Fig. 5 shows the decoded BER performance of the above mentioned methods. It is observed that the BER performance of LS method is still the worst. The BER performance of other methods has the same trend in Fig. 4, where the OMP

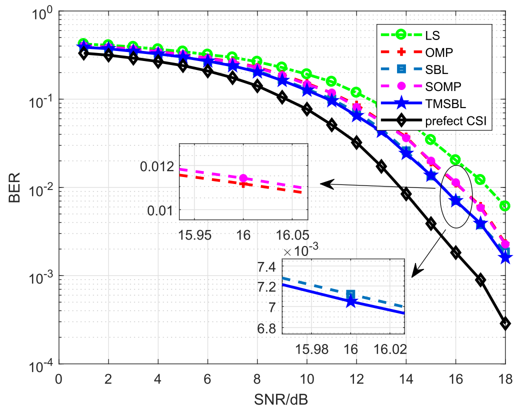


**FIGURE 5.** The comparison of decoded BER performance in weak temporal correlated channels.

method has close performance with SOMP method and the performance of the SBL method and the TMSBL method are also close. In the weak temporal correlated channels most path delays are unmatched the joint channel model so the advantage of the SOMP method and the TMSBL method has not been fully utilized. But we can notice that the SBL based methods hold better performance than CS based methods and have good robustness in weak temporal correlated channels.

Through the simulation results, we can summarize that the SBL based channel estimator can achieve better performance than conventional OMP method and even a little better than SOMP method. Especially when considering the channel temporal correlation of several consecutive OFDM blocks, the TMSBL algorithm cannot only exploit the sparse structure of channel paths but also use the temporal correlation from multiple received signals to achieve the best performance in strong temporal correlated channels. In weak temporal correlated channels, the performance of SOMP and TMSBL methods degrade to the OMP and SBL methods, and the TMSBL method exhibits good robustness with comparable performance. In the next section, we will show the results in sea experiment by using the above mentioned sparse channel estimators.

## V. EXPERIMENTAL RESULTS

### A. SYSTEM SETTINGS

As simulation results show that the sparse channel estimation methods above are effective in UWA channel, we next use real experimental data to further verify the methods. The data was recorded in the experiment in South China Sea in May 2014, in which we get the channel in Fig. 1. The UWA CP-OFDM system settings are shown in TABLE 2.

We consecutively send 12 frames of OFDM signal with 2-seconds interval between frames. The data subcarriers are encoded using convolutional code with QPSK constellation. We set a linear frequency modulation (LFM) signal before every frame for synchronization.

**TABLE 2.** UWA CP-OFDM settings.

| | | |
|---|---|---|
| Bandwidth | $B$ | 4kHz |
| Carrier frequency | $f_c$ | 8kHz |
| Sampling frequency | $f_s$ | 48kHz |
| No.subcarriers | $K$ | 681 |
| No.data subcarriers | $K_d$ | 571 |
| No.pilot subcarriers | $K_p$ | 86 |
| No.null subcarriers | $K_n$ | 24 |
| Symbol duration | $T$ | 170ms |
| Cyclic-prefix length | $T_{cp}$ | 20ms |
| Blocks in one frame | $N_b$ | 8 |

### B. RESULTS

In Fig. 6 we show four channel impulse response from $(1, 4, 7, 10)$# frames by the correlation of received LFM signals and local templates. We can find that the channel structures remain stable.

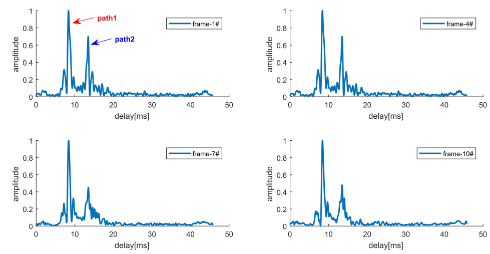


**FIGURE 6.** The impulse response of channel in sea trial.

Then we study four sparse channel estimation methods and compare their performance. For the OMP and SBL methods we estimate the channels block-by-block, and for SOMP and TMSBL methods we choose the first four consecutive blocks for joint channel estimation and do the same with the last four consecutive blocks in every frame. We use the same method in simulation to calculate the noise variance as input for SBL based methods, and we also set the positive scalar $\eta = 2$ in TMSBL method.

Besides we introduce the effective noise variance to evaluate channel estimation performance as

$$\hat{\sigma}_e^2 = E\{|\mathbf{Y} - \hat{\mathbf{H}}\mathbf{X}|^2\} \tag{30}$$

where the $\hat{\sigma}_e^2$ denotes the estimated effective noise variance which includes channel estimation error, the ambient noise and the residual Doppler shifting. And $\hat{\mathbf{H}}$ is obtained by the Fourier transform of the estimated $\hat{\mathbf{h}}$, $\mathbf{Y}$ is the received data and $\mathbf{X}$ is the transmitted data.

Fig. 7 shows the comparison results of effective noise variance under different channel estimators. Here we get the lowest effective noise variance by using SBL method with the help of all K subcarriers in each block and consider it as the benchmark for the effective noise variance comparisons, and also use the channel estimation results in the following Fig. 9, Fig. 10 and Fig. 11 for further analysis.

For the other four methods, the effective noise variance of TMSBL channel estimator is lower than others overall

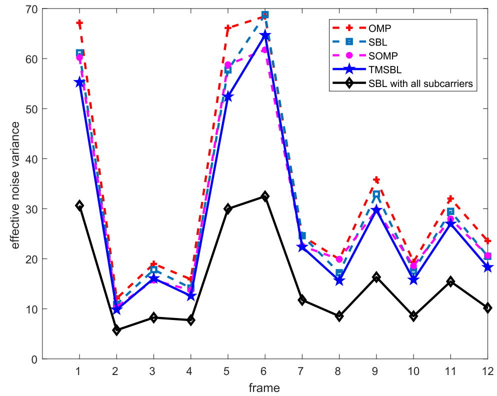


**FIGURE 7.** The comparison of effective noise variance in sea trial.

and followed by SBL and SOMP channel estimators, and the effective noise variance of OMP channel estimator is the highest overall. The results exhibit the similar trend as in simulations.

Fig. 8 shows the decoded BER performance. The BER of TMSBL method is still the lowest followed closely by SBL method and SOMP method, and the OMP method has the worst BER performance. The performance gap between TMSBL method and others is even more pronounced especially in (2, 11, 12)# frames, the BER of TMSBL achieves quite better performance than other methods. But in (6, 9)# frames, the BER for all four channel estimators are very close. Then we combine this BER results with channel parameters for analysis.

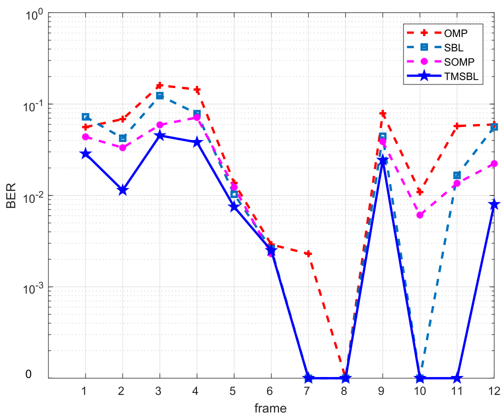


**FIGURE 8.** The comparison of decoded BER performance in sea trial.

Here we firstly figure the estimated time-varying delays in Fig. 9 and amplitudes in Fig. 10 of the first two strongest paths. Note that the delays of the two strongest paths have no significant changes and the amplitudes change within a small range for the four consecutive blocks in each frame, this provides strong support for the assumption of joint channel model.

In Fig. 11 we calculate the temporal correlation coefficients where we divide the all 8 blocks in every frame into two parts (the first 4 blocks and the back 4 blocks) and

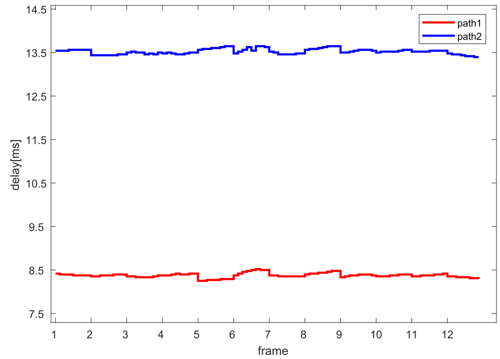


**FIGURE 9.** The time-varying delays of two strongest paths in sea trial.

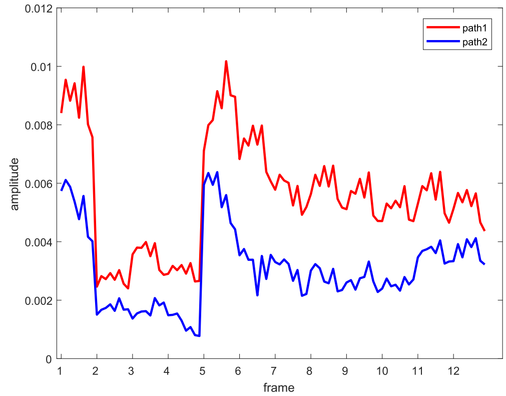


**FIGURE 10.** The time-varying amplitudes of two strongest paths in sea trial.

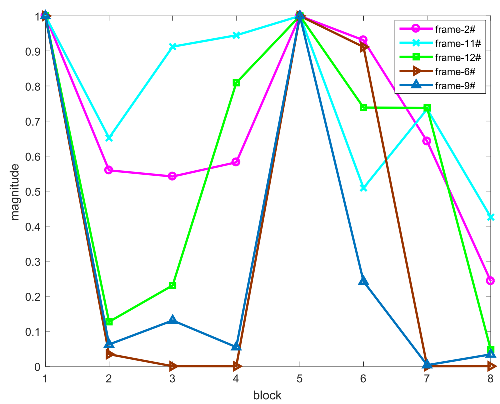


**FIGURE 11.** The temporal correlation coefficients in sea trial.

respectively calculate the $\eta(1, m)$ for $m \in [1, 4]$ and the $\eta(5, n)$ for $n \in [5, 8]$. We can find that in (2, 11, 12)# frames the temporal correlations are above 0.5 at most time, where we consider they are strong temporal correlated channels and can take full advantages of the channel temporal correlation to get pleasant results based on TMSBL method in Fig. 8. As a contrast the temporal correlations in (6, 9)# frames decline rapidly for consecutive blocks which are considered as weak temporal correlated channels, and the TMSBL based method is little better than the other methods and exhibits acceptable robustness.

## VI. CONCLUSION

This paper addresses the issue of channel estimation targeting to improve the system performance. Different from the traditional CS based methods, we focus on the SBL based channel estimators in this paper.

Firstly, we study the SBL framework for channel estimation in UWA OFDM communication systems, and design a SBL based channel estimator for block-by-block processing using the channel sparse structure independently in each block. Then we propose a joint channel model after Doppler compensation for multi-block joint processing where the delays of the channels for several consecutive blocks are similar and the path gains exhibit temporal correlation. And we denote a temporal correlation coefficient for path gains to evaluate the strength of the correlation. Further we propose the TMSBL based channel estimator to jointly estimate the channels by taking advantage of the channel coherence between consecutive OFDM blocks.

Results of numerical simulation and sea trial indicate superiority of the SBL and TMSBL channel estimator algorithms in time-varying UWA channel, which achieve better channel estimation performance and BER compared with the existing CS based methods, especially the TMSBL estimator achieves the best performance in strong temporal correlated channels and maintains robustness in weak temporal correlated channels.

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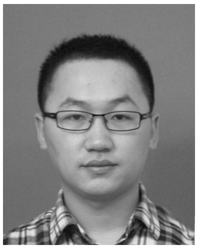

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