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# Inherence of Hard Decision Fusion in Soft Decision Fusion and a Generalized Radix-2 Multistage Decision Fusion Strategy

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**ABSTRACT** We consider the problem of decision fusion for binary event detection using a sensor network of nodes with non-identical detection and false-alarm probability pairs. We show that a soft decision fusion rule that is used to make a binary decision inherently possesses a hard decision fusion (HDF) part. Revelation of HDF part within the soft decision fusion rule, on the one hand, shows a connection between the two approaches and, on the other hand, enables straightforward computation of network-level detection and false-alarm probabilities. We consider the optimal soft decision fusion rule that minimizes the total probability of error and reveal its inherent HDF part for a network of two nodes. We subsequently use it to develop a radix-2 multistage decision fusion strategy for larger networks since revelation of HDF part for them is quite time-consuming. We consider spectrum sensing by a cognitive radio-enabled wireless sensor network to demonstrate the effectiveness of the proposed strategy. We show that the error performance of the proposed strategy is close to that exhibited by the optimal soft decision fusion rule and is better than many suboptimal hard and soft decision fusion strategies. The overall detection and false-alarm probabilities can be easily computed using the proposed strategy. We also show that the EXOR and EXNOR binary fusion rules are never optimal in minimizing the probability of error in decision making.

**INDEX TERMS** AND rule, detection probability, false-alarm probability, hard decision fusion, multistage decision fusion, non-identical nodes, OR rule, probability of error, soft decision fusion.

## I. INTRODUCTION

We begin with an analogy for cooperative decision making. Consider a parliamentary democracy with two Houses, namely, the Lower House (LH) and the Upper House (UH). The members of LH are directly elected by the people through voting where each vote is considered equal irrespective of the caste, creed, sex, or social, economical or educational background of the voter. The election of a candidate through such voting is an example of hard decision fusion (HDF). The members of UH, on the other hand, are elected indirectly by the people through voting by their representatives in the State Assemblies. In this case, each vote carries a weightage proportional to the number of persons, the voter represents. Such process of combining weighted decisions is called soft decision fusion (SDF). Looking at the UH election process, we realize that the 'softness' involved is ultimately to honor the 'hardness' at the common people level, which is the backbone of a democracy. In other words, this process, which is SDF on surface, has a HDF part underneath.

In this work, we consider cooperative decision making to detect a binary event. The optimal decision fusion rule that minimizes the total probability of error (POE) in decision making is Chair–Varshney (CV) rule, which is a SDF rule [1]. It is shown in the literature that for a network of identical sensor nodes, i.e., the nodes with identical detection and false-alarm probability (DP and FAP respectively) pairs, the CV rule becomes $K$-out-of-$N$ (KN) rule, which is a HDF rule [2]. We show in this work that for a network of non-identical sensor nodes, i.e., the nodes with non-identical DP and FAP pairs, also, the CV rule contains a HDF part, though its form may not be as simple as that of the KN rule. This concomitance of HDF and SDF rules is evident from the following observations—For a network of $N$ nodes, there exist $2^{2^N}$ possible HDF rules. One of these rules is optimum for minimizing the total POE [2]. For the same network, an optimum SDF rule (CV rule) also exists for minimizing the total POE [1]. Two rules, one HDF rule and another SDF rule, optimize the same quantity and both of them guarantee optimization; this implies that they are same. Thus, for a network of non-identical nodes, there exists a HDF equivalent of the optimum SDF rule.

For such a network of non-identical nodes, it seems natural to apply SDF, i.e., the individual nodes' decisions should not be treated equally and should be given weightages according to their DP and FAP values. For a network of identical nodes, on the other hand, all local decisions should be treated equally, and therefore a HDF must be employed. This is intuitive and is found in the literature also [2]–[14]. We, however, in this work, develop a scheme based on HDF for a network of non-identical nodes which exhibits performance close to that depicted by the optimal SDF rule in terms of the total POE. We emphasize that, in principle, it is possible to obtain a HDF equivalent of SDF rule, and we show it for a network of two non-identical nodes. For a network of $N$ nodes, in general, there exist $2^{2^N}$ possible HDF rules, which is a huge number even for small value of $N$, such as $N = 4$. For those detection tests at the individual nodes in which local DP is guaranteed to be greater than the local FAP always, the monotonicity property of Boolean fusion rules is followed. In such cases the optimum HDF rule cannot contain the complements of local decisions, and therefore the possible number of HDF rules reduces significantly from $2^{2^N}$ [2]. This number is still much larger for efficient computation of global DP and FAP. Therefore, for such networks obtaining the HDF equivalent, though straightforward, is quite time-consuming. We, therefore, use the HDF equivalent for two nodes as a basic unit for larger networks and develop a new multistage decision fusion (MDF) strategy, which we name as a generalized radix-2 MDF strategy, since this strategy works for a network of $2^v$ non-identical nodes, where $v$ is a positive integer. In a challenging scenario where the local DP may become smaller than the local FAP due to variations in the values of those parameters on which these quantities depend, the monotonicity property is not obeyed and the optimal HDF rule may contain complement of local decisions. We show for a network of two nodes, where in all 16 HDF rules are possible, that even in this situation the EX–OR and EX–NOR fusion rules can never be optimal. It is to be noted that computation of DP and FAP for these two rules require maximum number of multiplications among all HDF rules.

This work is motivated by two key requirements—one, to establish a connection between SDF and HDF rules; and two, to determine network-level performance in terms of DP and FAP along with the POE. Numerous HDF and SDF schemes, with their relative merits and demerits, exist for event detection using distributed systems. We investigate a common thread between these two approaches and show a relation between them, which can be revealed through some well-defined procedure. Another requirement is the computation of network-level DP and FAP which is extremely difficult for SDF rules, especially when the nodes of the network are non-identical [15]–[19]. However, for HDF, their computation is straightforward, though it may be very time-consuming for large networks. Even though the POE, as obtained by the CV rule, is optimum (minimum), the performance in terms of DP or FAP may not be acceptable. Depending upon the application, a minimum DP and a maximum FAP must be ensured. For a CR–WSN, for instance, the knowledge of network-level DP and FAP respectively gives information about the interference level with the PU and the spectrum opportunity available to the secondary users (SU). It is, therefore, required to compute the network-level DP and FAP, which is possible once the CV rule is expressed in the HDF form.

### A. RELATED WORK AND CONTRIBUTIONS

The methods proposed in the literature for binary event detection using data or decision fusion can be classified into two categories—one, in which the sensor nodes participating in the event detection have identical performance indexes; and two, in which the sensor nodes participating in the event detection have non-identical performance indexes, where the performance indexes are measured in terms of the DP and FAP pairs of the sensor nodes. These methods try to minimize the POE in event detection. However, computation of the POE and the corresponding overall DP and FAP is either very much time-consuming (for a network of identical nodes) or intractable (for a network of non-identical nodes). Computation of the overall DP, FAP and POE is required to assess the system-level performance.

The initial key work on fusion strategies was done by Chair and Varshney [1] and Tenney and Sandell, Jr., [20]. Tenney and Sandell, Jr., [20] focused on deriving the decision rules at individual nodes. Chair and Varshney [1], on the other hand, derived the optimal DF rule that minimizes the POE in decision making. Since then several multistage fusion strategies in the form of serial, parallel and tree topologies have been proposed in the literature [2], [19], [21]–[24]. In serial topology based strategies, each node combines its observations with that of the previous node and passes on the whole information to the next node. In parallel topology based strategies, all nodes transmit their observations about the event to a node with higher processing capability and memory, known as the fusion center (FC). In tree topology based strategies, two or more nodes transmit their observations to other nodes in a hierarchical manner. The last node in the hierarchy is usually a FC. The tree topology can be based on a relay structure also in which intermediate nodes act as relays and combine the decisions of previous nodes or relays and pass them on to the successive relays without making their own observations [22], [23]. Such a mechanism can also be implemented within a FC in a parallel topology [25], [26].

Zhang *et al.* [22], [23] consider a network of identical nodes and focus on computing probability error bounds using AND and OR rules as the size of the network increases. Gupta *et al.* [25], [26] propose MDF strategies to minimize the POE and at the same time to efficiently compute the POE and the system-level DP and FAP. In their first work [25], they consider a network of identical nodes, whereas in their second work [26], they consider a network of non-identical nodes. The MDF strategy considered for a network of identical nodes, is a radix-2 strategy in which the decisions of nodes are fused in pairs as per AND or OR rule and the process continues in a hierarchical manner for a large network resulting in a POE performance which is close to the optimal one. In the MDF strategy considered for a network of non-identical nodes, a cluster of four randomly chosen non-identical sensor nodes is formed. The decisions of the sensors of this cluster are fused as per a two-stage decision fusion strategy, in which, within the cluster, sensors' decisions are fused in pairs as per AND or the OR rule, and the two decisions are then fused as per the OR or the AND rule. The process is repeated for other clusters of four sensors, and then these decisions are clustered in the groups of four. The process continues until a final decision is obtained. This scenario, which considers a network of non-identical sensor nodes, forms a more realistic and practical problem. Therefore, we consider it in the present work, with a different approach. In the previous work [26], there is no attempt to get the optimal fusion at the cluster level. However, in the present work, we propose a method which ensures optimal fusion at the cluster level. It is known that the optimal fusion rule for non-identical sensors network is a SDF rule, which incorporates a weighted combination of the performance indexes of individual sensor nodes. Though such a fusion guarantees the optimal performance, the performance per se cannot be evaluated. We propose in this work a systematic procedure to express the SDF in terms of AND and OR fusion of the local decisions and their complements, which not only guarantees the optimal performance, but ensures the evaluation of the performance as well. We implement this approach over a large number of sensors in a multistage fashion. Since the kernel of the present approach is different from the previously proposed methods [25], [26], the resulting multistage approach, in effect, is different from them, even though it has a similar structure. It can be viewed as the generalization of the previously proposed radix-2 MDF strategy [25], for a network of non-identical nodes. The performance of the proposed strategy is close to that of the CV rule and in certain practical scenarios, such as in CR–WSN, the present approach works better than the previously proposed approaches. Specifically, it performs better than many other HDF rules such as AND, OR, HV, majority, and best KN rules; and SDF rules such as maximal ratio combining (MRC) and equal gain combining (EGC) rules [7], [27], and another MDF strategy [26] in terms of POE and computational complexity in obtaining system-level DP, FAP and POE. In developing the proposed strategy, we also prove that EX–OR and EX–NOR Boolean fusion rules can never be optimal in minimizing the POE.

The rest of the paper is organized as follows: Section II presents the system model and introduces basic relations on POE and CV rule. Section III explains how the CV rule can be expressed in terms of HDF rule and does it for a two-node network in general. It also explains it for a four-node network with an example. Section IV proposes a generalized radix-2 MDF strategy for a network of $2^v$ non-identical nodes, where $v$ is a positive integer, by using the optimal HDF equivalent of CV rule for two nodes. It also presents the complexity analysis of the proposed strategy and compares it with other HDF and SDF techniques. Section V presents the simulation results for a CR–WSN for various scenarios. A summary and discussion of the work is presented in Section VI along with the possible future research direction.

## II. SYSTEM MODEL

With $v$ as a positive integer, we consider a network of $N = 2^v$ sensor nodes deployed for detecting a binary event with non-identical DP and FAP pairs. We refer to such nodes as non-identical nodes. Let $(P_{di}, P_{fi})$ denote the DP and FAP pair for $i$th node. DP and FAP denote the probability with which the event is detected when the event actually occurs and when the event actually does not occur respectively. We assume that the FC can estimate these quantities along with the prior probability of event's presence. Such estimation is possible with high accuracy in real time as reported in the literature [28]–[34].

For increasing the reliability of the decision, cooperation in event detection is used [35], [36]. Combining the local decisions (decisions of individual nodes) by a FC is known as decision fusion (DF). It can be either HDF or SDF. In HDF, local decisions are combined directly without considering local DP and FAP values; whereas in SDF a weighted combination of decisions of individual nodes is considered where the weights are functions of the nodes' DP and FAP values.

Various counting rules described in terms of the $K$-out-of-$N$ (KN) rule belong to HDF. KN rule implies that for event detection by the FC, at least $K$ out of $N$ nodes should detect the event. Specific cases of KN rule include OR rule ($K = 1$), AND rule ($K = N$), half-voting (HV) rule ($K = \lceil N/2 \rceil$) and majority rule ($K > \lceil N/2 \rceil$).

For KN rule, closed form expressions are available to compute the global DP and FAP, which we denote by $Q_d$ and $Q_f$ respectively [26]. If we denote the a priori probability of the event to be present by $\alpha$, then the POE, denoted by $P_e$, in decision making by the FC is given by (1).

$$P_e = 1 - [\alpha Q_d + (1-\alpha)(1-Q_f)] \tag{1}$$

The optimal fusion rule, which minimizes POE is CV rule [1], which can be written as (2).

$$\Lambda = \frac{\alpha}{(1-\alpha)} \prod_{\substack{i \\ (u_i=1)}} \frac{P_{di}}{P_{fi}} \prod_{\substack{i \\ (u_i=0)}} \frac{(1-P_{di})}{(1-P_{fi})} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} 1 \tag{2}$$

Here, $\Lambda$ denotes the decision statistic, and $\mathcal{H}_1$ and $\mathcal{H}_0$ respectively denote the hypotheses 'event detected' and 'event not detected' as decided by the FC. Decision of the $i$th node is denoted by $u_i$, which is 1 or 0 for the presence or absence of the event respectively. Denoting $\mathcal{H}_1$ and $\mathcal{H}_0$ also as 1 and 0 respectively, (2) can be written as (3). The CV rule assumes that all nodes' observations are independent.

$$\Lambda \underset{0}{\overset{1}{\gtrless}} 1 \tag{3}$$

## III. HARD DECISION FUSION WITHIN SOFT DECISION FUSION

We begin by noting the fundamental difference between HDF and SDF for a WSN with a FC. In HDF, binary decisions of the individual nodes are combined without considering their local DP and FAP values, even though the local DP and FAP values and the fusion rule together decide the global DP and FAP values. In SDF, on the other hand, as we note in the CV rule, local DP and FAP values along with the local decisions make the integral part of the fusion rule and is therefore rightly expected to give better performance. Here, we ask a basic question—Can we break a SDF rule, such as the CV rule, into a HDF part and the SDF part? That is to say, can we express the CV rule as HDF, i.e., in terms of local binary decisions only, given local DP and FAP values? We show in this section with an example of a two-node network that it is indeed possible as the final decision is a binary (hard) decision.

### A. CV RULE AS A HDF RULE FOR A TWO-NODE NETWORK

Referring to (2) and denoting the factor $\alpha/(1-\alpha)$ by $\mu$, the decision statistic $\Lambda$ takes the form as given in Table 1 where the decisions of two nodes denoted by $u_1$ and $u_2$ may be 1 or 0 to indicate the presence or absence of the event respectively. For a given set of $(P_d, P_f)$ pairs for two nodes, the final decision ($D$) can be obtained as 1 or 0 using Table 1 and (3). Once the complete listing of $u_1$, $u_2$ and $D$ is known, the CV rule, which is a SDF rule, can be expressed as a Boolean fusion rule [2], which is a HDF rule.

**TABLE 1. Decision statistic ($\Lambda$) for a network of two nodes as per CV rule. Decisions of two nodes ($u_1$ and $u_2$) may be 0 or 1 to denote the absence or presence of the event respectively. Accordingly, the decision of the FC may be 0 or 1 as per the (DP, FAP) pair of the two nodes, viz., ($P_{d1}, P_{f1}$) and ($P_{d2}, P_{f2}$).**

| $u_1$ | $u_2$ | $\Lambda$ | $D$ |
|---|---|---|---|
| 0 | 0 | $\mu[(1-P_{d1})(1-P_{d2})]/[(1-P_{f1})(1-P_{f2})]$ | 0 or 1 |
| 0 | 1 | $\mu[(1-P_{d1})P_{d2}]/[(1-P_{f1})P_{f2}]$ | 0 or 1 |
| 1 | 0 | $\mu[P_{d1}(1-P_{d2})]/[P_{f1}(1-P_{f2})]$ | 0 or 1 |
| 1 | 1 | $\mu[P_{d1}P_{d2}]/[P_{f1}P_{f2}]$ | 0 or 1 |

For a two-node network, the decision set ($\mathcal{D}$) has sixteen possibilities, from $\mathcal{D}_0 = \{0000\}$ to $\mathcal{D}_{15} = \{1111\}$ as per Table 1. Out of these sixteen possible decision sets, $\mathcal{D}_0$ and $\mathcal{D}_{15}$ represent situations with final decisions respectively as 0 and 1 always and they do not depend upon the local decisions $u_1$ and $u_2$. The decision set $\mathcal{D}_1 = \{0001\}$ indicates the final decision to be 1 when both $u_1$ and $u_2$ are 1, which is same as the AND rule. The final decision $D$ in this case, can therefore be written as the intersection of $u_1$ and $u_2$, i.e., $D = u_1 \cap u_2$. More precisely, we can write

$$\{D=1|H_j\} = \{u_1 = 1|H_j\} \cap \{u_2 = 1|H_j\} \tag{4}$$

where $j$ can be 1 or 0. Computing probability of (4) yields

$$Q_x \triangleq \Pr[D=1|H_j] = \Pr[\{u_1 = 1|H_j\} \cap \{u_2 = 1|H_j\}] \tag{5}$$

where $Q_x$ denotes network-level DP ($Q_d$) for $j = 1$ and network-level FAP ($Q_f$) for $j = 0$. This is the general approach of computing global DP and FAP and it can be applied to a network of any number of nodes, by expressing the final decision as the unions and/or intersections of local decisions and/or their complements, and then computing the probabilities as per the probability axioms [37]. The complete list of possible global decisions in terms of local decisions and the corresponding global DP and FAP is shown in Table 2.

**TABLE 2. Global decisions $D_0$ through $D_{15}$ as unions and/or intersections of local decisions and/or their complements (represented by a prime); and the corresponding global detection and false-alarm probabilities. $Q_x$ represents global probability and $P_{xi}$ represents local probability of $i$th node, where $i = 1, 2$. Subscript $x$ represents $d$ for detection and $f$ for false-alarm.**

| Decision | Probability ($Q_x$) |
|---|---|
| $D_0 = 0$ | 0 |
| $D_1 = u_1 \cap u_2$ | $P_{x1}P_{x2}$ |
| $D_2 = u_1 \cap u_2'$ | $P_{x1}(1-P_{x2})$ |
| $D_3 = u_1$ | $P_{x1}$ |
| $D_4 = u_1' \cap u_2$ | $(1-P_{x1})P_{x2}$ |
| $D_5 = u_2$ | $P_{x2}$ |
| $D_6 = (u_1 \cap u_2') \cup (u_1' \cap u_2)$ | $P_{x1}(1-P_{x2}) + P_{x2}(1-P_{x1})$ $-P_{x1}P_{x2}(1-P_{x1})(1-P_{x2})$ |
| $D_7 = u_1 \cup u_2$ | $P_{x1} + P_{x2} - P_{x1}P_{x2}$ |
| $D_8 = (u_1 \cup u_2)'$ | $1-(P_{x1} + P_{x2} - P_{x1}P_{x2})$ |
| $D_9 = (u_1 \cap u_2) \cup (u_1' \cap u_2')$ | $P_{x1}P_{x2} + (1-P_{x1})(1-P_{x2})$ $-P_{x1}P_{x2}(1-P_{x1})(1-P_{x2})$ |
| $D_{10} = u_2'$ | $1-P_{x2}$ |
| $D_{11} = u_1 \cup u_2'$ | $1-P_{x2}+P_{x1}P_{x2}$ |
| $D_{12} = u_1'$ | $1-P_{x1}$ |
| $D_{13} = u_1' \cup u_2$ | $1-P_{x1}+P_{x1}P_{x2}$ |
| $D_{14} = (u_1 \cap u_2)'$ | $1-P_{x1}P_{x2}$ |
| $D_{15} = 1$ | 1 |

An example for four nodes follows.

*Example 1:* Consider four nodes with their $(P_d, P_f)$ pairs as (0.8, 0.2), (0.7, 0.1), (0.9, 0.3), and (0.6, 0.2), and their local decisions as $u_i$ with $i = 1, \ldots, 4$. In general, for four nodes there are 65, 536 possible decision sets from $\mathcal{D}_0$ through $\mathcal{D}_{65,535}$. This example yields $\mathcal{D}_{831} = \{0000001100111111\}$. For this case, the final decision $D$ can be expressed as

$$\begin{aligned}\{D=1|H_j\} = {} & \{u_1 = 1|H_j\} \cap \{u_2 = 1|H_j\} \\ & \cup \{u_1 = 1|H_j\} \cap \{u_3 = 1|H_j\} \\ & \cup \{u_2 = 1|H_j\} \cap \{u_3 = 1|H_j\}\end{aligned} \tag{6}$$

which we rewrite, for convenience, as

$$D = (u_1 \cap u_2) \cup (u_1 \cap u_3) \cup (u_2 \cap u_3) \tag{7}$$

assuming $H_j$. We note here that $u_i$'s are independent but not mutually exclusive events. Accordingly, the global DP and FAP can be computed as

$$Q_x = P_{x1}P_{x2} + P_{x1}P_{x3} + P_{x2}P_{x3} - 2P_{x1}P_{x2}P_{x3} \quad (8)$$

where '$x$' represents '$d$' for $j = 1$ and '$f$' for $j = 0$. Using $P_d$ and $P_f$ values of the nodes along with (8) and (1) assuming $\alpha = 0.5$, the performance metrics at the FC, viz., $Q_d$, $Q_f$, $Q_d - Q_f$ and $P_e$ are respectively obtained as 0.9020, 0.0980, 0.8040, and 0.0980.

*Remark 1:* This example happens to be a 2-out-of-3 rule as applied to $u_1$, $u_2$, and $u_3$, with $u_4$ being redundant. This approach can be applied to KN rule in general and thus we see that all counting rules can be expressed as unions of intersections of local decisions. It may further be noted that the union represents OR operation and the intersection represents AND operation. We thus see that the KN rule can be completely represented in terms of AND and OR rules (also called unanimity rules) as applied to local decisions.

*Remark 2:* As we see in case of a two-network node, that the HDF rule need not always be a counting rule, even in that case the final decision can always be expressed as the unions of intersections of local decisions or their complements. Thus we see that any HDF rule can be expressed in terms of AND and OR rules of local decisions or their complements. For detection tests at individual nodes when local DP is always greater than local FAP, monotonicity property of Boolean fusion rules holds and even the complements do not appear [2]. We have already seen that a SDF rule such as a CV rule can be expressed as a HDF rule; thus all DF rules, whether HDF or SDF, can be expressed in terms of AND and OR rules.

*Remark 3:* The corresponding global DP and FAP can always be computed as per probability axioms applied to representation of the global decision as in *Remark 1* and *Remark 2*.

## IV. GENERALIZED RADIX-2 MULTISTAGE DECISION FUSION

We now use the optimum HDF strategy for two nodes derived from the CV rule in the previous section to develop a generalized radix-2 MDF strategy. The scheme requires the network to have $N = 2^v$ nodes, where $v$ is a positive integer. The scheme works as follows: The decisions of $N$ nodes are collected in pairs randomly by the FC. Using the knowledge of the local DP and FAP values, the FC applies the HDF equivalent of the CV rule to each pair. The resulting decisions are again fused in pairs using the resultant DP and FAP values in the same manner. This process continues forming a multistage approach until a final decision about the presence or absence of the event is obtained along with the corresponding global DP and FAP. The entire process works as per Algorithm 1 and is depicted in Fig. 1. It should be noted here that changing the decision pairings would yield a different result. However, obtaining the best pairing would increase the complexity of the scheme tremendously for large $N$, without significantly improving the performance. We, therefore, propose to pair the decisions randomly and hence no complexity is involved with this step.

**Algorithm 1** RADIX2NONIDENT

**Require:** $N$, $P_d = [P_{di}]_{i=1,\ldots,N}$, $P_f = [P_{fi}]_{i=1,\ldots,N}$, $D_{\text{local}} = [D_{\text{local}_i}]_{i=1,\ldots,N}$, $\alpha$

$\mu = \alpha/(1-\alpha)$
**for** $i = 1$ to $\log_2 N$ **do**
  $j = 1, k = 1$
  **while** $j \neq N + 1$ **do**
    $(Q_{dk}, Q_{fk}, D_k) =$ CVTOBOOLEAN($P_d(j:j+1)$, $P_f(j:j+1)$, $D_{\text{local}}(j:j+1)$, $\mu$)
    $j = j + 2, k = k + 1$
  **end while**
  $P_d = Q_d, P_f = Q_f, D_{\text{local}} = D$
  $N = N/2$
**end for**
$Q_d = Q_{d1}, Q_f = Q_{f1}, D_0 = D_1$
**return** $(D_0, Q_d, Q_f)$

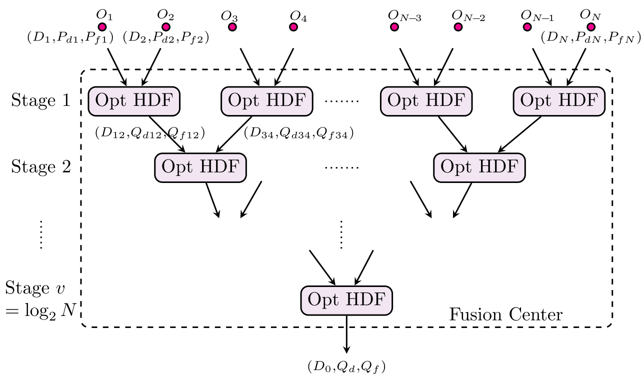


**FIGURE 1.** Generalized radix-2 multistage decision fusion strategy for $N = 2^v$, '$v$' a positive integer, non-identical nodes. $D_i$, $P_{di}$, and $P_{fi}$ respectively denote 1-bit decision, DP and FAP of the $i$th node $O_i$ for $i = 1, \ldots, N$; $D_{jk}$, $Q_{djk}$ and $Q_{fjk}$, '$j$' and '$k$' suitable integers, denote these quantities respectively for intermediate stages; and $D_0$, $Q_d$, and $Q_f$, respectively denote them for the final stage.

The function CVTOBOOLEAN works as follows:

1) Generate the truth-table for CV rule for 2 nodes as per Table 1
2) As per the output of the truth-table (one of the decision sets from $\mathcal{D}_0$ to $\mathcal{D}_{15}$), compute the global DP and FAP ($Q_d$ and $Q_f$) and the final decision ($D$) as per Table 2.

We next consider *Example 2* to illustrate the procedure of radix-2 MDF strategy, in which we again consider the same network of four nodes as considered in *Example 1*.

*Example 2:* We assume that the nodes are paired in the order they appear in *Example 1*. The first pair of nodes with $(P_d, P_f)$ pairs (0.8, 0.2) and (0.7, 0.1) yields the truth-table of OR rule, for which the $Q_d$ and $Q_f$ values of the pair are obtained as 0.94 and 0.28 respectively. Similarly, for another pair of nodes with $(P_d, P_f)$ pairs (0.9, 0.3) and (0.6, 0.2), the corresponding truth-table follows the decision of the first node in the pair. Accordingly the $Q_d$ and $Q_f$ values of this pair of nodes become 0.9 and 0.3 respectively. At the second stage, these intermediate DP and FAP pairs, viz., (0.94, 0.28) and (0.9, 0.3) yield the truth-table of AND rule, which results in the overall $Q_d$, $Q_f$, $Q_d - Q_f$ and $P_e$ as 0.846, 0.084, 0.762 and 0.119. Here again, we assume $\alpha = 0.5$.

*Example 3:* To illustrate the effectiveness of the proposed radix-2 MDF strategy, we compare it with another radix-2 MDF strategy proposed by Gupta *et al.* [25], which works on the principle of combining the decisions of a node pair as per the AND or the OR rule whichever yields higher value of $(Q_d - Q_f)$ for that node pair. Accordingly, for the present example, OR rule is applied for both the pairs, which results in intermediate DP and FAP pairs as (0.94, 0.28) and (0.96, 0.44). Similarly, the second stage also applies the OR rule. The overall $Q_d$, $Q_f$, $Q_d - Q_f$ and $P_e$ are respectively obtained as 0.9976, 0.5248, 0.4728 and 0.2636. As before, $\alpha = 0.5$ is assumed.

Thus, *Example 1*, *Example 2* and *Example 3* comprehensively illustrate the procedure to express the CV rule in terms of the HDF rule, which enables the computation of overall DP, FAP and POE; and the procedure to apply the generalized radix-2 MDF strategy and its comparison with another radix-2 strategy based on a different principle [25]. The POE values, which are 0.0980, 0.119, and 0.2636, respectively, for the three approaches, demonstrate the closeness of the proposed strategy with the optimal strategy, and its superiority over another radix-2 strategy available in the literature.

### A. COMPUTATIONAL COMPLEXITY OF GENERALIZED RADIX-2 MULTISTAGE DECISION FUSION STRATEGY

The proposed MDF strategy yields a global decision about the presence or absence of the binary event under consideration, along with the global DP and FAP values in $v = \log_2 N$ successive stages. The computation of DP and FAP in every stage requires certain computations in terms of multiplications, divisions, additions and subtractions. Apparently multiplications and divisions consume more resources compared to additions and subtractions. We, therefore, compute the complexity of the proposed strategy in terms of the number of multiplications and divisions. Moreover, for convenience, we consider division operations equivalent to multiplication operations.

Each stage of the radix-2 MDF requires multiple calls to `CVTOBOOLEAN` function, which in turn generates the truth-table as per Table 1, and subsequently computes the global DP and FAP as per Table 2. To generate the truth-table once, 12 multiplications and 4 divisions are required. Counting number of divisions also under multiplications, generating the truth-table once requires 16 multiplications. Table 2 lists all possible decisions, $D_0$ through $D_{15}$ for two inputs. However, assuming DPs to be greater than the corresponding FAPs at the node level, which is true for all practical local detection tests, following the monotonicity property, the optimum fusion cannot involve complement of the local decisions [2]. Thus, there remain only six possible fusion rules, namely, $D_0$, $D_1$, $D_3$, $D_5$, $D_7$ and $D_{15}$. Among them $D_2$ and $D_7$ require one multiplication each to compute DP and one multiplication each to compute FAP. $D_0$, $D_3$, $D_5$ and $D_{15}$ do not require any mathematical operation to compute DP and FAP. Since, only one of these six possible fusion rules is optimum, to compute DP and FAP for combining two decisions as per optimum fusion rule, at maximum 2 multiplications are required. Combining it with 16 multiplications required to generate the truth-table, total number of multiplications required for each call to function `CVTOBOOLEAN` are 18. In all, the function `CVTOBOOLEAN` is called $2^v - 1$ times, which is evident from Algorithm 1. In addition, 1 division is required to compute $\mu$. Thus, the total number of multiplications required by the proposed radix-2 MDF strategy is $18 \times (2^v - 1) + 1$ or $18 \times (2^{\log_2 N} - 1) + 1$. For the special case of $\alpha = 0.5$, i.e., for equiprobable binary event, $\mu = 1$, therefore, the number of multiplications reduces to $14 \times (2^{\log_2 N} - 1)$. Clearly, the computational complexity of the proposed strategy in terms of number of multiplications is $\Theta(N)$.

In the general case, when DPs at the node level are not necessarily greater than the corresponding FAPs, and may become smaller due to changing parameters, the monotonicity property of the Boolean fusion rules does not hold and complements of local decisions may be involved in the optimum global decision. However, we show in A that $D_6$ and $D_9$, which involve maximum number of multiplications in computing global DP and FAP, are never optimal. Incidentally, $D_6$ and $D_9$ respectively represent EX–OR and EX–NOR Boolean operations. Thus, we show that EX–OR and EX–NOR fusion rules are never optimal in minimizing the total POE. Among remaining 14 possible DF rules from $D_0$ to $D_{15}$, maximum number of multiplications required to compute global DP and FAP are two, same as that for the case when the local DPs are always greater than the corresponding local FAPs. Thus, in the general case also, the total number of multiplications required in computing global DP and FAP are same as that for the special case of local DP greater than the corresponding local FAP for all nodes.

### B. COMPARISON OF COMPUTATIONAL COMPLEXITY WITH COUNTING RULES

Computational complexity, in terms of the multiplications required, of the optimal counting rule for cooperative event detection using a network of $N$ non-identical nodes, which results in the minimum POE among all counting rules, as computed by Gupta *et al.* [26] is $2 \times \big(3(N/2) - 1)2^N + N + 1\big)$. Following a similar approach, the number of multiplications required for HV rule can be evaluated to be $\binom{N}{N/2} + (2N + 1)2^N - 2^{N+2} - 2N + 6$. It can be similarly evaluated for other counting rules such as majority rule, and except for the AND and OR rules, is found to be of the same order. Thus, in general, the complexity of counting rules in evaluating global DP and FAP is extremely high even for a network of moderate size. Gupta *et al.* [26] propose another MDF for such networks, which requires $M$ multiplications to compute global DP and FAP, where $M$ is given by $[118(N - 1)/3]$, $[118(N - 1)/3] + 4$, and

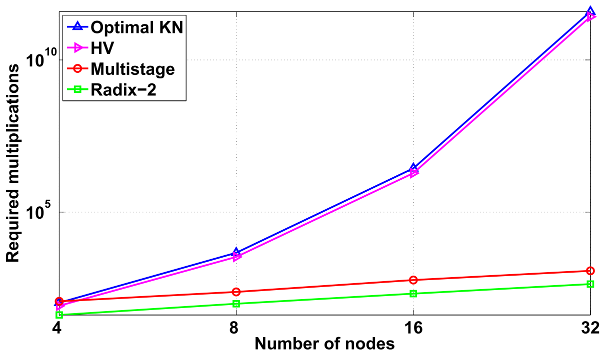


**FIGURE 2.** **Comparison of computational complexity of optimal KN, MDF proposed by Gupta *et al.* [26] and generalized radix-2 MDF strategy in terms of multiplication required for $N = 4, 8, 16$, and 32 nodes.**

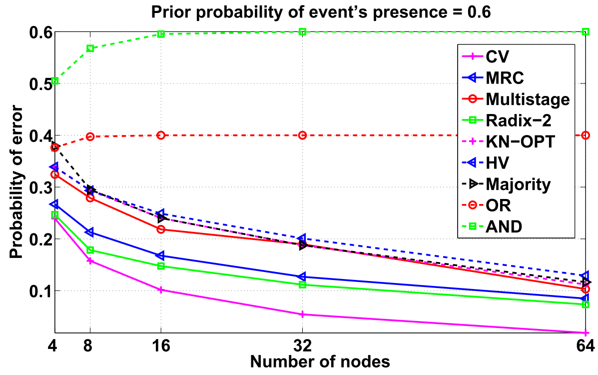


**FIGURE 3.** **Comparison of probability of error in binary event detection of radix-2 MDF strategy with the optimal (CV) rule and some other suboptimal soft and hard decision fusion rules, namely MRC based rule, optimal KN, HV, majority, OR, and AND rules, and a hard decision fusion based multistage scheme proposed by Gupta *et al.* [26] for $N = 4, 8, 16, 32$, and 64 nodes for Scenario I. Prior probability of the presence of event ($\alpha$) is considered to be 0.6.**

$[118(N-1)/3] + 32$, when $N$, $N/2$, and $N/3$ are divisible by 4 respectively. Figure 2 shows the comparison of computational complexity of the proposed generalized radix-2 MDF with that of the optimal counting rule and the MDF proposed by Gupta *et al.* [26]. We observe that the complexity of the optimal KN rule and the HV rule increases exponentially with the number of nodes, whereas it increases linearly for the other two schemes. It can be seen that the proposed strategy is the least complex among them.

## V. SIMULATION RESULTS

We now present the simulation results to show the efficacy of the proposed generalized radix-2 MDF strategy. We consider two simulation scenarios.

1) Scenario I: A CR–WSN performing spectrum sensing.
2) Scenario II: $P_d \sim U(0.25, 0.75)$, $P_f \sim U(0, 0.5)$, where '$U$' indicates uniform distribution.

In Scenario I, we consider a CR–WSN which performs spectrum sensing. The individual CR nodes perform energy detection (ED) for spectrum sensing [11], whereas the cooperative DF is performed following the proposed generalized radix-2 MDF strategy. The simulation scenario considered is as follows: The time–bandwidth product ($u$) is considered to be 5, the energy detection threshold ($\lambda$) is chosen as 3, the signal-to-noise ratio (SNR) ($\gamma$) is taken as uniformly distributed between 5 dB to 8 dB, and the noise variance ($\sigma_n^2$) is assumed to be uniformly distributed between 0.01 to 0.1. The DP ($P_{di}$) and FAP ($P_{fi}$) of the $i$th node are respectively computed in terms of Gaussian $\mathbb{Q}$-function as [38]

$$P_{xi} = \mathbb{Q}\left(\frac{\lambda - 2u(\gamma_{xi}+1)\sigma_n^2}{\sqrt{4u}(\gamma_{xi}+1)\sigma_n^2}\right) \tag{9}$$

where '$x$' denotes '$d$' for detection and '$f$' for false-alarm. Further, $\gamma_{xi}$ is $\gamma_i$ when '$x$' is '$d$' and is 0 when '$x$' is '$f$'.

Fig. 3 compares the POE in binary event detection of the generalized radix-2 MDF with the optimal SDF, namely, the CV rule [2], two suboptimal SDF approaches based on equal gain combining (EGC), which happens to be majority rule under the simulation scenario considered, and maximal ratio combining (MRC) [7], [27], and another MDF approach recently proposed by Gupta *et al.* [26]. Comparison is also done with all the counting rules. The counting rules considered cover entire range from OR rule to AND rule, including HV and majority rules. Also considered is the best of the counting rules, viz., optimal KN rule, which computes the POE for all counting rules (from $K = 1, \ldots, N$) and selects that value of $K$, which yields the smallest POE. The binary event is assumed to be present with a probability of 0.6. The comparison is made for $N = 4$, 8, 16, 32, and 64 nodes in the network. For simulation, 1000 instances of binary event are generated and the results are averaged over 100 trials. It can be observed from the figure that the proposed generalized radix-2 MDF shows lower POE among the other suboptimal strategies and its performance is slightly inferior to the optimal CV rule. Other suboptimal approaches show almost similar performance with the multistage approach of [26] showing better performance than that of MRC and EGC (majority rule) based approaches. Similar results are obtained for different prior probabilities of the presence of the event. Comparing with counting rules, we observe that the proposed strategy shows the best performance with the smallest POE. We further observe that in general, the performance of all the approaches improve with the increase in number of nodes, except for the AND and OR rules for which the performance deteriorates since the POE increases. Similar observations are made with other prior probabilities of the presence of the binary event.

Fig. 4 compares the proposed strategy with optimal CV rule and other suboptimal SDF and HDF rules as done in Fig. 3. However, now the comparison is for varying prior probability of event's presence from 0.1 to 0.9 in steps of 0.1. Number of nodes considered in the network are 8. It is again observed that the proposed strategy shows the POE performance very close to that exhibited by the CV rule and superior to all other suboptimal SDF and HDF rules.

As expected, the optimal CV rule exhibits the best performance with the least POE. The proposed strategy is close

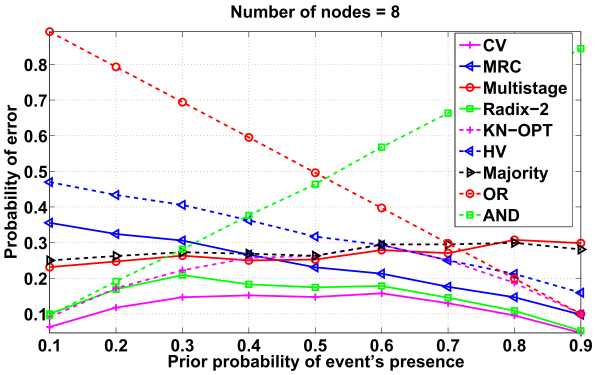


**FIGURE 4.** Comparison of probability of error in binary event detection of radix-2 MDF strategy with the optimal (CV) rule and some other suboptimal soft and hard decision fusion rules, namely MRC based rule, optimal KN, HV, majority, OR, and AND rules, and a hard decision fusion based 'multistage' scheme proposed by Gupta *et al.* [26] for prior probability of event's presence, ($\alpha$) varying from 0.1 to 0.9 in steps of 0.1 for Scenario I. The number of nodes $N$ is considered to be 8.

to it with the POE somewhat inferior to that of CV rule. However, at the cost of slightly poor POE compared to CV rule, the proposed strategy has the advantage of straightforward computation of global DP and FAP with very low computational complexity. Evaluation of these performance indexes is extremely complex for other strategies except for the MDF strategy of Gupta *et al.* [26] and the elementary AND and OR rules. However, the POE performance for AND and OR rules is very poor in general. For the MDF strategy of Gupta *et al.* [26] it is much better, however, the proposed strategy does better still.

Table 3 shows the $Q_d$ and $Q_f$ values also along with $P_e$ (the POE) for a network of 8 nodes and another network of 64 nodes for an equiprobable binary event. A closer look at this table reveals that the CV rule is better than the proposed strategy not only in terms of POE, but also in terms of $Q_d$ and $Q_f$,

**TABLE 3.** Comparison of radix-2 MDF with other soft and hard decision fusion approaches in terms of global detection probability ($Q_d$), global false-alarm probability ($Q_f$) and probability of error ($P_e$) for prior probability of the binary event's presence ($\alpha = 0.5$) and networks of 8 and 64 nodes.

| | | $Q_d$ | $Q_f$ | $P_e$ |
|---|---|---|---|---|
| $N = 8$ | CV | 0.8745 | 0.1690 | 0.1472 |
| | MRC | 0.9205 | 0.3814 | 0.2305 |
| | Multistage | 0.6866 | 0.1915 | 0.2524 |
| | Radix-2 | 0.8316 | 0.1800 | 0.1742 |
| | Opt KN | 0.7195 | 0.2452 | 0.2629 |
| | HV | 0.8726 | 0.5061 | 0.3168 |
| | Majority | 0.7195 | 0.2452 | 0.2629 |
| | OR | 0.9991 | 0.9910 | 0.4959 |
| | AND | 0.0738 | 0.0014 | 0.4638 |
| $N = 64$ | CV | 0.9814 | 0.0197 | 0.0192 |
| | MRC | 0.9934 | 0.2016 | 0.1041 |
| | Multistage | 0.8978 | 0.1034 | 0.1028 |
| | Radix-2 | 0.9323 | 0.0820 | 0.0748 |
| | Opt KN | 0.8478 | 0.0611 | 0.1066 |
| | HV | 0.9282 | 0.2118 | 0.1418 |
| | Majority | 0.9060 | 0.1458 | 0.1199 |
| | OR | 1.0000 | 1.0000 | 0.5000 |
| | AND | 0.0000 | 0.0000 | 0.5000 |

however, for the former these quantities are not easily computable. It then suggests to compute these quantities using the proposed strategy, and when the specified performance is met, the CV rule should be applied as it will yield a better performance. However, if the desired performance is not met with the proposed strategy, it may not also be achieved with the CV rule as their performances are close enough. In this case, more number of nodes should be involved in cooperative decision making until the desired performance is achieved. It may be noted that more nodes will consume more power, which must be kept in check; therefore, minimum number of nodes which achieve set performance should be selected.

To demonstrate the effectiveness of the proposed radix-2 MDF strategy in general, we consider an application independent scenario, Scenario II, wherein the DP and FAP of the nodes are assumed to be uniformly distributed in the intervals (0.25, 0.75) and (0, 0.5) respectively. These intervals are chosen to emphasize that, in general, the DP at the sensor node is greater than the corresponding FAP. However, to account for occasional violation of this rule during the sensor's working, which may be caused because of the changing parameters on which these probabilities depend, the corresponding intervals are made to overlap to some extent. Fig. 5 compares the performance of the generalized radix-2 MDF with other HDF and SDF approaches for a network of four nodes as the prior probabilities of the event's presence vary over a wide range from 0.1 to 0.9. Simulations are done by generating 1000 instances of the binary event and the results are averaged over 100 trials. We observe from the simulation curves that the POE of the proposed strategy is close to that of the optimal (CV) rule and is better than other suboptimal HDF and SDF strategies.

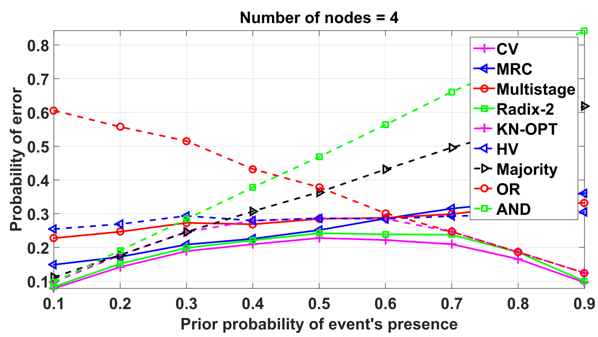


**FIGURE 5.** Comparison of probability of error in binary event detection of radix-2 MDF strategy with the optimal (CV) rule and some other suboptimal soft and hard decision fusion rules, namely MRC based rule, optimal KN, HV, majority, OR, and AND rules, and a hard decision fusion based 'multistage' scheme proposed by Gupta *et al.* [26] for prior probability of event's presence, ($\alpha$) varying from 0.1 to 0.9 in steps of 0.1 for Scenario II. The number of nodes $N$ is considered to be 4.

## VI. CONCLUSION

We proposed a generalized radix-2 multistage decision fusion strategy for binary event detection with a view to achieve a low probability of error in decision making and to enable efficient computation of overall detection and false-alarm probabilities at the same time for a network of nodes with non-identical local detection and false-alarm probability pairs. The strategy followed a 'divide and conquer' approach by dividing the number of nodes in the network in pairs and applying the optimal hard decision fusion strategy on those pairs and repeating this approach in successive stages, which ensures a low probability of error at the network-level. The optimal hard decision fusion applied to node pairs is indeed a hard decision fusion equivalent of the optimal soft decision fusion rule, which is more commonly known as the Chair–Varshney (CV) rule. Establishing the connection between the optimal hard decision fusion and the optimal soft decision fusion which minimize the probability of error and revealing this connection for a network of two nodes were other important aspects of this work, which made the basis for the proposed strategy. Through complexity analysis of various soft and hard decision fusion strategies, we observed that the proposed strategy is least complex for very small to large size networks in computing global detection and false-alarm probabilities, whose knowledge may be necessary in different situations. In addition, Monte Carlo simulations revealed the superiority of the proposed strategy over other suboptimal hard and soft decision fusion strategies in terms of the probability of error. In general, the analysis provided in this work revealed that the soft decision fusion simplifies to expressing it as a Boolean expression, which is the form of a hard decision fusion. Revelation of this simplicity, which engenders new multistage approaches for decision fusion with a potential to outperform the existing methods, is the crux of the present work. It should be noted that if we could obtain the hard decision fusion equivalent of the CV rule for any network of $N$ nodes, the performance of the proposed scheme would be identical to that of the CV rule. Though, in principle, it is possible to do so following the procedure proposed in this work, it would increase the complexity enormously for large $N$. We, therefore, attempt to achieve a performance close to that of the CV rule by a multistage decision fusion approach. The scheme proposed in this work is based on the assumption that the fusion center is aware of the detection and false-alarm probabilities of the individual nodes. Though this assumption seems somewhat idealistic, there are situations in which this information can be obtained in real-time through fast estimation algorithms. Invention of efficient and reliable fusion strategies that work for blind situations will be an interesting work.

## APPENDIX

Non-optimality of EX-OR and EX-NOR Fusion Rules From (3) and Table 1, in general, for $\mathcal{D}_6 = \{0110\}$, which is the EX-OR Boolean fusion rule, to hold, or in particular, the decision to be $D_6$, conditions (10) through (13) must hold.

$$\mu[(1-P_{d1})(1-P_{d2})]/[(1-P_{f1})(1-P_{f2})] < 1 \quad (10)$$
$$\mu[(1-P_{d1})P_{d2}]/[(1-P_{f1})P_{f2}] > 1 \quad (11)$$
$$\mu[P_{d1}(1-P_{d2})]/[P_{f1}(1-P_{f2})] > 1 \quad (12)$$
$$\mu[P_{d1}P_{d2}]/[P_{f1}P_{f2}] < 1 \quad (13)$$

We assume, without loss of generality, that $P_{d1} \leq P_{d2}$ and $P_{f1} \leq P_{f2}$. Then, there exist following six cases for consideration:

Case 1: $0 \leq P_{d1} \leq P_{d2} \leq P_{f1} \leq P_{f2} \leq 1$
Case 2: $0 \leq P_{d1} \leq P_{f1} \leq P_{f2} \leq P_{d2} \leq 1$
Case 3: $0 \leq P_{d1} \leq P_{f1} \leq P_{d2} \leq P_{f2} \leq 1$
Case 4: $0 \leq P_{f1} \leq P_{d1} \leq P_{d2} \leq P_{f2} \leq 1$
Case 5: $0 \leq P_{f1} \leq P_{d1} \leq P_{f2} \leq P_{d2} \leq 1$
Case 6: $0 \leq P_{f1} \leq P_{f2} \leq P_{d1} \leq P_{d2} \leq 1$

We now show that for all these cases one of the test conditions from (10) through (13) does not hold, therefore the decision set $\mathcal{D}_6$, which is the EX-OR rule, never satisfies the CV rule and therefore is never optimal in minimizing the total POE. For illustration, we choose $\mu = 1$ $(\alpha = 0.5)$ . It can similarly be shown for any value of $\mu$.

*Case 1*:

$$\begin{aligned}
&0 \leq P_{d1} \leq P_{d2} \leq P_{f1} \leq P_{f2} \leq 1\\
&\Leftrightarrow 0 \leq (1-P_{f2}) \leq (1-P_{f1}) \leq (1-P_{d2}) \leq (1-P_{d1}) \leq 1\\
&\Leftrightarrow (1-P_{d1})(1-P_{d2}) \geq (1-P_{f1})(1-P_{f2})\\
&\Leftrightarrow [(1-P_{d1})(1-P_{d2})]/[(1-P_{f1})(1-P_{f2})] \geq 1
\end{aligned}$$

which violates test condition (10).

*Case 2*:

$$\begin{aligned}
&0 \leq P_{d1} \leq P_{f1} \leq P_{f2} \leq P_{d2} \leq 1\\
&\Leftrightarrow 0 \leq (1-P_{d2}) \leq (1-P_{f2}) \leq (1-P_{f1}) \leq (1-P_{d1}) \leq 1\\
&\Leftrightarrow 0 \leq (1-P_{d2}) \leq (1-P_{f2}) \leq P_{d1} \leq P_{f1} \leq 1\\
&\Leftrightarrow P_{d1}(1-P_{d2}) \leq P_{f1}(1-P_{f2})\\
&\Leftrightarrow [P_{d1}(1-P_{d2})]/[P_{f1}(1-P_{f2})] \leq 1
\end{aligned}$$

which violates test condition (12).

*Case 3*:

$$\begin{aligned}
&0 \leq P_{d1} \leq P_{f1} \leq P_{d2} \leq P_{f2} \leq 1\\
&\Leftrightarrow 0 \leq (1-P_{f2}) \leq (1-P_{d2}) \leq (1-P_{f1}) \leq (1-P_{d1}) \leq 1\\
&\Leftrightarrow (1-P_{d1})(1-P_{d2}) \geq (1-P_{f1})(1-P_{f2})\\
&\Leftrightarrow [(1-P_{d1})(1-P_{d2})]/[(1-P_{f1})(1-P_{f2})] \geq 1
\end{aligned}$$

which violates test condition (10).

*Case 4*:

$$\begin{aligned}
&0 \leq P_{f1} \leq P_{d1} \leq P_{d2} \leq P_{f2} \leq 1\\
&\Leftrightarrow 0 \leq (1-P_{f2}) \leq (1-P_{d2}) \leq (1-P_{d1}) \leq (1-P_{f1}) \leq 1\\
&\Leftrightarrow 0 \leq (1-P_{d1}) \leq P_{f2} \leq P_{d2} \leq (1-P_{f1}) \leq 1\\
&\Leftrightarrow (1-P_{d1})P_{d2} \leq (1-P_{f1})P_{f2}\\
&\Leftrightarrow [(1-P_{d1})P_{d2}]/[(1-P_{f1})P_{f2}] \leq 1
\end{aligned}$$

which violates test condition (11).

*Case 5*:

$$\begin{aligned}
&0 \leq P_{f1} \leq P_{d1} \leq P_{f2} \leq P_{d2} \leq 1\\
&\Leftrightarrow P_{d1}P_{d2} \geq P_{f1}P_{f2}\\
&\Leftrightarrow [P_{d1}P_{d2}]/[P_{f1}P_{f2}] \geq 1
\end{aligned}$$

which violates test condition (13).

*Case 6*:

$$\begin{aligned} &0 \leq P_{f1} \leq P_{f2} \leq P_{d1} \leq P_{d2} \leq 1 \\ &\Leftrightarrow P_{d1}P_{d2} \geq P_{f1}P_{f2} \\ &\Leftrightarrow [P_{d1}P_{d2}]/[P_{f1}P_{f2}] \geq 1 \end{aligned}$$

which violates test condition (13).

It can similarly be shown for any value of $\mu$ (all $0 \leq \alpha \leq 1$) that one of the test conditions from (10) through (13) is never satisfied. Thus, EX-OR can never be an optimal fusion rule. On the similar lines, it can be shown that EX-NOR $[\mathcal{D}_9 = \{1001\}]$ can also never be an optimal fusion rule.

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