Received July 7, 2018, accepted August 14, 2018, date of publication September 17, 2018, date of current version October 12, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2869282

A Novel Approach of Option Portfolio Construction Using the Kelly Criterion

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The work of M.-E. Wu was supported by the Ministry of Science and Technology, Taiwan, under Grant MOST 106-2221-E-027-145 and Grant MOST 106-3114-E-001-006. The work of W.-H. Chung was supported in part by the Ministry of Science and Technology, Taiwan, under Grant 107-3017-F-009-001, Grant 104-2221-E-007-147-MY3, and Grant 105-2221-E-007-144-MY3 and in part by the Digital Economy Advancement Project under Grant 106-2218-E-002-014-MY4.

ABSTRACT Money management is one of the most important issues in financial trading. Many skills of money managements are based on the Kelly criterion, which is a theoretical optimization of bidding an optimal fraction for position sizing. However, there is still a large gap between the theory and the real trading for money management. In this paper, we design an option trading strategy via Kelly criterion. While the price movements of options are highly volatile, various options' portfolio can be formed by long or short at different strike prices to pre-lock the losses and profits; then we have a fixed profit and loss distribution via holding an option portfolio. Consequently, the Kelly criterion can be applied to the options' trading for calculating the optimal bidding fraction. We propose a method for option trading, in finding the profitable option portfolio by bidding optimal fraction. Compared with prior works, our proposed model is a novel approach for options' trading with the money management of position sizing. Experiments are conducted to show the feasibility and profitability of our method in practical scenarios. Future works are provided in the final section.

INDEX TERMS Money management, Kelly criterion, option spread trading, profitable gamble, optimal fraction.

I. INTRODUCTION

The most important issues for financial trading is money management. The origin of money management was studied by Kelly [1] in 1956, who proposed the Kelly criterion to solve the bidding problem with imperfect information by communications errors; it was later used to calculate the optimal bidding fraction [3]–[6], [14]. Other application areas include casino games, such as BlackJack [2] and Texas Hold'em Poker, (also referred as just Hold'em or Holdem), and money management [7]–[10] while trading financial instruments, such as stocks, futures, options, and currencies [21], [22]. The Kelly criterion can be interpreted as to optimize the bidding fraction of total assets for infinite rounds of bids [11]–[13]. For example, we consider a game (e.g., coin-tossing) with win rate & odds; the game is to be played for infinite rounds. In each round, we decide the bidding fraction of our total assets. If we lose the game at some round, the bid amount is lost. Otherwise, we win the game at some round, the earning profit calculated according to the odds will be returned. The Kelly criterion is a method to find the optimal bidding fraction for pursuing the maximal asset growing rate.

Nevertheless, the Kelly criterion may not be applicable to practical scenarios due to the gap between its assumptions and the markets. There are lots of researches studying on the drawbacks of Kelly criterion [5], [17], [18]. The most serious two issues are shown in the following.

Firstly, the Kelly criterion assumes that the games can be repeated infinitely, which is impractical [12]. In the derivation of the Kelly formula, we must set the number of playing games to be infinity such that the proportion of winning times approach to the win rate according to the law of large numbers. Then, we find the optimal value to maximize the growth rate by using the skill of calculus and then derive the Kelly formula. In the real world for gambling or trading, although the number of playing times must be finite, the difference of the returns between the finite games and the infinite games can be measured by KL divergence and the result depends on the binomial distribution. In fact, if the proportion of winning times in finite games is close to the win rate, we still can use the optimal fraction derived from Kelly formula to maximize the growth rate.

Secondly, Kelly criterion may not be applicable to real financial instruments. In real trading, the true distribution of profit\loss and the win rate are unavailable. Note that the concepts of profit\loss distribution in trading is like the odds distribution in traditional gamble. We use these two terms mixed in this paper. The most significant difference in financial trading and traditional gambling is the known win rate and the odds in conventional gamble of casino games. For example, in a coin-toss game, the win rate is 50% and odds is 2, but the win rate and the odds are not fixed in financial trading. For this challenge, Ralph Vince provided the concept of holding period return (HPR) in his work [7], [8], which obtained the optimal bidding fraction (Opt. f) via empirical outcomes. The Opt. f can be interpreted as the extension of the Kelly criterion, rendering the Kelly formula as a special scenario with a single set of win rate and odds. Vince's optimal fraction is found to possess better applicability to real-world trading.

Although the Vince's method considers the optimal bidding fraction under multiple odds, it still cannot be totally perfectly applied to real-world trading. For the case of real trading in financial markets, the win rates and the odds distributions vary as time goes on. One of the skills to fix the odds distribution is using the trading mechanisms, such as cut the loss and stop the profit. However, even with the pre-set stop-profit and stop-loss mechanisms, which seem to fix the odds a priori, the win rate still changes with different threshold for stop-loss or stop-profit.

The above issues may cause the traders suffer the huge risks when using Vince's Opt. f. Therefore, some studies suggest using the half Kelly fraction [17], with a compromise on optimality. As a result, the theoretical aspects of the optimal bidding fraction are widely discussed but there are few real-world applications. The fundamental unsolved difficulty is the unpredictability of odds distributions in all kinds of trading strategies. Besides, it is not practical to assume the trader can play the infinite number of trading. The odds and win rate may also not be consistent with actual outcomes as time goes by for a long time.

Although we use the mathematical optimization to calculate the optimal bidding fraction for gambling or trading in theory. there is still a large gap between traditional gambling and real trading. The main difference is the odds distribution between gamble and trading. For most games in traditional gambles, such as poker in casino, horse racing, sport betting, and so on, the odds distributions are fixed in advance. We can calculate the optimal bidding fraction for such kind of games. However, for the profit\loss in most trading strategies, the odds distributions are varying under the different time periods. We cannot predict the odds distributions accurately under some fixed time periods. In this work, we propose an approach to apply Vince' optimal fraction to options trading. One of the advantages of option trading is the fixed profit & loss distribution in the beginning of building the option portfolio. Thus, we can reasonably compute the optimal bidding fraction for all kinds of option combinations with different strike prices. Of course, we may find the most profitable combination of option portfolio with the index distribution estimated. This is the first study about how to apply the theoretical money management methods to the real option trading in practice, which is the main originality of this work.

This work provides an options trading strategy which can apply the theoretical optimization for money management. In options trading, there exist various options portfolio methods, such as bull spread, and bear spread utilizing the different strike prices of Call or Put options. Once the portfolio is set, the risks are moderately controlled; the odds distribution is known and fixed if we hold the positions till the settlement date. To apply the Kelly criterion in our trading model, the only unknown part is the distribution of the underlying index prices upon settlement. The strategy proposed in this work can operate without human intervention. The construction of the portfolio and the bidding fraction are completely based on the historical data, and the quotes on the underlying options.

In addition, the other important issue for financial trading is portfolio. The most well-known portfolio management is Markowitz's modern portfolio theory (MPT) [21]. With the estimations of mean and variance, we can maximize the return under the constraints of minimum deviation according to efficient frontier. However, the MPT is suitable for the portfolio of different assets, which is not the same case for the options portfolio with different strike prices. In the paper, the composition of options with different strike prices can be seen as a single asset with the fixed distributions of profit\loss. Thus, we just use the Ralph Vince's method to find the optimal bidding fraction for option portfolio. The remaining task is that we must estimate the odds distribution of market index instead of the mean and variance in MPT. Usually, financial investment companies adapted lots of information for their stock evaluations, such as P\E ratio, annual return, macro analysis, industry analysis, technical analysis and so on. Moreover, most companies use Markowitz's modern portfolio theory for their asserts allocation and money management. In this work, we use the skill of gambling to study the investing issues. For each trading action, we would like to compute the "optimal" bidding fraction to maximize the growth return. The skill depends on the win rate and the odds at each bidding time step, just like the coin tossing game with 50% win rate and odds 2. Of course, we still can consider the information to improve the prediction of the win rate and odds. Once we obtain the accurate win rate and odds, or the profit\loss distribution in trading cases, we can use the formula mentioned in the paper to find the optimal fraction. The formula actually works well under the assumption of accurate profit\loss distributions.

II. PRELIMINARIES

A. KELLY CRITERION

A game with win rate p and odds b is considered, where odds b represents, without loss of generality, the player's bidding of 1 dollar will lose 1 dollar in the case of losing, and will win $1 + b$ dollars in the case of a win. In other words, the net profit is b in the case of bidding 1 dollar. Assuming the initial asset is A_0, and the asset at t-th step is A_t, the player bids the fraction f of his total asset in each step, where $0\% < f < 100\%$. We can derive the following asset growth/decrease pattern:

$$A_t = A_{t-1}(1 + bf), \quad \text{for a win in the } (t-1) - \text{th step; or}$$
$$A_t = A_{t-1}(1 - f), \quad \text{for a loss in the} (t-1) - \text{th step.}$$

After T rounds of plays, we denote the number of wins as T and number of losses as L, i.e., $T = W + L$. Then, by the above derivations, we obtain

$$A_T = A_0(1 + bf)^W(1 - f)^L.$$

By optimizing A_T, the solution of f is obtained by dividing the total trading rounds T and taking a log function. That is

$$\log\frac{A_T}{A_0} = \frac{W}{T}\log(1 + bf) + \frac{L}{T}\log(1 - f).$$

Since the win rate is p, letting T approaching to infinity we have

$$\lim_{T\to\infty}\log\frac{A_T}{A_0} = p\log(1 + bf) + (1 - p)\log(1 - f).$$

To obtain the maxima of the asset growth rate, we take the derivative of the above equation and obtain the optimal fraction as

$$f = \frac{p(1 + b) - 1}{b}.$$

However, the realized number of wins and losses depends on the binomial distribution created during the process of playing T rounds. For example, 100 gambles with a win-rate of 50% will not always be composed of 50 wins and 50 losses. According to the binomial theorem, there is a probability of $C_k^{100} 50\%^k \times 50\%^{100-k}$ of winning k times and losing $100-k$ times. Therefore, there is a difference between theory and practice. Due to the limited number of games in real trading, the issue needs to be resolved. Next, we take a coin game as an example. In the coin game, the probability of getting a head/tail is 50%. If a head turns out in a round, the odds is 2, that is, the bid of one dollar will win three dollars (net profit is two dollars), as illustrated below.

Consider playing the above game for 40 rounds. By the Kelly criterion, the bidding fraction will result in an expected return of

$$A_{40}(f) = (1 + 2f)^{20} \times (1 - f)^{20}.$$

Via the above equation and considering 1 dollar as the initial asset, the final asset versus various bidding fractions are shown in the Fig. 2.

FIGURE 1. Coin-tossing with win rate 50% and odds 2.

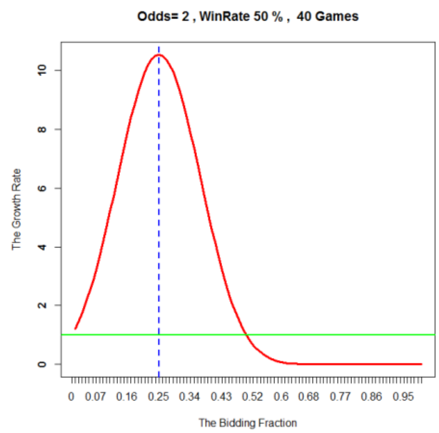

FIGURE 2. The returns after 40 rounds coin-tossing under different bidding fractions 0% ∼ 100%.

Via the above computations with Kelly criterion, a game with win rate 50% and odds 2 will lead to its optimal fraction (which obtains its fastest asset growth rate) as 25%, and its final asset will be 10.5 times its initial asset theoretically.

B. VINCE'S EXTENSION

Kelly criterion is just a special case for traditional gambles such as coin-tossing. Usually we use the gamble with binary outcomes (win & loss) to model and investigate Kelly's theory. However, this is not applicable in practical trading with invariant win rates and odds. Ralph Vince extends Kelly criterion and considers the bidding fraction under multiple sets of outcomes, instead of binary outcomes, i.e., a game with the following outcomes is considered:

$$(b_1, b_2, \ldots, b_n), \quad \text{where } b_i \in \mathbb{Z}.$$

The holding period return of each profit\loss is defined as:

$$\text{HPR}_i(f) = (1 - f\frac{-b_i}{L}),$$

where $L = \min\{b_1, b_2, \ldots, b_n\}$ denotes the biggest loss and the default of L is negative ($L < 0$). Under the fraction f, we multiply each term of HPR_i, which represents each b_i is realized once in the game. Vince then define the Terminal Wealth Relative (TWR) as the geometric mean of the under such realizations, as follows:

$$\text{TWR}(f) = (\text{HPR}_1(f) \times \text{HPR}_2(f) \ldots \times \text{HPR}_n(f))^{1/n}.$$

The above equation sustains in the case of equal probabilities for each outcome. For unequal probabilities, each b_i is associated with a probability p_i, and the TWR is defined as

$$\text{TWR}(f) = \prod_{i=1}^{n} (\text{HPR}_i(f))^{p_i}.$$

By plugging $f = 1\%, 2\%, \ldots, 100\%$ into the above equation, we can generate the TWRs corresponding to different f's. The f value corresponding to the maximum TWR is considered the optimal fraction. For example, we consider the game of profit/loss distribution as (3, −5, −2, 15), by TWR, we can obtain the growth rate versus the bidding fraction, as follows.

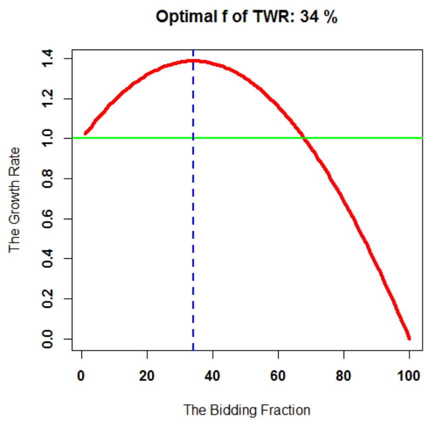

FIGURE 3. The returns of bidding different fractions on profit vector (3, −5, −2, 15). In this case the optimal bidding fraction is 34%.

In Vince' work, the Kelly criterion can be seen as a special case, and Vince' work obtains a more generalized framework for money management. Vince' work is more applicable to the real trading because of its incorporation of multiple outcomes, while some challenges remain. The major challenge is that Vince' work assumes a fixed set of its profit/loss outcomes, which may not be practical. The win rate and odds evolve and vary with time. In Vince' work, the probabilities associated with each outcome are calculated after collecting a certain amount of outcome realizations, which require a certain time elapsed. This method of calculation from hindsight often leads to overly large bidding fraction. Therefore, in this work, we investigate the optimal fraction for options portfolio. One major character for options is that its profit/loss distribution is known a priori, and therefore our investigation only needs to focus on the probability distributions.

III. OPTIMAL FRACTION OF OPTION TRADING

One of the property of option trading is the fixed distribution of profit and loss. We apply this property to better fit the usage of the Kelly criterion. In our proposed model, we do not find the traditional trading signal instead of investigating the size of holding positions for options with different strike prices.

A. FINDING THE FAVORABLE OPTIONS SPREAD

We consider a real case for example. At 2017-01-13 Friday, PM 13:30, the Taiwan Stock Exchange Index (short for TAIEX) closes at 9378. The quote for the weekly option to be cleared at Wednesday 2017-01-18 is as follows.

TABLE 1. Call and put for weekly options of TAIEX (PM 13:30, 2017-01-13).

Call	Strike Price	Put
90	9300	14.5
54	9350	29.5
29.5	9400	53
13.5	9450	89
5.5	9500	130

Consider a position of "long 9300 Call @90", and "short 9350Call @54". The portfolio is a bull spread and the distribution of profit and loss is distributed as follows.

From Fig. 4, the biggest loss of this portfolio is 36 points, to occur at TAIEX closing below 9300; the largest profit is 14 points, to occur at TAIEX closing above 9350. We illustrate it profits and losses as in Table II.

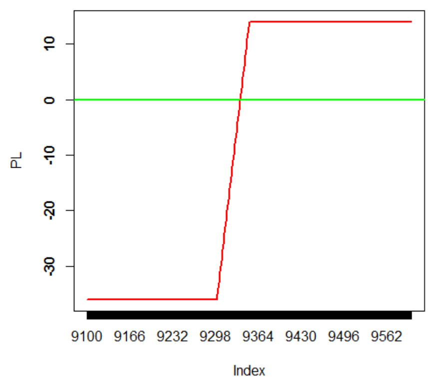

FIGURE 4. The distribution of profit and loss of a bull spread (Long 9300 Call @ 90; Short 9350 Call @ 54).

Table II shows the similar profit and loss structure as in conventional games, where the profits and losses to be adopted in the fraction calculation are based on the outcomes realized. Therefore, the Vince's optimal can be applicable to the profit and loss structure in Table II. Up to this step, the prediction of the TAIFEX at the closing date (2017-01-18) remains to be tackled. Since we construct the portfolio Friday PM 13:30, to predict the distribution of TAFEX at the following Wednesday, an intuitive technique is to use the adopt the historical TAIFEX rising/falling points from Friday PM 13:30 to its following closing prices on Wednesday.

TABLE 2. The profit vecotr of "Long 9300 Call @ 90; Short 9350 Call @ 54".

TAIEX	Profit or Loss
≤ 9300	-36
9301	-35
9302	-34
…	…
9348	+12
9349	+13
≥ 9350	+14

The empirical distributions (say PM 13:30, 2017-01-13 ∼ PM 13:30, 2017-01-18) can be treated as the distributions to be used in the fraction calculations. In this work, we collect data from 2007-01-5 to 2017-1-11 of TAIEX, as shown in Table III.

TABLE 3. The returns of TAIEX from friday to next wednesday during 2007/01/05∼2017/01/11.

Duration	Return
2007-01-05 ~ 2007-01-10	+3.04%
2007-01-12 ~ 2007-01-17	+1.56%
…	
2016-12-30 ~ 2017-01-04	-0.27%
2017-01-06 ~ 2017-01-11	-0.34%

There are 490 observations, which are summarized and shown in the form of empirical probability density distribution and histogram in Fig. 5.

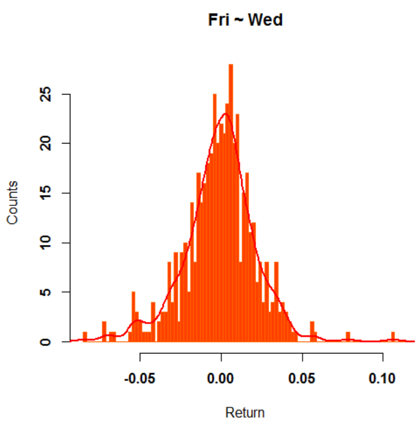

FIGURE 5. The histogram of TAIEX returns from Friday to Wednesday during 2007-01-05 ∼ 2017-01-11.

The distributions in Fig. 5 is used as the rise/fall return predictions for 2017-01-11 to PM 13:30, 2017-01-18 TAIEX. The TAIEX at PM 13:30, 2017-01-11 is 9378, which is used in the formula 9378 × (1+ Returns in Table III), and we can obtain the distributions of PM 13:30, 2017-01-18 TAIEX. The outcomes are demonstrated in Table IV.

TABLE 4. The estimated TAIEX on PM 13:30, 2017-01-18.

Duration	Return	Estimated TAIEX
2007-01-05 ~ 2007-01-10	+3.04%	9663
2007-01-12 ~ 2007-01-17	+1.56%	9524
…	…	…
2016-12-30 ~ 2017-01-04	-0.27%	9353
2017-01-06 ~ 2017-01-11	-0.33%	9347

We use Tables II and IV, along with the market index distribution, to calculate the profit and loss structure of the bull spread. The result is shown in the following Table.

The Fig. 6 and Fig. 7 are the histogram and distribution of the potential profit or loss in Table V, respectively.

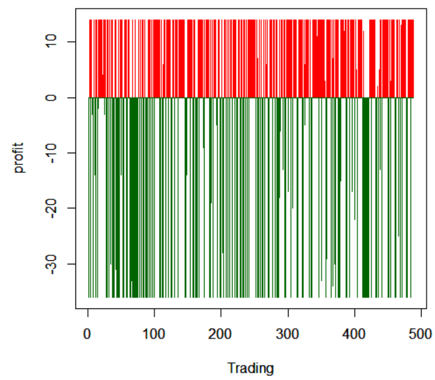

FIGURE 6. The barchart of possible profit or loss for "Long 9300 Call @ 90; Short 9350 Call @ 54" on PM 13:30, 2017-01-18.

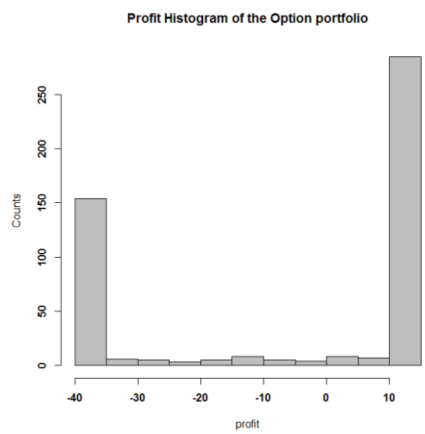

FIGURE 7. The histogram of possible profit andloss for "Long 9300 Call @ 90; Short 9350 Call @ 54" on PM 13:30, 2017-01-18.

Based on the profit/loss vector in Table V, we obtain the returns for each bidding fraction, as in Table VI.

In Table VI, all the returns are smaller than 1. In other words, this game has negative expected returns; no bidding fractions are profitable. The returns are shown in Fig. 8.

Although the above portfolio is not profitable, there exist the reverse positions which may be profitable. Since the

TABLE 5. The possible profit and loss for "Long 9300 Call @ 90; Short 9350 Call @ 54" on PM 13:30, 2017-01-18.

Return	Estimated TAIEX	Possible Profit and Loss
+3.04%	9663	+14
+1.56%	9524	+14
…	…	…
-0.27%	9352	+14
-0.33%	9347	+11

TABLE 6. The return of bidding various fraction for "Long 9300 Call @ 90; Short 9350 Call @ 54" on PM 13:30, 2017-01-18.

Bidding Fraction	1%	2%	…	50%	…	99%	100%
Return	0.998	0.997		0.875		0.269	0

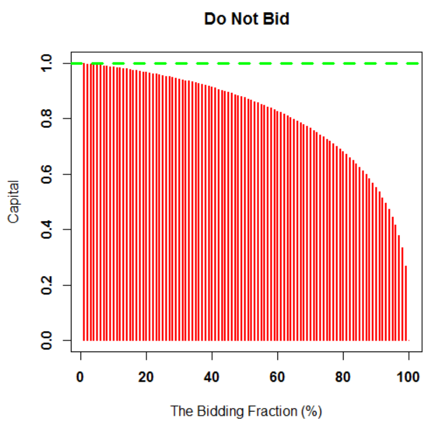

FIGURE 8. The returns of bull spread "Long 9300 Call @ 90; Short 9350 Call @ 54" on PM 13:30, 2017-01-18 for bidding different fraction.

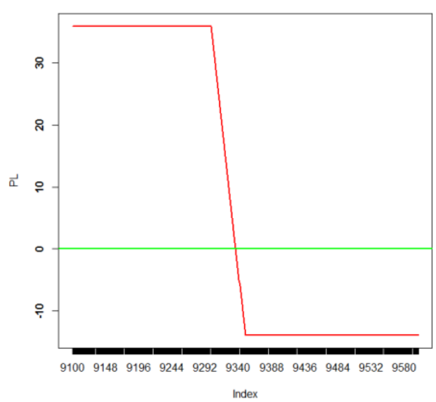

FIGURE 9. The profit and loss distribution bear spread for "Short 9300 Call @ 90; Long 935 0Call @ 54".

position of bull spread is not profitable, the reverse position, i.e., bear spread, shall be profitable. We consider the portfolio of "Short 9300 Call@90; Long 9350 Call @ 54" with the profit/loss as follows.

We repeat the various bidding fractions in Table V and Table VI, and obtain its optimal bidding fraction as 13% and return as 1.01945. Fig. 10 shows its returns in various bidding fractions.

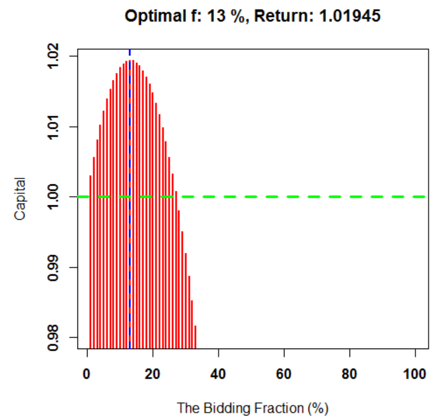

FIGURE 10. The returns of bear spread "Short 9300 Call @ 90; Long 9350 Call @ 54" on PM 13:30, 2017-01-18 for bidding different fraction.

B. THE MOST PROFITABLE OPTION PORTFOLIO

In the last section, we investigate the construction of a favorable option portfolio. However, there exist many option portfolios, including strangle, butterfly, and so on. In this section, we investigate the search for the most favorable option portfolios. Combined with the previous section, we propose the following algorithm.

Input:(Strike Prices, Holding Period)

1. Strike Price: The objects of Call options and Put options with the strike prices $\{p_1, p_2, \ldots, p_n\}$, where $p_i < p_j$ if $i < j$.

2. Holding Period: The holding period of the option portfolio. In our example, we let the holding period to be the time before the market close, say PM 13:43, on Friday to next Wednesday, say PM 13:30. We denoted as $\rho = \text{Fri1343} \sim \text{Wed1330}$

Step 1. Calculate the profit and loss for all possible outcomes of option portfolios in bull spread (or bear spread). i.e, long p_i and short p_j for each pair $i, j \in \{1, 2, \ldots, n\}$ and $i < j$. The example is shown in Fig. 4 and Table II.

Step 2. From the historical data, calculate the market index distribution for Holding Periods ρ. Note that in this work, we just use the naïve method to find the historical rising/falling distribution and regard it as the prediction for the market index distribution. The example is shown in Fig. 5 and Table III.

Step 3. According to the result of prediction in Step 2, we apply historical rise/fall return to current price of TAIEX (The price on PM 13:30, Friday) Thus, we get the estimated market index distribution on the expired day (PM 13:30 next Wednesday). The example is shown in Table IV.

Step 4. Calculate the possible profit\loss points of option portfolio depending on the estimated settlement price on expired day. Thus, we get the possible profit\loss distribution of the holding option portfolio. The example is shown in Table V, Fig. 7 and Fig. 6

Step 5. Take the possible return distribution in Step 4 to calculate the terminal wealth return (TWR) based on Vince' optimal fraction. Then, we calculate the return of each option portfolio by bidding fraction from 1% to 100%.Then, choose the most profitable portfolio with the max expected geometric return.

Output: (The most profitable option portfolio. For example, long Call p_i and short Call p_j)

The above algorithm can be applied to various option portfolios, and calculate its geometric returns. Returns of all the portfolios can be calculated and the most profitable portfolio can be selected. Next we show the experiments of finding the most profitable option spread.

IV. EXPERIMENTS

We use the TAIEX index value at the closing of 01/13/2017. We consider the options at the money as examples (refer to Table I, and intention to find the most profitable portfolio. Note that the reverse of bull spread is the bear spread. So we consider the bull spread, without loss of generality. There are 10 possible combinations, given as follows.

Bull Spread	Max Profit	Max Loss
Long 9300Cal @90 Short 9350Call@54	14	-36
Long 9300Call@90 Short 9400Call@29.5	39.5	-60.5
Long 9300Call@90 Short 9450Call@13.5	73.5	-76.5
Long 9300Call@90 Short 9500Call@5.5	115.5	-84.5

Bull Spread	Max Profit	Max Loss
Long 9350 Call@54 Short 9400 Call@29.5	25.5	-24.5
Long 9350Call @54 Short 9450Call@13.5	59.5	-40.5
Long 9350Cal @54 Short 9500Call@5.5	101.5	-48.5

Bull Spread	Max Profit	Max Loss
Long 9400Call@29.5 Short 9450Call@13.5	34	-16
Long 9400Call@29.5 Short 9500Call@5.5	76	-24

Bull Spread	Max Profit	Max Loss
Long 9450Cal @13.5 Short 9500Call@5.5	42	-8

We individually calculate the return distributions of all the 10 portfolios shown above. Note that in the cases where the return is less than 1, the games are not favorable. In such cases, we will calculate the returns of the corresponding bear spread, that is, the "Long" is changed to "Short", and the "Short" is changed to "Long". Our experimental outcomes are as follows.

Because the "Long 9300 Call @ 90; Short 9350 Call @ 54" is calculated as unfavorable in Section III, we calculate the bear spread positions, "Short 9300 Call @ 90; Long 9350 Call @ 54", as shown in Figure 11.

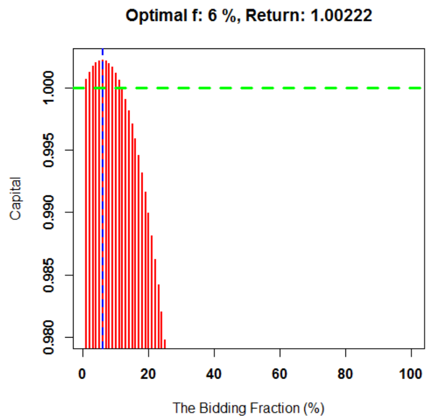

FIGURE 11. The returns of bear spread "Short 9300 Call @ 90; Long 9400 Call @ 29.5" on PM 13:30, 2017-01-18 for bidding different fraction.

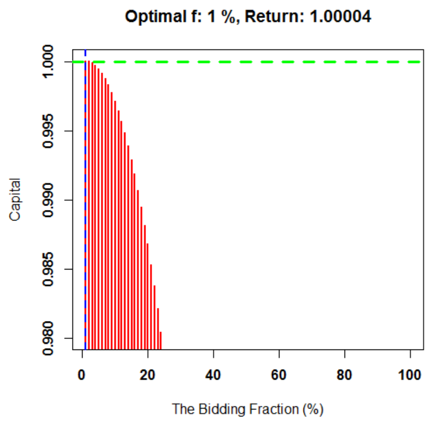

FIGURE 12. The returns of bull spread "Long 9300 Call @ 90; Short 9450 Call @ 13.5" on PM 13:30, 2017-01-18 for bidding different fraction.

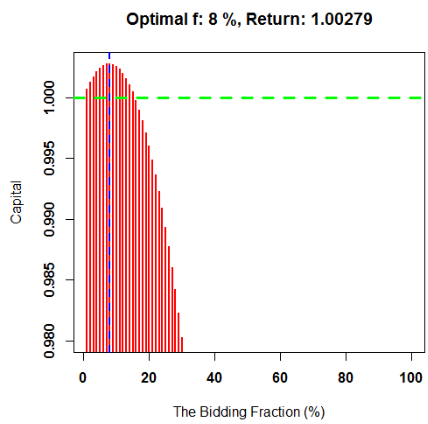

FIGURE 13. The return of bull spread "Long 9300 Call @ 90; Short 9500 Call @ 5.5" on PM 13:30, 2017-01-18 for bidding different fraction.

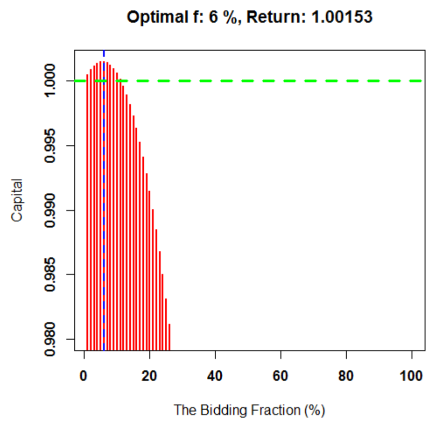

FIGURE 14. The returns of bull spread "Long 9350 Call @ 54; Short 9400 Call @ 29.5" on PM 13:30, 2017-01-18 for bidding different fraction.

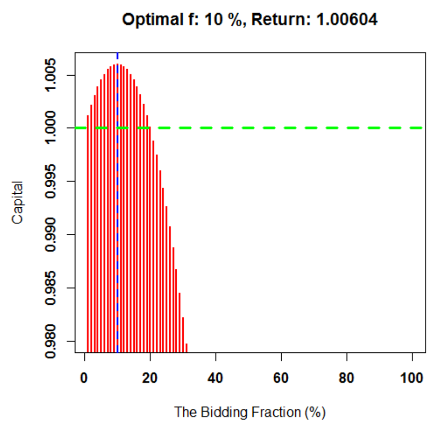

FIGURE 15. The returns of bull spread "Long 9350 Call @ 54; Short 9450 Call @13.5" on PM 13:30, 2017-01-18 for bidding different fraction.

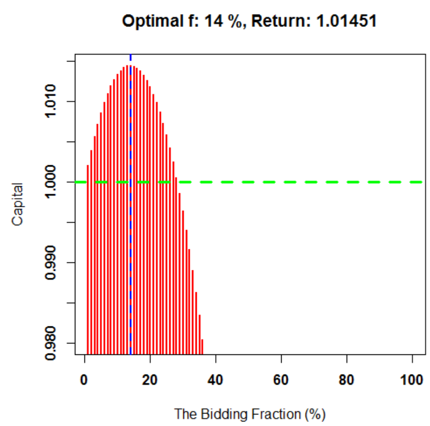

FIGURE 16. The returns of bull spread "Long 9350 Call @ 54; Short 9500 Call @ 5.5 on PM 13:30, 2017-01-18 for bidding different fraction.

The returns of other portfolios are shown in Figures 12–19.

In all portfolios for bull and bear spreads at money, the set of "Long 9450 Call @13.5; Short 9500 Call @ 5.5" has the highest return of 1.04288, with the recommended bidding fraction of 14%. Note that we may not use 14% in real trading, which leads to very high risks. In summary, our proposed

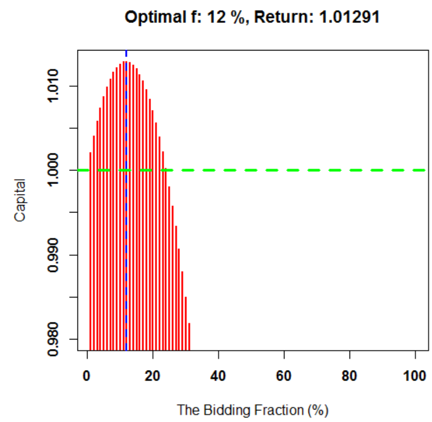

FIGURE 17. The returns of bull spread "Long 9400 Call @ 29.5; Short 9450 Call @13.5" on PM 13:30, 2017-01-18 for bidding different fraction.

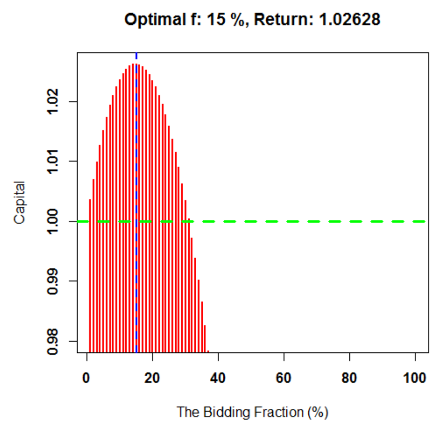

FIGURE 18. The returns of bull spread "Long 9400 Call @ 29.5; Short 9500 Call @ 5.5" on PM 13:30, 2017-01-18 for bidding different fraction.

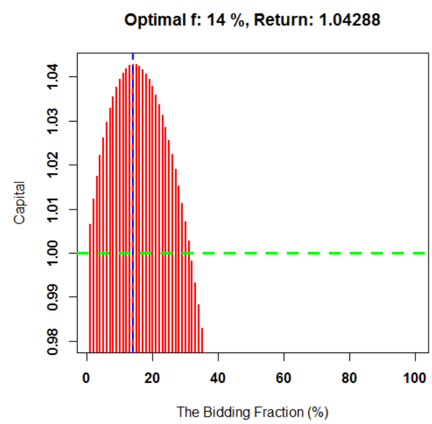

FIGURE 19. The returns of bull spread "Long 9450 Call @ 13.5; Short 9500 Call @ 5.5" on PM 13:30, 2017-01-18 for bidding different fraction.

approach provides a framework for comparing Spread portfolios.

V. CONCLUSIONS AND FUTURE WORK

We propose a novel approach for options trading based on Kelly criterion. We avoid the challenges in finding trading signals of traditional strategies. Instead, we adopt the optimal fraction in the profitable options portfolio and hold the positions until the expiration date. Note that we do not consider stop loss & stop profit in our model for the reason of reducing the uncertainty in trading strategies. In fact the idea in this work still can be applied to arbitrary trading time periods by stop-loss & stop-profit instead of cleaning positions at the expiration date. We may adopt the option pricing model such as Black-Scholes to estimate the value of time-decay, and apply the results to predict the distributions of rising/falling points. Consequently, we may choose the proper time periods to back-test the historical data for finding the distributions of rising/falling points. Based on the quotes, we calculate the empirical profit and loss distribution, and use Kelly criterion to obtain the optimal bidding fraction on option portfolio.

One of the advantages of trading option spread is the fixed distribution of profit & loss. We may select the most profitable portfolio once we know the distribution of the market index. However, no one can accurately predict the distribution of the market index. We just try our best to find the estimated distribution and hope it is close to the real market distribution. The error between estimated and real distributions causes the loss in the trading. Consequently, the performance of our method depends on the estimated market distribution and all the work about investing strategies is to predict the market distribution instead of investing traditional trading strategies (indicator, signal, and rules). In fact, there are lots of methods, such as neutral network, machine learning skills, to predict the market index distribution. We leave it to the future work of this study.

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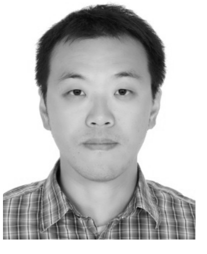

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