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Industrial Internet of Things-Based Prognostic Health Management: A Mean-Field Stochastic Game Approach

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ABSTRACT Recent advances in industrial Internet of Things (IIoT) have dramatically leveraged prognostic health management for industrial systems. Indeed, the cognitive and communication capabilities of IIoT empower their integration in the industrial systems maintenance workflow to ease the transition toward industry 4.0. In this paper, we study a mean field stochastic game for IIoT-based CBM of industrial facilities formulated to favor grouped maintenance for cost reduction. We provide an analytical analysis of the proposed game to characterize its equilibrium operating point: mean-field equilibrium (MFE). We design a learning algorithm to reach the MFE based on a local adjustment of the maintenance rate and the global health state distribution of the monitored components. Numerical evaluation validates the proposed game and ensures maintaining a high fraction of the components in a healthy state by acting on preventive and corrective replacement rates.

INDEX TERMS Prognostic health management, industrial Internet of Things, mean-field stochastic games, mean-field equilibrium, H-learning, Markov chain.

I. INTRODUCTION

Prognostic Health Management (PHM) for maintenance of industrial systems has attracted great research efforts from the reliability community [22]–[24]. Their efforts cover health state monitoring and prediction of industrial systems, deterioration processes modeling and maintenance procedures proposal [11], [17], [18]. The PHM aim is to spare the industrial system operators from incurring the consequences of failures. By providing alarms of eminent failures enough time before their occurrence, operators can set effective preventive and corrective maintenance operations. Industrial maintenance is formed by three major categories: Breakdown Maintenance (BM), Time-based Preventive Maintenance (TPM) and Condition Based Maintenance (CBM). While BM is realized on systems upon failure occurrence, the PTM and CBM try to operate on systems to expect and eventually prevent their failure [20]. Time-based preventive maintenance operates by setting periodic deadlines for maintenance operations. To avoid scheduling unnecessary maintenance operations, CBM [19], [21] plans maintenance operations according to the monitored components health state. CBM can detect early signs of potential failure of components according to a developed predictive model combined with inspections. Therefore, it reduces maintenance cost and industrial systems down times incurred from maintenance operations or failure and increases production rates.

Industrial Internet of Things (IIoT) [25]–[28] exhibits promising opportunities to develop novel applications around powerful industrial systems. Indeed, breakthroughs in key enabling technologies: wireless communications, sensor devices and hardware miniaturization have enlarged IIoT domains of application. For instance, a wide range of IIoT applications have been proposed and deployed in several industrial fields such as transportation, logistics, robotics and infrastructure assets monitoring.

IIoT-based CBM [9] is an emerging research field with a vast potential of industrial applications as discussed in [9], [12], and [13]. Ubiquity, sensing, interconnection and data fusion are inherent characteristics that empower IoT integration to existing CBM frameworks for industrial systems. In Condition Based Maintenance, deployed IIoT allow automated periodic (respectively on-demand) monitoring of systems and their health states reporting. Thus, developed preventive models could be updated and correct maintenance operations conducted.

Xanthopoulos *et al.* [16] address the problem of maintaining high service level while carrying as low inventory as possible in a deteriorating manufacturing system. At every time epoch the agent controlling the manufacturing facility has three possible decisions, namely: produce, maintain or remain idle. The formulated problem is solved through a Reinforcement Learning (RL) based approach. The proposed method is compared to standard production and maintenance policies. Active and preventive maintenance are investigated in [6]. The proposed approach combines the collection of manufacturing process bigdata off-line prediction with a method based on a neural network that computes the components lifetime under specific processing conditions. Yan *et al.* [7], based on device electrocardiogram and deep learning, propose a Remaining Useful Life (RUL) prediction algorithm. The proposed approach reduces the dependence w.r.t experts' decisions by incorporating Artificial Intelligence (IA) in the prediction loop.

Feng *et al.* [14] study an aircraft fleet maintenance planning problem for which they developed a two-stage dynamic decision-making model. To reduce the problem scale, they divided the fleet into dispatch and standby sets and formulated a heuristic hybrid game to investigate it.

In [15], two operators are making prognostics-based replacement decisions for identical systems with common manufacturer. They are involved in a strategic decision-making process to acquire the needed replacement parts from two suppliers. The studied problem is cast into a hierarchical game framework with one of the operators acting as a leader and the other serving as a follower.

PHM for aircraft power generators using Kalman filtering is investigated in [10]. The authors develop a state estimator such that model parameters are updated sequentially when new observations are available.

In this paper, we address the problem of setting optimal preventive and corrective maintenance rates for components of an industrial facility. To achieve these objectives, we rely on an IIoT to monitor the health state of each component and provide a global health statistics of the facility to its operators. Thus, maintenance operations could be planned and eventually grouped to reduce the costs. For industrial facilities with a considerable number of components, operating on a per-component basis is not optimal. We instead take advantage of the similarities between components regarding their deterioration process and need for preventive/corrective replacement. The components are competing against each other to undergo maintenance operations as quickly as possible to counteract the deterioration process effects. Meanwhile, to reduce their maintenance costs, delaying the maintenance time to group with others ensures cost sharing. Therefore, a balance or equilibrium has to be established.

We notice first that the rewards and costs of each component undergoing a maintenance operation do not depend solely on its actions; maintenance rates, but on the choices of other components as well. Besides, no inter-components cooperation is taking place while choosing the maintenance rates. Finally, the number of components to be monitored could evolve easily from hundreds to several thousand depending on the industrial plant size. Therefore, the IIoT-based CBM fits perfectly within the Mean-Field Game (MFG) framework [8]. One fundamental assumption in Mean-Field Game theory is the indistinguishability per class of the agents. This means that agents (players) could be assumed to have similar behavior (controls) within the same class. The other assumption states that the influence of a single agent on the large coupled interaction system is negligible. These assumptions lead to a more tractable decision problem described from a generic agent point of view with a generic control.

Our contribution can be summarized as follows: We provide a rigorous description of the microscopic transition to microscopic level for the IIoT-based CBM problem. The latter is formulated as a mean-field stochastic game (MFSG) between industrial facility components subject of a Markovian deterioration process. Then, we characterize the mean field limit as a solution of a Kolmogorov forward equation. For each component, an individual dynamic optimization problem whose objective is to maximize its expected finite horizon payoff subject to own stochastic dynamics and mean field limit is proposed. This formulation leads to a coupled system of forward-backward Partial Differential Equations (PDEs). The existence, uniqueness and computation of Mean-Field Equilibria (MFE) solution to the MFSG, as well as the H-learning algorithm for reaching them, are discussed. To the best of our knowledge, this study is among the first to formulate the IIoT-based CBM as a mean field stochastic game.

The remainder of this paper is organized as follows. In Section II, we state the IIoT-based prognostic management problem in industrial facilities. In Section III, we develop the mean-field game model of the studied problem and provide existence and uniqueness results for the game MFE. We also provide a Learning algorithm for the MFE. Numerical investigation results are presented and discussed in Section IV. Finally, in Section V, a brief conclusion and envisions of future development for MFG-based CBM are provided.

II. PROBLEM STATEMENT

We consider a network formed by $\mathcal{N} = \{1, \ldots, n\}$ devices that form an IIoT assisting the CBM operations of components within an industrial facility. Each device monitors the health state of a component whose state evolves according to a Markov chain composed of three states: Healthy(H), Failing(F) and Dead(D). When the component is fully operational (state H), no maintenance is authorized. Due to various endogenous and exogenous factors, the components become faulty with probability ρ. In the failing state (F), the component is still functioning but not fully operational, only preventive maintenance (action m) is allowed and brings the component back to its healthy state H. If no preventive maintenance (action $\overline{m}$) is realized, the component becomes dead (state D) with probability ν. In this state, only corrective maintenance (m) could bring the component back to its healthy state. The preventive (respectively corrective) maintenance occurs with probability $\delta(t)$ (respectively $\eta(t)$). The associated costs are C_{PR} and C_{CR} respectively. Taking into consideration the fact that the corrective maintenance is more costly than the preventive one, we further impose the following constraint: $C_{pr} < C_{cr}$. We are interested in modeling the IIoT-based CBM problem over a finite time interval $[0, t_f]$.

III. MFG MODEL FOR IIoT-BASED CBM

Let us consider that the IIoT serving as a backbone for the PHM is very dense ($n \to \infty$). The components seen as players are trying to share their maintenance costs by grouping maintenance operation with one another. Unfortunately, since the deterioration processes are independent, at a given time epoch, components have different health states. Therefore, for some components affording a maintenance delay is not an option. Consequently, each component aims to balance its need for maintenance with its interest in reducing the associated costs by grouping with other ones to undergo joint maintenance operations.

Given a particular component, referred to as the generic component, we model its health state evolution with a three-state Markov chain as depicted in FIGURE 1.

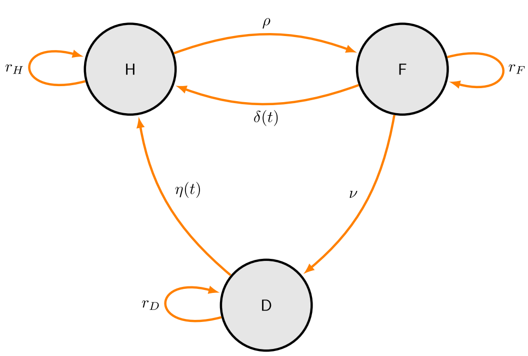

FIGURE 1. Markov chain representation: the parameters r_i are the complement of the other transitions.

Denote by $D(t)$, $F(t)$ and $H(t)$ the number of components in states D, F and H respectively. The health state repartition of components at time epoch t respects the following equality constraint: $n = D(t) + F(t) + H(t)$. Furthermore, their occupancy measure corresponding to the frequency vector of the states is denoted by:

TABLE 1. Probabilities, states ($H(t), D(t), F(t)$), actions.

Case	Transition probability	$\mathbf{M}^n(t+1) - \mathbf{M}^n(t)$	Actions
$H \xrightarrow{\rho} F$	$H^n(t) \times \rho$	$(0, \frac{1}{n}, -\frac{1}{n})$	$\emptyset$
$F \xrightarrow{\delta} H$	$F^n(t) \times \delta(t)$	$(0, -\frac{1}{n}, \frac{1}{n})$	$\{m, \overline{m}\}$
$F \xrightarrow{\nu} D$	$F^n(t) \times \nu$	$(\frac{1}{n}, -\frac{1}{n}, 0)$	$\emptyset$
$D \xrightarrow{\eta} H$	$D^n(t) \times \eta(t)$	$(-\frac{1}{n}, 0, \frac{1}{n})$	$\{m, \overline{m}\}$

$$\mathbf{M}^n(t) = \begin{pmatrix} \frac{D(t)}{n} \\ \frac{F(t)}{n} \\ \frac{H(t)}{n} \end{pmatrix} \doteq \begin{pmatrix} D^n(t) \\ F^n(t) \\ H^n(t) \end{pmatrix} \tag{1}$$

TABLE 1 summarizes the impact of each individual component state change on the per-epoch evolution of the occupancy measure $\mathbf{M}^n(t)$.

Given an occupancy measure $\mathbf{M} = \begin{pmatrix} d \\ f \\ h \end{pmatrix}$, the expected change of $\mathbf{M}^n$ in one time epoch, called drift is:

$$\phi^n(\mathbf{M}) = n\mathbb{E}(\mathbf{M}^n(t+1) - \mathbf{M}^n(t) | \mathbf{M}^n(t) = \mathbf{M}). \tag{2}$$

Then, its limit $\phi(\mathbf{M})$ for a highly dense IIoT is described by the following system of ordinary differential equations:

$$\phi(\mathbf{M}) = \begin{pmatrix} \dot{d} \\ \dot{f} \\ \dot{h} \end{pmatrix} = \begin{pmatrix} f\nu - \eta(t)d \\ h\rho - f(\delta(t) + \nu) \\ -h\rho + f\delta(t) + \eta(t)d \end{pmatrix} \tag{3}$$

The system simplifies further by noticing that:

$$\forall t \in [0, t_f],\ h(t) = 1 - f(t) + d(t).$$

In the remainder of the paper and without loss of generality, we restrict our analysis to $\mathbf{M}$ first two components denoted by $\mathbf{m} = \begin{pmatrix} d \\ f \end{pmatrix}$ with the following evolution dynamics:

$$\phi(\mathbf{m}) = \begin{pmatrix} \dot{d} \\ \dot{f} \end{pmatrix} = \begin{pmatrix} f\nu - \eta d \\ \rho - f(\delta + \nu + \rho) - d\rho \end{pmatrix} \tag{4}$$

We aim to minimize the proportion of components in states F and D by mean of control $\mathbf{u}^i(t) = (\delta^i(t), \eta^i(t)) \in \mathcal{U} = [0, 1]^2$ while encouraging grouped preventive and corrective maintenance. This will naturally lead to maximizing the proportion of healthy components while reducing maintenance costs.

Let us define the per-epoch maintenance cost for the generic component as follows:

$$g(\mathbf{m}(t), \mathbf{u}(t), t) = \frac{\delta(t)^2 + \eta(t)^2}{2} + (1 - f(t)\delta(t))C_{pr} + (1 - d(t)\eta(t))C_{cr} \tag{5}$$

Notice that by MFG assumptions stated above we drop the player index i and consider a generic one. Also, costs of grouped preventive (respectively corrective) maintenance are reduced by a factor of $(1 - \delta(t)f(t))$ (respectively $(1-\eta(t)d(t))$). The intuition behind this formulation is the following: the greater the number of components to be replaced is ($f(t) \nearrow$ and/or $d(t) \nearrow$), the more the maintenance costs will decrease.

We also express the objective functional to be minimized cooperatively as follows:

$$\mathcal{J}(\mathbf{u}(t), \mathbf{m}(t)) = \int_0^{t_f} g(\mathbf{m}(t), \mathbf{u}(t), t)dt. \tag{6}$$

Notice that the utility of each component $\mathcal{J}(\mathbf{u}(t), \mathbf{m}(t))$ is influenced by its action $\mathbf{u}(t)$ and by the mean field limit of the state distribution $\mathbf{m}(t)$ of other components. This mean field interaction leads naturally to a mean field stochastic game formulation.

Therefore, for a given occupancy measure $\mathbf{m} = \begin{pmatrix} d \\ f \end{pmatrix}$, the mean-field stochastic game is defined by the following coupled system:

$$\begin{cases} \min\limits_{\mathbf{u}(t)\in\mathcal{U}} \mathcal{J}(\mathbf{u}(t), \mathbf{m}(t)) \\ \dot{\mathbf{m}} = \phi(\mathbf{m}(t), \mathbf{u}(t), t) \\ \mathbf{m}(0) = \mathbf{m}_0 \end{cases} \tag{7}$$

Let us denote by $v(\mathbf{m}(t), t) = \int_0^{t_f} g(\mathbf{m}(t), \mathbf{u}(t), t)$ the generic component value function to be minimized. Taking into account the key MFG theory assumption leads to the following system of coupled backward Hamilton-Jacobi-Bellman and forward Fokker-Planck equations:

$$\begin{cases} -\partial_t v(t, \mathbf{m}(t)) = \min\limits_{\mathbf{u}(t)\in\mathcal{U}} g(\mathbf{m}(t), \mathbf{u}(t), t) \\ \quad + < \phi(\mathbf{m}(t)), \nabla v(t, \mathbf{m}(t)) > \\ \dot{\mathbf{m}} = \phi(\mathbf{m}(t), \mathbf{u}(t), t) \\ v(t_f, m) = 0, \ \mathbf{m}(0) = \mathbf{m}_0 \end{cases} \tag{8}$$

A. DYNAMICAL SYSTEM ANALYSIS

We will investigate the existence and uniqueness of solutions of the dynamical system $\dot{\mathbf{m}} = \phi(\mathbf{m})$ describing components states occupancy measure evolution. The main results are stated in **Theorem 1**.

Theorem 1: The controlled system $\dot{\mathbf{m}}(t) = \phi(\mathbf{m})$ that satisfies the initial conditions $\mathbf{m}(0) = \mathbf{m_0}$ has a unique solution.

Proof: Let $\mathbf{m} = \begin{pmatrix} d(t) \\ f(t) \end{pmatrix}$ and $\partial(\mathbf{m}) = \begin{pmatrix} \frac{d\,d(t)}{dt} \\ \frac{d\,f(t)}{dt} \end{pmatrix}$, so the state dynamics are rewriting in the following form:

$$\partial(\mathbf{m}) = A\,\mathbf{m} + b. \tag{9}$$

Where

$$A = \begin{pmatrix} -\eta(t) & \nu \\ -\rho & -\nu - \delta(t) - \rho \end{pmatrix} \text{ and } b = \begin{pmatrix} 0 \\ \rho \end{pmatrix}.$$

Then, given two states m^1 and m^2 the following inequality holds:

$$\|\partial(m^1) - \partial(m^2)\| \le \|A\|.\|m^1 - m^2\| \tag{10}$$

Thus, it follows that the function ∂ is uniformly Lipschitz continuous. Therefore, from the definition of the control $\mathbf{u}(t)$ and the restriction on $0 \le d(t) \le 1$ and $0 \le f(t) \le 1$. By [5], we realize that a unique solution of the controlled system $\dot{\mathbf{m}} = \phi(\mathbf{m})$ exists. □

We are interested in finding the equilibrium points of the dynamical system described by (4). Thus, for any MFE $\mathbf{u}^*$ solution to the MFSG (7), we solve the system $\phi(\mathbf{m}) = \mathbf{0}$. The unique rest point of the system is given by:

$$d = \frac{\nu\,\rho}{(\delta^* + \eta^* + \rho)\eta^* + \nu\,\rho}$$

and

$$f = \frac{\rho\,\eta^*}{(\delta^* + \nu + \rho)\eta^* + \nu\,\rho}$$

To investigate the stability of the system we compute its Jacobian matrix:

$$\mathbb{J}(\mathbf{u}^*) = \begin{bmatrix} -\eta^* & 0 \\ -\rho & -\delta^* - \nu - \rho \end{bmatrix}$$

Since $Tr(\mathbb{J}) = -(\eta^* + \delta^* + \nu + \rho) < 0$, we conclude that the system is stable. As depicted by FIGURE 2, starting from the initial occupancy measure $\mathbf{m}(0) = (0, 0)$, corresponding to all monitored components being in the state H, the system converges to its equilibrium point $\mathbf{m}(200) = (0.16, 0.24)$ after 200 time epochs.

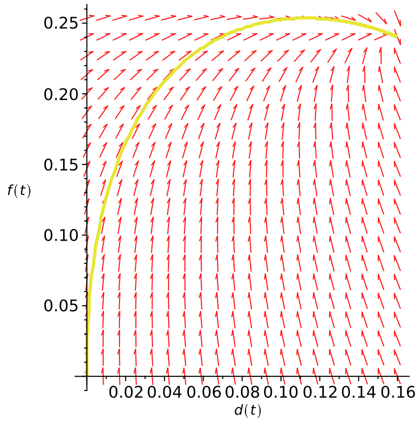

FIGURE 2. System $\dot{\mathbf{m}} = \phi(\mathbf{m})$ trajectory Simulation $\mathbf{u}^* = (0.3, 0.3)$, $\rho = 0.2$, $\nu = 0.2$.

Let us consider the augmented Hamiltonian $\mathcal{H}$ with penalty terms for the control constraints given by:

$$\mathcal{H}(\mathbf{m}(t), \mathbf{u}(t), \lambda(t), \mathbf{w}(t), t) = g(\mathbf{m}(t), \mathbf{u}(t), t) + \lambda.\dot{\mathbf{m}} \\ - \mathbf{w}(t). \begin{pmatrix} \delta(t) \\ (1 - \delta(t)) \\ \eta(t) \\ (1 - \eta(t)) \end{pmatrix} \tag{11}$$

where $\mathbf{w} = (w_1(t), w_2(t), w_3(t), w_4(t))$ is a vector of component wise positive penalty multipliers satisfying for $\mathbf{u}^*$ the following equality constraint:

$$\mathbf{w}. \begin{pmatrix} \delta^*(t) \\ (1 - \delta^*(t)) \\ \eta^*(t) \\ (1 - \eta^*(t)) \end{pmatrix} = 0 \tag{12}$$

B. EXISTENCE & CHARACTERIZATION OF THE MEAN-FIELD EQUILIBRIUM

Theorem 2: Consider the Mean-Field Game (7). There exists a mean-field equilibrium $\mathbf{u}^\star(\mathbf{t}) = (\delta^\star(t), \eta^\star(t)) \in \mathcal{U}$ *such that* $\mathcal{J}(\mathbf{u}^\star) = \min_{\mathbf{u}\in\mathcal{U}} \mathcal{J}(\mathbf{u})$*. Also, there exists a vector of adjoint variables* $\lambda(t) = \{\lambda_i\}_{i\in\{1,2\}}$ *satisfying the following:*

$$\begin{aligned} -\dot{\lambda_1} &= \frac{\partial \mathcal{H}(\mathbf{m}(t), \mathbf{u}(t), \lambda(t), \mathbf{w}(t), t)}{\partial d} \\ -\dot{\lambda_2} &= \frac{\partial \mathcal{H}(\mathbf{m}(t), \mathbf{u}(t), \lambda(t), \mathbf{w}(t), t)}{\partial f} \end{aligned} \tag{13}$$

with boundary conditions: $\forall i \in \{1, 2\}$, $\lambda_i(t_f) = 0$ *and* $f(0) = f_0, d(0) = d_0$*. Further,* $\mathbf{u}^*(t) = (\delta^*(t), \eta^*(t))$ *can be represented by:*

$$\delta^*(t) = \max\{0, \min\{1, f(t) \times (C_{pr} + \lambda_2(t))\}\} \tag{14}$$

and

$$\eta^*(t) = \max\{0, \min\{1, d(t) \times (C_{cr} + \lambda_1(t))\}\} \tag{15}$$

Proof: The existence of the mean-field equilibrium can be proved using a result by Fleming and Rishel [3]:

First, an existence result provided by [14, Th. 9.2.1] for the state system with bounded coefficients is invoked. Then, the control space $\mathcal{U} = \{(\delta, \eta)/\ \delta$ and η are measurable, $0 \leq \delta(t), \eta(t) \leq 1, t \in [0, T]\}$ is convex and closed by definition. We notice that the right-hand side of the state system (4) can be written as follows

$$\begin{aligned} \phi\left(\begin{pmatrix} d \\ f \end{pmatrix}\right) &= A.\begin{pmatrix} d \\ f \end{pmatrix} + b \\ &= \begin{pmatrix} -\eta & \nu \\ -\rho & -\nu - \delta - \rho \end{pmatrix}.\begin{pmatrix} d \\ f \end{pmatrix} + \begin{pmatrix} 0 \\ \rho \end{pmatrix} \end{aligned} \tag{16}$$

Let us find the upper bounds of the first part of the right hand side of equality (16).

$$\begin{aligned} \left|\begin{pmatrix} -\eta & \nu \\ -\rho & -\nu - \delta - \rho \end{pmatrix}\begin{pmatrix} d \\ f \end{pmatrix}\right| &\leq \left|\begin{pmatrix} 0 & \nu \\ 0 & 0 \end{pmatrix}\begin{pmatrix} d \\ f \end{pmatrix}\right| \\ &\leq C\left(\left|\begin{pmatrix} d \\ f \end{pmatrix}\right|\right) \end{aligned}$$

where C incorporates the upper bound of the given constant matrices: $\begin{pmatrix} 0 & \nu \\ 0 & 0 \end{pmatrix}$. Thus

$$\phi\left(\begin{pmatrix} d \\ f \end{pmatrix}\right) \leq C\left(\left|\begin{pmatrix} d \\ f \end{pmatrix}\right|\right) + |b|$$

This upper-bound is a linear combination of the control, state vectors and time. It is straightforward to notice that the integrand of the functional is convex on $\mathcal{U}$. Finally, $\exists\, c_1, c_2 > 0$, $\beta > 1$ such as $g(\mathbf{m}(t), \mathbf{u}(t), t) \geq c_1 \|\mathbf{u}\|^\beta - c_2$. For instance, for $c_1 = \frac{1}{2}, C_{pr} + C_{cr}$ and $\beta = 2$ the previous inequality holds.

The conditions of [3, Th. III.4.1] are satisfied. Then, we conclude that there exists a mean-field equilibrium for the MFSG (7).

Next, the maximum principle gives existence of the adjoint variables satisfying (13). To complete the representation for $\mathbf{u}^*$ we analyze the optimality conditions. The mean-field equilibrium $\mathbf{u}^*$ and system state trajectory is obtained by solving the following system:

$$\begin{cases} \dfrac{\partial \mathcal{H}}{\partial \delta} = 0 \\ \dfrac{\partial \mathcal{H}}{\partial \eta} = 0 \end{cases} \Leftrightarrow \begin{cases} \delta^* = f(t) \times (C_{pr} + \lambda_2(t)) + w_1 - w_2 \\ \eta^* = d(t) \times (C_{cr} + \lambda_1(t)) + w_3 - w_4 \end{cases}$$

For the controls δ we consider three cases:

1) On the set $\{t|0 < \delta^*(t) < 1\}$, $w_1(t) = 0 = w_2(t)$. Hence the mean-field equilibrium is:
$$\delta^*(t) = f(t) \times (C_{pr} + \lambda_2(t))$$
2) On the set $\{t|\delta^*(t) = 1\}$, $w_1(t) = 0$. Hence the mean-field equilibrium is:
$$1 = \delta^*(t) = f(t) \times (C_{pr} + \lambda_2(t)) - w_2(t)$$
This implies that $f(t) \times (C_{pr} + \lambda_2(t)) \geq 1$ since $w_2 \geq 0$ and consequently
$$\delta^* = 1 \leq f(t) \times (C_{pr} + \lambda_2(t))$$
3) On the set $\{t|\delta^*(t) = 0\}$, $w_2(t) = 0$. Hence the mean-field equilibrium is:
$$0 = \delta^*(t) = f(t) \times (C_{pr} + \lambda_2(t)) + w_1(t)$$
This implies that $f(t) \times (C_{pr} + \lambda_2(t))$ is negative since $w_1 \geq 0$. Consequently
$$\delta^* = 0 \geq f(t) \times (C_{pr} + \lambda_2(t))$$

Combining these three cases we characterize the MFE preventive maintenance probability $\delta^*(t)$ as follows:

$$\delta^*(t) = \max\{0, \min\{1, f(t) \times (C_{pr} + \lambda_2(t))\}\}$$

Using similar arguments, we can also obtain the MFE corrective maintenance probability:

$$\eta^*(t) = \max\{0, \min\{1, d(t) \times (C_{cr} + \lambda_1(t))\}\}$$

□

C. LEARNING THE MEAN-FIELD EQUILIBRIUM

In this section we propose a learning algorithm allowing convergence to the mean-field equilibrium with mean field limit. The proposed algorithm is Learning under the Hamiltonian function namely: H-Learning [29]. Let us first introduce the H-function defined by:

$$H(\mathbf{u}(t), \mathbf{m}(t), p) = g(\mathbf{u}(t), \mathbf{m}(t)) + < p, \phi(m(t)) >$$

Algorithm 1 summarizes the H-Learning algorithm steps:

Algorithm 1 H-Learning for IIot-Based CBM

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1) Initialize $\mathbf{m_0}$
2) At iteration l use $\mathbf{m}_l$ to compute the payoff $g(\mathbf{m}_l, .)$.
3) Use the Hamilton-Jacobi-Bellman-Fleming optimality (8) to obtain the value function approximation $\hat{v}(l)$ at stage l.
4) compute the mean-field equilibrium $\mathbf{u}^*(l)$ via the H-function (Pontryagin maximum principle)
5) Use $\mathbf{u}^*(l)$ to get $\mathbf{m}_l^*$ solution of the Fokker-Planck-Kolmogorov forward equation.

Each component will try to solve the coupled PDEs given the mean-field limit of the occupancy measure and its local action $\mathbf{u}(l)$. The algorithm operates iteratively to converge to the MFE of the mean-field game.

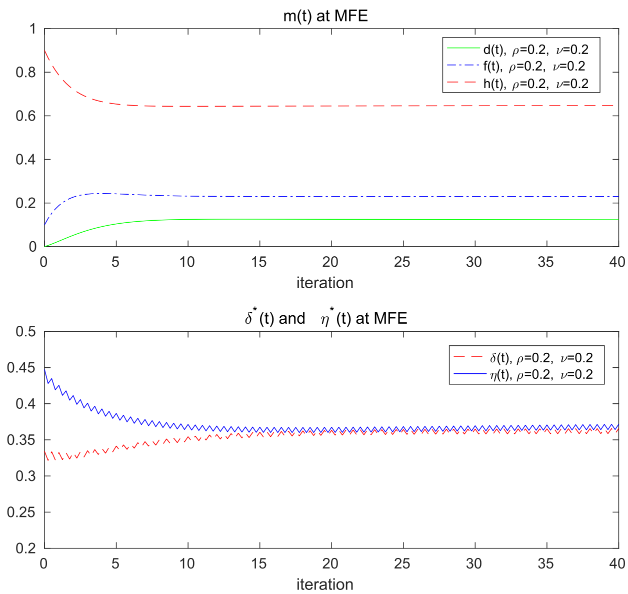

FIGURE 3. Components Health states distribution evolution & MFE Controls (NLP).

IV. NUMERICAL INVESTIGATION:

We consider the following scenario: $\rho = 0.2$, $\nu = 0.2$, $C_{pr} = 2$ and $C_{cr} = 4$, $t_f = 40$ time epochs, $\mathbf{m(0)} = (0, 0.1, 0.9)$. To derive a solution for the MFG and compute the Mean-Field Equilibrium (MFE) one must deal with solving HBJ-FPK PDEs depending on predefined boundary conditions. We proceed firstly by a direct approach [2] that transforms the infinite dimensional problem (7) into a finite dimensional Non Linear Optimization Problem (NLP). FIGURE 3 illustrates the maintenance policies at the MFE along with the components health state occupancy measure evolution. Starting from the initial occupancy measure $\mathbf{m}(0)$ the components states converges after 15 iteration to the the occupancy measure $\mathbf{m}(15) = (0.1, 0.2, 0.6)$ with the MFE $\mathbf{u}(15) = (\delta(15), \eta(15)) = (0.36, 0.36)$.

The previous scenario is investigated by letting each component learn its MFE using the H-Learning algorithm. As depicted by FIGURE 4, The learned equilibrium controls and components health state occupancy measure evolution matches the previously obtained results based on the direct approach.

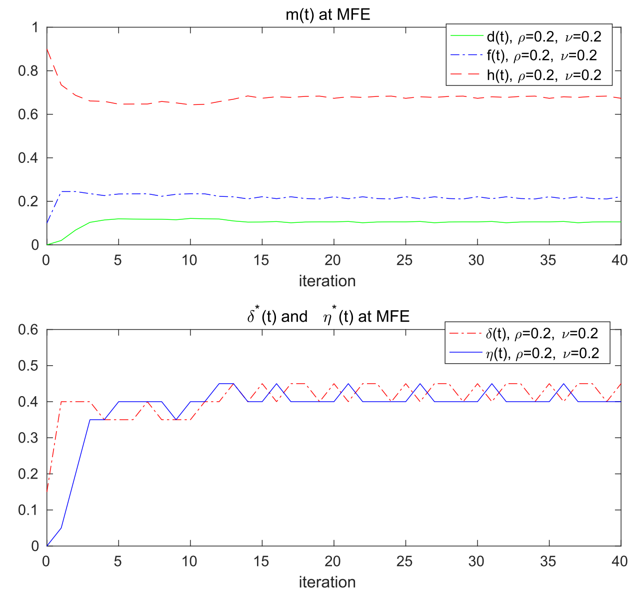

FIGURE 4. Components Health states distribution evolution & MFE Controls (H-Learning).

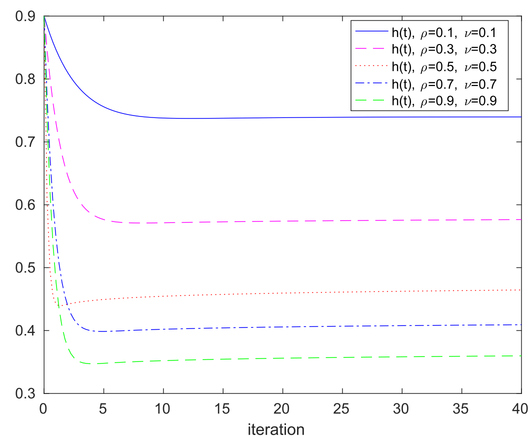

FIGURE 5. ***h(t)*** **at MFE for different** ρ **and** ν **values.**

To study the impact of the component's uncontrolled on state transition probabilities(respectively ρ and ν) on the health state occupancy measure, we compute the MFE for different parameters values. The corresponding components health state distribution and MFE controls are given in FIGURE 5 and FIGURE 6 respectively. It can be noticed that as deterioration probabilities increase, components preventive and corrective maintenance probabilities also increase. Thus, the equilibrium policy tries to bring as much as possible components to their healthy state by increasing the maintenance probabilities.

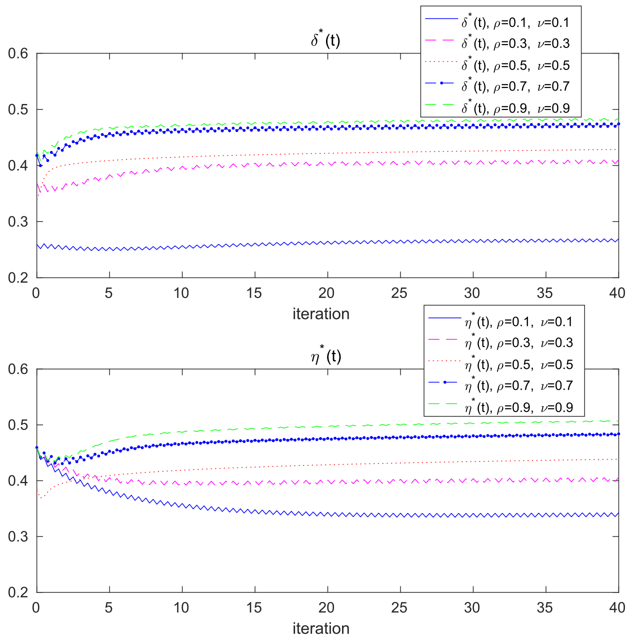

FIGURE 6. $\delta^*(t)$ and $\eta^*(t)$ at MFE for different ρ and ν values.

V. CONCLUSION

In this paper, we proposed a Mean-Field Stochastic game for IIoT-based CBM in Industrial facilities. Leveraging the frameworks of MFG and learning theories, we analytically characterized the equilibrium preventive and corrective maintenance rates. The proposed game allows keeping a considerable fraction of the monitored components in a healthy state even under severe deterioration probabilities while favoring grouped maintenance for cost reduction. Future work will generalize the modeling of the deterioration process of components with various deterioration probabilities.

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