IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received August 12, 2018, accepted September 7, 2018, date of publication September 18, 2018, date of current version October 17, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2870861

A Low Complexity Precoding Algorithm Based on Parallel Conjugate Gradient for Massive MIMO Systems

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This work was supported in part by the National Natural Science Foundation of China under Grants 61701284, 61571108, and 61701285, in part by the China Postdoctoral Science Foundation funded project under Grant 2017M622233, in part by the Application Research Project for Postdoctoral Researchers of Qingdao, in part by the Scientific Research Foundation of the Shandong University of Science and Technology for Recruited Talents under Grant 2016RCJJ010, in part by the Sci. & Tech. Development Fund of Shandong Province of China under Grants 2016ZDJS02A11, 2014GGX101035, and ZR2017MF027, in part by the Taishan Scholar Climbing Program of Shandong Province, in part by the SDUST Research Fund under Grant 2015TDJH102, and in part by the Open Foundation of the State Key Laboratory of Networking and Switching Technology, Beijing University of Posts and Telecommunications, under Grant SKLNST-2016-2-14.

ABSTRACT Linear precoding algorithms with low complexity in massive multi-in multi-out system have always been a hot research topic to solve the problem of inter-cell interference. In this paper, we proposed a conjugate gradient-based regularized zero-forcing (CG-RZF) precoding algorithm, with which the base station can directly obtain the transmitted signal after RZF precoding and avoid directly solving the inverse matrix in RZF. Moreover, an RZF precoding algorithm based on a parallel conjugate gradient (Parallel-CG-RZF) is also proposed, which can optimize initial values and iterative process of the aforementioned CG-RZF precoding algorithm. The simulation results have shown that the proposed CG-RZF and the optimized Parallel-CG-RZF precoding algorithm can significantly improve the performance of bit error rate with fast convergence speed compared with other precoding algorithms and can reduce the number of global communications. Meanwhile, the calculation complexity of the proposed CG-RZF and Parallel-CG-RZF precoding algorithm is much lower than the optimized Chebyshev iteration algorithm.

INDEX TERMS Massive MIMO systems, parallel conjugate gradient, RZF, bit error rate, calculation complexity.

I. INTRODUCTION

With the development of wireless communication technology, the number of data transmission and access devices has grown exponentially in the communication networks [1]. As one of the key and promising technologies of 5G [2], massive MIMO technology can support higher data transmission rates, enhance system capacity, and improve power efficiency and spectrum efficiency, and thus it has broad prospects in application. However, the use of large number of antennas in the massive MIMO systems may cause series of limitations to the mobile communications networks, such as cell interference, pilot pollution, and multi-user interference. Precoding technique [3] can be used to overcome above limitations effectively based on kinds of precoding algorithms. The precoding algorithm can obtain a related precoding matrix based on the acquired channel state information (CSI), with which the transmitted signal can be precoded before transmission, and thus the received signal by the users can avoid inter-user interference in the cell.

According to different modes of creating the precoding matrix, the precoding algorithms are classified into non-linear precoding algorithms [4] and linear precoding algorithms, respectively. The linear precoding algorithms have much lower complexity and are easier to obtain the precoding matrix than the non-linear precoding algorithms [5], and thus becoming more suitable for use in the massive MIMO systems. In linear precoding algorithms, the Regularized Zero-Forcing (RZF) precoding algorithm is improved form

Zero-Forcing (ZF) precoding algorithm, and can obtain a much better performance as the number of transmitting antennas increases, and therefore, this paper proposed a lower complexity precoding algorithm based on the RZF precoding algorithm.

Linear precoding algorithms are designed to obtain the approximate solution of an inverse matrix in the precoding algorithm indirectly by some proposed methods, rather than calculating the inverse matrix directly to reduce the calculation complexity. These algorithms are mainly divided into three categories based on series expansion [6], iterative methods [7] and gradient methods [8]. In [9], the inverse matrix in the precoding algorithm is developed based on Kapteyn series which can be truncated to reduce the calculation complexity. In [10], Symmetric Successive Over Relaxation (SSOR) iteration method is proposed to obtain the estimation of the inverse matrix to lower the direct calculation complexity. After several iterations, the obtained iterative results are approximately equal to the required inverse matrix. In [11] a mixed iteration method combined with conjugate gradients is proposed, i.e. joint Conjugate Gradient and Jacobi iteration (CGJC), which can speed up convergence and thus reduce bit error rate in the precoding algorithm. In [12], a novel low-complexity linear precoding algorithm based on Jacobi method (JM) is proposed to avoid calculating the matrix inversion, which can achieve the near-optimal performance and capacity-approaching of ZF precoding with a reduced number of iterations. In [13], a low-complexity precoding scheme based on dirty paper coding and zero-forcing is proposed to improve the downlink sum rate for a multi-cell massive MIMO system and combines a reduced form of QR decomposition and an orthogonal projection by applying a quasi-Newton algorithm per iteration. In [14], a low peak-to-average power ratio precoding scheme based on an approximate message passing algorithm is proposed to minimize multiuser interference in massive multiuser MIMO systems, which exhibits fast convergence and low complexity characteristics. In [15], the precoding algorithms of iterative discrete estimation and IDE2 are developed for a downlink massive MU-MIMO system with finite-alphabet precoding based on the alternating direction method of multipliers framework.

In this paper, a low complexity RZF precoding algorithm based on the conjugate gradient method for Massive MIMO Systems is proposed. The main contributions of this paper are as follows. Firstly, a conjugate gradient based RZF precoding algorithm (CG-RZF) is proposed, with which the base station can directly obtain the transmitted signal after RZF precoding and avoid directly solving the inverse matrix in RZF and thus reducing the calculation complexity. Secondly, a parallel conjugate gradient based RZF precoding algorithm (Parallel-CG-RZF) is also proposed by optimizing initial values and iterative process of the aforementioned conjugate gradient based RZF precoding algorithm, which can not only accelerate the convergence speed, but also reduce the number of global communications overhead. Thirdly, the calculation complexity of both conjugate gradient and parallel conjugate gradient based RZF precoding algorithm is reduced greatly, meanwhile, they can improve the performance of the Massive MIMO Systems in terms of BER significantly.

The remainder of this paper is organized as follows. Section II describes the system model. Section III presents the proposed RZF precoding algorithm based on parallel conjugate gradient and parallel conjugate gradient, respectively, and then also analyzes the complexity of both proposed algorithms. Section IV investigates the performance of the proposed algorithms and shows simulation results in terms of the BER for CG-RZF precoding algorithm, Parallel-CG-RZF precoding algorithm and other RZF precoding algorithms, respectively. Section V concludes this paper.

II. SYSTEM MODEL

As shown in Fig. 1, the defined massive MIMO system consists of a multi-antenna (M-antennas) Base Station (BS) and several single-antenna (S-antenna) users. Let M denote the number of transmit antennas of the BS, and K denote the number of single-antenna users, respectively. In the system, the real channel state information (CSI) matrix between the BS and the users can be denoted by [16]

$$\mathbf{H} = [\mathbf{h}_1, \ldots \mathbf{h}_K], \mathbf{h}_k \in \mathbb{C}^{M \times 1} \tag{1}$$

where $\mathbf{h}_k$ is the real channel matrix from the k-th single antenna user to the BS, which subjects to an average of 0 Gaussian distribution.

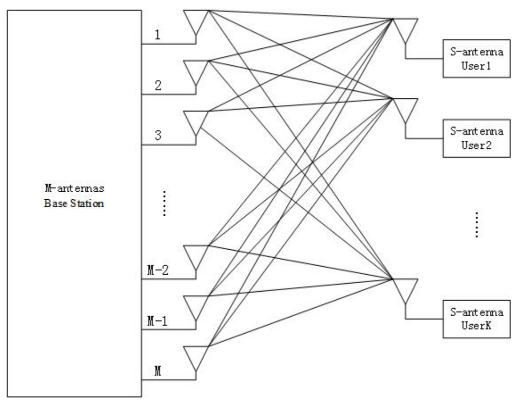

FIGURE 1. System model.

The estimated channel matrix obtained by the BS based on the real CSI matrix in equation (1) is calculated as

$$\hat{\mathbf{H}} = \left[\hat{\mathbf{h}}_1, \ldots \hat{\mathbf{h}}_K\right] \in \mathbb{C}^{M \times K} \tag{2}$$

where $\hat{\mathbf{h}}_k \in \mathbb{C}^{M \times 1}$ is the estimated channel matrix between the BS and the k-th single-antenna user calculated by the BS. Suppose that $\hat{h}_k$ subjects to Gaussian distribution, i.e. $\hat{\mathbf{h}}_k \sim CN\left(\mathbf{0}_{M \times 1}, \mathbf{\Phi}\right)$ where $\Phi \in \mathbb{C}^{M \times M}$ is the channel covariance matrix. However, in the actual channel environment, the BS may not be able to accurately obtain the estimated CSI matrix from equation (2), so this paper uses the general Gaussian

Markov model [17] to estimate channel matrix as follows.

$$\hat{\mathbf{h}}_k = \sqrt{1-\tau^2}\mathbf{h}_k + \tau \mathbf{n}_k \tag{3}$$

In equation (3), $\tau \in [0, 1]$ is the channel estimation parameter, and $\tau = 0$ means that the estimated channel matrix is the same as the real channel matrix, which denotes the channel estimation is quite accurate. In this model, the channel noise is also estimated by $\mathbf{n}_k$, which follows the Gaussian distribution $\mathbf{n}_k \sim CN\left(0, \sigma^2\right)$ and is distributed with the matrix $\mathbf{h}_k$ independently and identically.

The signal received by the k-th user can be expressed as

$$\mathbf{y}_k = \hat{\mathbf{h}}_k^{\mathrm{H}}\mathbf{x} + \mathbf{z}_k \tag{4}$$

where matrix $\mathbf{x}$ denotes the transmitted signal from the BS, and $\mathbf{z}_k$ denotes additive white Gaussian noise at the k-th user, which satisfies $\mathbf{z}_k \sim CN(0, \sigma^2)$.

Let $\mathbf{s} = [s_1, \ldots, s_K]^{\mathrm{T}} \sim CN\left(\mathbf{0}_{K\times 1}, \mathbf{I}_{K\times 1}\right)$ denote the transmitting data signal required by users from the BS, where s_k is the only desired signal for the k-th user. For mitigating these multi-user data interference, the transmitting data signal $\mathbf{s}$ should be preceded before being transmitted by the BS [18], and thus the transmitted signal $\mathbf{x}$ in equation (4) can be calculated by

$$\mathbf{x} = \mathbf{Gs} \tag{5}$$

where matrix $\mathbf{G} = [\mathbf{g}_1, \ldots, \mathbf{g}_K] \in \mathbb{C}^{M\times K}$ is the precoding matrix for the proposed precoding algorithm, which should satisfy the power constraint condition $tr\left(GG^H\right) = P$ and P is the overall transmitting power at the BS.

The proposed RZF precoding algorithm based on conjugate gradient is to obtain the transmitted signal matrix $\mathbf{x}$ in equation (4) directly and can avoid calculating the specific precoding matrix in equation (5). In this paper the Bit Error Rate (BER) is used to evaluate the performance of the proposed algorithms. The estimated received signal after signal detection can be expressed as

$$\tilde{\mathbf{x}} = \mathbf{Wy} \tag{6}$$

This paper adopts MMSE signal detection algorithm [19] to decode the received signal and the detection matrix $\mathbf{W}$ can be denoted by

$$\mathbf{W} = \left(\mathbf{H}^{\mathbf{H}}\mathbf{H} + \sigma^{\mathbf{2}}\mathbf{I}\right)^{-1} \tag{7}$$

Thus, the estimated received signal $\tilde{\mathbf{x}}$ is compared statistically with the original transmitted signal $\mathbf{x}$ to obtain the performance of the BER.

III. RZF PRECODING ALGORITHM BASED ON CONJUGATE GRADIENT

This section first makes a clear explanation to the proposed conjugate gradient based RZF precoding algorithm (CG-RZF), and then proposes a parallel conjugate gradient based RZF precoding algorithm (Parallel-CG-RZF) by further optimizing initial values and iterative process of the aforementioned CG-RZF precoding algorithm, and thus improving the performance in terms of Bit Error Rate (BER) and reducing the number of global communications overhead.

A. THE CONJUGATE GRADIENT BASED RZF PRECODING ALGORITHM (CG-RZF)

According to [20], the RZF precoding matrix can be expressed as

$$\mathbf{G} = \beta\hat{\mathbf{H}}\left(\hat{\mathbf{H}}^{\mathbf{H}}\mathbf{H} + \xi\mathbf{I}_K\right)^{-1} \tag{8}$$

Suppose $\mathbf{A} = \hat{\mathbf{H}}^{\mathbf{H}}\mathbf{H} + \xi\mathbf{I}_K$, and substitute matrix A into equation (8), we can obtain

$$\mathbf{G} = \beta\hat{\mathbf{H}}\hat{\mathbf{A}}^{-1} \tag{9}$$

Then substituting equation (9) into equation (5), we can have the transmitted signal $\mathbf{x}$ calculated by

$$\mathbf{x} = \beta\hat{\mathbf{H}}A^{-1}\mathbf{s} \tag{10}$$

Finally, let $\mathbf{t} = \mathbf{A}^{-\mathbf{1}}\mathbf{s}$ and we can obtain the linear equation as follows.

$$\mathbf{At} = \mathbf{s} \tag{11}$$

From the above, by using the conjugate gradient method to calculate the vector $\mathbf{t}$ based on the equation (11), the transmitted signal can be obtained directly, and avoid solving the specific precoding matrix. The detailed procedure of the proposed RZF precoding algorithm based on conjugate gradient is presented as follows.

Next, we discuss the convergence of the proposed CG-RZF precoding algorithm. According to the convergence theorem of classical conjugate gradient method proved in [21] and [22], the convergence condition of the iterative vector $\mathbf{t}_k$ in the above Algorithm 1 can be expressed as follows.

$$0 \le \left\|\mathbf{t}_k - \mathbf{t}^*\right\|_{\mathbf{A}} \le 2\left(\frac{\sqrt{\lambda_1}-\sqrt{\lambda_n}}{\sqrt{\lambda_1}+\sqrt{\lambda_n}}\right)^k \left\|\mathbf{t}_0 - \mathbf{t}^*\right\|_{\mathbf{A}} \tag{12}$$

where $\mathbf{A} = \left(\hat{\mathbf{H}}^{\mathrm{H}}\hat{\mathbf{H}} + \xi\mathbf{I}_k\right)$ in the Algorithm 1 denotes an n-order real symmetric positive definite matrix, and the maximum and minimum eigenvalues of $\mathbf{A}$ are denoted by $\lambda_1 > 0$

Algorithm 1 The Proposed RZF Precoding Algorithm Based on Conjugate Gradient

(Input $\hat{\mathbf{H}}$, $\mathbf{s}$; Output $\mathbf{x}$)

1. $\mathbf{A} = \left(\hat{\mathbf{H}}^{\mathrm{H}}\hat{\mathbf{H}} + \xi\mathbf{I}_k\right)$
2. $\mathbf{t}_0 = 0,\ \mathbf{r}_0 = \mathbf{s} - \mathbf{A}\mathbf{t}_0,\ \mathbf{p}_0 = \mathbf{r}_0$
3. for k = 0:n-1
4. $\mu_k = (\mathbf{r}_k, \mathbf{r}_k) / \left(\mathbf{p}_j, \mathbf{A}\mathbf{p}_j\right)$
5. $\mathbf{t}_{k+1} = \mathbf{t}_k + \mu_k\mathbf{p}_k$
6. $\mathbf{r}_{k+1} = \mathbf{r}_k - \mu_k\mathbf{A}\mathbf{p}_k$
7. $\eta_k = (\mathbf{r}_{k+1}, \mathbf{r}_{k+1}) / (\mathbf{r}_k, \mathbf{r}_k)$
8. $\mathbf{p}_{k+1} = \mathbf{r}_k + \eta_k\mathbf{p}_k$
9. end
10. $\mathbf{x} = \beta\hat{\mathbf{H}}t_n$

and $\lambda_n > 0$, respectively. In addition, the accurate value of the iterative vector $\mathbf{t}_k$ is denoted by $\mathbf{t}^*$ and the initial value of the iterative vector $\mathbf{t}_k$ is denoted by $\mathbf{t}_0$.

For the convergence, the right of the equation (12) can be converged to zero when the iteration times k is infinite, i.e.

$$\lim_{k\to\infty} \left\{2\left(\frac{\sqrt{\lambda_1}-\sqrt{\lambda_n}}{\sqrt{\lambda_1}+\sqrt{\lambda_n}}\right)^k \left\|\mathbf{t}_0-\mathbf{t}^*\right\|_{\mathbf{A}}\right\} = 0$$

$$\text{s.t. } \lim_{k\to\infty}\left(\frac{\sqrt{\lambda_1}-\sqrt{\lambda_n}}{\sqrt{\lambda_1}+\sqrt{\lambda_n}}\right)^k = 0 \text{ and } \frac{\sqrt{\lambda_1}-\sqrt{\lambda_n}}{\sqrt{\lambda_1}+\sqrt{\lambda_n}} < 1 \quad (13)$$

According to Squeeze Theorem, we can obtain the equation (14) as follows.

$$\lim_{k\to\infty} \left\|\mathbf{t}_k-\mathbf{t}^*\right\|_{\mathbf{A}} = 0 \quad (14)$$

This means that with the increase of the iteration times k, $\mathbf{t}_k$ will get closer to $\mathbf{t}^*$, and we have $\lim_{k\to\infty}\mathbf{t}_k = \mathbf{t}^*$ when the iteration times k is infinite. As a result, the proposed algorithm is converged.

Furthermore, it is also seen from the equation (12)

that the initial value $\mathbf{t}_0$ can significantly affect the convergence rate of the proposed algorithm, and $\mathbf{t}_0$ which is closer to the accurate value $\mathbf{t}^*$ can accelerate the convergence rate. Moreover, in the proposed CG-RZF precoding algorithm $\mathbf{t}_0$ is set to zero for simplicity, while in the proposed Parallel-CG-RZF precoding algorithm, we let $\mathbf{t}_0 = \mathbf{D}^{-1}\mathbf{s}$ after optimizing the initial values $\mathbf{t}_0$. The discussion of the convergence rate of the proposed algorithm is mainly presented from the perspective of the simulation results in Section VI.

B. THE PARALLEL CONJUGATE GRADIENT BASED RZF PRECODING ALGORITHM (PARALLEL-CG-RZF)

As the ratio of the number of the BS antennas to users increases, the diagonal elements of the matrix $\hat{\mathbf{H}}^{\mathrm{H}}\hat{\mathbf{H}}$ are much larger than those of non-diagonal elements, which tends to be diagonally dominant [23]. Therefore, matrix $\mathbf{A}$ is characterized by diagonally dominant property, and is approximately equal to its main diagonal matrix $\mathbf{\Lambda}$ when the number of antennas is very large, and thus equation (11) can be approximated as follows.

$$\mathbf{\Lambda t} = \mathbf{s} \quad (15)$$

Therefore, the initial value of $\mathbf{t}$ can be optimized and calculated as

$$\mathbf{t}_0 = \mathbf{\Lambda}^{-1}\mathbf{s} \quad (16)$$

The standard CG consists of two inner product calculations. In a distributed storage parallel system [24], the first inner product must be calculated before the second inner product data is required, so they need two separate global communications. Since global communications are relatively expensive in the current distributed storage parallel system, it is desirable to combine the two global communications into one to reduce communication overhead [25]. Here is a rearrangement of the calculation order of the CG method to reduce the number of global communications.

According to the intrinsic properties of CG process: orthogonality of residual vectors [26], we can obtain

$$\frac{(r_k, r_{k+1})}{(r_k, r_k)} \equiv \frac{(p_k, Ap_{k+1})}{(p_k, Ap_k)} = 0 \quad (17)$$

$$(r_i, Ar_j) = 0, \quad |i-j| > 1 \quad (18)$$

Rearrangement can be done by substituting $p_k = r_k + \eta_k p_{k-1}$ into (p_k, Ap_k), and thus we have

$$\begin{aligned}\sigma_k &= (p_k, v_k) = (p_k, Ap_k)\\ &= (r_k + \eta_k p_{k-1}, Ar_k + \eta_k v_{k-1})\\ &= (r_k, Ar_k) + \eta_k (r_k, v_{k-1}) + \eta_k (p_{k-1}, Ar_k)\\ &\quad + \eta_k^2 (p_{k-1}, v_{k-1})\\ &= (r_k, Ar_k) + 2\eta_k (r_k, v_{k-1}) + \eta_k^2 \sigma_{k-1}\end{aligned} \quad (19)$$

According to the equation (17) and considering about $\mathbf{r}_k = \mathbf{r}_{k-1} - \mu_{k-1}\mathbf{Ap}_{k-1} = \mathbf{r}_{k-1} - \mu_{k-1}v_{k-1}$, we can obtain

$$\begin{aligned}(r_k, r_k) &= (r_k, r_{k-1}) - \mu_{k-1}(r_k, v_{k-1})\\ \gamma_k &= 0 - \mu_{k-1}(r_k, v_{k-1})\end{aligned} \quad (20)$$

According to equation (19), equation (20) and $\eta_k = (r_{k+1}, r_{k+1}) / (r_k, r_k)$, and denoting $\delta_k = (r_k, Ar_k)$, thus we can have

$$\sigma_k = (r_k, Ar_k) + 2\eta_k(-\gamma_k/\mu_{k-1}) + \eta_k^2\sigma_{k-1} = \delta_k - \eta_k^2\sigma_{k-1} \quad (21)$$

The rearrangement from the above makes a correction to the standard CG method on the reduced global communication. And it can also be shown that this method is stable. The proposed algorithm is started by performing a one-step standard algorithm to obtain the initialization of v_1 and σ_1 as follows.

C. THE COMPLEXITY ANALYSIS

This part analyzes the complexity of the proposed CG-RZF precoding algorithm and the proposed Parallel-CG-RZF precoding algorithm, respectively. Here, the complexity is defined as the number of additions and multiplications required for the user data signal to be converted into signal transmitted by the antenna of the base station in equation (5).

According to the RZF precoding calculation procedures based on the conjugate gradient, it is assumed that $\hat{\mathbf{H}}$ and $\xi\mathbf{I}_k$ are known matrices. The transmitted signal is $\mathbf{x} = \beta\hat{\mathbf{H}}t_1$ after a single iteration of the conjugate gradient method. The overall calculation procedures of the proposed algorithms can be presented as follows.

1) The matrix $\hat{\mathbf{H}}^H$ multiplied by matrix $\hat{\mathbf{H}}$ and comes to $\hat{\mathbf{H}}^H\hat{\mathbf{H}}$, and the corresponding calculation complexity is $(2M-1)K^2$;

2) The matrix $\hat{\mathbf{H}}^H\hat{\mathbf{H}}$ added by diagonal matrix $\xi\mathbf{I}_K$ and comes to $\hat{\mathbf{H}}^H\hat{\mathbf{H}} + \xi\mathbf{I}_K$, and the corresponding calculation complexity is K;

3) The matrix $\mathbf{A}$ multiplied by vector $\mathbf{t}_0$ and comes to $\mathbf{At}_0$, and the corresponding calculation complexity is $(2K-1)K$;

TABLE 1. Calculation complexity of proposed CG-RZF algorithm and parallel-CG-RZF algorithm.

Iterations times/ N iterations	Addition times	Multiplication times	Calculation Complexity
CG-RZF Algorithm	$MK^2-2K+N\left(K^2+MK+4K-M-2\right)$	$MK^2+K^2-2K-1+N\left(K^2+MK+5K+M+2\right)$	$2MK^2+K^2-4K-1+N\left(2K^2+2MK+9K\right)$
Parallel-CG-RZF Algorithm	$MK^2+K^2+K+KM-M-2+N\left(K^2+5K-1\right)$	$MK^2+2K^2+KM+2K+2+N\left(K^2+6K+4\right)$	$2MK^2+3K^2+2KM+3K-M+N\left(2K^2+11K+3\right)$

Algorithm 2 The Proposed RZF Precoding Algorithm Based on Parallel Conjugate Gradient

(Input $\hat{\mathbf{H}}$, $\mathbf{s}$; Output $\mathbf{x}$)

1. $\mathbf{A}=\left(\hat{\mathbf{H}}^{\mathbf{H}}\hat{\mathbf{H}}+\xi\mathbf{I}_K\right)$
2. $\mathbf{D}=diag\left(diag\left(\hat{\mathbf{H}}^{\mathbf{H}}\hat{\mathbf{H}}+\xi\mathbf{I}_k\right)\right)$
3. $\mathbf{t}_0=\mathbf{D}^{-1}\mathbf{s},\ \mathbf{r}_1=\mathbf{s}-\mathbf{A}\mathbf{t}_0, \gamma_1=(\mathbf{r}_1,\mathbf{r}_1)\,\mathbf{p}_1=\mathbf{r}_1,\ \mathbf{v}_1=\mathbf{A}\mathbf{p}_1$
4. $\sigma_1=(\mathbf{p}_1,\mathbf{v}_1)\,,\ \mathbf{t}_2=(\gamma_1/\sigma_1)\,\mathbf{p}_1$
5. for k = 2:n-1
6. $\mathbf{s}_k=\mathbf{A}\mathbf{r}_k$
7. $\gamma_k=(\mathbf{r}_k,\mathbf{r}_k)\,,\ \delta_k=(\mathbf{r}_k,\mathbf{s}_k)$
8. $\eta_k=\gamma_k/\gamma_{k-1}$
9. $\mathbf{p}_k=\mathbf{r}_k+\eta_k\mathbf{p}_{k-1}$
10. $\mathbf{v}_k=\mathbf{s}_k+\eta_k\mathbf{v}_{k-1}$
11. $\sigma_k=\delta_k-\eta_k^2\sigma_{k-1}$
12. $\mu_k=\gamma_k/\sigma_k$
13. $\mathbf{t}_{k+1}=\mathbf{t}_k+\mu_k\mathbf{p}_k$
14. $\mathbf{r}_{k+1}=\mathbf{r}_k-\mu_k\mathbf{v}_k$
15. end
16. $\mathbf{x}=\beta\hat{\mathbf{H}}t_n$

4) The vector $\mathbf{s}$ added by vector $\mathbf{A}\mathbf{t}_0$ and comes to $\mathbf{r}_0=\mathbf{s}-\mathbf{A}\mathbf{t}_0$, and the corresponding calculation complexity is K;

5) The vector $\mathbf{r}_0^T$ multiplied by vector $\mathbf{r}_0$ and comes to $\mathbf{r}_0^T\mathbf{r}_0$, and the corresponding calculation complexity is $2K-1$;

6) The matrix $\mathbf{A}$ multiplied by vector $\mathbf{p}_0$ and comes to $\mathbf{A}\mathbf{p}_0$, and the corresponding calculation complexity is $(2K-1)\,K$;

7) The vector $\mathbf{p}_0^T$ multiplied by vector $\mathbf{A}\mathbf{p}_0$ and comes to $\mathbf{p}_0^T\mathbf{A}\mathbf{p}_0$, and the corresponding calculation complexity is $2K-1$;

8) The constant $\mathbf{r}_0^T\mathbf{r}_0$ divided by constant $\mathbf{p}_0^T\mathbf{A}\mathbf{p}_0$ and comes to $\mu_0=\frac{\mathbf{r}_0^T\mathbf{r}_0}{\mathbf{p}_0^T\mathbf{A}\mathbf{p}_0}$, and the corresponding calculation complexity is 1;

9) The constant μ_0 multiplied by vector $\mathbf{p}_0$ and then added by vector $\mathbf{t}_0$, thus coming to $\mathbf{t}_1=\mathbf{t}_0+\mu_0\mathbf{p}_0$, and the corresponding calculation complexity is K;

10) The matrix $\hat{\mathbf{H}}$ multiplied by vector $\mathbf{t}_1$ and comes to $\hat{\mathbf{H}}\mathbf{t}_1$, and the corresponding calculation complexity is $(2K-1)\,M$;

11) The constant β multiplied by vector $\hat{\mathbf{H}}\mathbf{t}_1$ and comes to $\beta\hat{\mathbf{H}}\mathbf{t}_1$, and the corresponding calculation complexity is M.

According to the above calculation procedures, it can be seen that the calculation complexity of the proposed CG-RZF precoding algorithm is $2MK^2+2MK+3K^2+5K-1$ after only a conjugate gradient iteration. Additionally, we can also obtain the calculation complexity of the proposed CG-RZF precoding algorithm after N iterations, denoted by $2MK^2+K^2-4K-1+N(2K^2+2MK+9K)$. Moreover, the calculation complexity of the proposed Parallel-CG-RZF precoding algorithm after N iterations can be denoted by $2MK^2+3K^2+2KM+3K-M+N\left(2K^2+11K+3\right)$.

The analysis results of calculation complexity of proposed CG-RZF Algorithm and Parallel-CG-RZF Algorithm are clearly shown in Table 1.

IV. SIMULATION AND ANALYSIS

In this section, we analyze and evaluate the performance and calculation complexity of the proposed conjugate gradient based RZF precoding algorithm (CG-RZF) and the proposed parallel conjugate gradient based RZF precoding algorithm (Parallel-CG-RZF), respectively. Compared with other RZF precoding algorithms, the proposed CG-RZF and Parallel-CG-RZF precoding algorithms can directly finds the transmit signal matrix and do not need to obtain the specific precoding matrix, and thus can avoid solving the inverse matrix in RZF precoding algorithm, so the bit error rate (BER) is used to appropriately evaluate the performance of the proposed algorithms in this section. In the following simulations, it is assumed that the number of transmitting antennas of the BS M equals to 256, the number of users K equals to 32. Moreover, 4QAM modulation mode is used by the BS, and the transmitted signal power is also normalized.

Fig. 2 shows the BER with the SNR using the proposed conjugate gradient based RZF precoding algorithm and other four precoding algorithms when iteration times N = 1. It is assumed that the channel estimation is imperfect, and the channel estimation parameter τ equals to 0.1. It is seen that the BER with the RZF precoding algorithm decreases much faster than that with other four algorithms. The reason is that the RZF precoding algorithm calculates the precoding matrix directly with high calculation complexity and thus its performance can be regarded as a reference to the proposed algorithms for evaluation. Moreover, it is also seen that the BER using Newton-RZF precoding algorithm is much smaller than that of using other three precoding algorithms after only one

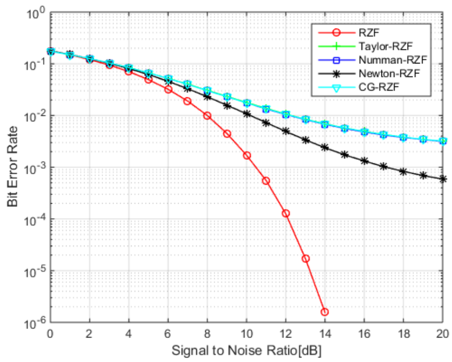

FIGURE 2. Bit error rate VS. SNR (M = 256, K = 32, τ = 0.1, N = 1).

iteration, and meanwhile the BER of CG, Taylor, Numman algorithms is almost the same and all improved slightly because of insufficient iteration times.

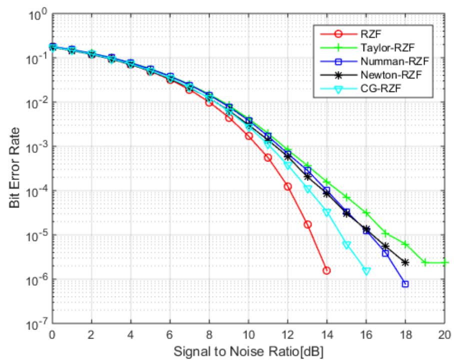

FIGURE 3. Bit error rate VS. SNR (M = 256, K = 32, τ = 0.1, N = 2).

Fig. 3 shows the BER with the SNR using the proposed CG-RZF precoding algorithm and other precoding algorithms when iteration times $N = 2$ and $\tau = 0.1$. It is obviously seen that, the BER with the four precoding algorithms performs much better after two iterations than after only an iteration shown in Fig. 2, and with increase of the SNR, the BER decreases much faster compared with the performance after only an iteration. Moreover, the proposed CG-RZF precoding algorithm has quite faster convergence speed, which can be explained that with the proposed CG-RZF algorithm the base station can directly obtain the transmitted signal after RZF precoding and avoid directly solving the inverse matrix in RZF and thus reducing the calculation complexity. Besides, after two iterations, the BER performance of the proposed algorithm is obviously superior to the Neumann-RZF and the Taylor-RZF precoding algorithm, and it also performs much better than Newton-RZF precoding algorithm in terms of BER, respectively.

Fig. 4 shows the BER with the SNR using the proposed Parallel-CG-RZF and CG-RZF precoding algorithms compared to that of the referred RZF precoding algorithm when iteration times $N = 1$. It can be seen that the BER with the proposed Parallel-CG-RZF algorithm is quite smaller than

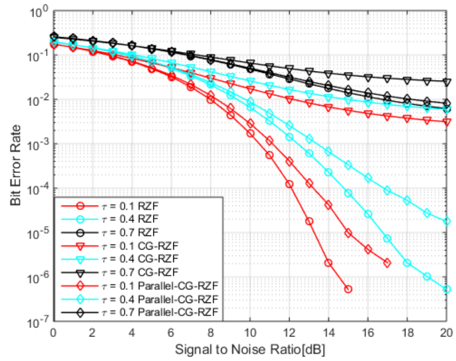

FIGURE 4. Bit error rate VS. SNR (M = 256, K = 32, N = 1).

that with the proposed CG-RZF algorithm and approaches the RZF precoding algorithm after only one iteration. Moreover, it also can be seen that the convergence speed of the proposed Parallel-CG-RZF precoding algorithm is obviously accelerated compared with the proposed CG-RZF precoding algorithms. This is because the Parallel-CG-RZF algorithm can optimize initial values and iterative process of the CG-RZF algorithm, and thus the BER and convergence speed are meanwhile improved significantly.

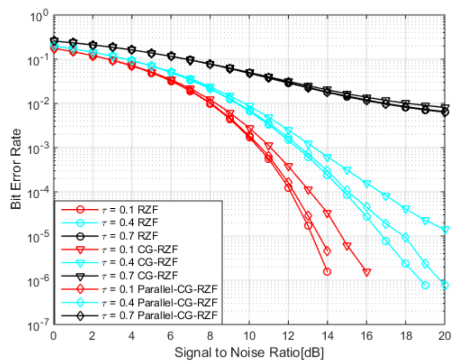

FIGURE 5. Bit error rate VS. SNR (M = 256, K = 32, N = 2).

Fig. 5 shows the BER with the SNR using the proposed Parallel-CG-RZF and CG-RZF precoding algorithms when iteration times $N = 2$. It is noticed that the proposed Parallel-CG-RZF and CG-RZF precoding algorithms can obtain much lower BER after two iterations than after only an iteration shown in Fig. 4. It also can be clearly seen that the BER of the Parallel-CG-RZF precoding algorithm gets nearly close to that of the RZF precoding algorithm, and the Parallel-CG-RZF precoding algorithm still performs much better than the CG-RZF precoding algorithm in term of BER and convergence speed, respectively. Moreover, the proposed Parallel-CG-RZF precoding algorithm has a significantly faster convergence speed after two iterations because of the reduced calculation complexity from the analysis in Section III. Additionally, the Parallel-CG-RZF precoding algorithm after only one iteration achieves the same performance as other precoding algorithms after two iterations,

TABLE 2. Complexity of proposed algorithms VS. Chebyshev-RZF algorithms.

Algorithms	One iteration Complexity	Two iterations Complexity
Chebyshev-RZF Algorithm	$2MK^2+K^2+2MK$ $-K+1$	$2MK^2+9K^2+2MK$ $+2K+1$
Optimized-Chebyshev-RZF Algorithm	$2MK^2+3K^2+2MK+K$	$2MK^2+11K^2+2MK$
CG-RZF Algorithm	MK^2+MK+K^2 $+2K-M-2$	$2MK^2+5K^2+4MK$ $+14K-1$
Parallel-CG-RZF Algorithm	$2MK^2+5K^2+2KM$ $+14K-M+3$	$2MK^2+7K^2+2KM$ $+25K-M+6$

which are shown in Fig. 4 and Fig. 3, respectively. Therefore, the proposed Parallel-CG-RZF precoding algorithm performs much better than other precoding algorithms in terms of BER and calculation complexity, respectively.

Moreover, form Fig. 4 and Fig. 5 it is seen that the BER performance of the proposed precoding scheme can be affected by the channel estimation parameter τ, and the BER performance of the proposed algorithm is getting much better with the value of τ is becoming smaller. The reason is that the smaller τ means the channel estimation is more accurate, based on which the performance of the proposed schemes can perform better.

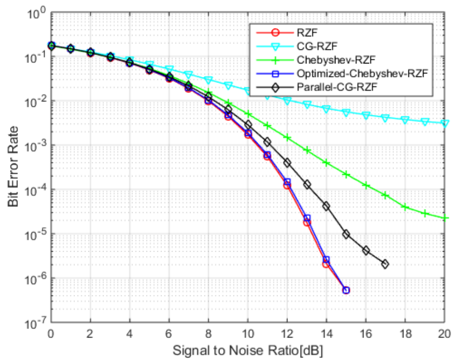

FIGURE 6. Bit error rate VS. SNR (M = 256, K = 32, τ = 0.1, N = 1).

In Fig. 6 and Fig. 7, we compare the proposed Parallel-CG-RZF and CG-RZF precoding algorithms with Chebyshev iteration [20] in terms of the performance of BER. It can be seen that when the number of iterations is 1, the proposed Parallel-CG-RZF precoding algorithms is obviously superior to the Chebyshev iteration algorithm proposed by Zhang *et al.* [20], but performs slightly less than the optimized Chebyshev iteration algorithm. Moreover, when the number of iterations is 2, the performance of the proposed Parallel-CG-RZF precoding algorithm is quite similar to the optimized Chebyshev iteration algorithm. And the complexity of the optimized Chebyshev iteration algorithm at N = 2 can be calculated as $2MK^2+11K^2+2MK$, however, the complexity of the Parallel-CG iteration algorithm is $2MK^2 + 7K^2 + 2KM + 25K - M + 6$. The comparison results are clearly shown in Table 2. Thus it can be see that the complexity of the Parallel-CG-RZF and CG-RZF precoding algorithms

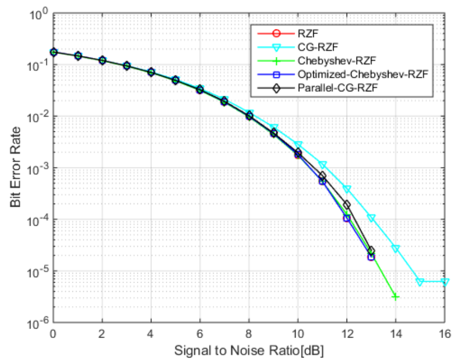

FIGURE 7. Bit error rate VS. SNR (M = 256, K = 32, τ = 0.1, N = 2).

are much lower than the optimized Chebyshev iteration algorithm by directly obtaining the transmitted signal and avoiding solving the inverse matrix in RZF and thus reducing the calculation complexity.

V. CONCLUSIONS

In this paper, we proposed a low complexity RZF precoding algorithm based on the conjugate gradient for Massive MIMO Systems. By using the conjugate gradient method, the base station can directly obtain the transmitted signal after RZF precoding and avoid directly solving the inverse matrix in RZF and thus reducing the calculation complexity. Moreover, a parallel conjugate gradient based RZF precoding algorithm is also proposed by optimizing initial values and iterative process of the aforementioned conjugate gradient based RZF precoding algorithm, which can speed up the iterations and reduce the number of global communications overhead. The simulation results have shown that the proposed CG-RZF precoding algorithm can significantly improve BER after two iterations with fast convergence speed. More importantly, the optimized Parallel-CG-RZF precoding algorithm can obtain much better performance of BER compared with other precoding algorithms, meanwhile, the calculation complexity of which is much lower than the Chebyshev iteration algorithm and optimized Chebyshev iteration algorithm, respectively.

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