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Multiuser Space–Time Line Code With Optimal and Suboptimal Power Allocation Methods

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ABSTRACT In this paper, a multiuser space-time line code (STLC) system is proposed. In the proposed multiuser STLC system, a zero-forcing (ZF)-based precoder decomposes multiuser multiple-input multiple-output (MIMO) channels into multiple single-user MIMO channels, and the STLC is performed with the effective single user MIMO channel independently for each user. Here, to maximize the sum rate, an optimal power allocation method is designed based on a water-filling strategy. Also, a simple suboptimal method called fairness-aware per-user (FAPU) power allocation is devised. It is analytically and numerically verified that each user of the proposed ZF-based STLC system asymptotically achieves the maximum of single-user achievable rate as the number of transmit antennas increases. The numerical results of rigorous simulation justify that the proposed ZF-based STLC with FAPU power allocation provides near optimal performance and outperforms the existing ZF-based maximum Eigen beamforming scheme and conventional multiuser STLC schemes. Furthermore, two simple search algorithms to find the optimal number of users that maximizes sum rate are devised. Numerical results verify that the proposed algorithms can effectively find the optimal number of users by reducing the search interval.

INDEX TERMS Multiuser MIMO, space–time line code, spatial-diversity gain, zero-forcing precoder.

I. INTRODUCTION

Recently, a space–time line code (STLC) scheme that achieves full spatial diversity gain was proposed for a multiple-input multiple-output (MIMO) system [1]. In an STLC transmitter, two consecutive (*time*) information symbols are weighted by channel gains (*space*) and combined before transmission. The two STLC symbols are then transmitted through M transmit antennas and simply combined at the receiver using two receive antennas. Here, full spatial diversity gain is achieved at the receiver. Furthermore, in [1], it is shown that the STLC achieves asymptotically optimal performance as M increases, in terms of the received signal-to-noise ratio (SNR) and spectral efficiency.

The single-user STLC scheme has been extended to support multiple users in [2]. Here, greedy-based transmit antenna allocation methods are proposed to improve the average signal-to-interference-plus-noise ratio (SINR) and quality-of-experience (QoE). If M is exceedingly large, e.g., a few hundred antennas, the antenna allocation methods can effectively reduce the multiuser interference (MUI) effect in the combined STLC signals, so that the multiuser STLC can improve the average SINR and QoE. However, if the number of transmit antennas is moderately large, e.g., a few tens of antennas [3], [4], strong MUI seriously degrades the performance.

In this study, zero-forcing (ZF)-based precoding (see [5] and references therein) is employed at the STLC transmitter to eliminate the MUI regardless of the number of transmit antennas. The STLC is then applied to each user's signal by using the decomposed single-user MIMO channels. On top of the ZF-based precoding, the optimal and suboptimal power allocation methods are designed to maximize the sum rate of the multiuser STLC systems. It has been revealed that the optimal power allocation among the users follows the water-filling (WF) power allocation strategy. Also, it has been shown that the simple suboptimal power allocation strategy, called fairness-aware per-user (FAPU) power allocation, can achieve near optimal performance. It has been analytically and numerically verified that each user in the proposed ZF-based STLC system asymptotically achieves the maximum single-user achievable rate, which is the ideal case of multiuser system without the multiuser interferences, as the number of transmit antennas M increases. Moreover, simulation results justify that the proposed ZF-based STLC scheme with FAPU power allocation outperforms the existing ZF-based maximum eigen beamforming (MEB) scheme, which is optimal in terms of the received SNR after the ZF [6], and the conventional multiuser STLC schemes in [2]. Furthermore, from the observations that there exists an optimal number of users that maximizes the sum rate of the multiuser STLC system and that the sum rate function is concave with respect to the number of users, two simple search algorithms to find the optimal number of users are devised. The proposed search algorithms effectively reduce the search interval and find the optimal number of users.

The main contribution of this study is summarized as follows:

- A ZF-based multiuser STLC method has been proposed.
- The optimal and suboptimal power allocation strategies have been designed to maximize the sum rate of the multiuser systems.
- It has been analytically shown that the proposed ZF-based STLC system asymptotically achieves the maximum single-user achievable rate as the number of transmit antennas increases.
- A simple algorithm has been devised to find the optimal number of users in the ZF-based multiuser STLC systems.

The rest of the paper is organized as follows. In Section II, we propose a ZF-based multiuser STLC system that includes transmit precoding, encoding, decoding, and optimal and suboptimal power control methods. Section III shows numerical results that verify the proposed system. In Section IV, the optimal number of users is obtained by the proposed line search algorithms. Section V concludes this paper.

Notation: The superscripts T, H, and $*$ denote the transposition, Hermitian transposition, and complex conjugate, respectively, for any scalar, vector, or matrix. E stands for the expectation of a random variable x. For any scalar x, vector $\boldsymbol{x}$, and matrix $\boldsymbol{X}$, the notations $|x|$, $\|\boldsymbol{x}\|$, and $\|\boldsymbol{X}\|_F$ denote the absolute value of x, the two norm of $\boldsymbol{x}$, and the Frobenius-norm of $\boldsymbol{X}$, respectively. $\text{tr}(\boldsymbol{X})$ denotes the trace operation of a matrix $\boldsymbol{X}$. $\boldsymbol{I}_{m\times n}$ and $\boldsymbol{0}_{m\times n}$ represent an m-by-n identity and zero matrices, respectively. Complex random variable x conforms to a normal distribution with a zero mean and variance σ^2 is denoted by $x \sim \mathcal{CN}(0, \sigma^2)$.

II. PROPOSED MULTIUSER STLC SYSTEM

We consider a multiuser STLC system, in which one STLC transmitter with M transmit antennas supports K users, each of which has two receive antennas, as shown in Fig. 1. The results obtained in this study can be readily extended to multiple users with more than two receive antennas by using low-rate STLC in [1]. The kth user is denoted by U_k, where $k \in \mathcal{K} = \{1, \ldots, K\}$. The independent channel gain from transmit antenna m to receive antenna n of user k is represented by $h_{n,m,k}$, where $n \in \{1, 2\}$ and $m \in \{1, \ldots, M\}$. Each channel conforms to an independent complex Gaussian distribution. Precisely, $h_{n,m,k} = \sqrt{\rho_k}\bar{h}_{n,m,k}$, where ρ_k is a large-scale fading factor and $\bar{h}_{n,m,k}$ is the small-scale fading channel, that is an independent and identically distributed (i.i.d.) random variable with a $\mathcal{CN}(0, 1)$ distribution, i.e., Rayleigh fading channels. The matrix form of the MIMO channel of U_k is written as follows:

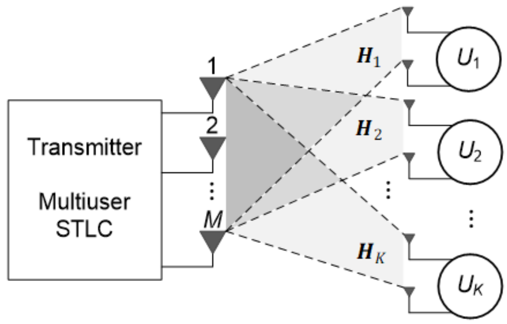

FIGURE 1. A multiuser STLC system model, where an STLC transmitter with M transmit antennas transmits information to K users, denoted by U_k, with two receive antennas.

$$\boldsymbol{H}_k = \begin{bmatrix} h_{1,1,k} & \cdots & h_{1,M,k} \\ h_{2,1,k} & \cdots & h_{2,M,k} \end{bmatrix} \in \mathcal{C}^{2\times M}. \quad (1)$$

Herein, by virtue of the channel reciprocity in a time division duplex system, the channels can be estimated at the transmitter by using the pilot/training signals from the users. Thus, it is assumed that the transmitter knows $\{\boldsymbol{H}_k\}$, $\forall k \in \mathcal{K}$, while U_k knows its own channel $\boldsymbol{H}_k$ only.

A. ENCODING AND TRANSMISSION

The transmitter generates an MUI suppression precoding matrix, $\boldsymbol{W}_k$, before the STLC transmission. To this end, a leakage channel matrix of U_k, which is denoted by $\boldsymbol{H}_{\bar{k}}$, is defined as follows:

$$\boldsymbol{H}_{\bar{k}} \triangleq \left[\boldsymbol{H}_1^H \cdots \boldsymbol{H}_{k-1}^H \boldsymbol{H}_{k+1}^H \cdots \boldsymbol{H}_K^H\right]^H \in \mathcal{C}^{2(K-1)\times M}. \quad (2)$$

Then, the ZF precoding matrix $\boldsymbol{W}_k$, which fulfills a ZF property (no leakage) such that

$$\boldsymbol{H}_{k'}\boldsymbol{W}_k = \boldsymbol{0}_{2\times L}, \quad \forall k' \neq k \in \mathcal{K} \quad (3)$$

is obtained as follows[1]:

$$\boldsymbol{W}_k = null(\boldsymbol{H}_{\bar{k}}) \in \mathcal{C}^{M\times L}, \quad (4)$$

where $null(\boldsymbol{X})$ provides the span of the nullspace of matrix $\boldsymbol{X}$, and L is the dimension of the nullspace derived as

$$L = M - 2K + 2, \quad (5)$$

under the assumption that $\boldsymbol{H}_k$ is an full-rank matrix.

In (4), the dimension of the nullspace should be greater than or equal to one, i.e., $L \geq 1$, to transmit at least one STLC symbol through the channel. From this condition, the number of transmit antennas is conditioned as follows:

$$M \geq 2K - 1. \quad (6)$$

[1]No leakage is equivalent to no MUI, i.e., $\boldsymbol{H}_k\boldsymbol{W}_{k'} = \boldsymbol{0}_{2\times L}$.

Note that the condition in (6) is stringent compared to the condition for multiuser SLTC in [2], i.e., $M \geq K$. In other words, the proposed ZF-based STLC requires at least $(K-1)$ additional antennas to eliminate MUIs.

Let $s_{t,\ell,k}$ be the STLC symbol of U_k, which is transmitted at time t through the ℓth dimension of the nullspace. User k's STLC symbol is precoded through $\boldsymbol{w}_{\ell,k}$ as $\boldsymbol{w}_{\ell,k} s_{t,\ell,k}$ and $\boldsymbol{w}_{\ell,k}$ is the ℓth column vector of $\boldsymbol{W}_k$, where $\ell \in \mathcal{L} = \{1, \ldots, L\}$. The precoded STLC symbols of all users are combined together and transmitted simultaneously to all users. Denoting the STLC symbol matrix by $\boldsymbol{S}_k$, defined as

$$\boldsymbol{S}_k = \begin{bmatrix} \boldsymbol{s}_{1,k} & \boldsymbol{s}_{2,k} \end{bmatrix} = \begin{bmatrix} s_{1,1,k} & \cdots & s_{1,L,k} \\ s_{2,1,k} & \cdots & s_{2,L,k} \end{bmatrix}^T \in \mathcal{C}^{L\times 2},$$

the received signals at U_k can be written as follows:

$$\begin{aligned} \boldsymbol{R}_k &= \boldsymbol{H}_k \sum_{k'\in\mathcal{K}} \eta_{k'} \boldsymbol{W}_{k'} \boldsymbol{S}_{k'} + \boldsymbol{Z}_k \\ &= \eta_k \boldsymbol{H}_k \boldsymbol{W}_k \boldsymbol{S}_k + \boldsymbol{Z}_k, \end{aligned} \tag{7}$$

where $\boldsymbol{R}_k \in \mathcal{C}^{2\times 2}$ is a received signal matrix whose (n,t)th element, denoted by $r_{n,t,k}$, is the receive signal at receive antenna n at time t; η_k is a transmit power normalization factor of U_k; and $\boldsymbol{Z}_k$ is an additive white Gaussian noise (AWGN) matrix, whose (n,t)th element is an AWGN with a zero mean and σ_z^2 variance in the received signal $r_{n,t,k}$, i.e., $z_{n,t,k} \sim \mathcal{CN}(0, \sigma_z^2)$. The second equality in (7) follows the ZF property in (3). From (7), it is clear that the MUIs are perfectly cancelled out and the effective channel matrix for U_k is then $\boldsymbol{H}_k \boldsymbol{W}_k$. Thus, STLC encoding for $\boldsymbol{S}_k$ can be performed with the effective channels, $\{\boldsymbol{G}_k\}$, defined as

$$\boldsymbol{G}_k = \begin{bmatrix} g_{1,1,k} & \cdots & g_{1,L,k} \\ g_{2,1,k} & \cdots & g_{2,L,k} \end{bmatrix} \triangleq \boldsymbol{H}_k \boldsymbol{W}_k \in \mathcal{C}^{2\times L}. \tag{8}$$

Following [1] and [2] the STLC encoding is performed as follows:

$$s_{1,\ell,k} = g_{1,\ell,k}^* x_{1,k} + g_{2,\ell,k}^* x_{2,k}^*, \tag{9a}$$
$$s_{2,\ell,k} = g_{2,\ell,k}^* x_{1,k}^* - g_{1,\ell,k}^* x_{2,k}, \tag{9b}$$

where $x_{t,k}$ is the tth data symbol of U_k with $\mathrm{E}\,|x_{t,k}|^2 \triangleq E_x$.

Substituting $s_{t,\ell,k}$ into (7), each element of received signal matrix $\boldsymbol{R}_k$ can be expressed as follows:

$$r_{1,1,k} = \eta_k \sum_{\ell\in\mathcal{L}} \left(g_{1,\ell,k} (g_{1,\ell,k}^* x_{1,k} + g_{2,\ell,k}^* x_{2,k}^*) \right) + z_{1,1,k}, \tag{10a}$$
$$r_{1,2,k} = \eta_k \sum_{\ell\in\mathcal{L}} \left(g_{1,\ell,k} (g_{2,\ell,k}^* x_{1,k}^* - g_{1,\ell,k}^* x_{2,k}) \right) + z_{1,2,k}, \tag{10b}$$
$$r_{2,1,k} = \eta_k \sum_{\ell\in\mathcal{L}} \left(g_{2,\ell,k} (g_{1,\ell,k}^* x_{1,k} + g_{2,\ell,k}^* x_{2,k}^*) \right) + z_{2,1,k}, \tag{10c}$$
$$r_{2,2,k} = \eta_k \sum_{\ell\in\mathcal{L}} \left(g_{2,\ell,k} (g_{2,\ell,k}^* x_{1,k}^* - g_{1,\ell,k}^* x_{2,k}) \right) + z_{2,2,k}. \tag{10d}$$

B. COMBINING AND DECODING

For the coherent detection of $x_{t,k}$ from (10), U_k needs to estimate $g_{1,\ell,k}$, $g_{2,\ell,k}$, and η_k. However, by following the STLC combining in [1] and [2] only partial information of $g_{1,\ell,k}$, $g_{2,\ell,k}$, and η_k are required as will be shown shortly. U_k first combines the received signals in (10) as follows:

$$r_{1,1,k} + r_{2,2,k}^* = \eta_k \gamma_k x_{1,k} + z_{1,k}, \tag{11a}$$
$$r_{2,1,k}^* - r_{1,2,k} = \eta_k \gamma_k x_{2,k} + z_{2,k}, \tag{11b}$$

where γ_k is the effective channel gain that is derived as

$$\gamma_k = \sum_{\ell\in\mathcal{L}} \left(|g_{1,\ell,k}|^2 + |g_{2,\ell,k}|^2 \right) = \|\boldsymbol{G}_k\|_F^2; \tag{12}$$

$z_{1,k} = z_{1,1,k} + z_{2,2,k}^*$; and $z_{2,k} = z_{2,1,k}^* - z_{1,2,k}$. From (11), a maximum-likelihood (ML) detector detects independently $x_{1,k}$ and $x_{2,k}$ with the knowledge of $\eta_k\gamma_k$. Here, $\eta_k\gamma_k$ can be estimated by using the blind SNR estimation techniques (see [7] and references therein). Note that, individual $g_{1,\ell,k}$, $g_{2,\ell,k}$, and η_k are not required and, for optimal phase-shift-keying (PSK) symbol detection, even $\eta_k\gamma_k$ is not required for the ML detection.

The received SNR can be readily derived from (11) as follows:

$$\mathsf{SNR}_k = \frac{\eta_k^2 \gamma_k^2 E_x}{2\sigma_z^2}, \tag{13}$$

and by using it, the achievable rate of U_k is written as follows:

$$R_k = \log_2\left(1 + \mathsf{SNR}_k\right). \tag{14}$$

C. OPTIMAL POWER ALLOCATION

To maximize the sum achievable rate of the multiuser STLC system, power allocation factor η_k is designed by solving an optimization problem below:

$$\{\eta_k^o\} = \arg\max_{\{\eta_k\}} \sum_{k\in\mathcal{K}} R_k \tag{15a}$$
$$\texttt{s.t.}\ \frac{1}{K} \sum_{k\in\mathcal{K}} \mathrm{E}\left\| \eta_k \boldsymbol{W}_k \boldsymbol{s}_{t,k} \right\|^2 \leq P, \tag{15b}$$

where the constraint (15b) is for the limit of the average maximum transmit power, given by P. Note that the expectation in (15b) is performed over the transmitted symbols for a given k. Using (13) and (14) to (15a), and from the facts that i) $\boldsymbol{W}_k^H \boldsymbol{W}_k = \boldsymbol{I}_L$, ii) $s_{t,\ell,k}$ is independent of $s_{t,\ell,\bar{k}}$, where $\bar{k} \in \mathcal{K}$ and $\bar{k} \neq k$, and iii) the average STLC symbol power is derived as $\mathrm{E}\,\|\boldsymbol{s}_{t,k}\|^2 = \gamma_k E_x$, the optimization problem (15) can be rewritten as follows:

$$\{\eta_k^o\} = \arg\max_{\{\eta_k\}} \sum_{k\in\mathcal{K}} \log_2\left(1 + \frac{\eta_k^2\gamma_k^2 E_x}{2\sigma_z^2}\right) \tag{16a}$$
$$\texttt{s.t.}\ \sum_{k\in\mathcal{K}} \eta_k^2 \gamma_k \leq \frac{KP}{E_x}. \tag{16b}$$

The problem (16) is a typical water-filling (WF) problem [8], and its optimal solution can be derived by using a Lagrangian method as follows:

$$\eta_k^o = \sqrt{\left(\frac{1}{\lambda\gamma_k \ln 2} - \frac{2\sigma_z^2}{\gamma_k^2 E_x}\right)^+}, \quad \forall k \in \mathcal{K}, \tag{17}$$

where $(x)^+ = \max\{x, 0\}$ and the Lagrange multiplier λ is chosen such that the transmit power constraint is fulfilled as

$$\sum_{k\in\mathcal{K}} \left(\frac{1}{\lambda \ln 2} - \frac{2\sigma_z^2}{\gamma_k E_x}\right)^+ = \frac{KP}{E_x}. \tag{18}$$

As a special case, when $K = 1$, i.e., there exists only one user U_k, λ in (18) can be derived in a closed form as

$$\lambda = \frac{\gamma_k E_x}{\ln 2(P\gamma_k + 2\sigma_z^2)}. \tag{19}$$

By using (19), the optimal power normalization factor in (17) is derived as

$$\eta_k^o = \sqrt{\frac{P}{\gamma_k E_x}}, \quad k \in \mathcal{K}. \tag{20}$$

By substituting η_k^o in (20) into (13), the single user SNR of U_k is derived as follows:

$$\begin{aligned}\mathsf{SNR}_{SU} &= \frac{\gamma_k P}{2\sigma_z^2} \\ &= \frac{P\|\boldsymbol{G}_k\|_F^2}{2\sigma_z^2} \\ &= \frac{P\|\boldsymbol{H}_k\boldsymbol{W}_k\|_F^2}{2\sigma_z^2} \\ &= \frac{P\|\boldsymbol{H}_k\|_F^2}{2\sigma_z^2}, \quad k \in \mathcal{K}. \end{aligned} \tag{21}$$

Here, it should be noted that $\boldsymbol{W}_k = \boldsymbol{I}_M$ in (4) when $K = 1$, and SNR_{SU} is the SNR of the single-user STLC system in [2]. From (21), we see that the proposed multiuser STLC provides the same performance as the single-user STLC in [2], if there is a single user, i.e., $K = 1$.

As other special case, when $K = 2$ and the SNRs of two users are sufficiently large, such that non-zero power is allocated to both the users, the water level in (18) is readily derived as

$$\lambda = \frac{\gamma_k \gamma_{\bar{k}} E_x}{\left(\sigma_z^2(\gamma_k + \gamma_{\bar{k}}) + P\gamma_k\gamma_{\bar{k}}\right)\ln 2}. \tag{22}$$

By using (22), the optimal power allocation factors in (17) are obtained as follows:

$$\eta_k^o = \sqrt{\frac{P}{\gamma_k E_x} + \frac{(\gamma_k - \gamma_{\bar{k}})\sigma_z^2}{\gamma_k^2\gamma_{\bar{k}} E_x}}, \quad k, \bar{k} \in \mathcal{K}, \ k \neq \bar{k}. \tag{23}$$

Using η_k^o in (23) into (13), the SNRs of the two users are derived as follows:

$$\mathsf{SNR}_k^o = \frac{\gamma_k P}{2\sigma_z^2} + \frac{1}{2}\left(\frac{\gamma_k}{\gamma_{\bar{k}}} - 1\right), \quad k, \bar{k} \in \mathcal{K}, \ k \neq \bar{k}. \tag{24}$$

The SNR analysis in (24) is intuitively true, as the WF power allocation strategy allocates more power to the relatively better conditioned channel, resulting in higher SNR. In other words, if $\gamma_k > \gamma_{\bar{k}}$, the resultant SNRs after the optimal WF power allocation fulfill $\mathsf{SNR}_k^o > \mathsf{SNR}_{\bar{k}^o}$ in (24).

D. FAIRNESS-AWARE PER-USER (FAPU) POWER ALLOCATION

The power allocation in (17) is optimal in terms of the sum achievable rate; however, the allocated power to each user could be significantly different if the effective channel gains, i.e., γ_k's in (12), are significantly different for different users. The significant unfair power allocation causes unfairness among the users. To mitigate the unfair power allocation, a simple fairness-aware per-user (FAPU) power allocation method is proposed. The proposed FAPU power allocation ensures fairness in terms of the allocated power to users [9].

The FAPU is realized by designing a power normalization factor η_k. The power normalization factor is directly derived from (7) by using (9) and also using the fact that $\boldsymbol{W}_k^H\boldsymbol{W}_k = \boldsymbol{I}_L$, such that $\mathrm{E}\left\|\eta_k\boldsymbol{W}_k\boldsymbol{s}_{t,k}\right\|^2 = P$, as follows:

$$\eta_k^f = \sqrt{\frac{P}{\gamma_k E_x}}, \quad \forall k \in \mathcal{K}. \tag{25}$$

Here, the FAPU power allocation in (25) can be interpreted as the generalized structure of the single-user optimal power allocation in (20) to K users.

The fairness-aware normalization factor η_k^f allocates equal amounts of power to all users, i.e., P. By substituting (12) and (25) into (13), the received SNR of STLC with a FAPU power allocation strategy is derived as follows:

$$\mathsf{SNR}_k^f = \frac{\gamma_k P}{2\sigma_z^2}. \tag{26}$$

Comparing (26) with (21), it is shown that the single user SNR of STLC with FAPU power allocation in (26) is identical to the optimal SNR, i.e., $\mathsf{SNR}_k^f = \mathsf{SNR}_{SU,k}^o$ when $K = 1$. On the other hand, when $K = 2$, the second term on the right-hand side of (24) disappears in (26), which increases the power allocation fairness between the two users.

Note that the FAPU power allocation with η_k^f provides not only fairness in terms of power allocation, but also low-complexity power allocation compared to the optimal power allocation. Furthermore, we can obtain an interesting theorem form FAPU. The theorem is difficult to be derived from the optimal power allocation strategy in (17).

Theorem 1: For a given number of users, K, as the number of transmit antenna increases, i.e., $M \to \infty$, or equivalently L increases, a multiuser STLC system with FAPU power allocation provides the optimal single-user achievable rate to all users.

Proof: The SNR of STLC with FAPU power allocation in (26) is further derived as follows:

$$\begin{aligned}\mathsf{SNR}_k^f &= \frac{\gamma_k P}{2\sigma_z^2} \\ &= \frac{P\|\boldsymbol{G}_k\|_F^2}{2\sigma_z^2} \\ &= \frac{P\|\boldsymbol{H}_k\boldsymbol{W}_k\|_F^2}{2\sigma_z^2} \\ &= \frac{P\,\mathrm{tr}(\boldsymbol{H}_k\boldsymbol{W}_k\boldsymbol{W}_k^H\boldsymbol{H}_k^H)}{2\sigma_z^2} \\ &= \frac{P\,\mathrm{tr}(\boldsymbol{H}_k(\boldsymbol{W}_k\boldsymbol{W}_k^H - \boldsymbol{I}_M + \boldsymbol{I}_M)\boldsymbol{H}_k^H)}{2\sigma_z^2} \\ &= \frac{P}{2\sigma_z^2}\,\mathrm{tr}(\boldsymbol{H}_k\boldsymbol{E}\boldsymbol{H}_k^H) + \frac{P}{2\sigma_z^2}\,\mathrm{tr}(\boldsymbol{H}_k\boldsymbol{H}_k^H), \end{aligned} \quad (27)$$

where $\boldsymbol{E} \triangleq \boldsymbol{W}_k\boldsymbol{W}_k^H - \boldsymbol{I}_M$. Here, denoting the (i,j)th element of $\boldsymbol{E}$ by $e_{i,j}$, we can show that the average power of elements of $\boldsymbol{E}$ goes to a zero as M increases for given K as follows:

$$\begin{aligned}\lim_{M\to\infty} \mathrm{E}\,|e_{i,j}|^2 &= \lim_{M\to\infty} \frac{1}{M^2}\sum_{i=1}^{M}\sum_{j=1}^{M}|e_{i,j}|^2 \\ &= \lim_{M\to\infty} \frac{1}{M^2}\|\boldsymbol{E}\|_F^2 \\ &= \lim_{M\to\infty} \frac{1}{M^2}\,\mathrm{tr}\left((\boldsymbol{W}_k{\boldsymbol{W}_k}^H - \boldsymbol{I}_M)\right. \\ &\qquad \left.\times(\boldsymbol{W}_k\boldsymbol{W}_k^H - \boldsymbol{I}_M)^H\right) \\ &\overset{(a)}{=} \lim_{M\to\infty} \frac{1}{M^2}\,\mathrm{tr}\left(-\boldsymbol{W}_k\boldsymbol{W}_k^H + \boldsymbol{I}_M\right) \quad (28a) \\ &= \lim_{M\to\infty} \frac{1}{M^2}\left(-\,\mathrm{tr}\left(\boldsymbol{W}_k^H\boldsymbol{W}_k\right) + \mathrm{tr}\left(\boldsymbol{I}_M\right)\right) \\ &\overset{(b)}{=} \lim_{M\to\infty} \frac{-(M-2(K-1))+M}{M^2} \quad (28b) \\ &= \lim_{M\to\infty} \frac{2(K-1)}{M^2}. \\ &= 0. \quad (28c)\end{aligned}$$

The equalities (a) and (b) in (28a) and (28b) follow the fact that $\boldsymbol{W}_k^H\boldsymbol{W}_k = \boldsymbol{I}_{(M-2(K-1))}$ from the othornormality of a nullspace matrix in (4). From (27), since $\lim_{M\to\infty}\mathrm{E}\,|e_{i,j}|^2 = 0$, $\lim_{M\to\infty}\mathrm{E}\,|e_{i,j}| = 0$ and $\lim_{M\to\infty}\boldsymbol{E}_M = \boldsymbol{0}_M$ (see also the numerical verification in Fig. 2.). Accordingly, as M increases the SNR of a multiuser STLC system with FAPU power allocation is asymptotically identical to the SNR of a single-user STLC system in (21) as follows:

$$\begin{aligned}\lim_{M\to\infty}\mathsf{SNR}_k^f &= \lim_{M\to\infty}\frac{P}{2\sigma_z^2}\,\mathrm{tr}(\boldsymbol{H}_k\boldsymbol{E}\boldsymbol{H}_k^H) \\ &\quad + \lim_{M\to\infty}\frac{P}{2\sigma_z^2}\,\mathrm{tr}(\boldsymbol{H}_k\boldsymbol{H}_k^H) \\ &\simeq \lim_{M\to\infty}\frac{P}{2\sigma_z^2}\,\mathrm{tr}(\boldsymbol{H}_k\boldsymbol{0}_M\boldsymbol{H}_k^H) \\ &\quad + \lim_{M\to\infty}\frac{P}{2\sigma_z^2}\,\mathrm{tr}(\boldsymbol{H}_k\boldsymbol{H}_k^H) \\ &= \lim_{M\to\infty}\frac{P}{2\sigma_z^2}\,\mathrm{tr}(\boldsymbol{H}_k\boldsymbol{H}_k^H) \\ &= \lim_{M\to\infty}\mathsf{SNR}_{SU,k}^o. \end{aligned} \quad (29)$$

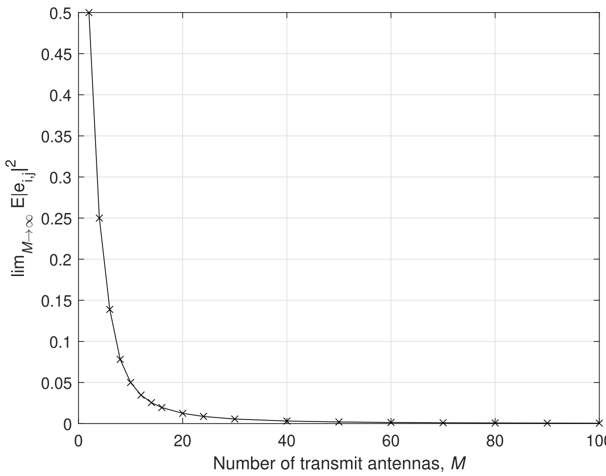

FIGURE 2. Numerical verification of the asymptotic result that $\lim_{M\to\infty}\mathrm{E}\,|e_{i,j}|^2 = 0$.

From (14) and (29), we can readily show that SNR_k^f asymptotically approches $\mathsf{SNR}_{SU,k}^o$ as $M \to \infty$, and the proof is completed. ■

From Theorem 1, the following proposition is derived.

Proposition 1: With sufficient number of transmit antennas, the multiuser STLC with FAPU power allocation system can provide full spatial diversity gain to each user with an order S, such that $2M - 4K + 4 \leq S \leq 2M$.

Proof: As M increases, each user achieves the single user SNR, which is identical to the SNR in [1]. Therefore, each user in the multiuser STLC with FAPU power allocation system can achieve full spatial diversity gain with an order of $2M$. If M is insufficiently large, the maximum diversity order obtained from 2-by-L effective channel matrix $\boldsymbol{H}_k\boldsymbol{W}_k$ is $2L$, where $L = M - 2K + 2$. This completes the proof. ■

III. PERFORMANCE EVALUATION AND DISCUSSION

In this section, the achievable rate performance of the proposed ZF-based STLC systems is evaluated and compared to existing multiuser MIMO and multiuser STLC systems. In our simulation, the system SNR parameter is defined as the average SNR of a single user system per transmit antenna, as follows:

$$\mathsf{SNR}_{sys} \triangleq \frac{\mathrm{E}[\mathsf{SNR}_{SU}]}{M} = \frac{\mathrm{E}[P\|\boldsymbol{H}\|_F^2]}{2\sigma_z^2 M} = \frac{\sigma_h^2\, P}{\sigma_z^2}. \quad (30)$$

Without loss of generality, σ_h^2 and σ_z^2 are set to be one, and the system SNR is adjusted according to the transmit power P to clarify the difference between the WF and FAPU power allocation effects. The system SNR varies between -20 dB

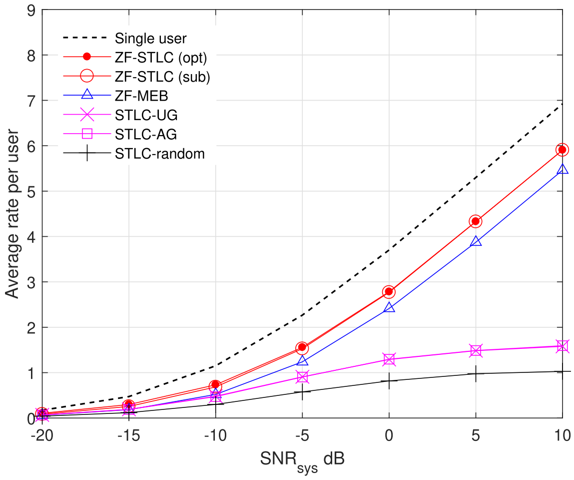

FIGURE 3. Comparison of average achievable rate per user over system SNR when $M = 12$ and $K = 4$.

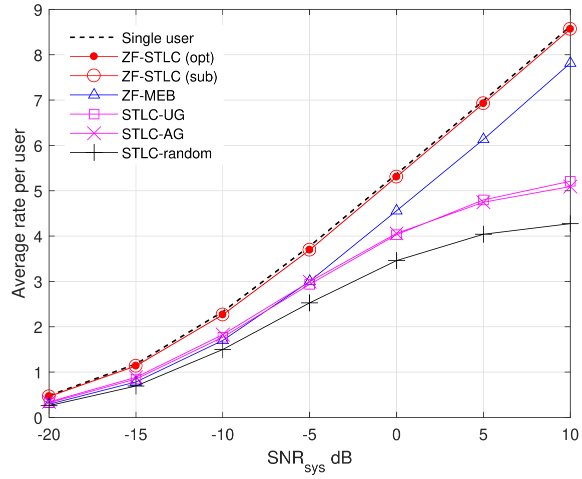

FIGURE 4. Comparison of average achievable rate per user over system SNR when $M = 40$ and $K = 2$.

and 10 dB, such that the received SNR ranges between around -10 dB to 25 dB, which is a reasonable range for the received SNR in practical wireless communication systems.

The following systems are compared in the simulation:

- Single user: as a reference, the $M \times 2$ single-user STLC system in [2] is compared.
- ZF-MEB: as a benchmarking system, the maximum eigen beamforming (MEB)-based scheme is employed for each user after ZF-based preprocessing. Power allocation follows WF, such that sum rate is maximized [10], [11].
- ZF-STLC (opt): the proposed multiuser STLC system with WF power allocation.
- ZF-STLC (sub): the proposed multiuser STLC system with FAPU power allocation.
- STLC-AG: the multiuser STLC system in [2] with antenna-greedy (AG) antenna allocation.
- STLC-UG: the multiuser STLC system in [2] with user-greedy (UG) antenna allocation.
- STLC-Random: the multiuser STLC system in [2] with random antenna allocation.

Figs. 3 and 4 show the average rate per user across the system SNR, SNR_{sys}. In Fig. 3, four users are supported by an STLC transmitter having 12 antennas, namely, $M = 12$ and $K = 4$. As shown in the results, the proposed ZF-STLC achieves the largest achievable rate compared to other multiuser schemes. Here, the dimension of null space is six, i.e., $L = M - 2K + 2 = 6$, which is equivalent to the number of effective transmit antennas. Thus, each STLC user can obtain a spatial diversity gain of order $2L$. However, the gap from the single-user scheme is not negligible. On the other hand, the antenna allocation schemes, namely STLC-UG, STLC-AG, and STLC-random, provide poor performance due to the strong residual MUIs. As reported in [2], the multiuser STLC based on antenna allocation is relevant for systems with extremely large number of transmit antennas (equivalently, a very large size of L). When SNR_{sys} is low, or equivalently, when the transmit power is insufficient, the WF power allocation strategy achieves better performance than the FAPU power allocation, yet the improvement is marginal. In Fig. 4, a similar observation can be made for the systems with $M = 12$ and $K = 4$ in Fig. 3, except the proposed ZF-STLC systems. Each user of the proposed ZF-STLC systems achieves almost the single-user achievable rate. Here, it should be noted that L is increased to 38 from six in Fig. 3. From these results, Theorem 1 can be verified.

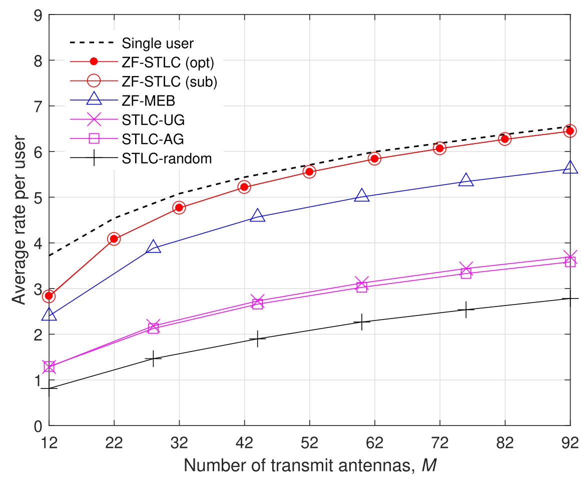

FIGURE 5. Comparison of average achievable rate per user over the number of transmit antennas when $\mathsf{SNR}_{sys} = 0$ dB and $K = 4$.

To further verify Theorem 1, in Figs. 5 and 6, the achievable rate per user is evaluated across the number of transmit antennas, M, when $\mathsf{SNR}_{sys} = 0$ dB. As expected and stated in Theorem 1, the achievable rate of the proposed ZF-STLC systems increases as M increases, and it approaches the performance of a single-user system. It is shown that the performance gap between the ZF-STLC and the antenna allocation schemes decreases as the antenna allocation schemes work well under the reduced multiuser-interference environment.

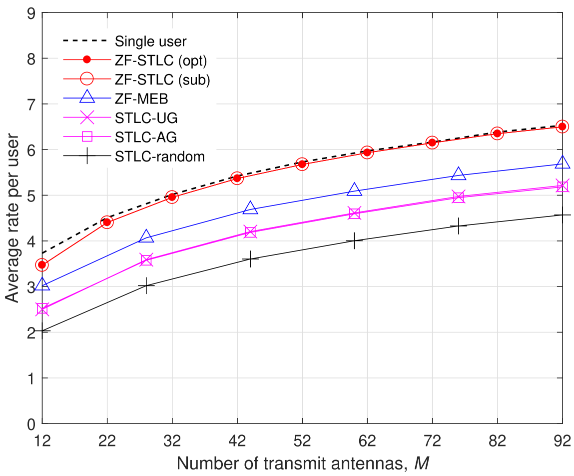

FIGURE 6. **Comparison of average achievable rate per user over the number of transmit antennas when** $\mathrm{SNR}_{sys} = 0$ **dB and** $K = 2$**.**

Note that the numbers of users are reduced from four to two in the results in Figs. 5 and 6, respectively.

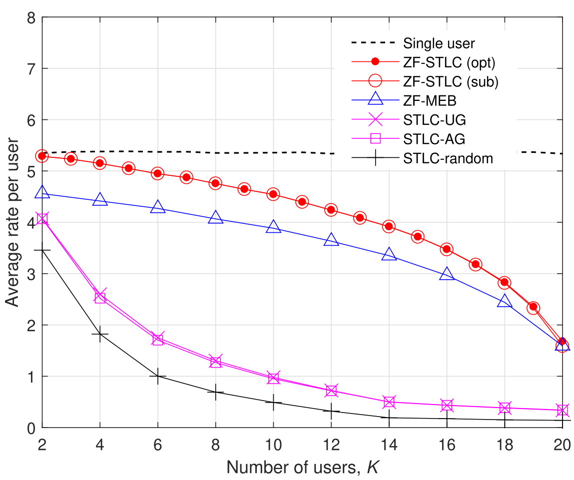

FIGURE 7. **Comparison of average achievable rate per user over the number of users when** $\mathrm{SNR}_{sys} = 0$ **dB and** $M = 40$**.**

The results in Figs. 7 and 8 show that the achievable rate per user for the multiuser schemes decreases as the number of users increases; equivalently, L decreases when the number of transmit antennas is fixed by 40 and 100, respectively. The performance gap between ZF-STLC and ZF-MEB also decreases and becomes almost identical to each other when $L = 2$, i.e., $K = 20$. Comparing the results with $L = 42-2K$ in Fig. 7 and the results with $L = 102-2K$ in Fig. 8, it can be surmised that the proposed ZF-STLC is more advantageous compared to the ZF-MEB scheme when L is moderately large.

In Figs. 9 and 10, the sum rates in (14) were compared. As an ideal performance bound, the K-times achievable rate of a single user STLC (denoted by 'Single user$\times K$') is included in the results. Note that this ideal bound is obtained through K single-user STLC systems that use K times greater orthogonal frequency resources compared to the multiuser systems. As shown in Fig. 9, there exists an optimal number

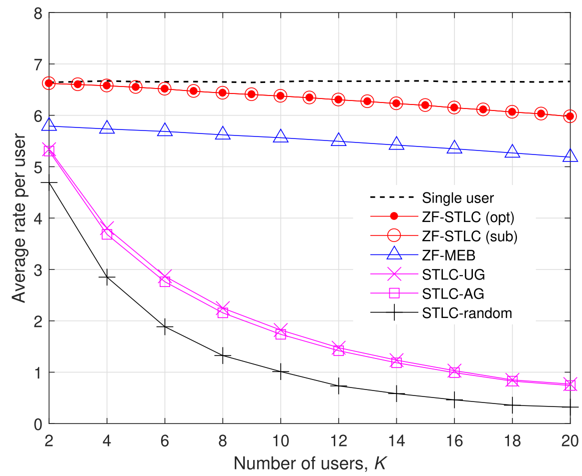

FIGURE 8. **Comparison of average achievable rate per user over the number of users when** $\mathrm{SNR}_{sys} = 0$ **dB and** $M = 100$**.**

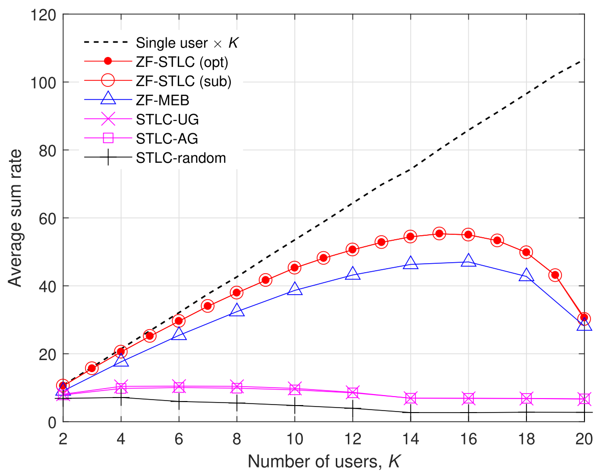

FIGURE 9. **Comparison of average achievable sum rate over the number of users when** $\mathrm{SNR}_{sys} = 0$ **dB and** $M = 40$**.**

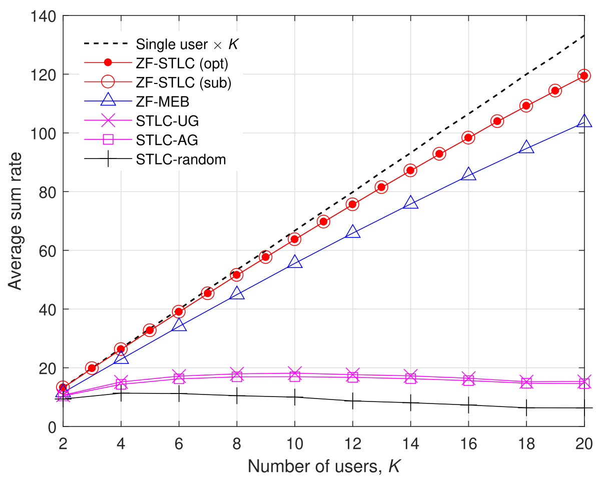

FIGURE 10. **Comparison of average achievable sum rate over the number of users when** $\mathrm{SNR}_{sys} = 0$ **dB and** $M = 100$**.**

of users that provides the maximum sum achievable rate for the ZF-based systems for a given number of transmit antennas. For example, 15 is the optimal number of users for an ZF-STLC system with 40 transmit antennas. Comparing the results with $M = 40$ in Fig. 7 and the results with $M = 100$ in Fig. 8, the sum achievable rate of the proposed ZF-STLC system approaches the ideal sum achievable rate of a single-user STLC systems as M increases. These results match well with the analysis in Theorem 1.

IV. OPTIMAL NUMBER OF USERS

In this section, the optimal number of users that can provide the maximum sum rate is designed for the proposed ZF-STLC systems. As observed in Fig. 9, there exists an optimal number of users with respect to the average sum rate. Since $L \geq 1$ to transmit data to each user, where $L = M - 2K + 2$, the number of users that can be supported by M transmit antennas is limited as follows:

$$K \leq \lfloor (M+1)/2 \rfloor \triangleq K_{max}, \tag{31}$$

where $\lfloor x \rfloor = \max\{m \in \mathbb{Z} | m \leq x\}$, and $\mathbb{Z}$ is the set of integers. The achievable rate of user k is written as $\log_2(1 + \mathsf{SNR}_k)$ [12], and from (14), the sum rate maximization problem to find the optimal number of users is formulated as follows:

$$K^o = \arg\max_K \mathrm{E}\left[\sum_{k=1}^{k=K} \log_2\left(1 + \frac{P\|\boldsymbol{H}_k \boldsymbol{W}_k\|_F^2}{2\sigma_z^2}\right)\right] \tag{32a}$$

$$\text{s.t. } 1 \leq K \leq K_{max}, \tag{32b}$$

where the average is over the T channel realizations. Because (32) does not admit a closed form solution, numerical search for K is needed to obtain the optimal solution. Since the objective function in (32a) is not differentiable, the descent-direction based line search algorithms, such as gradient descent and Newton's methods, are not applicable to solve this problem. Furthermore, section-based algorithms, such as golden-section and bisection searches [13], [14], are not directly appropriate for the integer space of search, i.e., the number of users.

A. PROPOSED SEARCH ALGORITHMS

Simple search algorithms are proposed to find the optimal K in ZF-STLC systems, i.e., the solution of (32), denoted by K^o. From numerical results in Fig. 9 (and refer to Fig. 12 in Section IV.B), it is observed that the average achievable rate is clearly a unimodal function, which has a unique maximum with respect to the number of users. By virtue of the unimodality of the sum achievable rate function, we can consider two simple line search algorithms, whose search intervals are bounded as in (32b). The first algorithm starts with the minimum number of supportable users, i.e., $K = 1$, and increases K until there is no increase of the sum rate. On the other hand, the second algorithm starts with the maximum number of supportable users, i.e., K_{max}, and keeps decreasing K until there is no increase of the sum rate. The former and latter are called user increasing line search (UiLS) and user decreasing line search (UdLS) algorithms, respectively, and they are summarized in Algorithm 1. Here, the UiLS

Algorithm 1 : UiLS and UdLS Algorithms

1. **Setup**: $K_1 = 1$ for UiLS and $K_1 = K_{max}$ for UdLS, $R_0 = 0$, and $i = 1$.
2. Calculate R_1 with randomly realized $K_1 T$ channel matrices, where T is averaging time in (32a)
3. **while** $R_i > R_{i-1}$ and $K_{low} \leq K_{up} - 1$ **do**
4. $\quad K_{i+1} = K_i + 1$ for UiLS and $K_{i+1} = K_i - 1$ for UdLS.
5. $\quad$ Calculate R_{i+1} from (32a) with K_{i+1} and $i = i + 1$.
6. **end while**
7. If $R_i \leq R_{i-1}$, $K^o = K_i - 1$ for UiLS and $K^o = K_i + 1$ for UdLS, and otherwise, $K^o = K_i$ for UiLS and $K^o = K_i$ for UdLS.

algorithm can employ a recursive nullspace calculation in each iteration by using the following property [15]:

Property 1: For $m_1 \times n$ matrix $\boldsymbol{A}$ and $m_2 \times n$ matrix $\boldsymbol{B}$, if $m_1 + m_2 < n$, then the following equality holds:

$$null([\boldsymbol{A}^T\ \boldsymbol{B}^T]^T) = null(\boldsymbol{A})\,(null(\boldsymbol{B}null(\boldsymbol{A}))). \tag{33}$$

Proof: By showing that the multiplication of $[\boldsymbol{A}^T\ \boldsymbol{B}^T]^T$ and the right-hand side of (33) gives a zero matrix, the proof can be completed. ■

For example, $\boldsymbol{W}_2 = null([\boldsymbol{H}_1])$ and $\boldsymbol{W}_1 = null([\boldsymbol{H}_2])$, which are calculated at $i = 2$, can be used to calculate $null([\boldsymbol{H}_2\ \boldsymbol{H}_3])$, $null([\boldsymbol{H}_1\ \boldsymbol{H}_3])$, and $null([\boldsymbol{H}_1\ \boldsymbol{H}_2])$ at $i = 3$. The UdLS algorithm cannot employ the recursive nullspace calculation because it starts with the largest dimension of the channel matrix, i.e., $K = K_{max}$. However, the UdLS algorithm may converge with the smaller number of iterations than the UiLS algorithm because K_o is typically biased to K_{max} as observed in Figs. 9 and 12. Thus, it is nontrivial to analytically compare the computational complexities between UiLS and UdLS algorithms.

To further reduce the computational complexity of both UiLS and UdLS algorithms, the search intervals are designed based on the observation of K^o with respect to the number of transmit antennas. In Fig. 11, the optimal number of users is numerically found, and the lower and upper guidelines are obtained by using a curve fitting method [16]. The curve fitting method is widely used to determine the experimental parameters [17]. The lower guideline is a first order polynomial function, i.e., $g_{low}(M) = 0.38M - 0.91$, with normalized mean squared error (NMSE) 6.61×10^{-5} when M is an odd number and $M \geq 3$. The upper guideline is also a first order polynomial function, i.e., $g_{up}(M) = 0.39M - 0.49$, with NMSE 2.98×10^{-5} when M is an even number and $M \geq 2$. From the numerical results, it is observed that the guidelines do not vary over the system SNR, P/σ_z^2. Therefore, the guideline can be applied to the proposed ZF-STLC systems with M transmit antennas regardless of the system SNR.

Applying these guidelines, i.e., $\lfloor g_{low}(M) \rfloor$ and $\lfloor g_{up}(M) \rfloor$, to the UiLS and UdLS algorithms, the search intervals can

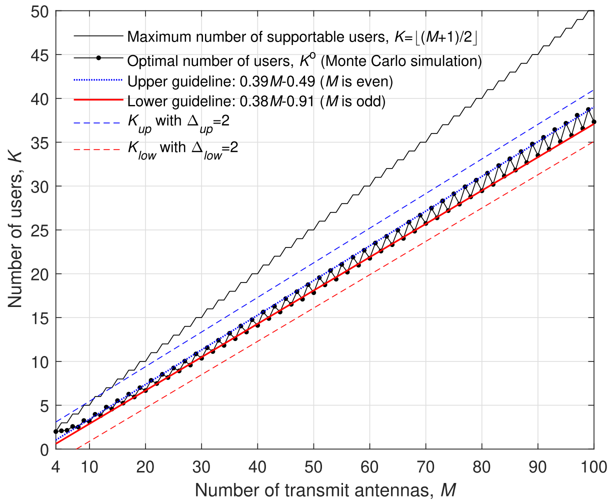

FIGURE 11. Numerical results on optimal number of users for various number of transmit antennas, when SNR_{sys} = 10 dB.

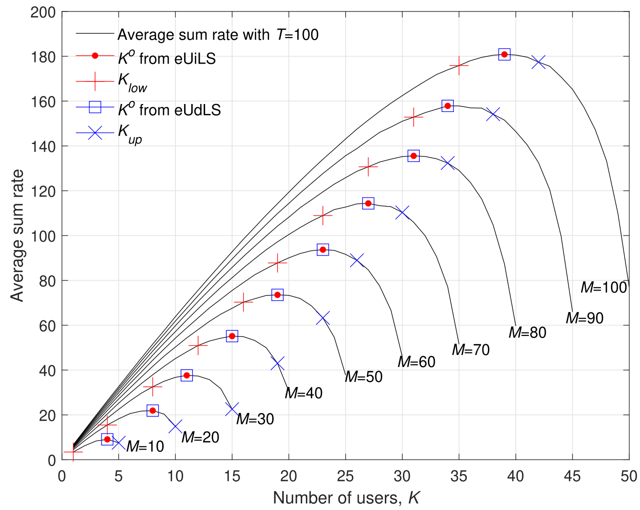

FIGURE 12. Average sum rate with $T = 100$, i.e., the objective function in (32), over the number of users when $P/\sigma_z^2 = 0$ dB, $\Delta_{low} = 2$, and $\Delta_{up} = 4$.

be effectively reduced, resulting in computational complexity reduction. Concretely, we set two limits for the search interval as follows:

$$K_{low} = \max\{\lfloor g_{low}(M)\rfloor - \Delta_{low}, 1\}, \tag{34}$$

$$K_{up} = \lfloor g_{up}(M)\rfloor + \Delta_{up}, \tag{35}$$

where $\Delta_{low} \geq 0$ and $\Delta_{up} \geq 0$ are the integer-value margins. With the reduced interval, we propose enhanced UiLS and UdLS algorithms, denoted by eUiLS and eUdLS, respectively. The eUiLS and eUdLS algorithms can be realized by modifying the initial setups of UiLS and UdLS, as shown in Algorithm 2.

Algorithm 2 : eUiLS and eUdLS Algorithms

1. **Setup**: $K_1 = K_{low}$ for eUiLS and $K_1 = K_{high}$ for eUdLS, $R_0 = 0$, and $i = 1$.
2. remaining steps are the same as the Algorithm 1.

B. EVALUATION OF THE PROPOSED SEARCH ALGORITHMS

The search performance of the proposed eUiLS and eUdLS algorithms are evaluated. Fig. 12 shows the results, in which the average sum rate (32) is obtained by averaging 100 channel realizations, i.e., $T = 100$, when $P/\sigma_z^2 = 0$ dB. Obviously, there exists a tradeoff between the computation complexity and fidelity of K^o obtained from the proposed algorithms. The larger integer-value margins, Δ_{low} and Δ_{up}, provide the better accuracy of K^o at the cost of computational complexity. In other words, obviously, there exits tradeoff between the accuracy and the computational complexity, which can be achieved by adjusting the integer-value margins, namely, Δ_{low} and Δ_{up}. In simulation, the integer-value margins are set as $\Delta_{low} = 2$ and $\Delta_{up} = 4$, for clear representation of the proposed algorithms. Here, the search interval is between K_{low} and K_{up}, marked by '+' and '×,' respectively. As shown in the results, both eUiLS and eUdLS find K^o, correctly, regardless of the number of transmit antennas, M.

V. CONCLUSION

In this study, a ZF-based STLC system was proposed to support multiple STLC users. For the proposed ZF-based STLC system, an optimal water-filling power allocation method was designed to maximize the sum achievable rate. For the allocated power fairness, a simple suboptimal FAPU power allocation method was also proposed. It was shown that the proposed ZF-based STLC with FAPU power allocation can provide near optimal performance. The analysis and rigorous simulation results verified that each user in the proposed ZF-based STLC system asymptotically achieves the single-user achievable rate as the number of transmit antennas increases, and that the proposed ZF-STLC outperforms the existing ZF-based eigen beamforming scheme and other multiuser STLC schemes. From the observation that the sum rate function is concave with respect to the number of users, two simple search algorithms, namely eUiLS and eUdLS, were proposed to effectively reduce the search interval while finding the optimal number of users that maximizes the sum rate. Numerical results have verified that the proposed eUiLS and eUdLS algorithms can effectively find the optimal number of users within the reduced search interval. The proposed ZF-based STLC can be readily extended to multiple users with more than two antennas.

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