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Doppler Frequency Shift Based Source Localization in Presence of Sensor Location Errors

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ABSTRACT In this paper, the Doppler frequency shift-based localization problem in the presence of sensor location errors is addressed. Based on the measurement model, we present two methods, an explicit solution and a semidefinite relaxation (SDR) technique, to estimate the position of the stationary source. In the two proposed algorithms, the non-accurate information on sensor position and velocity is considered. In the former algorithm, based on two best linear unbiased estimator, an explicit solution of the source position can be obtained, whereas in the latter one, the SDR technique is applied to relax the original maximum likelihood estimation into a convex semidefinite programming problem, which provides accurate estimate without postprocessing. Compared with the methods where the sensor location errors are not taken into consideration, the proposed algorithms are able to achieve a more accurate localization result. Simulations verify a better performance of these two proposed algorithms.

INDEX TERMS Doppler frequency shift (DFS), sensor location uncertainty, best linear unbiased estimator (BULE), semidefinite programming (SDP).

I. INTRODUCTION

The technology of passive source localization has received considerable interest due to its wide applications in various areas including radar, sonar, navigation and wireless communications [1]–[4]. In general, the most common technique is to measure the time information of the source signal from spatially separated sensors. Based on the time delay (TD) measurements, a set of hyperbolic equations are formed, and the solution of these nonlinear equations gives the estimate of the source location [5], [6]. By utilizing the relative movements between the source and moving platforms, the Doppler frequency shift (DFS) measurements derived from Doppler effect can be combined with TD to obtain a more accurate estimate [7]–[9].

However, in the scenarios with low synchronization accuracy or the narrow bandwidth of the signal, the time information will not be valid and the source localization accuracy will be degraded [10]. While in these cases, the DFS information can still be accurately estimated. Hence, carrying out the research of source localization based on DFS measurements is important and indispensable. On account of a higher nonlinearity between the measurement parameters and the solution space, it is nontrivial to convert frequency information into source position. In general, the DFS based localization methods are divided into two categories: direct and indirect methods. The direct methods estimate the source position via process all the observed signals simultaneously; while the indirect methods firstly extract all measurement parameters from received signals, and estimate the source position based on the extracted measurements subsequently. The direct methods, which generally employ exhaustive search in solution space (also named grid searching) to obtain the desired result [11], [12], are asymptotically optimal; however, the multidimensional grid searching algorithm will render tremendous computation pressure. To simplify and enhance the position computation process, some information fusion techniques can be used [13]. In contrast, the indirect methods, which normally apply iterative algorithm to address the locating problem after linearizing the relation between the measurements and the source position [14], [15], have the advantage of low computational complexity and satisfactory performance. Therefore, the indirect methods with iterative algorithm are more popular in source localization. However, the performance of the numerical iterative algorithm relies on a good initial value, which means the initial guess has to get close to the true value, or else the convergence of this algorithm is not assured. In fact, such a good initial value is difficult to obtain.

In order to avoid the above mentioned drawbacks, the explicit estimation, which has the advantages of lighter computational load and locating without initial guesses, has been proposed as an alternative method [16]. In order to express the relationship between measurement parameters and source position in a linear form, this solution only remains the linear terms, therefore its localization result cannot absolutely achieve the Cramer-Rao lower bound (CRLB). After obtaining the estimate, a numerical iterative method has to be implement to further improve the location result. Recently, the semidefinite relaxation (SDR) technique which relaxes the ML problem to obtain a convex semidefinite programming (SDP) problem and then solved by modern convex optimization algorithms, has been successfully applied and sparked some rapid developments in the area of source localization [17]–[20], and as well some other fields, such as waveform design [21]–[23] and beamforming [24], [25].

Nevertheless, whatever the position estimation methods are, the estimation accuracy is fairly sensitive to the information of the sensor positions and velocities [26]. A slight error of the sensors may cause degradation of source localization performance. Therefore, the inaccuracy of sensors has to be taken into account. Among the existing works, Yang and Ho analyzed TD based multiple disjoint sources localization in presence of sensor location errors [27]. Afterwards, Sun and Ho addressed the localization problem for multiple sources by jointly using the time and frequency information under the condition of sensor location errors [28], while few works in literatures addressed the constrained localization problem using frequency information only.

Different from previous works where the inaccuracy of sensors is not taken into account [16], [20], this work aims to employ DFS measurements to address the source localization problem in presence of sensor location errors. In this work, we add the sensor position errors into our received signal models. In accordance with the best liner unbiased estimator (BLUE) criterion, to obtain an explicit estimation, the first solution transforms the relationship between DFS measurements and the source position to a set of linear equations, and further improves the initial estimate accuracy through the use of the BLUE again. In this work, we propose another location method by relaxing the maximum likelihood estimation (MLE) of the source position into semidefinite programming (SDP) problem, which can provide an accurate estimate without post processing. With the location errors considered in the above proposed localization methods, the source location accuracy is improved. The CRLB is analyzed and simulations are provided to confirm the estimated accuracy of the proposed methods. The contributions of this paper can be summarized as follows:

1) We provide an explicit solution (marked as TB-SLE) and an SDR solution (marked as SDR-SLE) to address the DFS based source localization problem without initial position guesses in presence of sensor location errors.

2) The CRLB for DFS based source localization, under the condition of sensor location errors, are derived as the benchmark.

3) Simulation comparison is carried out to verify the effectiveness of the proposed algorithms. The simulation result shows that the TB-SLE and the SDR-SLE solutions outperform the algorithms without considering sensor location errors.

The remainder of this paper is organized as follows: Section II provides the DFS measurement models. Section III presents the TB-SLE solution and the SDR-SLE solution in the present of sensors location errors. After that, the CRLB accuracy is analyzed in this section. Section IV compares the location estimation performance of the proposed solutions to the previous methods as well as CRLB, and Section V is the conclusion.

Notation: uppercase and lowercase bold letters denote matrices and vectors, respectively. The i-th to j-th components of $\mathbf{x}$ is denoted by $\mathbf{x}(i{:}j)$. $\mathbf{X}(i{:}j)$ is the submatrix of $\mathbf{X}$ with corners (i, i), (i, j), (j, i), (j, j). The symbols $\|\cdot\|$ and $\mathrm{tr}(\cdot)$ denote the Frobenius norm and trace. $\mathbf{X} \succeq 0$ means $\mathbf{X}$ is positive semidefinite. $\mathbf{x}$ and $\hat{\mathbf{x}}$ denote the true value and the estimate value of $\mathbf{x}$, respectively.

II. PROBLEM STATEMENT

Assume the scenario with one stationary source whose position is denoted as $\mathbf{u}$, L moving sensors intercept the emitted signals N times. As a result, the intercepting number of frequency-shifted signals is $M = LN$. In each interception, the sensors' position and velocity are denoted as $\mathbf{s}_i\,(i = 1, \ldots, M)$ and $\mathbf{v}_i\,(i = 1, \ldots, M)$. Assume that the state of the sensors are constant within a short interval, the DFS measurements models of this scenario related to the source and sensors are expressed as

$$d_i = \frac{f_c}{c} \cdot \frac{\mathbf{v}_i^T(\mathbf{u} - \mathbf{s}_i)}{\|\mathbf{u} - \mathbf{s}_i\|}, \quad i = 1, \ldots, M, \tag{1}$$

where c is the speed of signal propagation, and f_c is the carrier frequency, assumed known. By defining $f_i \overset{\Delta}{=} cd_i/f_c$, (1) can be reformulated as

$$f_i = \frac{\mathbf{v}_i^T(\mathbf{u} - \mathbf{s}_i)}{\|\mathbf{u} - \mathbf{s}_i\|}, i = 1, \ldots, M. \tag{2}$$

Since the measurement errors are always present, we have to take the noise into account. Define the true values as $f_i = \hat{f}_i - \Delta f_i$, $\mathbf{s}_i = \hat{\mathbf{s}}_i - \Delta\mathbf{s}_i$, $\mathbf{v}_i = \hat{\mathbf{v}}_i - \Delta\mathbf{v}_i$, and (2) can be rewritten as

$$\hat{f}_i - \Delta f_i = \frac{\left(\hat{\mathbf{v}}_i^T - \Delta{\mathbf{v}_i}^T\right)\left(\mathbf{u} - \hat{\mathbf{s}}_i + \Delta\mathbf{s}_i\right)}{\left\|\mathbf{u} - \hat{\mathbf{s}}_i + \Delta\mathbf{s}_i\right\|}, \tag{3}$$

where $\hat{f}_i$, $\hat{\mathbf{s}}_\mathbf{i}$ and $\hat{\mathbf{v}}_\mathbf{i}$ are estimate values. Δf_i is frequency measurement noise, we express the M measurement noises as a vector form

$$\Delta\mathbf{f} = \left[\,\Delta f_1 \quad \Delta f_2 \quad \ldots \quad \Delta f_M\,\right]^T, \tag{4}$$

$$\hat{f}_i^2\|\mathbf{u}-\hat{\mathbf{s}}_i\|^2 = \left(\hat{\mathbf{v}}_i^T\mathbf{u}\right)^2 + k_i^2 - 2k_i\hat{\mathbf{v}}_i^T\mathbf{u} + 2\hat{r}_i\hat{\mathbf{v}}_i^T\mathbf{u}\Delta f_i - 2k_i\hat{r}_i\Delta f_i - 2\left(\hat{f}_i^2(\mathbf{u}-\hat{\mathbf{s}}_i)^T + \left(k_i - \hat{\mathbf{v}}_i^T\mathbf{u}\right)\hat{\mathbf{v}}_i^T\right)\Delta\mathbf{s}_i - 2\left(k_i\left(\hat{\mathbf{s}}_i^T - \mathbf{u}^T\right) + \hat{\mathbf{v}}_i^T\mathbf{u}\left(\mathbf{u}^T - \hat{\mathbf{s}}_i^T\right)\right)\Delta\mathbf{v}_i. \tag{8}$$

$$\hat{f}_i^2\left(x^2+y^2-2x_ix-2y_iy+\hat{\mathbf{s}}_i^T\hat{\mathbf{s}}_i\right) = a_i^2x^2+b_i^2y^2+2a_ib_ixy-2k_ia_ix-2k_ib_iy+k_i^2+2\hat{r}_i\hat{\mathbf{v}}_i^T\mathbf{u}\Delta f_i-2k_i\hat{r}_i\Delta f_i - 2\left(\hat{f}_i^2(\mathbf{u}-\hat{\mathbf{s}}_i)^T + \left(k_i-\hat{\mathbf{v}}_i^T\mathbf{u}\right)\hat{\mathbf{v}}_i^T\right)\Delta\mathbf{s}_i - 2\left(k_i\left(\hat{\mathbf{s}}_i^T-\mathbf{u}^T\right)+\hat{\mathbf{v}}_i^T\mathbf{u}\left(\mathbf{u}^T-\hat{\mathbf{s}}_i^T\right)\right)\Delta\mathbf{v}_i. \tag{9}$$

which is assumed to be zero-mean Gaussian with covariance $\mathrm{E}\left[\Delta\mathbf{f}\Delta\mathbf{f}^T\right] = \mathbf{Q}_f$. $\Delta\mathbf{s}_i$ and $\Delta\mathbf{v}_i$ are the errors of position and velocity (sensor location errors), the error vectors are respectively denoted as

$$\Delta\mathbf{s} = \begin{bmatrix} \Delta\mathbf{s}_1^T & \Delta\mathbf{s}_2^T & \ldots & \Delta\mathbf{s}_M^T \end{bmatrix}^T, \tag{5}$$
$$\Delta\mathbf{v} = \begin{bmatrix} \Delta\mathbf{v}_1^T & \Delta\mathbf{v}_2^T & \ldots & \Delta\mathbf{v}_M^T \end{bmatrix}^T. \tag{6}$$

Referring to the previous literatures [26]–[28], the errors of position and velocity are modeled as zero-mean Gaussian variables and independent of Doppler frequency measurement noise. The covariance matrixes are $\mathrm{E}\left[\Delta\mathbf{s}\Delta\mathbf{s}^T\right] = \mathbf{Q}_s$ and $\mathrm{E}\left[\Delta\mathbf{v}\Delta\mathbf{v}^T\right] = \mathbf{Q}_v$, respectively.

Based on (3), the maximum-likelihood (ML) estimate of the source position can be obtained. Nevertheless, the nonlinear relationship between the DFS measurements and the source position implies that the global minimum is difficult to achieve.

III. SOURCE LOCALIZATON IN PRESENCE OF SENSOR LOCATION ERRORS

A. TB-SLE SOLUTION

In this section, we present an efficient solution to estimate the source position using the DFS measurements only. We elaborate the TB-SLE solution method in 2D scenario and this method can be extended to 3D-space straightforwardly. Notice that (3) can be reformulated as

$$\hat{f}_i\left\|\mathbf{u}-\hat{\mathbf{s}}_i+\Delta\mathbf{s}_i\right\| = \Delta f_i\left\|\mathbf{u}-\hat{\mathbf{s}}_i+\Delta\mathbf{s}_i\right\| + \left(\hat{\mathbf{v}}_i^T-\Delta\mathbf{v}_i^T\right)\left(\mathbf{u}-\hat{\mathbf{s}}_i+\Delta\mathbf{s}_i\right). \tag{7}$$

Squaring both side and neglecting second-order error terms, we get formula (8), as shown at top of this page, where $k_i = \hat{\mathbf{v}}_i^T\hat{\mathbf{s}}_i$. $\hat{r}_i = \left\|\mathbf{u}-\hat{\mathbf{s}}_i\right\|$ is the measurement distance from the source to the i-th sensor. Defining $\mathbf{u} = [x, y]^T$, $\mathbf{s}_i = [x_i, y_i]^T$, $\mathbf{v}_i = [a_i, b_i]^T$, equation (8) can be reformulated as (9) which is shown below equation (8). Rewriting (9) as matrix form yields

$$\mathbf{A}\mathbf{q}-\mathbf{h} = \boldsymbol{\varepsilon}, \tag{10}$$

where

$$\mathbf{q} = \left[x, y, x^2, y^2, xy\right]^T, \tag{11}$$
$$\boldsymbol{\varepsilon} = \mathbf{B}\Delta\mathbf{f}+\mathbf{C}\Delta\mathbf{s}+\mathbf{D}\Delta\mathbf{v}. \tag{12}$$

The expression of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{h}$ can be found at the bottom of this page page. According to the Gauss-Markov theorem [29], the estimate of $\mathbf{q}$ can be obtained via the best linear unbiased estimator, as follows

$$\hat{\mathbf{q}} = \left(\mathbf{A}^T\mathbf{W}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{W}^{-1}\mathbf{h}, \tag{18}$$

where

$$\mathbf{W} = \mathbf{B}\mathbf{Q}_f\mathbf{B}^T+\mathbf{C}\mathbf{Q}_s\mathbf{C}^T+\mathbf{D}\mathbf{Q}_v\mathbf{D}^T, \tag{19}$$

and the corresponding covariance matrix of $\hat{\mathbf{q}}$ is

$$\mathrm{cov}\left(\hat{\mathbf{q}}\right) = \mathrm{E}\left[\Delta\mathbf{q}\Delta\mathbf{q}^T\right] = \left(\mathbf{A}^T\mathbf{W}^{-1}\mathbf{A}\right)^{-1} \tag{20}$$
$$\Delta\mathbf{q} = \mathbf{q}-\hat{\mathbf{q}}. \tag{21}$$

In (19), the weighting matrix $\mathbf{W}$ is related to the source position $\mathbf{u}$, therefore it cannot be directly obtained. For implementation purpose, assume $\mathbf{W}$=$\mathbf{Q}_f$ and substitute it into (18) to obtain an initial estimate of source position. Afterwards substitute this initial estimate to (19) to recalculate $\mathbf{W}$ and utilize the new weighting matrix to estimate a more accurate source position $\mathbf{u}$. It is acceptable to approximate the

$$\mathbf{A} = \begin{bmatrix} -2x_1\hat{f}_1^2+2k_1a_1 & -2y_1\hat{f}_1^2+2k_1b_1 & \hat{f}_1^2-a_1^2 & \hat{f}_1^2-b_1^2 & -2a_1b_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -2x_M\hat{f}_M^2+2k_Ma_M & -2y_M\hat{f}_M^2+2k_Mb_M & \hat{f}_M^2-a_M^2 & \hat{f}_M^2-b_M^2 & -2a_Mb_M \end{bmatrix}, \tag{13}$$

$$\mathbf{B} = \mathrm{diag}\left\{\begin{bmatrix} 2\hat{r}_1\hat{\mathbf{v}}_1^T\mathbf{u}-2k_1\hat{r}_1 & 2\hat{r}_2\hat{\mathbf{v}}_2^T\mathbf{u}-2k_2\hat{r}_2 & \cdots & 2\hat{r}_M\hat{\mathbf{v}}_M^T\mathbf{u}-2k_M\hat{r}_M \end{bmatrix}\right\}, \tag{14}$$

$$\mathbf{C} = \mathrm{diag}\left\{\begin{bmatrix} -2\left(\hat{f}_1^2(\mathbf{u}-\hat{\mathbf{s}}_1)^T+(k_1-\hat{\mathbf{v}}_1^T\mathbf{u})\hat{\mathbf{v}}_1^T\right) & \ldots & -2\left(\hat{f}_M^2(\mathbf{u}-\hat{\mathbf{s}}_M)^T+(k_M-\hat{\mathbf{v}}_M^T\mathbf{u})\hat{\mathbf{v}}_M^T\right) \end{bmatrix}\right\}, \tag{15}$$

$$\mathbf{D} = \mathrm{diag}\left\{\begin{bmatrix} -2\left(k_1\left(\hat{\mathbf{s}}_1^T-\mathbf{u}^T\right)+\hat{\mathbf{v}}_1^T\mathbf{u}\left(\mathbf{u}^T-\hat{\mathbf{s}}_1^T\right)\right) & \ldots & -2\left(k_M\left(\hat{\mathbf{s}}_M^T-\mathbf{u}^T\right)+\hat{\mathbf{v}}_M^T\mathbf{u}\left(\mathbf{u}^T-\mathbf{s}_M^T\right)\right) \end{bmatrix}\right\}, \tag{16}$$

$$\mathbf{h} = \begin{bmatrix} -\hat{f}_1^2\hat{\mathbf{s}}_1^T\hat{\mathbf{s}}_1+k_1^2 & -\hat{f}_2^2\hat{\mathbf{s}}_2^T\hat{\mathbf{s}}_2+k_2^2 & \cdots & -\hat{f}_M^2\hat{\mathbf{s}}_M^T\hat{\mathbf{s}}_M+k_M^2 \end{bmatrix}^T. \tag{17}$$

weighting matrix because of its little effects on the cost function. In fact, this method has been used in many other literatures [30], and has become a general approach in source localization.

The first two elements of $\hat{\mathbf{q}}$ contain the source position namely $\mathbf{u} = \hat{\mathbf{q}}(1{:}2)$. The other elements of $\mathbf{q}$ related to the source position is

$$\mathbf{q}^2(1) = \mathbf{q}(3), \tag{22}$$

$$\mathbf{q}^2(2) = \mathbf{q}(4), \tag{23}$$

$$\mathbf{q}(1)\,\mathbf{q}(2) = \mathbf{q}(5). \tag{24}$$

According to the definition of $\Delta\mathbf{q}$ in (21) and ignoring the higher-order error terms, (22)-(24) can be rewritten as

$$\hat{\mathbf{q}}^2(1) + 2\hat{\mathbf{q}}(1)\,\Delta\mathbf{q}(1) - \hat{\mathbf{q}}(3) = \Delta\mathbf{q}(3), \tag{25}$$

$$\hat{\mathbf{q}}^2(2) + 2\hat{\mathbf{q}}(2)\,\Delta\mathbf{q}(2) - \hat{\mathbf{q}}(4) = \Delta\mathbf{q}(4), \tag{26}$$

$$\hat{\mathbf{q}}(1)\,\hat{\mathbf{q}}(2) + \hat{\mathbf{q}}(1)\,\Delta\mathbf{q}(2) + \hat{\mathbf{q}}(2)\,\Delta\mathbf{q}(1) - \hat{\mathbf{q}}(5) = \Delta\mathbf{q}(5). \tag{27}$$

Define $\Delta\mathbf{u} \triangleq [\Delta\mathbf{q}(1), \Delta\mathbf{q}(2)]^T$, and reformulate (25)-(27) as a linear expression (matrix form)

$$\mathbf{C}\Delta\mathbf{u} - \mathbf{b} = \Delta\mathbf{q}, \tag{28}$$

where

$$\mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2\hat{\mathbf{q}}(1) & 0 \\ 0 & 2\hat{\mathbf{q}}(2) \\ \hat{\mathbf{q}}(2) & \hat{\mathbf{q}}(1) \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \hat{\mathbf{q}}(3) - \hat{\mathbf{q}}^2(1) \\ \hat{\mathbf{q}}(4) - \hat{\mathbf{q}}^2(2) \\ \hat{\mathbf{q}}(5) - \hat{\mathbf{q}}(1)\,\hat{\mathbf{q}}(2) \end{bmatrix}. \tag{29}$$

Basing on (28), the estimator of $\Delta\mathbf{u}$ is obtained

$$\Delta\hat{\mathbf{u}} = \left(\mathbf{C}^T \mathrm{cov}\left(\hat{\mathbf{q}}\right)^{-1}\mathbf{C}\right)^{-1}\mathbf{C}^T \mathrm{cov}\left(\hat{\mathbf{q}}\right)^{-1}\mathbf{b}. \tag{30}$$

The final solution of source position is

$$\hat{\mathbf{u}} = \hat{\mathbf{q}}(1{:}2) + \Delta\hat{\mathbf{u}}. \tag{31}$$

B. SDR-SLE SOLUTION

Note that the TB-SLE solution method neglects the second-order error terms twice in the process of constructing linear equations. This approximation may be reasonable at sufficiently low noise levels. However, the increase of noise level may affect the positioning accuracy.

In fact, the optimization problem can be constructed directly from (3), and then the SDP method is used to approximate the solution. The expression (3) can be rewritten as the following form

$$\hat{f}_i r_i = \Delta f_i r_i + \left(\hat{\mathbf{v}}_i^T - \Delta\mathbf{v}_i^T\right)\left(\mathbf{u} - \hat{\mathbf{s}}_i + \Delta\mathbf{s}_i\right), \tag{32}$$

where

$$r_i = \left\|\mathbf{u} - \hat{\mathbf{s}}_i + \Delta\mathbf{s}_i\right\| = \|\mathbf{u} - \mathbf{s}_i\|, \tag{33}$$

is the true range between the source and the i-th sensor, and it is only dependent on the value of the sensor position $\mathbf{s}_i$. Expanding $r_i = \|\mathbf{u} - \mathbf{s}_i\|$ in Taylor series and retaining only the linear terms give

$$r_i \approx \hat{r}_i - \boldsymbol{\rho}_{\mathbf{u},\hat{\mathbf{s}}_i}^T \Delta\mathbf{s}_i, \quad \hat{r}_i = \left\|\mathbf{u} - \hat{\mathbf{s}}_i\right\|, \tag{34}$$

where $\boldsymbol{\rho}_{\mathbf{x},\mathbf{y}} = (\mathbf{x} - \mathbf{y}) / \|\mathbf{x} - \mathbf{y}\|$. Substituting (34) into (32) and eliminating the second-order noise terms yield

$$\hat{f}_i\hat{r}_i - \hat{\mathbf{v}}_i^T\mathbf{u} + \hat{\mathbf{v}}_i^T\hat{\mathbf{s}}_i = \hat{r}_i\Delta f_i + \left(\hat{\mathbf{s}}_i^T - \mathbf{u}^T\right)\Delta\mathbf{v}_i + \left(\hat{f}_i\boldsymbol{\rho}_{\mathbf{u},\hat{\mathbf{s}}_i}^T + \hat{\mathbf{v}}_i^T\right)\Delta\mathbf{s}_i, \tag{35}$$

or, use vector notation

$$\mathbf{U}\mathbf{g} + \mathbf{k} = \boldsymbol{\eta}, \tag{36}$$

where

$$\boldsymbol{\eta} = \mathbf{R}_1\Delta\mathbf{f} + \mathbf{R}_2\Delta\mathbf{s} + \mathbf{R}_3\Delta\mathbf{v}, \tag{37}$$

$$\mathbf{g} = \left[\mathbf{u}^T, \left\|\mathbf{u} - \hat{\mathbf{s}}_1\right\|, \ldots, \left\|\mathbf{u} - \hat{\mathbf{s}}_M\right\|\right]^T, \tag{38}$$

$$\mathbf{k} = \left[\hat{\mathbf{v}}_1^T\hat{\mathbf{s}}_1, \hat{\mathbf{v}}_2^T\hat{\mathbf{s}}_2 \ldots, \hat{\mathbf{v}}_M^T\hat{\mathbf{s}}_M\right]^T, \tag{39}$$

$$\mathbf{U} = \begin{bmatrix} -\hat{\mathbf{v}}_1^T & \hat{f}_1 & 0 & 0 & 0 \\ -\hat{\mathbf{v}}_2^T & 0 & \hat{f}_2 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\hat{\mathbf{v}}_M^T & 0 & 0 & 0 & \hat{f}_M \end{bmatrix}. \tag{40}$$

The expression of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ can be found at the top of next page. Addressing the following optimization functions can provide the estimate of the source position

$$\min_{\mathbf{u}} \ (\mathbf{U}\mathbf{g} + \mathbf{k})^T\mathbf{Q}^{-1}(\mathbf{U}\mathbf{g} + \mathbf{k}), \tag{44}$$

where $\mathbf{Q} = \mathbf{R}_1\mathbf{Q}_f\mathbf{R}_1^T + \mathbf{R}_2\mathbf{Q}_s\mathbf{R}_2^T + \mathbf{R}_3\mathbf{Q}_v\mathbf{R}_3^T$. If function $\mathbf{Q}$ contains $\mathbf{u}$, the expression (44) is identical with a maximum likelihood estimate. For the purpose of solving (44), assume $\mathbf{Q} = \mathbf{Q}_f$ and get an initial estimate of $\mathbf{u}$. Then, (44) can be transformed into a constrained weighted least squares problem

$$\begin{aligned} &\min_{\mathbf{g},\mathbf{u}} \ (\mathbf{U}\mathbf{g} + \mathbf{k})^T\mathbf{Q}^{-1}(\mathbf{U}\mathbf{g} + \mathbf{k}) \\ &\text{s.t. } \mathbf{g}(n{+}i) = \left\|\mathbf{u} - \hat{\mathbf{s}}_i\right\|, \\ &\qquad \mathbf{g}(1{:}n) = \mathbf{u}, \end{aligned} \tag{45}$$

where n represents the dimension of the coordinate. Denoting $\mathbf{G}{=}\mathbf{g}\mathbf{g}^T$, (45) can be equivalently written as

$$\begin{aligned} &\min_{\mathbf{G},\mathbf{g},\mathbf{u}} \ \mathrm{tr}\left(\mathbf{U}^T\mathbf{Q}^{-1}\mathbf{U}\mathbf{G}\right) + 2\mathbf{k}^T\mathbf{Q}^{-1}\mathbf{U}\mathbf{g} \\ &\text{s.t. } \mathbf{G}(n{+}i, n{+}i) = \|\mathbf{u} - \mathbf{s}_i\|^2, \\ &\qquad \mathbf{g}(1{:}n) = \mathbf{u}, \\ &\qquad \mathbf{G} = \mathbf{g}\mathbf{g}^T. \end{aligned} \tag{46}$$

(46) is convex except the last constraint $\mathbf{G} = \mathbf{g}\mathbf{g}^T$. However, the non-convex constraint is equivalent to two constraints [31] as follows

$$\mathbf{G} = \mathbf{g}\mathbf{g}^T \Leftrightarrow \begin{cases} \begin{bmatrix} \mathbf{G} & \mathbf{g} \\ \mathbf{g}^T & 1 \end{bmatrix} \succeq 0, \\ \mathrm{rank}(\mathbf{G}) = 1. \end{cases} \tag{47}$$

$$\mathbf{R}_1 = \mathrm{diag}\left\{ \hat{r}_1 \quad \hat{r}_2 \quad \ldots \quad \hat{r}_M \right\}, \tag{41}$$

$$\mathbf{R}_2 = \mathrm{diag}\left\{\left[\hat{\mathbf{v}}_1^T + \hat{f}_1 \boldsymbol{\rho}_{\mathbf{u},\hat{\mathbf{s}}_1}^T \quad \hat{\mathbf{v}}_2^T + \hat{f}_2 \boldsymbol{\rho}_{\mathbf{u},\hat{\mathbf{s}}_2}^T \quad \ldots \quad \hat{\mathbf{v}}_M^T + \hat{f}_M \boldsymbol{\rho}_{\mathbf{u},\hat{\mathbf{s}}_M}^T \right]\right\}, \tag{42}$$

$$\mathbf{R}_3 = \mathrm{diag}\left\{\left[\hat{\mathbf{s}}_1^T - \mathbf{u}^T \quad \hat{\mathbf{s}}_2^T - \mathbf{u}^T \quad \ldots \quad \hat{\mathbf{s}}_M^T - \mathbf{u}^T \right]\right\}. \tag{43}$$

With the rank constraint dropped, the constraint $\mathbf{G}=\mathbf{g}\mathbf{g}^T$ can be relaxed as

$$\mathbf{G} = \mathbf{g}\mathbf{g}^T \Leftrightarrow \left\{ \begin{bmatrix} \mathbf{G} & \mathbf{g} \\ \mathbf{g}^T & 1 \end{bmatrix} \succeq 0. \right. \tag{48}$$

Therefore, (46) is relaxed into the following SDP

$$\begin{aligned} &\min_{\mathbf{G},\mathbf{g},\mathbf{u}} \ \mathrm{tr}\left(\mathbf{U}^T\mathbf{Q}^{-1}\mathbf{U}\mathbf{G}\right) + 2\mathbf{k}^T\mathbf{Q}^{-1}\mathbf{U}\mathbf{g} \\ &\text{s.t. } \mathbf{G}(n+i, n+i) = \mathrm{tr}\left(\mathbf{G}(1{:}n)\right) - 2\hat{\mathbf{s}}_i^T\mathbf{u} + \hat{\mathbf{s}}_i^T\hat{\mathbf{s}}_i, \\ &\quad \mathbf{g}(1{:}n) = \mathbf{u}, \\ &\quad \begin{bmatrix} \mathbf{G} & \mathbf{g} \\ \mathbf{g}^T & 1 \end{bmatrix} \succeq 0. \end{aligned} \tag{49}$$

Exploiting the interior-point methods [32], we can obtain the optimal solution, denoted as $\left(\hat{\mathbf{G}}, \hat{\mathbf{g}}, \hat{\mathbf{u}}\right)$. Notice that the estimate $\hat{\mathbf{G}}(1{:}n)$ also contains the information of source position. By employing the rank-one approximation method, the final estimate is obtained [33]. In addition, Gaussian randomization procedure can be an alternative method to find the final estimate when the dimension of the estimated parameter is not high [34].

C. CRLB ANALYSIS

The CRLB is employed in many estimation problems to establish a theoretical bound on the variance of the unbiased estimator. In this section, the CRLB of source localization algorithms under the condition of sensor location errors is derived, and it is employed in the simulation as the benchmark of the performance of the algorithms.

Note that the actual positions and velocities of the sensors are unknown, we put them into vector form $\boldsymbol{\alpha} = \left[\mathbf{s}^T, \mathbf{v}^T\right]^T$, and collect the source position $\mathbf{u}$, true sensor position $\mathbf{s}$ and velocity $\mathbf{v}$ as $\boldsymbol{\vartheta} = \left[\mathbf{u}^T, \mathbf{s}^T, \mathbf{v}^T\right]^T$. The observation sensor position vector $\hat{\mathbf{s}}$ and velocity vector $\hat{\mathbf{v}}$ are independent of the measurement vector $\hat{\mathbf{f}}$, and both obey the Gaussian distribution. Hence, the logarithm of the probability density function of the data vector $\hat{\boldsymbol{\psi}} = \left[\hat{\mathbf{f}}^T, \hat{\mathbf{s}}^T, \hat{\mathbf{v}}^T\right]^T$ is

$$\begin{aligned} \ln f\left(\hat{\boldsymbol{\psi}} \middle| \boldsymbol{\vartheta}\right) &= \ln f\left(\hat{\mathbf{f}} \middle| \boldsymbol{\vartheta}\right) + f\left(\hat{\boldsymbol{\alpha}} \middle| \boldsymbol{\vartheta}\right) \\ &= k - \frac{1}{2}\left(\hat{\mathbf{f}} - \mathbf{f}\right)^T \mathbf{Q}_f^{-1}\left(\hat{\mathbf{f}} - \mathbf{f}\right) \\ &\quad - \frac{1}{2}\left(\hat{\boldsymbol{\alpha}} - \boldsymbol{\alpha}\right)^T \mathbf{Q}_\alpha^{-1}\left(\hat{\boldsymbol{\alpha}} - \boldsymbol{\alpha}\right), \end{aligned} \tag{50}$$

where k is a constant independent of $\boldsymbol{\vartheta}$, and

$$\mathbf{Q}_\alpha = \begin{bmatrix} \mathbf{Q}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_v \end{bmatrix}, \tag{51}$$

According to (50) we express the Fishier Matrix with respect of $\boldsymbol{\vartheta}$ as following

$$\mathrm{FIM}_{\boldsymbol{\vartheta}} = -\mathrm{E}\left[\left(\frac{\partial^2 \ln f\left(\hat{\boldsymbol{\psi}}|\boldsymbol{\vartheta}\right)}{\partial\boldsymbol{\vartheta}\partial\boldsymbol{\vartheta}^T}\right)\right] = \begin{bmatrix} \mathbf{X} & \mathbf{Y} \\ \mathbf{Y}^T & \mathbf{Z} \end{bmatrix} \tag{52}$$

where

$$\mathbf{X} = -\mathrm{E}\left[\frac{\partial^2 \ln f\left(\hat{\boldsymbol{\psi}} \middle| \boldsymbol{\vartheta}\right)}{\partial\mathbf{u}\partial\mathbf{u}^T}\right], \tag{53}$$

$$\mathbf{Y} = -\mathrm{E}\left[\frac{\partial^2 \ln f\left(\hat{\boldsymbol{\psi}} \middle| \boldsymbol{\vartheta}\right)}{\partial\mathbf{u}\partial\boldsymbol{\alpha}^T}\right], \tag{54}$$

$$\mathbf{Z} = -\mathrm{E}\left[\frac{\partial^2 \ln f\left(\hat{\boldsymbol{\psi}} \middle| \boldsymbol{\vartheta}\right)}{\partial\boldsymbol{\alpha}\partial\boldsymbol{\alpha}^T}\right]. \tag{55}$$

The details in evaluating the partial derivatives are provided in Appendix A. The CRLB of $\boldsymbol{\vartheta}$ is equal to

$$\mathrm{CRLB}\left(\boldsymbol{\vartheta}\right) = \mathrm{FIM}_{\boldsymbol{\vartheta}}^{-1} = \begin{bmatrix} \mathbf{X} & \mathbf{Y} \\ \mathbf{Y}^T & \mathbf{Z} \end{bmatrix}^{-1}. \tag{56}$$

The partitioned matrix inversion [35] is applied in (56), and we have

$$\mathrm{CRLB}\left(\mathbf{u}\right) = \mathbf{X}^{-1} + \mathbf{X}^{-1}\mathbf{Y}\left(\mathbf{Z} - \mathbf{Y}^T\mathbf{X}^{-1}\mathbf{Y}\right)^{-1}\mathbf{Y}^T\mathbf{X}^{-1}. \tag{57}$$

Note that $\mathbf{X}^{-1}$ is the CRLB of the source location vector $\mathbf{u}$ when there is no sensor position and velocity errors. Hence, the second term in (57) represents the increase in CRLB in presence of sensor location errors.

The CRLB under the 2-D plane and 3-D space are illustrated as examples, respectively. In the first one, the covariance matrix of the measurement noise is $\mathbf{Q}_f = \sigma_f^2\mathbf{I}_M$, the sensor position and velocity errors covariance matrixes are $\mathbf{Q}_s = \sigma_s^2\mathrm{diag}\left\{8\mathbf{I}_{2N}, 10\mathbf{I}_{2N}, 2\mathbf{I}_{2N}\right\}$ and $\mathbf{Q}_v = 0.01\mathbf{Q}_s$. Three moving sensors $\mathbf{s}_1 = [1000, 0]^T$, $\mathbf{s}_2 = [10000, 10000]^T$, $\mathbf{s}_3 = [10000, 1000]^T$ with velocities $\mathbf{v}_1 = [300, 0]^T$, $\mathbf{v}_2 = [-300, 0]^T$, $\mathbf{v}_3 = [0, 300]^T$ are set to locate the source. The true position of the source is $\mathbf{u} = [6500, 4000]^T$. In the second scenario, the sensors' positions are $\mathbf{s}_1 = [1000, 0, 0]^T$, $\mathbf{s}_2 = [10000, 10000, 0]^T$, $\mathbf{s}_3 = [0, 1000, 1000]^T$ with velocities $\mathbf{v}_1 = [250, 100, 200]^T$, $\mathbf{v}_2 = [-200, 100, 100]^T$, $\mathbf{v}_3 = [100, -50, 200]^T$. The noise covariance matrixes of the measurements and sensor locations are $\mathbf{Q}_f = \sigma_f^2\mathbf{I}_M$, $\mathbf{Q}_s = \sigma_s^2\mathrm{diag}\left\{8\mathbf{I}_{3N}, 30\mathbf{I}_{3N}, 2\mathbf{I}_{3N}\right\}$ and $\mathbf{Q}_v = 0.01\mathbf{Q}_s$.

The source is placed at $\mathbf{u} = [6500, 4000, 2000]^T$. Each sensor observes the emitted signals 10 times in both scenarios.

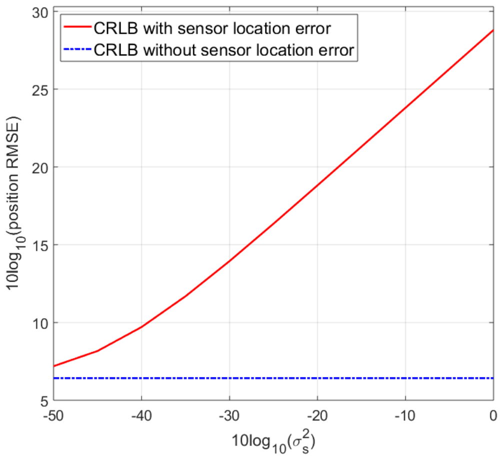

FIGURE 1. **Comparison of the CRLBs with and without sensor location errors in 2-D plane when the σ_s^2 increases.**

Fig. 1 shows the comparison of CRLBs with and without sensor location errors in 2-D plane. In this simulation, we fix the measurement noise variance at $\sigma_f^2 = -4$ dB. It can be seen that when the sensor location noise σ_s^2 increases, the estimation accuracy goes away from the noise-free CRLB. Fig. 2 shows a similar comparison result of CRLBs in 3-D space.

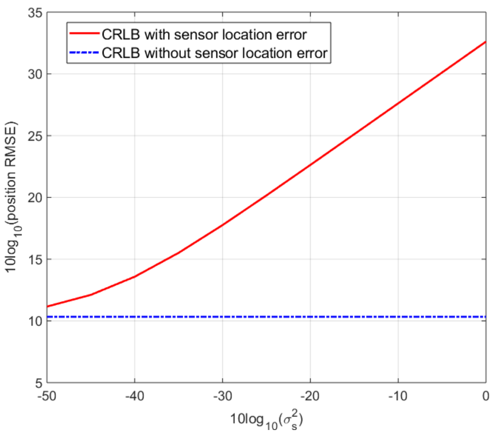

FIGURE 2. **Comparison of the CRLBs with and without sensor location errors in 3-D space when the σ_s^2 increases.**

In Fig. 3 and Fig. 4, we plot CRLBs with the increasing of σ_f^2 in 2-D and 3-D scenarios, the variance of sensor location errors is set to $\sigma_s^2 = -5$ dB. The simulation results show that when the measurement noise level is low, the sensor location noise is the main factor affecting the positioning accuracy. However, when the measurement noise level is high, it plays a major role in the localization accuracy.

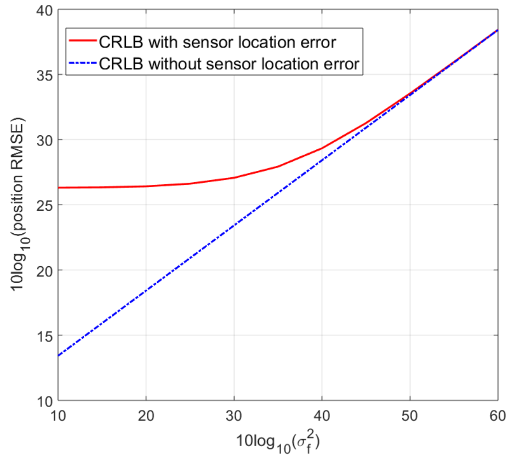

FIGURE 3. **Comparison of the CRLBs with and without sensor location errors in 2-D plane when the σ_f^2 increases.**

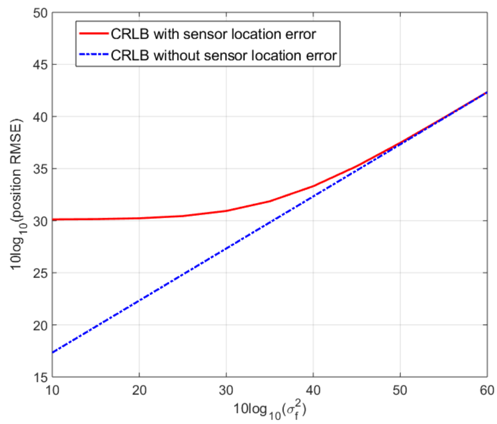

FIGURE 4. **Comparison of the CRLBs with and without sensor location errors in 3-D space when the σ_f^2 increases.**

IV. SIMULATIONS

In this section, we carry out the simulations to evaluate the performance of the proposed methods and compare it with the previous works (which didn't take the sensor location errors into consideration) [12], [16], [20] and CRLB. For convenience, we will mark these methods as DPD solution, TB solution and SDR solution in the simulation result figures. The location scenarios are corresponding to the above Subsection C. The solver SDPT3 in MATLAB CVX toolbox can be used to deal with the SDP problems [32]. the root mean squares error (RMSE) can be used to calculate the estimation accuracy

$$RMSE = \sqrt{\left(\sum_{k=1}^{K} \left\|\hat{\mathbf{u}}_k - \mathbf{u}\right\|^2\right)/K}, \tag{58}$$

where the number of the Monte Carlo run is $K = 1000$ and $\hat{\mathbf{u}}_k$ is the estimate of $\mathbf{u}$ at the k-th run. The total of the independent ensemble runs is $K = 1000$.

Fig. 5 and Fig. 6 show the estimation performance of the proposed methods, in the 2-D plane and 3-D space, respectively. It is straightforward to observe that by taking

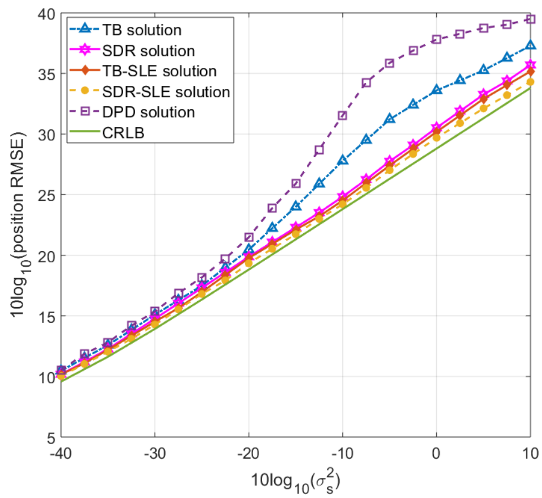

FIGURE 5. Comparisons of the CRLB with and source location accuracy in presence of sensors location errors in 2-D plane.

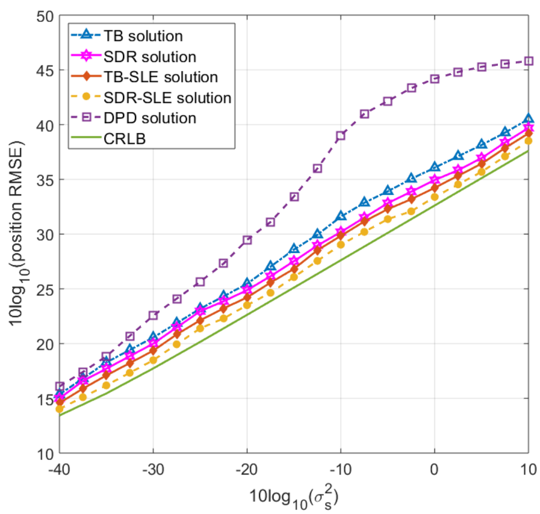

FIGURE 6. Comparisons of the CRLB with and source location accuracy in presence of sensors location errors in 3-D space.

into consideration of the sensor location errors, both of the proposed methods perform better than the localization algorithms which do not do anything with the sensor location errors. Specifically, when the noise of sensor location errors is much smaller compared to the measurement noise (which means that the sensor location errors are not the dominator of localization accuracy), all the compared algorithms perform similarly. Along with the sensor location errors increase, the algorithms without considering the sensor location errors show more performance decrease compared to the proposed algorithms. Furthermore, the DPD method is more sensitive to the sensor location errors compared to other algorithms. It should be noted that, the SDR-SLE solution shows a better positioning performance to the TB-SLE solution, due to the fact that the proposed TB-SLE solution neglects the second-order error terms twice.

V. CONCLUSION

The source localization problem based on the DFS measurements with sensor location errors is discussed. With the errors of both the location and measurements taken into account, a TB-SLE solution and an SDR-SLE solution are proposed. The former solution employs two BLUEs successively to produce an explicit estimate of the source position, and the latter one relaxes the original MLE into a convex SDP problem through the SDR technique. The simulation results show that the performance of the proposed methods is more accurate than that of the methods without the consideration of the location errors, and also confirm the validity of the previous theoretical analysis. The potential future work would be exploring the localization of multiple sources with the DFS measurements in presence of sensor location errors.

APPENDIX A
DERIVATION OF THE CRLB

The appendix provides the partial derivatives in (53)-(55). We rewrite matrix $\mathbf{Y}$ to a block matrix as

$$\mathbf{Y} = [\mathbf{Y}_1, \mathbf{Y}_2], \tag{59}$$

where

$$\mathbf{Y}_1 = -\mathrm{E}\left[\frac{\partial^2 \ln f\left(\hat{\boldsymbol{\psi}}\middle|\boldsymbol{\vartheta}\right)}{\partial \mathbf{u}\partial \mathbf{s}^T}\right] = \left(\frac{\partial \mathbf{f}}{\partial \mathbf{u}}\right)^T \mathbf{Q}_f^{-1}\left(\frac{\partial \mathbf{f}}{\partial \mathbf{s}}\right), \tag{60}$$

$$\mathbf{Y}_2 = -\mathrm{E}\left[\frac{\partial^2 \ln f\left(\hat{\boldsymbol{\psi}}\middle|\boldsymbol{\vartheta}\right)}{\partial \mathbf{u}\partial \mathbf{v}^T}\right] = \left(\frac{\partial \mathbf{f}}{\partial \mathbf{u}}\right)^T \mathbf{Q}_f^{-1}\left(\frac{\partial \mathbf{f}}{\partial \mathbf{v}}\right), \tag{61}$$

matrix $\mathbf{Z}$ is rewritten as

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_1 & \mathbf{Z}_2 \\ \mathbf{Z}_2^T & \mathbf{Z}_3 \end{bmatrix}, \tag{62}$$

where

$$\mathbf{Z}_1 = -\mathrm{E}\left[\frac{\partial^2 \ln f\left(\hat{\boldsymbol{\psi}}\middle|\boldsymbol{\vartheta}\right)}{\partial \mathbf{s}\partial \mathbf{s}^T}\right] = \left(\frac{\partial \mathbf{f}}{\partial \mathbf{s}}\right)^T \mathbf{Q}_f^{-1}\left(\frac{\partial \mathbf{f}}{\partial \mathbf{s}}\right) + \mathbf{Q}_s^{-1}, \tag{63}$$

$$\mathbf{Z}_2 = -\mathrm{E}\left[\frac{\partial^2 \ln f\left(\hat{\boldsymbol{\psi}}\middle|\boldsymbol{\vartheta}\right)}{\partial \mathbf{s}\partial \mathbf{v}^T}\right] = \left(\frac{\partial \mathbf{f}}{\partial \mathbf{s}}\right)^T \mathbf{Q}_f^{-1}\left(\frac{\partial \mathbf{f}}{\partial \mathbf{v}}\right), \tag{64}$$

$$\mathbf{Z}_3 = -\mathrm{E}\left[\frac{\partial^2 \ln f\left(\hat{\boldsymbol{\psi}}\middle|\boldsymbol{\vartheta}\right)}{\partial \mathbf{v}\partial \mathbf{v}^T}\right] = \left(\frac{\partial \mathbf{f}}{\partial \mathbf{v}}\right)^T \mathbf{Q}_f^{-1}\left(\frac{\partial \mathbf{f}}{\partial \mathbf{v}}\right) + \mathbf{Q}_v^{-1}. \tag{65}$$

The partial derivatives are expressed in (66)-(68), as shown at the top of the next page.

$$\frac{\partial \mathbf{f}^T}{\partial \mathbf{u}} = \left[\frac{(\mathbf{u}-\mathbf{s}_1)(\mathbf{u}-\mathbf{s}_1)^T\mathbf{v}_1}{r_1^3} - \frac{\mathbf{v}_1}{r_1} \quad \cdots \quad \frac{(\mathbf{u}-\mathbf{s}_M)(\mathbf{u}-\mathbf{s}_M)^T\mathbf{v}_M}{r_M^3} - \frac{\mathbf{v}_M}{r_M} \right] \tag{66}$$

$$\frac{\partial \mathbf{f}^T}{\partial \mathbf{s}} = \mathrm{diag}\left\{ \left[-\frac{(\mathbf{u}-\mathbf{s}_1)(\mathbf{u}-\mathbf{s}_1)^T\mathbf{v}_1}{r_1^3} + \frac{\mathbf{v}_1}{r_1} \quad \cdots \quad -\frac{(\mathbf{u}-\mathbf{s}_M)(\mathbf{u}-\mathbf{s}_M)^T\mathbf{v}_M}{r_M^3} + \frac{\mathbf{v}_M}{r_M} \right] \right\} \tag{67}$$

$$\frac{\partial \mathbf{f}^T}{\partial \mathbf{v}} = \mathrm{diag}\left\{ \left[\frac{(\mathbf{u}-\mathbf{s}_1)}{r_1} \quad \frac{(\mathbf{u}-\mathbf{s}_2)}{r_2} \quad \cdots \quad \frac{(\mathbf{u}-\mathbf{s}_M)}{r_M} \right] \right\} \tag{68}$$

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