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RSS-Based Cooperative Localization in Wireless Sensor Networks via Second-Order Cone Relaxation

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ABSTRACT In this paper, we consider the received signal strength-based cooperative localization problem in both known and unknown transmit power of target nodes. For the case of known transmit power, we treat the transmit power as a constant and derive a novel non-convex weighted least squares estimator which can be transformed into a second-order cone programming (SOCP) problem for reaching an efficient solution. For the case of unknown transmit power, we treat the transmit power as an additional unknown parameter and propose a hybrid maximum likelihood-SOCP algorithm to alternatively estimate the target node locations and transmit power. Simulation results confirm the effectiveness of the proposed methods in all considered settings.

INDEX TERMS Wireless sensor networks (WSNs), cooperative localization, received signal strength (RSS), second-order cone programming (SOCP), weighted least squares (WLS).

I. INTRODUCTION

Recently, wireless sensor networks (WSNs) attract more and more attention due to their wide applications in many fields like health-care monitoring, data networking, military applications, public safety and communications [1]. Target localization is one of the main tasks in WSNs since most network activities require the location information of network nodes. Generally, the locations of a number of sensor network nodes (called anchor nodes) are known, while the locations of some sensor network nodes (called target nodes) are unknown which need to be estimated. The main purpose of target localization is to determine the coordinates of target nodes via noisy measurements [2]. These measurements mainly include time-of-arrival (ToA) [3], [4], time-difference-of-arrival (TDoA) [5], [6], angle-of-arrival (AoA) [7], [8] and received signal strength (RSS) [9]–[15] or combinations of them [18]. Among the different types of measurements, RSS-based localization technique receives researchers' most attention thanks to its merits such as easy implementation, low-complexity and low-cost [14]. Therefore, we focus on the target localization problem using the RSS measurements information in WSNs. In order to obtain the useful information, it is necessary to enable nodes to communicate with each other. For this purpose, normally two different ways namely non-cooperative and cooperative are used. In the former case, target nodes only communicate with anchor nodes, where poor radio conditions may cause frequent link interruptions making it difficult for the traditional non-cooperative way to work effectively. While in the latter case, target nodes can communicate not only with anchor nodes as normal, but also with other target nodes within the communication range (target-target). This means that cooperative between any two nodes in WSNs can avoid frequent link interruptions and provide more information. Thus, by using cooperative localization, both estimation accuracy and robustness can be improved significantly [16]–[18].

The most popular cooperative localization method is the maximum likelihood (ML) estimator, which can provide the optimal solution asymptotically. However, solving the ML estimator of RSS-based localization problem is a difficult task, since it is highly non-convex and non-linear and can only be solved by iterative methods. In such methods, finding an appropriate initial point is crucial because a poor initialization normally leads to a poor estimation, making it difficult to converge, and unable to find the globally optimal solution. In order to guarantee the convergence of the algorithm, convex optimization techniques are extensively studied to apply on both cooperative and non-cooperative localization problems [18]. In [9], for the cooperative localization, based on the unscented transformation, RSS-based localization problem for known target transmit power case is formulated as a weighted least squares (WLS) problem, which can then be relaxed to a semi-definite programming / second-order cone programming (SD/SOCP) problem. In [13], based on convex optimization, RSS-based non-cooperative and cooperative localization problems are proposed. The authors derive new non-convex estimators, which can be relaxed to convex problems, for non-cooperative and cooperative localization problems, respectively, in both cases of known and unknown target transmit power. Furthermore, it is shown that the derived approaches work well in the case when both the target transmit power and the path loss exponent are unknown at the anchor nodes. In [16], Vaghefi *et al.* address the RSS-based cooperative localization problem when the target transmit power is time-varying and unknown. It shows that by applying a semi-definite programming (SDP) relaxation technique, the original ML localization problem can be transformed into a convex problem. In [18], by using RSS and AoA hybrid measurements, the authors address the target localization problems in both non-cooperative and cooperative 3D WSNs, for both cases of known and unknown target transmit power. It is shown that for cooperative localization, the developed estimator can be transformed into a convex problem by applying appropriate semi-definite programming relaxation techniques. Moreover, the proposed estimator for known target transmit power case can be effectively extended to the unknown target transmit power case.

In this paper, we solve the RSS-based cooperative localization problems by using convex relaxation, i.e., the second-order cone relaxation. Both cases of known and unknown target transmit power are considered. For the case of known transmit power, the transmit power is treated as a constant and a novel non-convex WLS estimator is formulated to estimate the target nodes. Then, we relax the non-convex problem by using the second-order cone relaxation technique to a SOCP problem to reach an efficient solution. For the case of unknown transmit power, the transmit power is treated as an additional unknown parameter. To cope with the complexities caused by the additional unknown parameter, we employ further approximation techniques. In addition, we also propose an effective hybrid ML-SOCP algorithm to alternatively estimate the target node locations and transmit power.

Notations: The following notations are adopted throughout the paper. Bold face lower case letters and bold face upper case letters denote the vectors and matrices, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the set of n-dimensional real column vectors and $n \times m$ real matrix. $\boldsymbol{I}_N$ denotes the $N \times N$ identity matrix. Q_{ij} denotes the (i, j)th entry of the matrix $\boldsymbol{Q}$. r_i denotes the ith entry of the vector $\boldsymbol{r}$. In addition, for any symmetric matrix $\boldsymbol{A}$, $\boldsymbol{A} \succeq 0$ means that $\boldsymbol{A}$ is positive semi-definite. $\|\cdot\|$ denote the ℓ_2-norm.

The remainder of this paper is organized as follows. In Section II, the RSS measurements model is briefly introduced and the non-convex target localization problem formulated. Section III derives the proposed methods in both known and unknown transmit power cases. In Section IV, complexity analysis is given. The corresponding CRLBs are provided in both known and unknown transmit power cases in Section V. Computer simulation results are presented in Section VI. Finally, in Section VII, the main conclusions are drawn.

II. SYSTEM MODEL

Consider a centralized WSNs with N anchor nodes and M target nodes in a two-dimensional (2D) cooperative localization scenario, where the locations of anchor nodes, noted as $\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_N$, are known, while the locations of the target nodes, noted as $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_M$, are unknown (where $\boldsymbol{s}_j$, $\boldsymbol{x}_i \in \mathbb{R}^2$, for $j = 1, 2, \ldots, N$, and $i = 1, 2, \ldots, M$). The RSS measurements for target/anchor and target/target path loss model can be denoted by [23]–[25]

$$L_{ij}^{\mathcal{A}} = L_0 + 10\gamma \log_{10} \frac{\|\boldsymbol{x}_i - \boldsymbol{s}_j\|}{d_0} + v_{ij}, \quad (i, j) \in \mathcal{A}, \tag{1a}$$

$$L_{ik}^{\mathcal{B}} = L_0 + 10\gamma \log_{10} \frac{\|\boldsymbol{x}_i - \boldsymbol{x}_k\|}{d_0} + w_{ik}, \quad (i, k) \in \mathcal{B}, \tag{1b}$$

where $L_{ij}^{\mathcal{A}}$ is the path loss from ith target node to jth anchor node, $L_{ik}^{\mathcal{B}}$ is the path loss from ith target node to kth target node, L_0 is the reference path loss value at the reference distance d_0, γ is the path loss exponent, v_{ij} and w_{ik} follow the identically independent distributed (i.i.d.) zero-mean Gaussian distribution, *i.e.*, $v_{ij} \sim \mathcal{N}(0, \sigma_{v_{ij}}^2)$ and $w_{ik} \sim \mathcal{N}(0, \sigma_{w_{ik}}^2)$, representing log-normal shadowing effect, the tuple sets $\mathcal{A} = \{(i, j)|d_0 \leq \|\boldsymbol{x}_i - \boldsymbol{s}_j\| \leq R, i = 1, 2, \ldots, M, j = 1, 2, \ldots, N\}$, and $\mathcal{B} = \{(i, k)|d_0 \leq \|\boldsymbol{x}_i - \boldsymbol{x}_k\| \leq R, i, k = 1, 2, \ldots, M, i \neq k\}$, where R is referred to as effective communication range for any pair of sensors, composing of the target/anchor and target/target connections index pairs, respectively. For simplicity and without loss of generality, we assume the target/target path loss measurements are symmetric, i.e., $L_{ik}^{\mathcal{B}} = L_{ki}^{\mathcal{B}}$ for $i \neq k$, all target nodes radiate with the same power, i.e., the reference value L_0 and communication distance R are the same for all target nodes.

Under a centralized processing mode for the localization, all sensors send their RSS measurements with respect to the target nodes to the central processor, during which the locations of all the nodes are assumed unchanged. Fig. 1 shows the ith target node, jth anchor node, and kth target node link in the cooperative WSNs.

Based on (1), the ML estimator of target nodes can be derived, which corresponds to solving the following

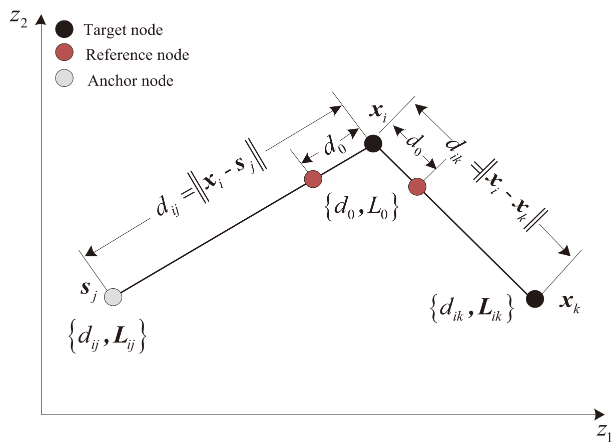

FIGURE 1. **The ith target node, jth anchor node, and kth target node link in the cooperative wireless sensor networks (WSNs).**

WLS problem, i.e.,

$$\min_{\{\boldsymbol{x}_i\}} \sum_{(i,j):(i,j)\in\mathcal{A}} \frac{1}{\sigma_{v_{ij}}^2}\left[L_{ij}^{\mathcal{A}} - L_0 - 10\gamma\log_{10}\frac{\|\boldsymbol{x}_i-\boldsymbol{s}_j\|}{d_0}\right]^2 + \sum_{(i,k):(i,k)\in\mathcal{B}} \frac{1}{\sigma_{w_{ik}}^2}\left[L_{ik}^{\mathcal{B}} - L_0 - 10\gamma\log_{10}\frac{\|\boldsymbol{x}_i-\boldsymbol{x}_k\|}{d_0}\right]^2. \quad (2)$$

Clearly, the ML estimator (2) is non-convex and non-linear in both known and unknown transmit power cases, which leads to difficult global optimization. Therefore, novel approaches are needed to solve this problem. In Section III-A and Section III-B, we propose central processor localization methods based on SOCP relaxation techniques to solve (2) in both known and unknown target transmit power cases, respectively.

III. PROPOSED METHODS

For the simplicity, in this section, we assume $\sigma_{v_{ij}} = \sigma_{w_{ik}} = \sigma$, and provide method for transforming the ML estimator into SOCP problem that can be solved efficiently.

A. COOPERATIVE LOCALIZATION IN KNOWN TRANSMIT POWER CASE

Both sides of (1) are divided by 10γ, and then take the power of 10, we have

$$10^{\frac{L_0-L_{ij}^{\mathcal{A}}}{10\gamma}}\|\boldsymbol{x}_i-\boldsymbol{s}_j\| = d_0 10^{-\frac{v_{ij}}{10\gamma}}, \quad (i,j)\in\mathcal{A}, \quad (3a)$$

$$10^{\frac{L_0-L_{ik}^{\mathcal{B}}}{10\gamma}}\|\boldsymbol{x}_i-\boldsymbol{x}_k\| = d_0 10^{-\frac{w_{ik}}{10\gamma}}, \quad (i,k)\in\mathcal{B}. \quad (3b)$$

When v_{ij} and w_{ik} are quite small ($|v_{ij}| \ll \frac{10\gamma}{\ln 10}$ and $|w_{ik}| \ll \frac{10\gamma}{\ln 10}$), it is reasonable to approximate the second factor on the right hand of (3) by using the first-order Taylor expansion as follows

$$10^{-\frac{v_{ij}}{10\gamma}} \approx 1 - \frac{\ln 10}{10\gamma}v_{ij}, \quad (4a)$$

$$10^{-\frac{w_{ik}}{10\gamma}} \approx 1 - \frac{\ln 10}{10\gamma}w_{ik}, \quad (4b)$$

where the higher-order terms are omitted.

We then substitute (4) into (3) resulting in

$$\alpha_{ij}^{\mathcal{A}}\|\boldsymbol{x}_i-\boldsymbol{s}_j\| \approx d_0 - d_0\frac{\ln 10}{10\gamma}v_{ij}, \quad (i,j)\in\mathcal{A}, \quad (5a)$$

$$\alpha_{ik}^{\mathcal{B}}\|\boldsymbol{x}_i-\boldsymbol{x}_k\| \approx d_0 - d_0\frac{\ln 10}{10\gamma}w_{ik}, \quad (i,k)\in\mathcal{B}, \quad (5b)$$

where $\alpha_{ij}^{\mathcal{A}} = 10^{\frac{L_0-L_{ij}^{\mathcal{A}}}{10\gamma}}$ and $\alpha_{ik}^{\mathcal{B}} = 10^{\frac{L_0-L_{ik}^{\mathcal{B}}}{10\gamma}}$.

Moving the second term on the right hand of (5) to the left hand, squaring both side and then ignoring the second-order noise term, we can obtain an approximation expression

$$\alpha_{ij}^{\mathcal{A}^2}\|\boldsymbol{x}_i-\boldsymbol{s}_j\|^2 + 2\mu\alpha_{ij}^{\mathcal{A}}\|\boldsymbol{x}_i-\boldsymbol{s}_j\|v_{ij} \approx d_0^2, \quad (i,j)\in\mathcal{A}, \quad (6a)$$

$$\alpha_{ik}^{\mathcal{B}^2}\|\boldsymbol{x}_i-\boldsymbol{x}_k\|^2 + 2\mu\alpha_{ik}^{\mathcal{B}}\|\boldsymbol{x}_i-\boldsymbol{x}_k\|w_{ik} \approx d_0^2, \quad (i,k)\in\mathcal{B}, \quad (6b)$$

where $\mu = d_0\frac{\ln 10}{10\gamma}$.

From (6), we have

$$v_{ij} \approx \frac{\mu^{-1}(d_0^2\alpha_{ij}^{\mathcal{A}^{-1}} - \alpha_{ij}^{\mathcal{A}}\|\boldsymbol{x}_i-\boldsymbol{s}_j\|^2)}{2\|\boldsymbol{x}_i-\boldsymbol{s}_j\|}, \quad (i,j)\in\mathcal{A}, \quad (7a)$$

$$w_{ik} \approx \frac{\mu^{-1}(d_0^2\alpha_{ik}^{\mathcal{B}^{-1}} - \alpha_{ik}^{\mathcal{B}}\|\boldsymbol{x}_i-\boldsymbol{x}_k\|^2)}{2\|\boldsymbol{x}_i-\boldsymbol{x}_k\|}, \quad (i,k)\in\mathcal{B}. \quad (7b)$$

Based on (7), the following WLS estimation problem is obtained

$$\min_{\{\boldsymbol{x}_i\}} \sum_{(i,j):(i,j)\in\mathcal{A}} \frac{\left[\mu^{-1}(d_0^2\alpha_{ij}^{\mathcal{A}^{-1}} - \alpha_{ij}^{\mathcal{A}}\|\boldsymbol{x}_i-\boldsymbol{s}_j\|^2)\right]^2}{4\sigma^2\|\boldsymbol{x}_i-\boldsymbol{s}_j\|^2} + \sum_{(i,k):(i,k)\in\mathcal{B}} \frac{\left[\mu^{-1}(d_0^2\alpha_{ik}^{\mathcal{B}^{-1}} - \alpha_{ik}^{\mathcal{B}}\|\boldsymbol{x}_i-\boldsymbol{x}_k\|^2)\right]^2}{4\sigma^2\|\boldsymbol{x}_i-\boldsymbol{x}_k\|^2}. \quad (8)$$

Problem (8) is very difficult to solve since it is non-convex. In the next part, we relax (8) to a SOCP problem, which is convex and can be solved efficiently using the interior point method [27].

For the sake of presentation, by stacking all the positions of the targets into a matrix $\boldsymbol{X}$, i.e. $\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_M] \in \mathbb{R}^{2\times M}$, then introduce a variable $\boldsymbol{y} = vec(\boldsymbol{X})$ and a vector $\boldsymbol{E}_i = [\boldsymbol{e}_{2i-1}, \boldsymbol{e}_{2i}]$, where $vec(\boldsymbol{X})$ denotes the column-wise vectorization of $\boldsymbol{X}$ and $\boldsymbol{e}_i$ denotes the ith column vectorization of the identity matrix $\boldsymbol{I}_{2M}$, the problem (8) can be equivalently written as

$$\min_{\boldsymbol{y}} \sum_{(i,j):(i,j)\in\mathcal{A}} \frac{\left[\mu^{-1}(d_0^2\alpha_{ij}^{\mathcal{A}^{-1}} - \alpha_{ij}^{\mathcal{A}}\|\boldsymbol{E}_i^T\boldsymbol{y}-\boldsymbol{s}_j\|^2)\right]^2}{4\sigma^2\|\boldsymbol{E}_i^T\boldsymbol{y}-\boldsymbol{s}_j\|^2} + \sum_{(i,k):(i,k)\in\mathcal{B}} \frac{\left[\mu^{-1}(d_0^2\alpha_{ik}^{\mathcal{B}^{-1}} - \alpha_{ik}^{\mathcal{B}}\|\boldsymbol{E}_i^T\boldsymbol{y}-\boldsymbol{E}_k^T\boldsymbol{y}\|^2)\right]^2}{4\sigma^2\|\boldsymbol{E}_i^T\boldsymbol{y}-\boldsymbol{E}_k^T\boldsymbol{y}\|^2}. \quad (9)$$

The objective function of the problem (9) is still non-convex. To progress, introduce two slack variables $h_{ij} \in \mathbb{R}^{N\times M}$ and $g_{ik} \in \mathbb{R}^{M\times M}$. Then, using the above slack variables, (9) can be rewritten as the following form

$$\min_{\boldsymbol{y},h,g} \sum_{(i,j):(i,j)\in\mathcal{A}} h_{ij} + \sum_{(i,k):(i,k)\in\mathcal{B}} g_{ik}, \tag{10a}$$

$$\text{s.t.} \ \frac{\left[\mu^{-1}(d_0^2\alpha_{ij}^{\mathcal{A}-1} - \alpha_{ij}^{\mathcal{A}}\|\boldsymbol{E}_i^T\boldsymbol{y}-\boldsymbol{s}_j\|^2)\right]^2}{4\sigma^2\|\boldsymbol{E}_i^T\boldsymbol{y}-\boldsymbol{s}_j\|^2} \le h_{ij}, \tag{10b}$$

$$\frac{\left[\mu^{-1}(d_0^2\alpha_{ik}^{\mathcal{B}-1} - \alpha_{ik}^{\mathcal{B}}\|\boldsymbol{E}_i^T\boldsymbol{y}-\boldsymbol{E}_k^T\boldsymbol{y}\|^2)\right]^2}{4\sigma^2\|\boldsymbol{E}_i^T\boldsymbol{y}-\boldsymbol{E}_k^T\boldsymbol{y}\|^2} \le g_{ik}. \tag{10c}$$

To convert the problem (10) into a convex SOCP problem, the non-convex and non-linear constraints (10b) and (10c) need to be relaxed. To cope with the difficulties, two auxiliary variables $d_{ij}^{\mathcal{A}} = \|\boldsymbol{E}_i^T\boldsymbol{y}-\boldsymbol{s}_j\|^2$ for $(i,j)\in\mathcal{A}$ and $d_{ik}^{\mathcal{B}} = \|\boldsymbol{E}_i^T\boldsymbol{y} - \boldsymbol{E}_k^T\boldsymbol{y}\|^2$ for $(i,k)\in\mathcal{B}$ are introduced. Apply SOCP relaxtion technique, we can obtain the following convex problem

$$\min_{\substack{\boldsymbol{Y},\boldsymbol{y},d^{\mathcal{A}}\\ d^{\mathcal{B}},h,g}} \sum_{(i,j):(i,j)\in\mathcal{A}} h_{ij} + \sum_{(i,k):(i,k)\in\mathcal{B}} g_{ik}, \tag{11a}$$

$$\text{s.t.} \ d_{ij}^{\mathcal{A}} = tr(\boldsymbol{E}_i^T\boldsymbol{Y}\boldsymbol{E}_i) - 2\boldsymbol{s}_j^T\boldsymbol{E}_i^T\boldsymbol{y} + \|\boldsymbol{s}_j\|^2, \tag{11b}$$

$$\left\|\left[2\mu^{-1}(d_0^2\alpha_{ij}^{\mathcal{A}-1} - \alpha_{ij}^{\mathcal{A}}d_{ij}^{\mathcal{A}});\, 4d_{ij}^{\mathcal{A}}\sigma^2 - h_{ij}\right]\right\| \le 4d_{ij}^{\mathcal{A}}\sigma^2 + h_{ij}, \quad for\ (i,j)\in\mathcal{A}, \tag{11c}$$

$$d_{ik}^{\mathcal{B}} = tr(\boldsymbol{E}_i^T\boldsymbol{Y}\boldsymbol{E}_i) - 2tr(\boldsymbol{E}_i^T\boldsymbol{Y}\boldsymbol{E}_k) + tr(\boldsymbol{E}_k^T\boldsymbol{Y}\boldsymbol{E}_k), \tag{11d}$$

$$\left\|\left[2\mu^{-1}(d_0^2\alpha_{ik}^{\mathcal{B}-1} - \alpha_{ik}^{\mathcal{B}}d_{ik}^{\mathcal{B}});\, 4d_{ik}^{\mathcal{B}}\sigma^2 - g_{ik}\right]\right\| \le 4d_{ik}^{\mathcal{B}}\sigma^2 + g_{ik}, \quad for\ (i,k)\in\mathcal{B}, \tag{11e}$$

$$\begin{bmatrix}\boldsymbol{Y} & \boldsymbol{y}\\ \boldsymbol{y}^T & 1\end{bmatrix} \succeq \boldsymbol{0}_{2M+1}. \tag{11f}$$

Problem (11) is a SOCP problem and can be efficiently solved. In the following paper, we refer this method to as "SOCP-K".

B. COOPERATIVE LOCALIZATION IN UNKNOWN TRANSMIT POWER CASE

In practice, it is normally impossible to acquire the perfect knowledge on the transmit power of the target nodes in WSNs, for which the transmit power (L_0) is an additional unknown parameter to estimate. To cope with the complexities introduced by the additional unknown parameter, we consider the following further approximation.

Based on (3) and (4), (3) is rewritten as

$$\beta_{ij}^{\mathcal{A}}\|\boldsymbol{x}_i - \boldsymbol{s}_j\| \approx d_0\eta_0 - d_0\eta_0\frac{\ln 10}{10\gamma}v_{ij}, \quad (i,j)\in\mathcal{A}, \tag{12a}$$

$$\beta_{ik}^{\mathcal{B}}\|\boldsymbol{x}_i - \boldsymbol{x}_k\| \approx d_0\eta_0 - d_0\eta_0\frac{\ln 10}{10\gamma}w_{ik}, \quad (i,k)\in\mathcal{B}, \tag{12b}$$

where $\beta_{ij}^{\mathcal{A}} = 10^{\frac{-L_{ij}^{\mathcal{A}}}{10\gamma}}$ and $\beta_{ik}^{\mathcal{B}} = 10^{\frac{-L_{ik}^{\mathcal{B}}}{10\gamma}}$, $\eta_0 = 10^{\frac{-L_0}{10\gamma}}$.

Squaring both sides and then ignoring the second-order noise term, we can obtain an approximation expression

$$v_{ij} \approx \frac{\mu^{-1}(d_0^2\eta - \beta_{ij}^{\mathcal{A}2}\|\boldsymbol{E}_i^T\boldsymbol{y}-\boldsymbol{s}_j\|^2)}{2d_0\eta}, \quad (i,j)\in\mathcal{A}, \tag{13a}$$

$$w_{ik} \approx \frac{\mu^{-1}(d_0^2\eta - \beta_{ik}^{\mathcal{B}2}\|\boldsymbol{E}_i^T\boldsymbol{y}-\boldsymbol{E}_k^T\boldsymbol{y}\|^2)}{2d_0\eta}, \quad (i,k)\in\mathcal{B}, \tag{13b}$$

where $\eta = \eta_0^2 = 10^{\frac{-L_0}{5\gamma}}$.

Based on (13), the following WLS estimation problem is obtained

$$\min_{\boldsymbol{y},\eta} \sum_{(i,j):(i,j)\in\mathcal{A}} \frac{[\mu^{-1}(d_0^2\eta - \beta_{ij}^{\mathcal{A}2}\|\boldsymbol{E}_i^T\boldsymbol{y}-\boldsymbol{s}_j\|^2)]^2}{4d_0^2\eta^2\sigma^2} + \sum_{(i,k):(i,k)\in\mathcal{B}} \frac{[\mu^{-1}(d_0^2\eta - \beta_{ik}^{\mathcal{B}2}\|\boldsymbol{E}_i^T\boldsymbol{y}-\boldsymbol{E}_k^T\boldsymbol{y}\|^2)]^2}{4d_0^2\eta^2\sigma^2}. \tag{14}$$

Using the similar idea as in the "SOCP-K" method, the following optimization problem can be obtained

$$\min_{\substack{\boldsymbol{Y},\boldsymbol{y},d^{\mathcal{A}},d^{\mathcal{B}}\\ h,g,\eta}} \sum_{(i,j):(i,j)\in\mathcal{A}} h_{ij} + \sum_{(i,k):(i,k)\in\mathcal{B}} g_{ik}, \tag{15a}$$

$$\text{s.t.} \ d_{ij}^{\mathcal{A}} = tr(\boldsymbol{E}_i^T\boldsymbol{Y}\boldsymbol{E}_i) - 2\boldsymbol{s}_j^T\boldsymbol{E}_i^T\boldsymbol{y} + \|\boldsymbol{s}_j\|^2, \tag{15b}$$

$$\left\|\left[2\mu^{-1}(d_0^2\eta - \beta_{ij}^{\mathcal{A}2}d_{ij}^{\mathcal{A}});\, 4d_0^2\eta^2\sigma^2 - h_{ij}\right]\right\| \le 4d_0^2\eta^2\sigma^2 + h_{ij}, \quad for\ (i,j)\in\mathcal{A}, \tag{15c}$$

$$d_{ik}^{\mathcal{B}} = tr(\boldsymbol{E}_i^T\boldsymbol{Y}\boldsymbol{E}_i) - 2tr(\boldsymbol{E}_i^T\boldsymbol{Y}\boldsymbol{E}_k) + tr(\boldsymbol{E}_k^T\boldsymbol{Y}\boldsymbol{E}_k), \tag{15d}$$

$$\left\|\left[2\mu^{-1}(d_0^2\alpha_{ik}^{\mathcal{B}-1} - \alpha_{ik}^{\mathcal{B}}d_{ik}^{\mathcal{B}});\, 4d_0^2\eta^2\sigma^2 - g_{ik}\right]\right\| \le 4d_0^2\eta^2\sigma^2 + g_{ik}, \quad for\ (i,k)\in\mathcal{B}, \tag{15e}$$

$$\begin{bmatrix}\boldsymbol{Y} & \boldsymbol{y}\\ \boldsymbol{y}^T & 1\end{bmatrix} \succeq \boldsymbol{0}_{2M+1}. \tag{15f}$$

It is noted that the constraints (15c) and (15e) are still non-convex since the L_0 (and hence η) is an unknown parameter. Introducing the auxiliary variable τ, by relaxing $\eta^2 = \tau$ to $\eta^2 \le \tau$, then the non-convex minimization problem (15) can be transformed into the following SOCP optimization problem

$$\min_{\substack{\boldsymbol{Y},\boldsymbol{y},d^{\mathcal{A}},d^{\mathcal{B}}\\ h,g,\eta,\tau}} \sum_{(i,j):(i,j)\in\mathcal{A}} h_{ij} + \sum_{(i,k):(i,k)\in\mathcal{B}} g_{ik}, \tag{16a}$$

$$\text{s.t.} \ d_{ij}^{\mathcal{A}} = tr(\boldsymbol{E}_i^T\boldsymbol{Y}\boldsymbol{E}_i) - 2\boldsymbol{s}_j^T\boldsymbol{E}_i^T\boldsymbol{y} + \|\boldsymbol{s}_j\|^2, \tag{16b}$$

$$\left\|\left[2\mu^{-1}(d_0^2\eta - \beta_{ij}^{\mathcal{A}2}d_{ij}^{\mathcal{A}});\, 4d_0^2\tau\sigma^2 - h_{ij}\right]\right\| \le 4d_0^2\tau\sigma^2 + h_{ij}, \quad for\ (i,j)\in\mathcal{A}, \tag{16c}$$

$$d_{ik}^{\mathcal{B}} = tr(\boldsymbol{E}_i^T\boldsymbol{Y}\boldsymbol{E}_i) - 2tr(\boldsymbol{E}_i^T\boldsymbol{Y}\boldsymbol{E}_k) + tr(\boldsymbol{E}_k^T\boldsymbol{Y}\boldsymbol{E}_k), \tag{16d}$$

Algorithm 1 Hybrid ML-SOCP

1: **Input**:
$\{L_{ij}\}$: path loss from ith target node to jth anchor node;
$\{L_{ik}\}$: path loss from ith target node to kth target node;
$\{\sigma_{ij}\}$: variances of the measurements noise from ith target node to jth anchor node;
$\{\sigma_{ik}\}$: variances of the measurements noise from ith target node to kth target node;
$\{d_0\}$: reference distance;
$\{L_0\}$: reference path loss value at the reference distance d_0;
$\{s_j\}$: sensor locations;
$\{\gamma\}$: path loss exponent;
2: Find an initial estimate $\hat{\boldsymbol{y}}$ in the feasible region of (16);
3: Compute $\hat{d}_{ij}^{\mathcal{A}} \leftarrow \|\boldsymbol{E}_i^T\hat{\boldsymbol{y}} - \boldsymbol{s}_j\|^2$ for $(i,j) \in \mathcal{A}$ and $\hat{d}_{ik}^{\mathcal{B}} \leftarrow \|\boldsymbol{E}_i^T\hat{\boldsymbol{y}} - \boldsymbol{E}_k^T\hat{\boldsymbol{y}}\|^2$;
4: Compute by (2) the ML estimate as

$$\hat{L}_0 = \frac{\sum\limits_{(i,j):(i,j)\in\mathcal{A}} (L_{ij}^{\mathcal{A}} - 10\gamma\log_{10}\frac{\|\boldsymbol{E}_i^T\hat{\boldsymbol{y}}-\boldsymbol{s}_j\|}{d_0})}{|\mathcal{A}|+|\mathcal{B}|} + \frac{\sum\limits_{(i,k):(i,k)\in\mathcal{B}} (L_{ik}^{\mathcal{B}} - 10\gamma\log_{10}\frac{\|\boldsymbol{E}_i^T\hat{\boldsymbol{y}}-\boldsymbol{E}_k^T\hat{\boldsymbol{y}}\|}{d_0})}{|\mathcal{A}|+|\mathcal{B}|},$$

where $|\mathcal{A}|$ and $|\mathcal{B}|$ represent the cardinalities of sets $\mathcal{A}$ and $\mathcal{B}$;
5: Compute $\hat{\alpha}_{ij}^{\mathcal{A}} = 10^{\frac{\hat{L}_0 - L_{ij}^{\mathcal{A}}}{10\gamma}}$ for $(i,j) \in \mathcal{A}$ and $\hat{\alpha}_{ik}^{\mathcal{B}} = 10^{\frac{\hat{L}_0 - L_{ik}^{\mathcal{B}}}{10\gamma}}$ for $(i,k) \in \mathcal{B}$;
6: Solving (11), and obtain an updated $\boldsymbol{y}$;
7: **Output**: the solution of step 6, $\boldsymbol{y}$, is the estimate of the target locations.

$$\left\|\left[2\mu^{-1}(d_0^2\alpha_{ik}^{\mathcal{B}^{-1}} - \alpha_{ik}^{\mathcal{B}}d_{ik}^{\mathcal{B}}); 4d_0^2\tau\sigma^2 - g_{ik}\right]\right\| \leq 4d_0^2\tau\sigma^2 + g_{ik}, \quad \textit{for } (i,k) \in \mathcal{B}, \tag{16e}$$

$$\|[2\eta; \tau - 1]\| \leq \tau + 1, \tag{16f}$$

$$\begin{bmatrix} \boldsymbol{Y} & \boldsymbol{y} \\ \boldsymbol{y}^T & 1 \end{bmatrix} \succeq \boldsymbol{0}_{2M+1}. \tag{16g}$$

Even though (16) can be efficiently solved by using the interior method [27], its performance may not be always good. Therefore, we can further improve its performance. In Algorithm 1, we propose a hybrid ML-SOCP method by alternatively estimating L_0 and target locations. This hybrid ML-SOCP method is referred to as "SOCP-U".

IV. COMPLEXITY ANALYSIS

For all discussed methods, there always exists a trade-off between the estimation accuracy and the implementation complexity. To evaluate this trade-off, we assume a network where all the nodes are inter-connected. The formula for computing the *worst-case* complexity of the mixed SD/SOCP [27] is used to analyze the complexities of the proposed methods and other discussed methods in this paper:

$$\mathcal{O}(\sqrt{L}(m\sum_{i=1}^{N_{sd}} n_i^{sd^3} + m^2\sum_{i=1}^{N_{sd}} n_i^{sd^2} + m^2\sum_{i=1}^{N_{soc}} n_i^{soc} + \sum_{i=1}^{N_{soc}} n_i^{soc^2} + m^3)), \tag{17}$$

where L is the number of iterations of the algorithm, m is the number of equality constraints, N_{sd}, N_{soc} is respectively the number of the semi-definite cone (SDC) and second order (SOC) constraints, and n_i^{sd}, n_i^{soc} is the number of dimensions of the ith SDC and ith SOC, respectively. Based on (17), we provide the complexity analysis of the proposed methods and other discussed methods in Table 1 and Table 3. Further analysis results together with the simulation results will be discussed in the following section.

TABLE 1. Summary of the considered methods in section VI-A.

Method	Description	Complexity
SD/SOCP	The SD/SOCP method in [9]	$\mathcal{O}\left(M^{0.5}\left(M^4\left(N+\frac{M}{2}\right)^2\right)\right)$
SDP1	The SDP1 method in [13]	$\mathcal{O}\left(M^{0.5}\left(4M^4\left(N+\frac{M}{2}\right)^2\right)\right)$
SOCP-K	The proposed SOCP method in Section III-A	$\mathcal{O}\left(M^{0.5}\left(4M^4\left(N+\frac{M}{2}\right)^2\right)\right)$

TABLE 2. Average running times of various methods (sec.).

Method	SD/SOCP	SDP1	SOCP-K
Time (s)	4.90	10.41	23.96

TABLE 3. Summary of the considered methods in section VI-B.

Method	Description	Complexity
SDP-URSS	The SDP method in [16]	$\mathcal{O}\left(M^{0.5}\left(M^4\left(N+\frac{M}{2}\right)^2\right)\right)$
SDP2	The SDP2 method in [13]	$\mathcal{O}\left(M^{0.5}\left(4M^4\left(N+\frac{M}{2}\right)^2\right)\right)$
SOCP-U	The proposed SOCP method in Section III-B	$\mathcal{O}\left(M^{0.5}\left(4M^4\left(N+\frac{M}{2}\right)^2\right)\right)$

V. CRAMER-RAO LOWER BOUND ANALYSIS

The Cramer-Rao Lower Bounds of the targets localization in cooperative WSNs are given in this section to compare the performance of the proposed methods.

For the target location $\boldsymbol{x}_i$, the ML estimator $\hat{\boldsymbol{x}}_i$ in (2) is an unbiased estimator, i.e., $E(\hat{\boldsymbol{x}}_i) = \boldsymbol{x}_i$, as it is essentially based on (1). Then, the covariance matrix of $\hat{\boldsymbol{x}}_i$ is subject to the CRLB as $\mathrm{VAR}(\hat{\boldsymbol{x}}_i) \succeq \boldsymbol{F}^{-1}$, where $\boldsymbol{F}$ is the Fisher information matrix (FIM).

In order to evaluate the accuracy performance of the unbiased estimation, the root mean square error (RMSE) is defined as

$$\mathrm{RMSE} = \sqrt{\left(\sum_{i=1}^{Mc} \| \frac{\hat{\boldsymbol{x}}_i - \boldsymbol{x}_i\|^2}{Mc}\right)}, \tag{18}$$

where $\hat{\boldsymbol{x}}_i$ is the estimation of the randomly generated target location $\boldsymbol{x}_i$ in the ith simulation, and Mc is the number of independent Monte carlo (Mc) simulation rounds.

Accordingly, we define the CRLB on RMSE by computing the trace of $\boldsymbol{F}^{-1}$. As proved in the following, the CRLB on RMSE for both known and unknown transmit power of targets is given, respectively, by

$$\mathrm{CRLB-K} = \mathrm{trace}\left\{\boldsymbol{F}_K^{-1}\right\}, \tag{19}$$

$$\mathrm{CRLB-U} = \mathrm{trace}\left\{\boldsymbol{F}_U^{-1}[1:2M, 1:2M]\right\}, \tag{20}$$

The FIM $\boldsymbol{F}_K$ and $\boldsymbol{F}_U$ are computed as

$$\boldsymbol{F}_K = \boldsymbol{G}_1^T\boldsymbol{Q}_1^{-1}\boldsymbol{G}_1 + \boldsymbol{G}_2^T\boldsymbol{Q}_2^{-1}\boldsymbol{G}_2, \tag{21}$$

$$\boldsymbol{F}_U = \boldsymbol{H}_1^T\boldsymbol{Q}_1^{-1}\boldsymbol{H}_1 + \boldsymbol{H}_2^T\boldsymbol{Q}_2^{-1}\boldsymbol{H}_2, \tag{22}$$

where $\boldsymbol{Q}_1 = diag\{\sigma_{v_{ij}}^2\}$, $\boldsymbol{Q}_2 = diag\{\sigma_{w_{ik}}^2\}$, $[\boldsymbol{G}_1]_{ij} = \boldsymbol{g}_{ij}$, $[\boldsymbol{G}_2]_{ik} = \boldsymbol{g}_{ik}$, $[\boldsymbol{H}_1]_{ij} = \boldsymbol{h}_{ij}$, $[\boldsymbol{H}_2]_{ik} = \boldsymbol{h}_{ik}$, with

$$\boldsymbol{g}_{ij} = \frac{10\gamma}{\ln 10}\frac{\boldsymbol{E}_i^T(\boldsymbol{E}_i^T\boldsymbol{y} - \boldsymbol{s}_j)^T}{\|\boldsymbol{E}_i^T\boldsymbol{y} - \boldsymbol{s}_j\|^2},$$

$$\boldsymbol{g}_{ik} = \frac{10\gamma}{\ln 10}\frac{(\boldsymbol{E}_i - \boldsymbol{E}_k)^T(\boldsymbol{E}_i^T\boldsymbol{y} - \boldsymbol{E}_k^T\boldsymbol{y})^T}{\|\boldsymbol{E}_i^T\boldsymbol{y} - \boldsymbol{E}_k^T\boldsymbol{y}\|^2},$$

$$\boldsymbol{h}_{ij} = \left[\frac{10\gamma}{\ln 10}\frac{\boldsymbol{E}_i^T(\boldsymbol{E}_i^T\boldsymbol{y} - \boldsymbol{s}_j)^T}{\|\boldsymbol{E}_i^T\boldsymbol{y} - \boldsymbol{s}_j\|^2}, 1\right]^T,$$

$$\boldsymbol{h}_{ik} = \left[\frac{10\gamma}{\ln 10}\frac{(\boldsymbol{E}_i - \boldsymbol{E}_k)^T(\boldsymbol{E}_i^T\boldsymbol{y} - \boldsymbol{E}_k^T\boldsymbol{y})^T}{\|\boldsymbol{E}_i^T\boldsymbol{y} - \boldsymbol{E}_k^T\boldsymbol{y}\|^2}, 1\right]^T,$$

for all $(i, j) \in \mathcal{A}$, $(i, k) \in \mathcal{B}$.

VI. SIMULATION RESULTS

In this section, computer simulation results are provided to compare the performance of the proposed SOCP-K and SOCP-U methods with that of the other discussed methods and the CRLBs in cooperative localization scenario. All presented methods are solved by using MATLAB package CVX [28], where the solver is SeDuMi [29]. The propagation model (1) is used to generate the RSS measurements. In all computer results presented hereafter, the reference distance $d_0 = 1\,\mathrm{m}$, the path loss $L_0 = 40\,\mathrm{dB}$, the path loss exponent $\gamma = 3$. 3000 Mc runs are used to compute the normalized root mean square error (NRMSE) defined as NRMSE$=\sqrt{\frac{1}{M}\sum_{i=1}^{Mc}\sum_{j=1}^{M}\frac{\|\boldsymbol{x}_{ij}-\hat{\boldsymbol{x}}_{ij}\|^2}{Mc}}$, where $\hat{\boldsymbol{x}}_{ij}$ is the estimated location of the jth target, in the ith Monte carlo runs. Unless stated otherwise, the anchor nodes are fixed at the locations (B, B), $(0, B)$, $(-B, B)$, $(-B, 0)$, $(-B, -B)$, $(0, -B)$, $(B, -B)$, $(B, 0)$, $(0, 0)$, where B will be given in each figure. Target nodes are randomly located inside the convex hull and only those within the effective communication range to the anchor nodes can be connected to each other.

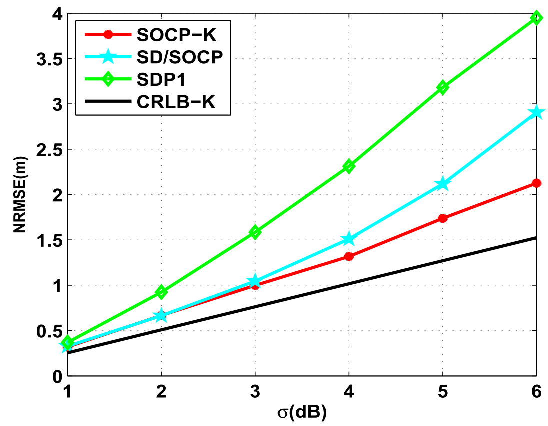

FIGURE 2. Simulation results for cooperative localization when L_0 is known: NRMSE versus the σ under the conditions of $N = 9, M = 30, \gamma = 3, d_0 = 1\,m, B = 15\,m, R = 8\,m$.

A. COOPERATIVE LOCALIZATION IN KNOWN TRANSMIT POWER CASE

Table 1 provides an overview of the discussed methods in known transmit power case, together with their complexities. From Table 1, we know that the proposed SOCP-K method has the same complexity as the SDP1 method and slightly higher than the SD/SOCP method. Simulation results in the following part will show that the proposed SOCP-K method will provide superior performance, but at the cost of higher computational complexity. Table 2 also shows the running times of the discussed methods.

Fig. 2 compares the NRMSE of the discussed methods versus the standard derivation (STD), σ, when $N = 9$, $M = 30, \gamma = 3, R = 8\,m$. This figure shows that the performance of the discussed methods is becoming worse as σ increases. Moreover, it is observed that the proposed SOCP-K method provides superior performance over the other discussed methods, and more closes to the CRLB-K in presence of high σ. The following two cases are considered for further illustration: $\sigma = 3\,dB$ and $\sigma = 6\,dB$. From Fig. 2, it can be seen that although the proposed SOCP-K method outperforms the SDP1 method, the gap between the proposed SOCP-K method and the SD/SOCP method is very small when $\sigma = 3\,dB$. However, the proposed SOCP-K method outperforms all the other discussed methods with large gaps when $\sigma = 6\,dB$. In summary, the proposed method outperforms the discussed methods in terms of the localization accuracy particularly when σ is high.

Fig. 3 shows the NRMSE versus N of the discussed methods when $M = 30, \sigma = 5\,dB, R = 8\,m$. The anchor nodes and target nodes are randomly located at a square region of length $2B$ in each Mc runs. As predicted, the NRMSE decreases as N increases for all the discussed methods, which means more available information can improve the performance. However, it can be seen that the proposed SOCP-K method always outperforms the other discussed methods although the NRMSE margin tends to become smaller in presence of higher N. It also shows that the

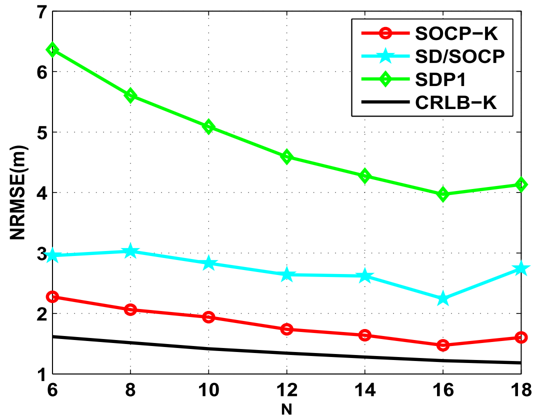

FIGURE 3. Simulation results for cooperative localization when L_0 is known: NRMSE versus the number of anchor nodes N under the conditions of $M = 30, \sigma = 5\,dB, \gamma = 3, d_0 = 1\,m, B = 15\,m, R = 8\,m$.

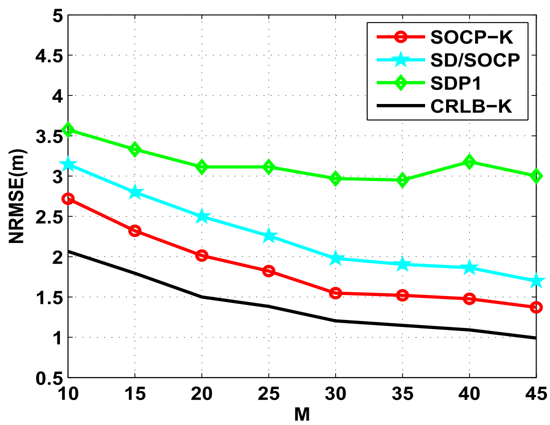

FIGURE 4. Simulation results for cooperative localization when L_0 is known: NRMSE versus the number of target nodes M under the conditions of $N = 9, \sigma = 5\,dB, \gamma = 3, d_0 = 1\,m, B = 15\,m, R = 8\,m$.

performance of SOCP-K is more close to that of CRLB-K for all chosen N. Finally, even though we derive the method by assuming small noise, Fig. 3 reveals that the proposed method can work effectively with presence of high noise, for instance as high as 5dB.

Fig. 4 shows the NRMSE versus M when $N = 9$, $\sigma = 5\,dB, R = 8\,m$. Similar to Fig. 3, the performance of the discussed methods can also be improved by increasing the number of target nodes. Fig. 4 confirms again that the proposed SOCP-K method outperforms the discussed methods and more close to CRLB-K for all chosen M.

B. COOPERATIVE LOCALIZATION IN UNKNOWN TRANSMIT POWER CASE

Table 3 provides an overview of the discussed methods in unknown transmit power case, together with their complexities. From Table 3, it can be seen that the proposed SOCP-U method has the same complexity as the SDP2 method and slightly higher complexity than the SDP-URSS method. Furthermore, the higher computational complexity of the proposed SOCP-U method can be justified by its superior

TABLE 4. Average running times of various methods (sec.).

Method	SDP-URSS	SDP2	SOCP-U
Time (s)	12.79	26.21	73.31

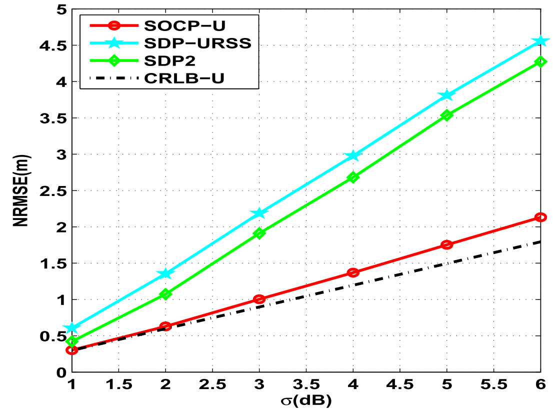

FIGURE 5. Simulation results for cooperative localization when L_0 is unknown: NRMSE versus the σ under the conditions of $N = 9, M = 30, \gamma = 3, d_0 = 1\,m, B = 15\,m, R = 8\,m$.

performance in the sense of estimation accuracy, as we will see in the following simulation results. Table 4 also shows the running times of the discussed methods.

Fig. 5 compares the NRMSE versus the STD, σ, when $N = 9, M = 30, R = 8\,m$. To the same conclusion as in the known transmit power case, the figure shows that the performance of the discussed methods becomes worse as σ increases. Moreover, the gaps between the proposed SOCP-U method and the other discussed methods become larger as σ increases. For instance, when $\sigma = 1\,dB$, the gaps of the performance are approximately 0.1 m and 0.3 m, when compared to the SDP2 method and SDP-URSS method, respectively; when $\sigma = 6\,dB$, they are 2.1 m and 2.3 m, respectively. On the other hand, the proposed SOCP-U method underperforms the CRLB-U by about 0.2 m when $\sigma = 5\,dB$. In summary, the proposed SOCP-U method provides superior performance over the other discussed methods, and more close to the CRLB-U even when σ is high.

Fig. 6 compares the NRMSE versus the number of the anchor nodes N when $M = 30, \sigma = 5\,dB, R = 8\,m$. Similar to known transmit power case, the anchor nodes and target nodes are still randomly located at a square region of length $2B$ in each Mc runs. Fig. 6 shows that the NRMSE decreases as N increases for all the discussed methods. This result shows that the performance of the proposed method can be improved with more reliable information available. The figure shows that the proposed SOCP-U method outperforms the other discussed methods and is more close to CRLB-U for all chosen N, e.g., for $N = 12$, the proposed SOCP-U method outperforms the other discussed methods by more than 3 m. Similar to SOCP-K, it shows that SOCP-U works effectively in presence of high noise.

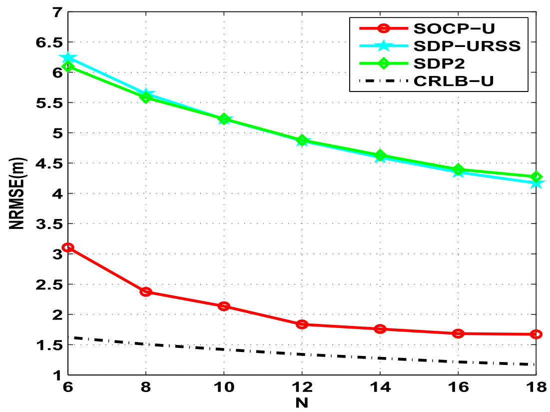

FIGURE 6. Simulation results for cooperative localization when L_0 is unknown: NRMSE versus the number of anchor nodes N under the conditions of $M = 30, \sigma = 5\,dB, \gamma = 3, d_0 = 1\,m, B = 15\,m, R = 8\,m$.

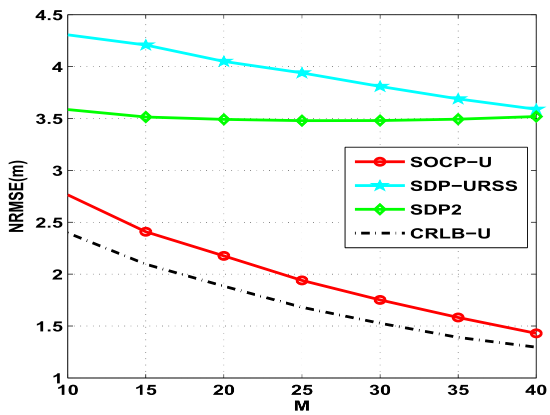

FIGURE 7. Simulation results for cooperative localization when L_0 is unknown: NRMSE versus the number of target nodes M under the conditions of $N = 9, \sigma = 5\,dB, \gamma = 3, d_0 = 1\,m, B = 15\,m, R = 8\,m$.

Fig. 7 compares the NRMSE versus the number of the target nodes M when $N = 9, \sigma = 5\,dB, R = 8\,m$. This figure confirms that the performance of the discussed methods can be improved by adding more target nodes. This is due to the fact that more communication links are set up when more target nodes can communicate. Furthermore, when more target nodes are added in the network, more estimation accuracy of transmit power is obtained, which improves the performance of the location estimation. Finally, this result shows that the proposed SOCP-U method outperforms the other discussed methods for the number of the target nodes M.

VII. CONCLUSION

We propose two second-order cone programming (SOCP) estimators for RSS-based cooperative localization problem in both known and unknown target transmit power cases. For the case of known transmit power, we treat the transmit power as a constant and derive a novel non-convex WLS estimator to estimate the target node locations. We then show that it can be transformed into a SOCP problem for reaching an efficient solution. For the case of unknown transmit power, we treat the transmit power as an additional unknown parameter and propose a hybrid ML-SOCP algorithm to alternatively estimate the target nodes and transmit power. Extensive computer simulations are carried out and the results depict that for all considered scenarios, the proposed estimators exhibited exceeding performance.

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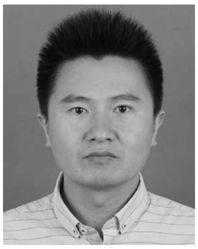

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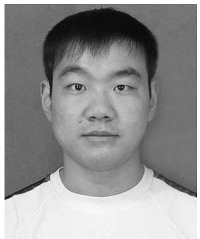

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