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Robust Adaptive Beamforming Based on Desired Signal Power Reduction and Output Power of Spatial Matched Filter

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ABSTRACT The performance of the conventional beamformers degrades in the presence of desired signal in the data samples and array steering vector (ASV) mismatch. Many beamformers have been proposed to improve the performance of standard Capon beamformer. However, the performance of these beamformers is affected by a number of factors, such as a number of data samples or sensors and signal-to-noise ratio. Moreover, the existing beamformers are also sensitive to the ASV mismatch of desired signal. In this paper, two robust adaptive beamformers are proposed to overcome the problems associated with these beamformers. The proposed beamformers have two pre-processing steps. First, the ASV of desired signal is estimated by computing the correlation between the nominal ASV and the eigenvectors corresponding to the dominant eigenvalues. Second, the power of desired signal in the sample covariance matrix is reduced by estimating the desired signal covariance matrix from the output power of spatial matched filter and noise covariance matrix. Subsequently, the matrix regularization is used to estimate the desired sample covariance matrix. In the first beamformer, the desired sample covariance matrix is constructed from the sample covariance matrix with the reduced desired signal power and the diagonal loading based on the output power of spatial matched filter, whereas in the second beamformer, the desired sample covariance matrix is constructed from the sample covariance matrix with the reduced desired signal power and the reconstructed interference-plus-noise matrix loaded with the output power of spatial matched filter. The proposed beamformers can provide a good performance in the presence of desired signal in the data samples and ASV mismatch as shown in the simulation results.

INDEX TERMS Steering vector estimation, spatial matched filter, matrix regularization, diagonal loading, covariance matrix reconstruction.

I. INTRODUCTION

Adaptive beamforming has been of great interest in array processing for the past couple of decades. It plays an important role in radar, sonar, satellite navigation, medical imaging and array microphone speech processing. Many beamformers have already been developed for adaptive beamforming. The standard Capon beamformer (SCB) [1] is one of the renown adaptive beamformers with good resolution and interference suppression capabilities. SCB is based on the covariance matrix computation using the received signal samples to maximize the array output signal-to-interference-plus-noise ratio (SINR). However, with finite number of snapshots, desired signal in the data samples and in presence of ASV mismatch, the performance of SCB degrades significantly. Therefore, a number of robust adaptive beamformers (RAB) [2]–[9] have been proposed to improve the performance of SCB.

Diagonal loading (DL) is one of the widely used approach to enhance the robustness of SCB. The performance of the beamformers based on DL is highly dependent on the selection of the optimal DL-factor. Most of them suffer from performance degradation in the presence of ASV mismatch and when the desired signal power is greater than interference power in the sample covariance matrix. The conventional DL beamformer [2] is based on a fixed DL-factor which can be taken as the noise power or the normalized constraint of weight vector. These parameters are hard to determine. Therefore, parameter free DL beamformers have been proposed to solve this problem. Hoerl, Kennard, and Baldwin (HKB) beamformer [5] based on the generalized sidelobe canceler re-parameterization of SCB. Spatial matched filter (SMF) beamformer [6] based on the output power of the spatial matched filter associated with the ASV obtained through eigen-analysis of the beampattern. General linear combination (GLC) based robust Capon beamformer [3], which is a shrinkage based beamformer utilizing the knowledge aided space time adaptive processing in [10]. Some other shrinkage based beamformers can also be found in [7] and [9].

The performance of these parameter free DL beamformers is dependent on the adaptation of their DL-factors. DL-factors should decrease as the number of snapshots increases for a fixed number of array elements, and increase as the number of array elements increases for a fixed number of snapshots [7]. However, the performance of HKB beamformer degrades when the number of snapshots increases, because of the DL factor increases with the increasing of snapshots [3]. On the other hand, GLC beamformer's performance degrades with the increasing number of snapshots, since its DL-factor rapidly converges to zero. In addition, both HKB and GLC as well as SCB suffer from signal self-cancellation when the desired signal is presence in the received snapshots. The problem of self-cancellation is however solved by the SMF beamformer. But its performance may degrade when the number of snapshots is very large since its DL-factor oscillates about the output power. Moreover, the performance of SMF beamformer is also affected by the number of antenna elements.

One of the general shortcomings of the DL beamformers is that the performance also degrades in the presence of the desired signal ASV mismatch. In the presence of ASV mismatch, it results into the array beam pattern distortion. A number of beamforming approaches have been investigated in conjunction with estimating the actual desired signal ASV [7]– [9], [11], [12]. The worst case beamformer (WCB) [12] is also a DL based beamformer that uses a convex second-order cone (SOC) programming approach to estimate the ASV and computing the weight vector. However, the computational complexity of optimization approach is huge and the algorithm is based on unknown deterministic variables. In [11], an approach to determine the desired signal ASV based on estimating the desired signal subspace was presented. This is achieved through finding the intersection between the signal-plus-interference subspace and a reference space covered by the angular region of the desired signal. In [9], the oracle approximating shrinkage (OAS) method is iteratively used to estimate the signal ASV in a set desired signal reference space. However, the iterative process in this algorithm increases the computation.

Therefore, two robust adaptive beamfomers are proposed to enhance the performance of SCB by reconstructing the sample covariance matrix. The proposed beamformers have two pre-processing steps. Firstly, the ASV of desired signal is estimated by computing the correlation between the nominal ASV and the eigenvectors corresponding to the dominant eigenvalues of the sample covariance matrix. This approach has also been employed for instance in [13] to eliminate the mainlobe interference through eigen decomposition. Secondly, the power of desired signal in the sample covariance matrix is reduced by estimating the desired signal covariance matrix from the output power of spatial matched filter and noise covariance matrix. Then, both the proposed beamformers use the matrix regularization to estimate desired sample covariance matrix as introduced in the knowledge aided space time adaptive processing [10], which is also adopted in [3] and [4]. In the first beamformer, the desired sample covariance matrix is constructed by the desired signal power reduction matrix and diagonal loading (DL) based on the output power of the spatial matched filter. In the second beamformer, the desired sample covariance matrix is computed by the desired signal power reduction matrix and the reconstructed interference-plus-noise (INC) matrix loaded by the output power of spatial matched filter. In INC-matrix reconstruction, the direction of arrival (DOA) [17]–[19] of the interferences are required. The proposed beamformers can provide good performance in the presence of desired signal in the data samples and ASV mismatch as shown in the simulation results.

The rest of the paper is organized as follows. The signal model and background are given in section II. The formulation of the proposed beamformers are described in section III. The performance of the proposed beamformers is analyzed in section IV. The simulation results and analysis are provided in section V. Finally, the paper is concluded in section VI.

II. SIGNAL MODEL AND BACKGROUND

Consider one desired signal and P uncorrelated sidelobe interferences incident on an antenna array with M elements, where $P + 1 \leq M$. Both the desired signal and interferences are narrowband signals. The nth snapshot of the received signal $\mathbf{x}(n) \in \mathbb{C}^{M\times 1}$ is expressed as

$$\mathbf{x}(n) = \mathbf{a}(\theta_0)\mathrm{s}_0(n) + \sum_{i=1}^{P} \mathbf{a}(\theta_i)\mathrm{s}_i(n) + \mathbf{n}(n), \tag{1}$$

where $\mathrm{s}_i(n)[i = 0, 1, \cdots P]$ is the corresponding complex envelope of the desired signal and the interferences. θ_i is the direction of arrival (DOA) of the incident signal. $\mathbf{n}(n) \in \mathbb{C}^{M\times 1}$ is a zero mean, uncorrelated spatially white noise. $\mathbf{a}(\theta_i) \in \mathbb{C}^{M\times 1}$ (with $\|\mathbf{a}(\theta_i)\|_2^2 = M$) is the nominal ASV of the ith signal. $\|\cdot\|_2$ denotes the $\mathcal{L}_2$-norm. The array output is linearly combined by the beamformer to form the desired output

$$\mathrm{y}(n) = \mathbf{w}^H\mathbf{x}(n), \tag{2}$$

where $\mathbf{w} \in \mathbb{C}^{M\times 1}$ is the complex weight vector of the beamformer and $(\cdot)^H$ denotes the conjugate transpose. In this implementation, the main aim is to maximize the signal-to-interference-plus-noise ratio (SINR)

$$\mathbf{SINR} = \frac{\sigma_0^2 |\mathbf{w}^H \mathbf{a}(\theta_0)|^2}{\mathbf{w}^H \mathbf{R}_{\text{IPN}} \mathbf{w}}, \tag{3}$$

where σ_0^2 is the desired signal power and $\mathbf{R}_{\text{IPN}}$ is the interference-plus-noise covariance matrix. Capon proposed an optimal beamformer in [1] with a true covariance (TC) matrix $\mathbf{R} \in \mathbb{C}^{M \times M}$ given by

$$\mathbf{R} = \sigma_0^2 \mathbf{a}(\theta_0)\mathbf{a}^H(\theta_0) + \mathbf{R}_{\text{IPN}}, \tag{4}$$

where its optimal weight vector is obtained by optimization of the linearly constrained quadratic equation in (5)

$$\min_{\mathbf{w}} \ \mathbf{w}^H \mathbf{R} \mathbf{w}, \quad \text{subject to: } \mathbf{w}^H \mathbf{a}(\theta_0) = 1. \tag{5}$$

The optimal weight vector of the optimization problem in (5) with accurate knowledge of the desired ASV $\mathbf{a}(\theta_0)$ is given by

$$\mathbf{w}_{\text{OPT}} = \frac{\mathbf{R}^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0)\mathbf{R}^{-1}\mathbf{a}(\theta_0)}, \tag{6}$$

where $(\cdot)^{-1}$ denotes the matrix inverse. Since in practice the true covariance matrix $\mathbf{R}$ is not known, it is normally replaced with the sample covariance (SC) matrix $\hat{\mathbf{R}}_{\text{X}}$ given by

$$\hat{\mathbf{R}}_{\text{X}} = \frac{1}{N}\sum_{n=1}^{N} \mathbf{x}(n)\mathbf{x}^H(n), \tag{7}$$

where N denotes the total number of snapshots. The weight vector of SCB is given by

$$\mathbf{w}_{\text{SCB}} = \frac{\hat{\mathbf{R}}_{\text{X}}^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0)\hat{\mathbf{R}}_{\text{X}}^{-1}\mathbf{a}(\theta_0)}, \tag{8}$$

Assuming that the nominal ASV $\mathbf{a}(\theta_0)$ is perfectly known, the SC-matrix $\hat{\mathbf{R}}_{\text{X}}$ of SCB converges to the TC-matrix $\mathbf{R}$ in (4). Therefore, the corresponding SINR value will approach the optimal value when the number of snapshots $N \rightarrow \infty$ under stationary assumptions [8]. On the other hand, the performance of SCB severely degrades when few snapshots are used and there exists a mismatch in the ASV of the desired signal [3]–[7]. Therefore, different beamformers based on DL have been employed to solve SCB's shortcomings. The complex weight vectors of the DL beamformers take the form

$$\mathbf{w}_{\text{DL}} = \frac{\left(\hat{\mathbf{R}}_{\text{X}} + \rho_{\text{DL}}\mathbf{I}\right)^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0)\left(\hat{\mathbf{R}}_{\text{X}} + \rho_{\text{DL}}\mathbf{I}\right)^{-1}\mathbf{a}(\theta_0)}, \tag{9}$$

where ρ_{DL} is the DL-factor to be estimated and $\mathbf{I}$ is the identity matrix. A constant ρ_{DL} approach was proposed in [2]. The DL-factor ρ_{DL} is taken to be a small loading factor e.g 0dB of noise power. However, the constant diagonal loading method is limited to how best ρ_{DL} value is chosen. Therefore, a number of parameter-free RABs have been proposed including the GLC, SMF, HKB and WCB beamformers.

A. GENERAL LINEAR COMBINATION-BASED (GLC) BEAMFORMER

GLC beamformer in [3] is a parameter-free RAB which is based on reconstructing the sample covariance matrix using the matrix regularization approach. The weight vector of the GLC beamformer is given by

$$\mathbf{w}_{\text{GLC}} = \frac{\left(\hat{\mathbf{R}}_{\text{X}} + \frac{\beta_0}{\alpha_0}\mathbf{I}\right)^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0)\left(\hat{\mathbf{R}}_{\text{X}} + \frac{\beta_0}{\alpha_0}\mathbf{I}\right)^{-1}\mathbf{a}(\theta_0)}, \tag{10}$$

where $\frac{\beta_0}{\alpha_0}$ is the DL-factor which can be denoted by ρ_{GLC}. The optimal regularization parameters α_0 and β_0 are determined as

$$\alpha_0 = \frac{\gamma}{\varepsilon + \gamma}, \tag{11}$$

$$\beta_0 = \nu(1 - \alpha_0) = \nu\frac{\varepsilon}{\varepsilon + \gamma}. \tag{12}$$

where $\varepsilon \triangleq \mathbb{E}\{\|\hat{\mathbf{R}}_{\text{X}} - \mathbf{R}\|_2^2\}$, $\nu = \text{tr}(\mathbf{R})/M$ and $\gamma = \|\nu\mathbf{I} - \mathbf{R}\|_2^2$. $\mathbb{E}\{\cdot\}$ denotes the expectation operator and $\text{tr}(\cdot)$ denotes the trace of a matrix. Therefore, the regularization parameters α_0 and β_0 depend on the unknown TC-matrix $\mathbf{R}$. A GLC based robust Capon beamformer is developed in [3] using parameters α_0 and β_0 estimated without the knowledge of $\mathbf{R}$. The derivation of the parameters can be found in [4].

B. HOERL, KENNARD AND BALDWIN (HKB) BEAMFORMER

HKB beamformer in [5] is another parameter-free RAB with weight vector given by

$$\mathbf{w}_{\text{HKB}} = \frac{\left(\hat{\mathbf{R}}_{\text{X}} + \rho_{\text{HKB}}\mathbf{I}\right)^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0)\left(\hat{\mathbf{R}}_{\text{X}} + \rho_{\text{HKB}}\mathbf{I}\right)^{-1}\mathbf{a}(\theta_0)}, \tag{13}$$

where ρ_{HKB} is the DL-factor expressed by

$$\rho_{\text{HKB}} = \frac{(M-1)\hat{\sigma}_{\text{LS}}^2}{\|\eta_{\text{LS}}\|_2^2}. \tag{14}$$

with

$$\hat{\sigma}_{\text{LS}}^2 = \mathbf{Q}\eta_{\text{LS}} - \left(\frac{\mathbf{R}_{\text{X}}^{1/2}\mathbf{a}(\theta_0)}{M}\right). \tag{15}$$

The standard least squares estimator is used to obtain η_{LS} as

$$\eta_{\text{LS}} = (\mathbf{Q}^H\mathbf{Q})^{-1}\mathbf{Q}^H\left(\frac{\mathbf{R}_{\text{X}}^{1/2}\mathbf{a}(\theta_0)}{M}\right), \tag{16}$$

where $\mathbf{Q} = \mathbf{R}_{\text{X}}^{1/2}\mathbf{B}$. $\mathbf{B} \in \mathbb{C}^{M \times (M-1)}$ is the semi-unitary blocking matrix which satisfies the constraints $\mathbf{B}^H\mathbf{a}(\theta_0) = 0$ and $\mathbf{B}^H\mathbf{B} = \mathbf{I}$. The columns of $\mathbf{B}$ can be computed as the eigenvectors corresponding to the $M-1$ non-zero eigenvalues of $\mathbf{I} - \mathbf{a}(\theta_0)\mathbf{a}^H(\theta_0)/M$ as used in [7].

C. SPATIAL MATCHED FILTER-BASED (SMF) BEAMFORMER

The spatial matched filter SMF beamformer [6] is a recently proposed robust adaptive beamformer. The weight vector of the SMF beamformer is given by

$$\mathbf{w}_{\mathrm{SMF}} = \frac{\left(\hat{\mathbf{R}}_{\mathrm{X}} + \rho_{\mathrm{SMF}}\mathbf{I}\right)^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0)\left(\hat{\mathbf{R}}_{\mathrm{X}} + \rho_{\mathrm{SMF}}\mathbf{I}\right)^{-1}\mathbf{a}(\theta_0)}. \quad (17)$$

where the DL-factor ρ_{SMF} of the beamformer is obtained through the eigen-analysis of the beam pattern. ρ_{SMF} was taken to be the output power of the SMF associated with the ASV $\mathbf{a}(\theta_0)$.

$$\rho_{\mathrm{SMF}} = \frac{1}{N}\left\| \frac{\mathbf{a}(\theta_0)}{\|\mathbf{a}(\theta_0)\|_2}\mathbf{X}\right\|_2^2, \quad (18)$$

where $\mathbf{X} \in \mathbb{C}^{M\times N}$ is the received signal matrix.

D. WORST CASE BEAMFORMER (WCB)

The worst case beamformer [12] is based on estimating the ASV of desired signal by solving the problem of ASV mismatch between the nominal and the actual signal ASVs. i.e. $\hat{\mathbf{a}}(\theta_0) = \mathbf{a}(\theta_0) + \Delta \neq \mathbf{a}(\theta_0)$. Where Δ is an unknown deterministic norm of the ASV distortion bounded by $\|\Delta\|_2 \leq \varepsilon$. $\varepsilon > 0$ is the bound mismatch vector.

For robustness, the actual desired signal ASV is assumed to belong to the set

$$\mathcal{A}(\varepsilon) \triangleq \{\mathbf{c}|\mathbf{c} = \mathbf{a}(\theta_0) + \mathbf{e}, \|\mathbf{e}\|_2 \leq \varepsilon\}, \quad (19)$$

it is assumed that, if $\mathbf{e} = \Delta$, then $\mathbf{c} = \hat{\mathbf{a}}(\theta_0)$ since $\hat{\mathbf{a}}(\theta_0)$ can be any vector in (19). The beamformer is intended to minimize the following minimization problem

$$\min_{\mathbf{w}} \ \mathbf{w}^H\hat{\mathbf{R}}_{\mathrm{X}}\mathbf{w},$$
$$\text{subject to: } |\mathbf{w}^H\mathbf{c}| \geq 1, \quad \mathbf{c} \in \mathcal{A}(\varepsilon). \quad (20)$$

The complex weight vector of WCB is given by

$$\mathbf{w}_{\mathrm{WCB}} = \frac{\lambda_o(\hat{\mathbf{R}}_{\mathrm{X}} + \lambda_o\varepsilon^2\mathbf{I})^{-1}\mathbf{a}(\theta_0)}{\lambda_o\mathbf{a}^H(\theta_0)(\hat{\mathbf{R}}_{\mathrm{X}} + \lambda_o\varepsilon^2\mathbf{I})^{-1}\mathbf{a}(\theta_0)}, \quad (21)$$

where λ_o is the Lagrange multiplier. Therefore, $\mathbf{w}_{\mathrm{WCB}}$ in (20) is iteratively computed using the SOC programming approach employed in [12].

III. PROPOSED BEAMFORMING ALGORITHMS

In this section, two beamformers are proposed to improve the performance of SCB in terms of convergence rate and SINR. The beamformers are based on the output power of the SMF associated with the ASV of desired signal direction [6], estimation of ASV of the desired signal, the signal power reduction and the matrix regularization [10]. The proposed method formulation is described firstly, and followed by the estimation of the signal steering vector through ASV mismatch compensation, then proceeded with the desired signal power reduction in SC-matrix. Finally, the two proposed beamformers based on diagonal loading (SMF_1-DL) and interference-plus-noise-covariance-matrix reconstruction (SMF_1-INC) are introduced.

A. ALGORITHM FORMULATION

In order to reconstruct the SC-matrix $\hat{\mathbf{R}}_{\mathrm{X}}$, a matrix regularization form in (22) is utilized to compute the desired covariance matrix $\breve{\mathbf{R}}_{\mathrm{X}}$. As introduced in [4], [9], and [10], $\breve{\mathbf{R}}_{\mathrm{X}}$ is calculated by linearly combining the SC-matrix $\hat{\mathbf{R}}_{\mathrm{X}}$ and its initial guess $\mathbf{R}_0$.

$$\breve{\mathbf{R}}_{\mathrm{X}} = \alpha\hat{\mathbf{R}}_{\mathrm{X}} + (1-\alpha)\mathbf{R}_0, \quad (22)$$

where $\alpha \in [0,1]$ is a regularization parameter. Assuming $\mathbf{R}_0 = \mathbf{I}$, the beamformer is considered to be the SCB when $\alpha = 1$ and a delay-and-sum beamformer (DSB) when $\alpha = 0$. In this work, the performance of SCB is improved thus, parameter α is considered to be less than but close to 1 ($\alpha \to 1$). According to [10], $\mathbf{R}_0$ is obtained from the physical cluttering model in which previous knowledge of clutter (interferences in this case) is used. The mathematical model of $\mathbf{R}_0$ is given as

$$\mathbf{R}_0 = \frac{1}{P}\sum_{i=1}^{P}\sigma_i^2\mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i), \quad (23)$$

where σ_i^2 is the interference power for a given previous scan. When (23) is substituted into (22) with no DOA mismatches and parameter α set to 0, the beamformer becomes the optimal one. However, due to the dynamic nature of the interferences, using their previous information to approximate $\mathbf{R}_0$ may not be a good choice in practice, as it may result into low performance due to DOA mismatches and power variations. The current received signal data $\mathbf{X}$ is therefore utilized to estimate $\mathbf{R}_0$.

The performance of the RABs also depends on the selection choice of the regularization parameter α in (22). Therefore, a variable α_{x} is also proposed given by

$$\alpha_{\mathrm{x}} = 1 - \frac{1}{KN}. \quad (24)$$

where K $[1 < K \leq M]$ is a constant integer that adds one degree of freedom [15], [16].

B. DESIRED SIGNAL ASV MISMATCH COMPENSATION

In practice, the actual ASV of the desired signal is usually hard to obtain by just using the nominal DOA. This is due to the complexity of the signal propagation environment [8]. Therefore, there is need for the compensation of the nominal ASV mismatch that may occur. This is achieved by employing correlation between the nominal ASV $\mathbf{a}(\theta_0)$ and the eigenvectors corresponding to the dominant eigenvalues of the desired signal and interference (SI) subspace. The eigenvalues with their corresponding eigenvectors are obtained through eigen-decomposition of the SC-matrix $\hat{\mathbf{R}}_{\mathrm{X}}$.

$$\hat{\mathbf{R}}_{\mathrm{X}} = \mathbf{U}_{\mathrm{X}}\Lambda_{\mathrm{X}}\mathbf{U}_{\mathrm{X}}^H, \quad (25)$$

where $\mathbf{U}_x \in \mathbb{C}^{MxM}$ is a square matrix having columns corresponding to eigenvectors $\mathbf{u}_m[m = 0,1,\cdots, M-1]$ of $\hat{\mathbf{R}}_X$. $\Lambda_X \in \mathbb{C}^{MxM}$ is a diagonal matrix. That is to say, $\Lambda_x = \text{diag}\{\mu_0, \mu_1, \cdots, \mu_{D-1}, \cdots, \mu_{M-1}\}$ with elements $\mu_m > 0$ in descending order are eigenvalues of $\hat{\mathbf{R}}_X$. D is the total number of the detected interferences plus the desired signal, obtained as the m that maximizes the coefficient [11]

$$\eta(m) = \max_m \left| \frac{\mu_{m-1}}{\mu_m} \right|, \tag{26}$$

for $m = 1, \cdots, M-1$. The correlation coefficient of any two column vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is generally computed [13]

$$\psi(\mathbf{v}_1, \mathbf{v}_2) \triangleq \frac{\mathbf{v}_1^H \mathbf{v}_2}{\|\mathbf{v}_1\|_2 \|\mathbf{v}_2\|_2}. \tag{27}$$

Therefore, the correlation coefficients of the nominal ASV $\mathbf{a}(\theta_0)$ and the eigenvectors $\mathbf{u}_m[m{=}0,\cdots,D{-}1]$ are calculated as in (28). The maximum correlation coefficient is achieved when $\mathbf{u}_m$ is the eigenvector of the desired signal $\mathbf{u}_d$.

$$\begin{aligned} |\psi(\mathbf{u}_d, \mathbf{a}(\theta_0))| &= \max_{\mathbf{u}_m} |\psi(\mathbf{u}_m, \mathbf{a}(\theta_0))|, \quad m{=}0,\cdots,M-1, \\ &= \max_{\mathbf{u}_m} \left| \frac{\mathbf{u}_m^H \mathbf{a}(\theta_0)}{\|\mathbf{u}_m\|_2 \|\mathbf{a}(\theta_0)\|_2} \right|. \end{aligned} \tag{28}$$

Therefore, the estimated desired signal ASV $\hat{\mathbf{a}}(\theta_0)$ is obtained by

$$\hat{\mathbf{a}}(\theta_0) = \sqrt{M} \frac{\mathbf{u}_d}{\|\mathbf{u}_d\|}. \tag{29}$$

C. DESIRED SIGNAL POWER REDUCTION IN SC-MATRIX

To reduce the signal power in SC-matrix, the desired signal covariance (DSC) matrix has to be eliminated from the SC-matrix $\hat{\mathbf{R}}_X$ as described below.

$$\bar{\mathbf{R}}_X = \hat{\mathbf{R}}_X - \bar{\mathbf{R}}0. \tag{30}$$

$\bar{\mathbf{R}}0$ is the DSC-matrix estimated as

$$\bar{\mathbf{R}}0 = \hat{\sigma}_0^2 \hat{\mathbf{a}}(\theta_0) \hat{\mathbf{a}}(\theta_0)^H, \tag{31}$$

where $\hat{\sigma}_0^2$ is the estimate of desired signal power derived as follows.

- Considering the received signal matrix $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N]$ written in the form as in (1)

$$\mathbf{X} = \mathbf{a}(\theta_0)\mathbf{s}_0 + \sum_{i=1}^{P} \mathbf{a}(\theta_i)\mathbf{s}_i + \mathbf{N}, \tag{32}$$

- Introducing the normalized ASV associated with the DOA of the desired signal on both sides of (32). For convenience we let $\mathbf{a}_0 = \mathbf{a}(\theta_0)$.

$$\frac{\mathbf{a}_0^H}{\sqrt{M}}\mathbf{X} = \frac{\mathbf{a}_0^H}{\sqrt{M}}\mathbf{a}_0\mathbf{s}_0 + \frac{\mathbf{a}_0^H}{\sqrt{M}} \sum_{i=1}^{P} \mathbf{a}(\theta_i)\mathbf{s}_i + \frac{\mathbf{a}_0^H}{\sqrt{M}}\mathbf{N}. \tag{33}$$

- On assumption that the $\mathbf{a}_0$ is uncorrelated with the interferences [9], the interference component in (33) can be approximated to zero.

$$\frac{\mathbf{a}_0^H}{\sqrt{M}}\mathbf{X} = \frac{\mathbf{a}_0^H}{\sqrt{M}}\mathbf{a}_0\mathbf{s}_0 + \frac{\mathbf{a}_0^H}{\sqrt{M}}\mathbf{N}. \tag{34}$$

  In practice, the assumption in (34) may not hold for small array sized structures. By considering the above assumption, this desired signal power estimation approach will be very sensitive to the ASV mismatch of the desired signal. Therefore, the nominal ASV $\mathbf{a}(\theta_0)$ in (34) with a mismatch is replaced with the estimated ASV $\hat{\mathbf{a}}(\theta_0)$ of the desired signal in (29) to overcome this shortcoming.

- We then normalize (34) to obtain the output power of the SMF as $\frac{1}{N}\left\|\frac{\hat{\mathbf{a}}(\theta_0)^H}{\sqrt{M}}\mathbf{X}\right\|_2^2$. The output power of the SMF can be denoted by, P_{SMF} and for convenience, we consider $\hat{\mathbf{a}}_0 = \hat{\mathbf{a}}(\theta_0)$.

$$\begin{aligned} \text{P}_{\text{SMF}} &= \frac{1}{N}\left\| \frac{\hat{\mathbf{a}}_0^H}{\sqrt{M}}\hat{\mathbf{a}}_0\mathbf{s}_0 + \frac{\hat{\mathbf{a}}_0^H}{\sqrt{M}}\mathbf{N} \right\|_2^2, \\ &= \frac{1}{N}\left\| \left(\frac{\hat{\mathbf{a}}_0^H}{\sqrt{M}}\hat{\mathbf{a}}_0\mathbf{s}_0 + \frac{\hat{\mathbf{a}}_0^H}{\sqrt{M}}\mathbf{N} \right) \right. \\ &\quad \left. \times \left(\frac{\hat{\mathbf{a}}_0^H}{\sqrt{M}}\hat{\mathbf{a}}_0\mathbf{s}_0 + \frac{\hat{\mathbf{a}}_0^H}{\sqrt{M}}\mathbf{N} \right)^H \right\|_2. \end{aligned} \tag{35}$$

- Assuming the desired signal and the noise are statistically independent, (35) can be manipulated and solved as

$$\begin{aligned} \text{P}_{\text{SMF}} &= \left(\underbrace{\frac{1}{N}\mathbf{s}_0\mathbf{s}_0^H}_{\hat{\sigma}_0^2} \left| \frac{\hat{\mathbf{a}}_0^H\hat{\mathbf{a}}_0}{\sqrt{M}} \right|^2 + \frac{\hat{\mathbf{a}}_0^H}{\sqrt{M}} \big(\underbrace{\frac{1}{N}\left\|\mathbf{N}\mathbf{N}^H\right\|_2}_{\hat{\sigma}_n^2} \big) \frac{\hat{\mathbf{a}}_0}{\sqrt{M}} \right) \\ &= \left(\underbrace{\frac{1}{N}\mathbf{s}_0\mathbf{s}_0^H}_{\hat{\sigma}_0^2} \left| \frac{\hat{\mathbf{a}}_0^H\hat{\mathbf{a}}_0}{\sqrt{M}} \right|^2 + \frac{\hat{\mathbf{a}}_0^H\mathbf{R}_N\hat{\mathbf{a}}_0}{M} \right). \end{aligned} \tag{36}$$

- By substituting for P_{SMF}, multiplying M on both sizes and reforming the left hand size of (36), it is obtained

$$\hat{\mathbf{a}}_0^H \hat{\mathbf{R}}_X \hat{\mathbf{a}}_0 = \underbrace{\frac{1}{N}\mathbf{s}_0\mathbf{s}_0^H}_{\hat{\sigma}_0^2} \left| \hat{\mathbf{a}}_0^H \hat{\mathbf{a}}_0 \right|^2 + \hat{\mathbf{a}}_0^H \mathbf{R}_N \hat{\mathbf{a}}_0. \tag{37}$$

- With straight forward manipulation, the estimated desired signal power $\hat{\sigma}_0^2$ is obtained as

$$\hat{\sigma}_0^2 = \frac{\hat{\mathbf{a}}_0^H \hat{\mathbf{R}}_X \hat{\mathbf{a}}_0 - \hat{\mathbf{a}}_0^H \mathbf{R}_N \hat{\mathbf{a}}_0}{\left| \hat{\mathbf{a}}_0^H \hat{\mathbf{a}}_0 \right|^2}. \tag{38}$$

The derivation is based on one used in [9]. $\mathbf{R}_N$ is the noise covariance (NC) matrix which can be estimated from the

TABLE 1. SMF_1-DL beamformer.

1) **Inputs:**
a) Array elements M
b) Received signal matrix $\mathbf{X}$
2) **Computation:**
a) Determine the number signal sources with (26)
b) Calculate the SC-matrix $\hat{\mathbf{R}}_{\text{X}}$
c) Desired signal ASV mismatch compensation
i) Eigen-decompose $\hat{\mathbf{R}}_{\text{X}}$ in (25)
ii) Estimate the required eigenvector $\mathbf{u}_{\text{d}}$ as in (28)
iii) Compute the signal ASV as in (29)
d) Desired signal power reduction
i) Estimate NC-matrix using (39)
ii) Calculate desired signal power σ_0^2 from (38)
e) Desired covariance matrix
i) Calculate initial estimate of $\hat{\mathbf{R}}_{\text{X}}$ from (30)
ii) Reconstruct $\hat{\mathbf{R}}_0$ from (40)
iii) Reconstruct $\breve{\mathbf{R}}_{\text{X}}$ from (41)
3) **Output:**
The weight vector $\mathbf{w}_{\text{SMF}_1\text{-D}}$ calculated using (42)

noise powers in the noise subspace of the eigen-decomposed SC-matrix in (25) as

$$\mathbf{R}_{\text{N}} = \left(\frac{1}{M-D}\sum_{m=D}^{M-1}\mu_m\right).\mathbf{I}. \tag{39}$$

D. PROPOSED SMF_1−DL BEAMFORMER

The SMF_1-DL beamformer is based on SMF and DL. From (22), $\breve{\mathbf{R}}_{\text{X}}$ is computed by replacing $\hat{\mathbf{R}}_{\text{X}}$ with $\bar{\mathbf{R}}_{\text{X}}$ in (30), and $\hat{\mathbf{R}}_0$ is estimated as an identity matrix $\mathbf{I}$ loaded with output power of the SMF as given in (40). The output power is computed using the $\mathcal{L}_1$-norm concept.

$$\begin{aligned}\hat{\mathbf{R}}_0 &= \frac{1}{N}\left\|\frac{\hat{\mathbf{a}}(\theta_0)}{\sqrt{M}}\mathbf{X}\right\|_1^2.\mathbf{I},\\ &= \sigma^2_{\text{SMF}_1}\mathbf{I},\end{aligned} \tag{40}$$

where $\sigma^2_{\text{SMF}_1}$ is the $\mathcal{L}_1$-norm output power of SMF associated with the normalized ASV of the desired signal DOA given by $\frac{\hat{\mathbf{a}}(\theta_0)}{\sqrt{M}}$. $\|\cdot\|_1$ denotes $\mathcal{L}_1$-norm of a matrix. Replacing $\mathbf{R}_0$ in (22) with $\hat{\mathbf{R}}_0$ in (40) and α with α_{x}, the following expression is obtained

$$\breve{\mathbf{R}}_{\text{X}} = \alpha_{\text{x}}\bar{\mathbf{R}}_{\text{X}} + (1-\alpha_{\text{x}})\sigma^2_{\text{SMF}_1}\mathbf{I}. \tag{41}$$

By substituting (24) into (41), the proposed SMF_1-DL based beamformer weight vector is given by

$$\mathbf{w}_{\text{SMF}_1\text{-D}} = \frac{\left(\bar{\mathbf{R}}_{\text{X}} + \frac{\sigma^2_{\text{SMF}_1}}{KN-1}\mathbf{I}\right)^{-1}\hat{\mathbf{a}}(\theta_0)}{\hat{\mathbf{a}}^H(\theta_0)\left(\bar{\mathbf{R}}_{\text{X}} + \frac{\sigma^2_{\text{SMF}_1}}{KN-1}\mathbf{I}\right)^{-1}\hat{\mathbf{a}}(\theta_0)}, \tag{42}$$

where $\frac{\sigma^2_{\text{SMF}_1}}{KN-1}$ is the loading factor which is denoted by $\rho_{\text{SMF}_1\text{-D}}$. A summary of the proposed SMF_1-DL beamformer is given in Table 1.

E. PROPOSED SMF_1−INC BEAMFORMER

The second proposed beamformer is based on SMF and INC-matrix reconstruction. Considering the regularization form in (22), $\breve{\mathbf{R}}_{\text{X}}$ is constructed as

$$\breve{\mathbf{R}}_{\text{X}} = \alpha_{\text{x}}\bar{\mathbf{R}}_{\text{X}} + (1-\alpha_{\text{x}})\hat{\mathbf{R}}_{\text{IPN}}, \tag{43}$$

where the interference-plus-noise (IPN) matrix $\hat{\mathbf{R}}_{\text{IPN}}$ is computed as $\mathbf{R}_0$ in (23). But the system noise is added to $\mathbf{R}_0$. The reconstructed $\hat{\mathbf{R}}_{\text{IPN}}$ is given by

$$\hat{\mathbf{R}}_{\text{IPN}} = \frac{1}{P}\sum_{i=1}^{P}\mu_0\mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i) + \mathbf{R}_{\text{N}}, \tag{44}$$

where μ_0 is the maximum power of the signals obtained through the eigen-decomposition of the SC-matrix in (25). $\mathbf{a}(\theta_i)$ is the ASV of the ith interference. The power of the signals can also be obtained as $\mu_0 = 1/(\mathbf{a}^H(\theta_i)\hat{\mathbf{R}}_{\text{X}}^{-1}\mathbf{a}(\theta_i))$ to overcome the effects of mismatches in the DOA of the interferences but with a higher computation complexity. Therefore, the reconstructed covariance matrix of the proposed SMF_1-INC beamformer is given by

$$\breve{\mathbf{R}}_{\text{X}} = \alpha_{\text{x}}\bar{\mathbf{R}}_{\text{X}} + (1-\alpha_{\text{x}})\sigma^2_{\text{SMF}_1}\hat{\mathbf{R}}_{\text{IPN}}. \tag{45}$$

The output power $\sigma^2_{\text{SMF}_1}$ is introduced in (45) to avoid the convergence of the covariance matrix $\breve{\mathbf{R}}_{\text{X}}$ very fast before reaching the TC-matrix, which is also analyzed in the next section. This is due to the tendency of the parameter $(1-\alpha_{\text{x}})$ to converge to a very small value as N increases. Meanwhile, the DOAs $\theta_i[i = 1, \cdots, P]$ of the interferences are determined using any DOA estimation method e.g Root-MUSIC [17], MUSIC-Like [18], as well as DOA estimation based on the concept of nulling antenna using the LMS algorithm [19]. To reconstruct $\hat{\mathbf{R}}_{\text{IPN}}$, the desired signal DOA θ_0 has to be removed from the estimated DOAs. Angle θ_0 should lie within the first null beam-width (FNBW) range. FNBW is estimated as

$$\text{FNBW} = 2\left(\frac{\pi}{2} - \cos^{-1}\left(\frac{\lambda}{Md}\right)\right). \tag{46}$$

Therefore, the desired DOA θ_0 uncertainty is bounded in the upper and lower bounds of $\theta_0 + \Delta\theta_0$, with $\Delta\theta_0$ limited by the inequality in (47) when $\frac{d}{\lambda} = 0.5$

$$-\sin^{-1}\left(\frac{2}{M}\right) \leq \Delta\theta_0 \leq \sin^{-1}\left(\frac{2}{M}\right). \tag{47}$$

The weight vector for the proposed SMF_1-INC beamformer is given as

$$\mathbf{w}_{\text{SMF}_1\text{-I}} = \frac{\left(\bar{\mathbf{R}}_{\text{X}} + \frac{\sigma^2_{\text{SMF}_1}}{KN-1}\hat{\mathbf{R}}_{\text{IPN}}\right)^{-1}\hat{\mathbf{a}}(\theta_0)}{\hat{\mathbf{a}}^H(\theta_0)\left(\bar{\mathbf{R}}_{\text{X}} + \frac{\sigma^2_{\text{SMF}_1}}{KN-1}\hat{\mathbf{R}}_{\text{IPN}}\right)^{-1}\hat{\mathbf{a}}(\theta_0)}. \tag{48}$$

where the loading factor $\frac{\sigma^2_{\text{SMF}_1}}{KN-1}$ is denoted by $\rho_{\text{SMF}_1\text{-I}}$, which is the same as the DL-factor $\rho_{\text{SMF}_1\text{-D}}$ of the proposed SMF_1-DL beamformer. A summary of the proposed SMF_1-INC algorithm is given in Table 2.

TABLE 2. SMF_1-INC algorithm.

1) **Inputs:**
 a) Array elements M
 b) Received signal matrix $\mathbf{X}$
 c) Estimate of interference DOA
2) **Computation:**
 a) Determine the number signal sources with (26)
 b) Calculate the SC-matrix $\hat{\mathbf{R}}_X$
 c) Desired signal ASV mismatch compensation
  i) Eigen-decompose $\hat{\mathbf{R}}_X$ in (25)
  ii) Estimate the required eigenvector $\mathbf{u}_d$ as in (28)
  iii) Compute the signal ASV as in (29)
 d) Desired signal power reduction
  i) Estimate NC-matrix using (39)
  ii) Calculate desired signal power σ_0^2 from (38)
 e) Desired covariance matrix
  i) Calculate initial estimate of $\hat{\mathbf{R}}_X$ from (30)
  ii) Determine DOA of the signal based on (46) and (47)
  ii) Reconstruct $\hat{\mathbf{R}}_{IPN}$ from (44)
  ii) Reconstruct $\check{\mathbf{R}}_X$ from (43)
3) **Output:**
 The weight vector $\mathbf{w}_{SMF_1\text{-I}}$ calculated using (48)

IV. PERFORMANCE ANALYSIS OF THE PROPOSED BEAMFORMERS

In this section, the performance of the proposed SMF_1-DL and SMF_1-INC beamformers is analyzed in terms of the convergence and computational complexity, and compared with the GLC, HKB, SMF and WCB beamformers.

A. CONVERGENCE ANALYSIS OF THE PROPOSED BEAMFORMERS

The convergence analysis of the proposed SMF_1-DL and SMF_1-INC beamformers is based on minimizing the mean square error (MSE) cost function $\mathbb{J}(\check{\mathbf{R}}_X)$ of the reconstructed covariance matrix $\check{\mathbf{R}}_X$.

$$\min_{\alpha_x}\left\{\mathbb{J}(\check{\mathbf{R}}_X)\right\} = \min_{\alpha_x}\mathbb{E}\left\{\left\|\check{\mathbf{R}}_X - \mathbf{R}\right\|_2^2\right\}. \tag{49}$$

Substituting $\check{\mathbf{R}}_X$ with the reconstructed covariance matrix in (41) or in (45) for the SMF_1-DL and SMF_1-INC beamformers respectively.

$$\min_{\alpha_x}\left\{\mathbb{J}(\check{\mathbf{R}}_X)\right\} = \min_{\alpha_x}\mathbb{E}\left\{\left\|\alpha_x\bar{\mathbf{R}}_X + (1-\alpha_x)\sigma^2_{SMF_1}\hat{\mathbf{T}}_0 - \mathbf{R}\right\|_2^2\right\}, \tag{50}$$

where $\hat{\mathbf{T}}_0$ represents the identity matrix $\mathbf{I}$ in (41) or the INC-matrix $\hat{\mathbf{R}}_{IPN}$ in (45). By substituting $\alpha_x = 1 - 1/(KN)$ into (50), we obtain

$$\min_{N}\left\{\mathbb{J}(\check{\mathbf{R}}_X)\right\} = \min_{N}\mathbb{E}\left\{\left\|\left(1-\frac{1}{KN}\right)\bar{\mathbf{R}}_X + \frac{\sigma^2_{SMF_1}}{KN}\hat{\mathbf{T}}_0 - \mathbf{R}\right\|_2^2\right\}. \tag{51}$$

The limits of (51) are determined when $N \to \infty$.

$$\begin{aligned}\lim_{N\to\infty}\mathbb{J}(\check{\mathbf{R}}_X) &= \lim_{N\to\infty}\mathbb{E}\left\{\left\|\left(1-\frac{1}{KN}\right)\bar{\mathbf{R}}_X + \frac{\sigma^2_{SMF_1}}{KN}\hat{\mathbf{T}}_0 - \mathbf{R}\right\|_2^2\right\}, \\ &= \lim_{N\to\infty}\mathbb{E}\left\{\left\|\bar{\mathbf{R}}_X - \frac{\bar{\mathbf{R}}_X}{KN} + \frac{\sigma^2_{SMF_1}}{KN}\hat{\mathbf{T}}_0 - \mathbf{R}\right\|_2^2\right\}, \\ &= \lim_{N\to\infty}\mathbb{E}\left\{\left\|\bar{\mathbf{R}}_X - \mathbf{R}\right\|_2^2\right\} \to 0, \end{aligned} \tag{52}$$

where $\bar{\mathbf{R}}_X$ is the SC-matrix $\hat{\mathbf{R}}_X$ with reduced desired signal power as given in (30). Therefore, the above inequality satisfies the convergence theorem in which the SC-matrix $\hat{\mathbf{R}}_X$ of SCB will only converge to the TC-matrix $\mathbf{R}$ i.e. $\hat{\mathbf{R}}_X \to \mathbf{R}$ when the ASV $\hat{\mathbf{a}}(\theta_0)$ is perfectly estimated [8]. The output power of the SMF $\sigma^2_{SMF_1}$ controls the loading factor from converging to zero when few snapshots are used. Therefore, the optimal solution will be attained even faster due to the reduced desired signal power in the SC-matrix. Thus, the convergence of the proposed beamformers is dependent on their loading factors $\rho_{SMF_1\text{-D}}$ in (42) and $\rho_{SMF_1\text{-I}}$ in (48), where the loading factors tends to zero as $N \to \infty$. With the minimum values of $K = 2$ and $N = 1$, the solutions of $\rho_{SMF_1\text{-D}} = \rho_{SMF_1\text{-I}} = \frac{\sigma^2_{SMF_1}}{KN-1}$ are tightly bounded by

$$0 \leq \rho_{SMF_1\text{-D}} \mid \rho_{SMF_1\text{-I}} \leq \sigma^2_{SMF_1} \tag{53}$$

B. COMPUTATIONAL COMPLEXITY OF THE PROPOSED BEAMFORMERS

In this subsection, the computational complexities of the proposed SMF_1-DL and SMF_1-INC beamformers are analyzed in comparison with the GLC, HKB, SMF and WCB beamformers. The computation cost of the SMF_1-DL beamformer is divided into three parts: a) Desired signal ASV compensation in (29), which involves eigen-decomposition of SC-matrix with $\mathcal{O}(M^3)$ complexity, and the estimation of the maximum correlation coefficient in (28) to obtain the eigenvector corresponding to the desired signal ASV with a complexity of $\mathcal{O}(DM)$ subject to $\|\mathbf{a}(\theta_0)\|_2 = \|\mathbf{u}_m\|_2 = \sqrt{M}$. b) DSC-matrix $\bar{\mathbf{R}}0$ estimation in (31), which involves the estimation of the desired signal power with a computational complexity of $\mathcal{O}(M^2)$. c) Estimating the DL-factor $\rho_{SMF_1\text{-D}}$ with $\mathcal{O}(MN)$ complexity. However, the computation cost of the SMF_1-INC beamformer is divided into four parts: a) Compensation of the ASV of the desired signal and estimation of the maximum correlation coefficient in (28) to obtain the eigenvector corresponding to the desired signal ASV with a total complexity of $\mathcal{O}(M^3+DM)$. b) Estimation of the DSC-matrix $\bar{\mathbf{R}}0$ in (31) with a complexity of $\mathcal{O}(M^2)$. c) Estimation of the loading factor $\rho_{SMF_1\text{-I}}$ with $\mathcal{O}(MN)$ complexity. d) Reconstruction of $\hat{\mathbf{R}}_{IPN}$ with $\mathcal{O}(PM)$. The computational complexities of Proposed SMF1-DL and SMF1-INC are summarized in Table 3, and compared with GLC, HKB, SMF and WCB.

TABLE 3. Computational complexity of the beamformers.

Beamformers	Multiplications
GLC	$\mathcal{O}(M^2(N+2)+M(N+1))$
HKB	$\mathcal{O}(4M^3+2M-1)$
SMF	$\mathcal{O}(MN+N+M)$
WCB	$\mathcal{O}(M^3L)$
Proposed SMF_1-DL	$\mathcal{O}(M^3+M^2+M(D+N))$
Proposed SMF_1-INC	$\mathcal{O}(M^3+M^2+M(2D+N))$

It is found from table 3 that the proposed SMF_1-DL and SMF_1-INC beamformers have lower computational complexities than HKB, WCB and GLC beamformers for a given number of array elements and snapshots. However, SMF beamformer has the least computational complexity, since it only employs the low complex sample matched filter to compute the diagonal loading factor.

V. SIMULATION RESULTS

In this section, the simulation results of the proposed SMF_1-DL and SMF_1-INC beamformers are analyzed with parameter K set to the minimum value of 2. The performance of the proposed beamformers is compared with the GLC, HKB, SMF and WCB beamformers. A ULA with 20 antenna elements having half-wavelength spacing between adjacent elements are considered. Four ($D = 4$) far-field narrowband signals are considered to incident onto the ULA from 2°, 30°, 45° and 60°. The signal incident from 2° is considered as the desired signal and the rest of the signals are considered as interferences. Both the desired signal, interferences and the noise are assumed to be spatially Gaussian random processes. Firstly, the loading factor of the proposed SMF_1-DL and SMF_1-INC in comparison with those of GLC, HKB and SMF are analyzed. Secondly, the SINR output performances with number of snapshots N and array elements M are evaluated. Also, the performance of the beamformers is analyzed when the SNR of desired signal is varied until SNR $\gg$ INR. Thirdly, the effect of imprecise estimation of D is analyzed for the estimation of the desired signal power $\hat{\sigma}_0^2$ in (38). Finally, the effect of the desired signal DOA mismatch on the SINR output is analyzed. The SeDuMi convex optimization MATLAB toolbox [14] is used to solve the SOC programming problems of WCB. The constant parameter ε and the loading factor in WCB are set to 3 and 10 respectively.

The experiments are performed under two conditions such as the ASV of desired signal is perfectly known or not. The imprecise ASV of desired signal is considered mainly due to the DOA mismatch and sensor element position perturbations. The DOA error is assumed to be uniformly distributed in $[-2°, 6°]$ and the element position perturbations are uniformly distributed in $[-0.05, 0.05]$. In the INC-matrix reconstruction of SMF_1-INC, the DOAs of three interferences are randomly and uniformly distributed in [26°, 34°], [41°, 49°] and [56°, 64°], respectively. All the angular sectors are sampled with the same angular interval of $\Delta\theta = 0.1°$. The SNR of the desired signal is set to 10dB and the interference-plus-noise ratio (INR) of interferences is set to 20dB. The SINR output is computed over an average of 100 independent Monte Carlo trials. Note that the optimal SINR obtained from the TC-matrix $\mathbf{R}$ is included for reference.

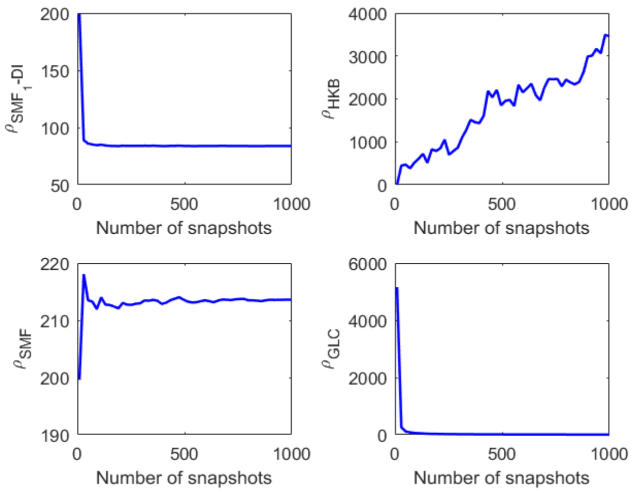

FIGURE 1. Loading factors the number of snapshots N with $M = 20$, SNR = 10dB and INR = 20dB. Where $\rho_{SMF_1\text{-DI}}$ is for the proposed SMF_1-DL and SMF_1-INC (*a*), ρ_{SMF} is for SMF (*b*), ρ_{HKB} for HKB (*c*), and ρ_{GLC} for GLC (*d*).

A. NUMERICAL ANALYSIS OF LOADING FACTORS

As already discussed in section IV, the loading factors of the proposed SMF_1-DL and SMF_1-INC beamformers are the same.i.e. $\rho_{SMF_1\text{-D}} = \rho_{SMF_1\text{-I}} = \sigma^2_{SMF_1}/(KN-1)$. Therefore, the loading factor is denoted as $\rho_{SMF_1\text{-DI}}$ for the both proposed methods. The effects of the number of snapshots and array elements on the loading factors for the proposed, GLC, HKB and SMF beamformers are analyzed. In this section, the ASV mismatch is not considered.

1) EFFECT OF NUMBER OF SNAPSHOTS ON LOADING FACTOR

The loading factor $\rho_{SMF_1\text{-DI}}$ of the proposed beamformers and ρ_{GLC} of the GLC beamformer decrease as N increases as shown in Fig.1(a) and Fig.1(d) respectively. However, the loading factor $\rho_{SMF_1\text{-DI}}$ of SMF_1-DL and SMF_1-INC tends to be much larger than that of GLC beamformer as N increases. This increases the possibility of proposed algorithm to converge to the optimal beamformer as $N \to \infty$. On the other hand, the loading factor of SMF beamformer tends to be constant as shown in Fig.1(b). The loading factor of HKB beamformer gradually increases with the increasing snapshots N as illustrated in Fig.1(c), which reduces the performance of HKB beamformer.

2) EFFECT OF NUMBER OF ANTENNA ELEMENTS ON LOADING FACTOR

The loading factors of the proposed SMF_1-DL and SMF_1-INC as well as SMF beamformer increase with

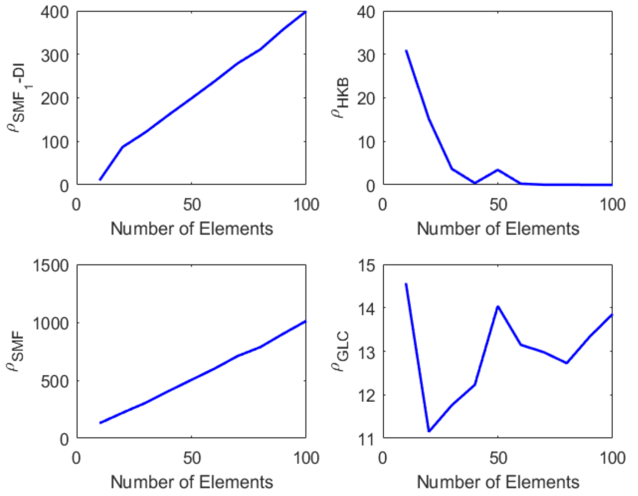

FIGURE 2. **Loading factors versus the number of array elements M with $N = 1000$, SNR = 10dB and INR = 20dB. Where $\rho_{SMF_1\text{-DI}}$ is for the proposed SMF$_1$-DL and SMF$_1$-INC (*a*), ρ_{SMF} is for SMF (*b*), ρ_{HKB} for HKB (*c*), and ρ_{GLC} for GLC (*d*).**

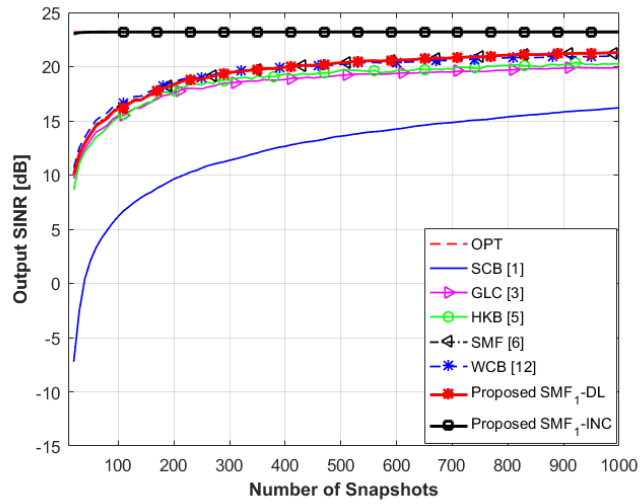

FIGURE 3. **Output SINRs of the beamformers versus the number of snapshots with $M = 20$, SNR = 10dB and INR = 20dB in absence of signal and interference ASV mismatches.**

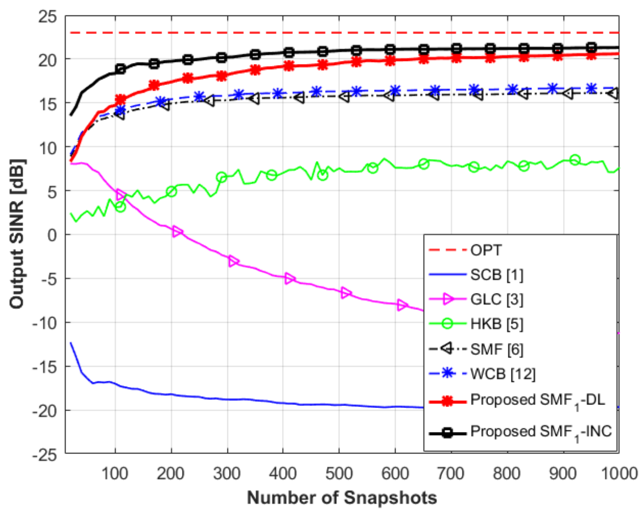

FIGURE 4. **Output SINRs of the beamformers versus the number of snapshots with $M = 20$, SNR = 10dB and INR = 20dB in presence of the mismatched signal DOA of 0°, and the mismatched DOAs of the interferences are 30.1°, 42.0° and 58.9°.**

increasing number of antenna elements M as shown in Fig. 2(a) and Fig. 2(b) respectively. This is due to their loading factors being dependent on the output power of the spatial matched filter, which increases with the increasing of number of antenna elements. The loading factor of GLC beamformer decreases with increasing M as shown in Fig. 2(d). Also the loading factor of HKB beamformer decreases with increasing M as shown in Fig. 2(c).

B. EFFECT OF NUMBER OF SNAPSHOTS

The effect of number of snapshots on output SINR is analyzed under the two scenarios. i.e. in absence and presence of ASV mismatches of the desired signal and the interferences as shown in Fig. 3 and Fig. 4 respectively. The mismatched DOA of the desired signal is randomly selected as 0°, and the DOAs of three interferences with mismatches are randomly selected as 30.1°, 42.0°, 58.9°, respectively. The SNR and INR are set to 10dB and 20dB respectively.

It is found from Fig. 3 that the proposed SMF$_1$-DL and SMF$_1$-INC SMF and WCB beamformers generally give larger output SINRs than the SCB, GLC and HKB beamformers, and SCB gives the smallest output SINR. The proposed SMF$_1$-INC beamformer gives the largest output SINR versus the number of snapshots. On the other hand, in presence of ASV mismatch as shown in Fig.4, the performance of SCB and GLC severally degrades. While, WCB gives better performance than SCB, GLC, HKB and SMF as the ASV mismatch is compensated at a certain extent. Meanwhile, the proposed SMF$_1$-DL and SMF$_1$-INC beamformers maintain a good performance due to the better compensation ability of the ASV mismatch.

C. EFFECT OF NUMBER OF ANTENNA ELEMENTS

The effect of the number of array elements on the output SINR is analyzed as shown in Fig. 6 and Fig. 6, respectively. The number of snapshots is 150, with SNR and INR at 10dB and 20dB respectively. In the presence of ASV mismatch, the DOA of desired signal with mismatch is given as 0° and those of interferences are given as 30.1°, 42.0° and 58.9°.

It is found from Fig. 6 that GLC and HKB beamformers give lower SINR outputs than SMF, SMF$_1$-DL and SMF$_1$-INC as M increases. This is because when the number of elements increase, so does the loading factors of the proposed SMF$_1$-DL and SMF$_1$-INC as well as SMF as depicted in Fig. 2. SCB has a severe degradation because of the increasing deviation between its SC-matrix and the TC-matrix. WCB gives the same output SINR with increasing array elements as in SMF$_1$-DL and SMF, but low for fewer array elements. The proposed SMF$_1$-INC beamformer shows

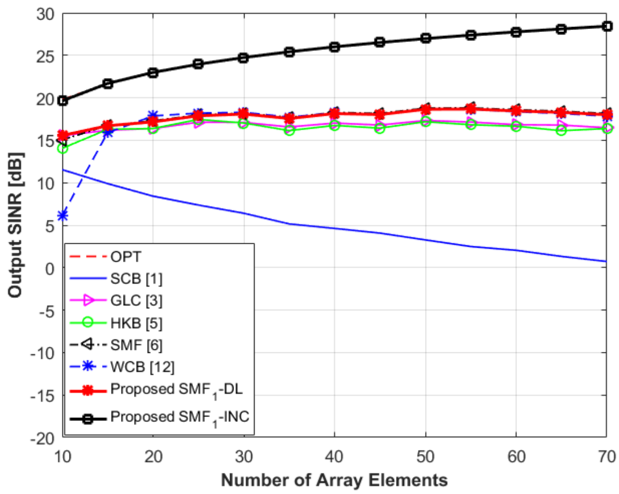

FIGURE 5. Output SINRs of the beamformers versus the number of antenna elements with $N = 150$, SNR = 10dB and INR = 20dB in absence of ASV mismatches.

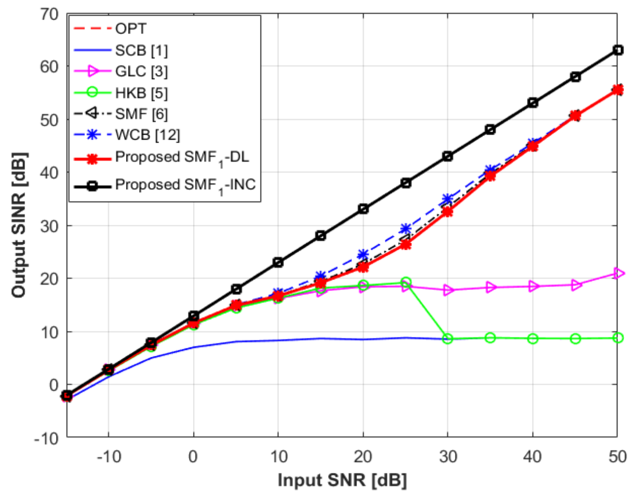

FIGURE 7. Output SINRs of the beamformers versus input SNR with $N = 150$ in of absence of signal and interference ASV mismatches.

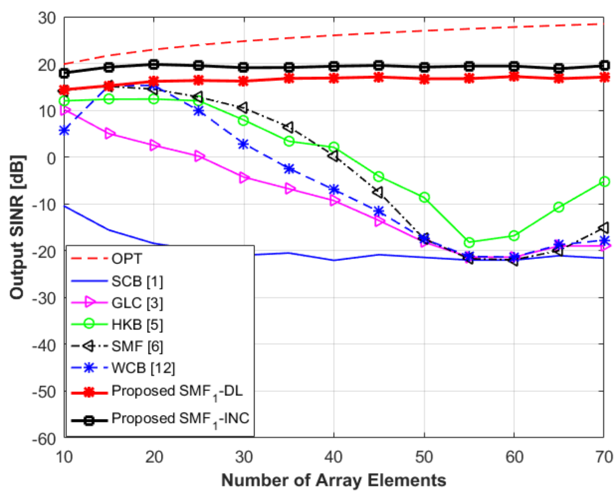

FIGURE 6. Output SINRs of the beamformers versus the number of antenna elements with $N = 150$, SNR =10dB and INR = 20dB in presence of the mismatched signal DOA of 0°, and the mismatched DOAs of the interferences are 30.1°, 42.0° and 58.9°.

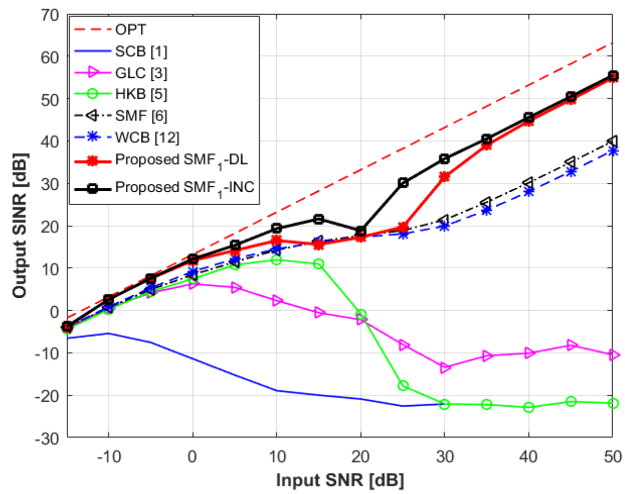

FIGURE 8. Output SINRs of the beamformers versus input SNR values with $N = 150$. In presence of mismatched signal DOA of 0°, and mismatched DOAs of the interferences are 30.1°, 42.0° and 58.9°.

a better performance. Considering with the presence of ASV mismatch as shown in Fig. 6, the performance of GLC, HKB, SMF and WCB beamformers severely deteriorates as SCB. While the proposed SMF$_1$-DL and SMF$_1$-INC beamformers maintain good performances.

D. EFFECT OF INPUT SNR

The effect of the input SNR on the output SINR of the beamformers is analyzed as shown in Fig. 7 and Fig. 8, respectively. The number of snapshots is 150, and INR is 20dB.

From Fig. 7 and Fig. 8, the proposed SMF$_1$-DL and SMF$_1$-INC beamformers as well as SMF and WCB give better performances compared to SCB, GLC and HKB beamformers. The output SINR of SCB, GLC and HKB beamformers decrease with input SNR when input SNR is larger than some values, because of desired signal self-cancellation. It can be found that the proposed SMF$_1$-INC beamformers give the best performances regardless of ASV mismatch or not as shown in Fig. 7 and Fig. 8.

E. EFFECT OF IMPRECISE ESTIMATION OF NUMBER OF SIGNAL SOURCES

During the estimation of the desired signal, the estimation of the number of signal sources is key, which can be reflected in the estimation of NC-matrix $\mathbf{R}_N$ and in the mismatch compensation of desired signal ASV $\hat{\mathbf{a}}_0$ in (38). The effect of imprecise estimation of the number of signal sources on the performance of the proposed SMF$_1$-DL and SMF$_1$-INC beamformers are analyzed in Fig. 9 and Fig. 10 in terms of output SINR versus the number of snapshots, where the estimated number of signal sources D_0 is assumed to be 2, 3, 4, 6 and 10 compared with the 4 actual signal sources.

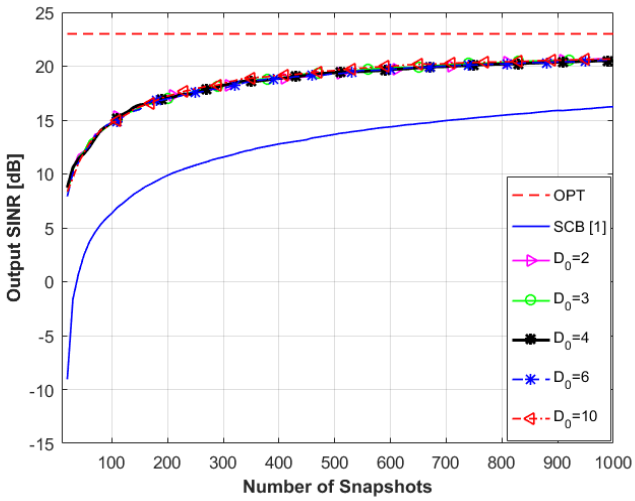

FIGURE 9. Output SINRs of the proposed SMF_1-DL beamformer versus the number of snapshots and the estimated number of signal sources D_0 with $M = 20$, SNR = 10dB and INR = 20dB.

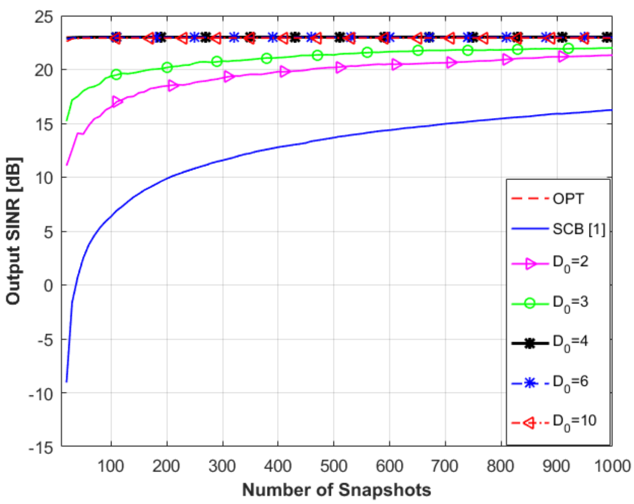

FIGURE 10. Output SINRs of the proposed SMF_1-INC beamformer versus the number of snapshots and the estimated number of signal sources D_0 with $M = 20$, SNR = 10dB and INR = 20dB.

From Fig. 9, the imprecise estimate of number of signal sources barely affects the performance of proposed SMF_1-DL beamformer. However, it can be found in Fig. 10 that when $D_0 < D$, the performance of proposed SMF_1-INC beamformer degrades. Meanwhile, when $D_0 > D$, the beamformer converges to the optimal performance of $D_0 = D = 4$. Therefore, it is better to overestimate than underestimate the number of signal sources for the proposed SMF_1-INC beamformer.

F. EFFECT OF DESIRED SIGNAL DOA MISMATCH

The presence of the ASV mismatch of desired signal has a great impact on the performance of beamformers as analyzed in the previous sections. Therefore, the effect of DOA mismatch of the desired signal on the output SINR is illustrated in Fig. 11 and Fig. 12. The absence and presence of the interference DOA mismatches are also investigated.

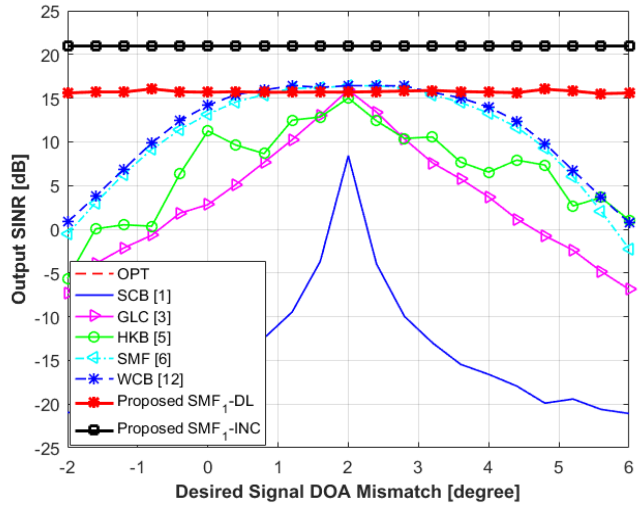

FIGURE 11. Output SINRs of the beamformers versus desired signal DOA mismatches with $N = 150$, $M = 20$, SNR = 10dB and INR = 20dB in absence of interference DOA mismatches.

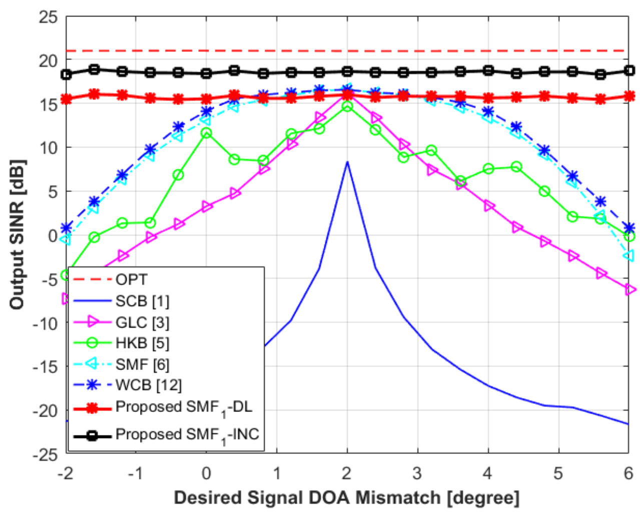

FIGURE 12. Output SINRs of the beamformers versus desired signal DOA mismatches with $N = 150$, $M = 20$, SNR = 10dBand INR = 20dB in presence of interference DOA mismatches in the SMF_1-INC beamformer.

From Fig. 11 and Fig. 12, it can be found that the proposed SMF_1-DL and SMF_1-INC beamformers are robust against ASV mismatches. On the other hand, SMF and WCB beamformers show better performances than GLC and HKB, whereas SCB has the worst performance in the presence of the desired signal ASV mismatch.

VI. CONCLUSION

In this paper, two robust adaptive beamformers have been proposed to improve the performance of the standard Capon beamformer and other existing adaptive beamformers. The proposed beamformers have two pre-processing steps. Firstly, the ASV of desired signal is estimated by computing the correlation between the nominal ASV and the eigenvectors corresponding to the dominant eigenvalues. Secondly, the power of desired signal in the sample covariance matrix is reduced by estimating the desired signal covariance matrix from the output power of spatial matched filter and noise covariance matrix. Then, the matrix regularization is used to estimate the desired sample covariance matrix. In the first beamformer, the desired sample covariance matrix has been constructed from the sample covariance matrix with the reduced desired signal power and the diagonal loading based on the output power of the spatial matched filter. In the second beamformer, the desired sample covariance matrix has been constructed from the sample covariance matrix with the reduced desired signal power and the reconstructed INC-matrix loaded with the output power of the spatial matched filter. The performance of proposed adaptive beamformers has been investigated by simulations. It has been found that the proposed beamformers can provide superior performance compared with others existing adaptive beamformers in the absence and presence of desired signal ASV mismatches. Moreover, the proposed beamformers have also comparatively low computational complexities.

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