IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received July 28, 2018, accepted September 3, 2018, date of publication September 10, 2018, date of current version October 8, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2869419

# A Student's T Mixture Cardinality-Balanced Multi-Target Multi-Bernoulli Filter With Heavy-Tailed Process and Measurement Noises

**MINGJIE WANG, HONGBING JI, (Senior Member, IEEE), YONGQUAN ZHANG, AND XIAOLONG HU**
School of Electronic Engineering, Xidian University, Xian 710071, China

Corresponding author: Hongbing Ji (hbji@xidian.edu.cn)

This work was supported by the National Natural Science Foundation of China under Grant 61871301 and Grant 61503293.

**ABSTRACT** The cardinality-balanced multi-target multi-Bernoulli (CBMeMBer) filter is a promising solution for multi-target tracking. However, the performance of the CBMeMBer filter will be degraded severely by outliers in the presence of heavy-tailed process noise and measurement noise. To address this challenging issue, a novel CBMeMBer filter called the Student's t mixture CBMeMBer (STM-CBMeMBer) filter is proposed in this paper, by assuming that the joint probability density function (pdf) of the state and process noise and the joint pdf of the state and measurement noise follow joint Student's t distributions. Following that, a closed-form solution of the CBMeMBer recursion is obtained by approximating the probability density parameter of the multi-Bernoulli as a STM. The proposed algorithm is a generalization of existing Gaussian mixture CBMeMBer (GM-CBMeMBer) filter, and it reduces to the GM-CBMeMBer filter in some special cases. Simulation results demonstrate that robust multi-target tracking can be achieved in the presence of outliers in process and measurement noises.

**INDEX TERMS** Multi-Bernoulli, multi-target tracking, outlier, random finite set, Student's t mixture.

## I. INTRODUCTION

Due to the random birth and die pattern of different targets in multi-target tracking, the filters are required to estimate both the state and time-varying number of multiple targets. Multi-target tracking has attracted intensive research interests over the past decades. Generally, the joint probabilistic data association (JPDA) filter [1], [2], the multiple hypothesis tracking (MHT) [3], [4], and the random finite set (RFS) theory [5] are the most commonly used approaches for multi-target tracking. The RFS provides an elegant Bayesian multi-target framework by modeling the states and measurements at each moment as a state RFS and a measurement RFS, respectively. Due to the complicated combinatorial nature and multiple integration, the RFS is usually mathematically intractable. To address this issue, the probability hypothesis density (PHD) [6] and cardinalized PHD (CPHD) [7] filters have been proposed. The PHD filter achieves multi-target tracking by propagating the multi-target moments, on the other hand, the CPHD recursively calculating the multi-target moments and distributions of the number of the targets. Note that both of the filters can be implemented by Gaussian mixture [8], [9] and particle methods [10], [11]. So far, both of the filters and their modified versions [12]–[16] have been applied to deal with different filtering problems. In addition, another Bayesian multi-target approximation, named the multi-target multi-Bernoulli (MeMBer) filter, was proposed in [5] by recursively propagating the multi-target posterior density. Unlike the PHD and CPHD filters, the multi-target state and number can be obtained by recursively predicting and updating the parameters of a multi-Bernoulli set in the MeMBer filter. However, there exists a cardinality bias in the MeMBer measurement update, which usually causes an overestimate of the multi-target cardinality. Thus VO etc. proposed a cardinality-balanced MeMBer (CBMeMBer) filter in [17], where the calculation of the multi-Bernoulli parameter was modified in measurement update procedure to eliminate the posterior cardinality bias. So far, the existing implementation of the CBMeMBer mainly includes

Gaussian mixture CBMeMBer (GM-CBMeMBer) [17], particle CBMeMBer filter (SMC-CBMeMBer) [17] and their modified versions [18]–[21]. The GM-CBMeMBer filter is proposed based on the Gaussian assumption of both process and measurements noise. Unfortunately, the GM-CBMeMBer filter is only suitable to linear Gaussian system and it may become ineffective in some practical scenarios with outliers in the process and measurement noises. An outlier which usually has heavy tails can be regarded as an observation that lies outside of an overall pattern of distribution [22], [23]. Intuitively, outliers may be samples that deviate from the positions where they are supposed to be. In multi-target tracking, unanticipated environmental disturbances and unreliable sensors may cause outliers in process and measurement noises. The process outlier may cause a target maneuver with an abrupt change in target position and velocity. The measurement outlier may result in a negligible weight of the target due to the lightweight tail of Gaussian distribution. The performance of the GM-CBMeMBer filter may be degraded by the process and measurement outliers.

Recently, the Student's t distribution is found to be capable to handle the process and measurement outliers due to its heavy tailed characteristics. A large number of Student's t based filters and smoothers [24]–[26] have been proposed for heavy-tailed process and measurement noises. In these filters and smoothers, the process and measurement noises are modelled as Student's t distributions, meanwhile, the posterior distribution is approximated as Gaussian distribution. Then the state and noise parameters are jointly estimated based on the variational Bayesian approach. In addition, another robust Kalman filter [27] has been proposed to handle the non-Gaussian heavy-tailed and/or skewed state and measurement noises through modeling the state and measurement noises as Gaussian scale mixtures distributions. Unfortunately, some fixed-point iterations need to be utilized to calculate the coupled variational parameters in variational Bayesian approach, which incurs higher computational complexities. Another class of Student's t based filter [28] has also been proposed by modeling both the process and measurement noises as Student's t distributions and approximating the posterior probability density as a Student's t distribution to obtain a closed-form solution for linear single target tracking problem in the presence of heavy tailed process and measurement noises. Furthermore, the Student's t filter [28] is extended to the nonlinear system through different numerical integration methods [29]–[32]. However, the aforementioned methods above are only applicable for single target tracking. To the best of our knowledge, the CBMeMBer filter based on Student's distribution has not been reported in the literature, which motivates this work.

This paper presents a linear multi-target filter based on the Student's t distribution and CBMeMBer recursion, referred to as the Student's t mixture CBMeMBer (STM-CBMeMBer) filter, to handle multi-target tracking in linear system with process and measurement outliers. In addition, we extend this technique to nonlinear system via the unscented transform. In the proposed filter, the joint density of the process noise and the state is approximated by a joint Student's distribution, then the multi-target predicted density is approximated by a multi-Bernoulli distribution whose probability parameters are represented by a Student's t mixture. Meanwhile, the joint density of the measurement noise and the state is also approximated by a joint Student's distribution, then the multi-target posterior density is approximated as a multi-Bernoulli distribution in which each Bernoulli's probability parameter is represented by a Student's t mixture. Following that, the CBMeMBer filter based Student's mixture is derived in closed form. It is shown that the GM-CBMeMBer filter is a special case of the proposed filter. Simulation results show that it can achieve comparable performance with the GM-CBMeMBer filter in linear Gaussian case without outliers, moreover outperforms the GM-CBMeMBer filter in scenarios where outliers occur in process and measurement noises.

The rest of this paper is organized as follows. The system model and problem statement including the properties of the Student's t distribution and the CBMeMBer filter are presented in Section II. Following that, the STM-CBMeMBer filter is proposed and derived in Section III. Simulation results are shown in Section IV. Finally, conclusions are given in Section V.

## II. SYSTEM MODEL AND PROBLEM STATEMENT

Consider the state evolution and the measurement equation as follows

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{w}_k, \tag{1}$$

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k, \tag{2}$$

where $\mathbf{x}_k \in \mathbb{R}^{d_x}$ is the target state at time step $k$, $\mathbf{y}_k \in \mathbb{R}^{d_y}$ is the measurement vector generated from $\mathbf{x}_k$, $\mathbf{F}_k$ is the state transition matrix, $\mathbf{H}_k$ is the measurement matrix, $\mathbf{w}_k$ and $\mathbf{v}_k$ are the process noise and the measurement noise, respectively. Note that the system matrices $\mathbf{F}_k$ and $\mathbf{H}_k$ are assumed to be known, and the process noise and measurement noise are assumed to be independent with each other. Furthermore, $\mathbf{w}_k$ and $\mathbf{v}_k$ are assumed to have heavy tails and admit the Student's t distributions described by

$$p(\mathbf{w}_k) = St(\mathbf{w}_k; \mathbf{0}, \mathbf{Q}_k, \upsilon_1), \tag{3}$$

$$p(\mathbf{v}_k) = St(\mathbf{v}_k; \mathbf{0}, \mathbf{R}_k, \upsilon_2), \tag{4}$$

where $St(\mathbf{x}; \mu, \Sigma, \upsilon)$ represents a student's t probability density function (pdf) with mean $\mu$, scale matrix $\Sigma$, and degree of freedom (dof) $\upsilon$. Note that $\Sigma$ is not the covariance of the Student's t distribution. The relationship between the covariance $\mathbf{P}$ and the scale matrix $\Sigma$ of student's t random variable is $\mathbf{P} = \frac{\upsilon}{\upsilon-2}\Sigma$ [28].

### A. STUDENT'S T DISTRIBUTION

Suppose that $V > 0$, $V \in \mathbb{R}$, $V \sim \text{Gamma}(\frac{\upsilon}{2}, \frac{\upsilon}{2})$, where $\text{Gamma}(\alpha, \beta)$ represents the Gamma distribution with shape $\alpha > 0$, and rate $\beta > 0$. Let $\mathbf{z} \in \mathbb{R}^d$ be a random vector, which admits the Gaussian distribution $\mathcal{N}(\mathbf{0}, \Sigma)$ with zero mean and covariance $\Sigma$. Then

$$\mathbf{x} = \mu + \frac{1}{\sqrt{V}}\mathbf{z} \tag{5}$$

obeys the multivariate t distribution [28], [34] whose pdf can be expressed as

$$p(\mathbf{x}) = \frac{\Gamma\left(\frac{\upsilon+2}{2}\right)}{\Gamma\left(\frac{\upsilon}{2}\right)} \frac{1}{(\upsilon\pi)^{d/2}} \frac{1}{\sqrt{\det(\Sigma)}} \left(1 + \frac{\Delta^2}{\upsilon}\right)^{-\frac{\upsilon+2}{2}}. \tag{6}$$

For the sake of convenience, we use $St(\mathbf{x}; \mu, \Sigma, \upsilon)$ to represent $p(\mathbf{x})$ in (6) in the rest of this paper.

The Student's t distribution can be regarded as a generalized Gaussian distribution [33], which will reduce to the Gaussian distribution when the dof $\upsilon$ approaches infinity. Similar to the Gaussian distribution, the Student's t distribution has several convenient properties [28], which can be used to facilitate the following derivation of the proposed filter. In the following, we review three important properties of the Student's t distribution.

#### 1) LINEAR TRANSFORMATION

Similar to the Gaussian distribution, for $\mathbf{x} \sim St(\mathbf{x}; \mu, \Sigma, \upsilon)$, the pdf of its linear transformation $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{b}$ can be expressed as

$$p(\mathbf{y}) = St(\mathbf{y}; \mathbf{A}\mu + \mathbf{b}, \mathbf{A}\Sigma\mathbf{A}^T, \upsilon). \tag{7}$$

#### 2) MARGINAL DENSITY

Assume that $\mathbf{x}_1 \in \mathbb{R}^{d_1}$ and $\mathbf{x}_2 \in \mathbb{R}^{d_2}$ are random vectors with joint Student's t distribution

$$p(\mathbf{x}_1, \mathbf{x}_2) = St(\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{bmatrix}; \begin{bmatrix}\mu_1\\ \mu_2\end{bmatrix}, \begin{bmatrix}\Sigma_{11} & \Sigma_{12}\\ \Sigma_{21} & \Sigma_{22}\end{bmatrix}, \upsilon), \tag{8}$$

then the marginal pdf of $\mathbf{x}_1$ is

$$p(\mathbf{x}_1) = St(\mathbf{x}_1; \mu_1, \Sigma_{11}, \upsilon). \tag{9}$$

Note that this property can be obtained by a linear transformation to (8) with appropriate matrix $\mathbf{A} = [\mathbf{I}\ \mathbf{0}]$, where $\mathbf{I}$ is a unit matrix.

#### 3) CONDITIONAL DENSITY

Given that $\mathbf{x}_1$ and $\mathbf{x}_2$ follow the joint Student's t distribution described by (8), and $\mathbf{x}_2$ admits Student's t distribution $p(\mathbf{x}_2) = St(\mathbf{x}_2; \mu_2, \Sigma_{22}, \upsilon)$. Then the pdf of $\mathbf{x}_1$ conditioned on $\mathbf{x}_2$ can be expressed

$$p(\mathbf{x}_1|\mathbf{x}_2) = \frac{p(\mathbf{x}_1, \mathbf{x}_2)}{p(\mathbf{x}_2)} = St(\mathbf{x}_1; \mu_{1|2}, \Sigma_{1|2}, \upsilon_{1|2}), \tag{10}$$

where

$$\upsilon_{1|2} = \upsilon + d_2, \tag{11}$$

$$\mu_{1|2} = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(\mathbf{x}_2 - \mu_2), \tag{12}$$

$$\Sigma_{1|2} = \frac{\upsilon + (\mathbf{x}_2 - \mu_2)^T \Sigma_{22}^{-1}(\mathbf{x}_2 - \mu_2)}{\upsilon + d_2}(\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}^T). \tag{13}$$

The proofs of the above properties are omitted here, readers may refer [28] and [34] for more details.

### B. THE CBMeMBer FILTER

The MeMBer filter [5] can be used to track targets through propagating the approximated posterior density recursively, which is represented by a multi-Bernoulli parameter set $\pi = \{(r^{(i)}, p^{(i)})\}_{i=1}^M$ with $r^{(i)}$ representing the existence probability and $p^{(i)}$ representing the probability density, respectively. However, the MeMBer filter may result in a cardinality over-estimation problem. To solve this problem, the CBMeMBer filter was proposed in [17] through modifying the parameter estimation of the multi-Bernoulli in the measurement update step. In the following, we will give a brief description of the CBMeMBer filter.

#### 1) PREDICTION

We assume that the multi-target posterior density of the target RFS at time $k-1$ is represented by a multi-Bernoulli distribution

$$\pi_{k-1} = \{(r_{k-1}^{(i)}, p_{k-1}^{(i)})\}_{i=1}^{M_{k-1}}. \tag{14}$$

Then the multi-target predicted density of the target RFS at time $k$ can be expressed by the union of the survival multi-target multi-Bernoulli set $\{(r_{P,k|k-1}^{(i)}, p_{P,k|k-1}^{(i)})\}_{i=1}^{M_{k-1}}$ and the spontaneous births multi-Bernoulli set $\{(r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)})\}_{i=1}^{M_{\Gamma,k}}$, i.e.,

$$\pi_{k|k-1} = \{(r_{P,k|k-1}^{(i)}, p_{P,k|k-1}^{(i)})\}_{i=1}^{M_{k-1}} \cup \{(r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)})\}_{i=1}^{M_{\Gamma,k}}. \tag{15}$$

Note that the survival multi-Bernoulli parameter can be calculated as

$$r_{P,k|k-1}^{(i)} = r_{k-1}^{(i)}\langle p_{k-1}^{(i)}, p_{S,k}\rangle, \tag{16}$$

$$p_{P,k|k-1}^{(i)}(x) = \frac{\langle f_{k|k-1}(\mathbf{x}|\cdot), p_{k-1}^{(i)} p_{S,k}\rangle}{\langle p_{k-1}^{(i)}, p_{S,k}\rangle}, \tag{17}$$

where $f_{k|k-1}(\mathbf{x}|\cdot)$ represents the single-target transition probability density from time $k-1$ to $k$, $p_{S,k}$ represents the survival probability. $\langle v, h\rangle \triangleq \int v(\mathbf{x})h(\mathbf{x})d\mathbf{x}$, where $v(\mathbf{x})$ is density function, $h(\mathbf{x})$ is referred as test function (see [5, pp. 371]).

#### 2) UPDATE

Suppose that the multi-target predicted density of the target RFS at time $k$ is expressed by a multi-Bernoulli distribution

$$\pi_{k|k-1} = \{(r_{k|k-1}^{(i)}, p_{k|k-1}^{(i)})\}_{i=1}^{M_{k|k-1}}. \tag{18}$$

Then the multi-target posterior density at time $k$ can be approximated by a multi-Bernoulli distribution which is represented by the union of the detected targets $\{(r_{U,k}(\mathbf{z}), p_{U,k}(\mathbf{x};\mathbf{z}))\}_{\mathbf{z}\in\mathbf{Z}_k}$ and undetected targets $\{(r_{L,k}^{(i)}, p_{L,k}^{(i)})\}_{i=1}^{M_{k|k-1}}$, i.e.,

$$\pi_k \approx \{(r_{L,k}^{(i)}, p_{L,k}^{(i)})\}_{i=1}^{M_{k|k-1}} \cup \{(r_{U,k}(\mathbf{z}), p_{U,k}(\mathbf{x};\mathbf{z}))\}_{\mathbf{z}\in\mathbf{Z}_k}, \tag{19}$$

where $\mathbf{Z}_k$ is the measurement RFS at time $k$,

$$r_{L,k}^{(i)} = r_{k|k-1}^{(i)} \frac{1 - \langle p_{k|k-1}^{(i)}, p_{D,k} \rangle}{1 - r_{k|k-1}^{(i)} \langle p_{k|k-1}^{(i)}, p_{D,k} \rangle}, \tag{20}$$

$$p_{L,k}^{(i)} = p_{k|k-1}^{(i)}(\mathbf{x}) \frac{1 - p_{D,k}(\mathbf{x})}{1 - \langle p_{k|k-1}^{(i)}, p_{D,k} \rangle}, \tag{21}$$

$$r_{U,k}(\mathbf{z}) = \frac{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}(1-r_{k|k-1}^{(i)})\langle p_{k|k-1}^{(i)}, \psi_{k,\mathbf{z}} \rangle}{(1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)}, p_{D,k} \rangle)^2}}{\kappa_k(\mathbf{z}) + \sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)}, \psi_{k,\mathbf{z}} \rangle}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)}, p_{D,k} \rangle}}, \tag{22}$$

$$p_{U,k}(\mathbf{x};\mathbf{z}) = \frac{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}}{1-r_{k|k-1}^{(i)}} p_{k|k-1}^{(i)}(\mathbf{x}) \psi_{k,\mathbf{z}}(\mathbf{x})}{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}}{1-r_{k|k-1}^{(i)}} \langle p_{k|k-1}^{(i)}, \psi_{k,\mathbf{z}} \rangle}, \tag{23}$$

$$\psi_{k,z}(\mathbf{x}) = g_k(\mathbf{z}|\mathbf{x}) p_{D,k}(\mathbf{x}), \tag{24}$$

$g_k(\mathbf{z}|\mathbf{x})$ represents the measurement likelihood, $p_{D,k}(\mathbf{x})$ represents the single-target detection probability, $\kappa_k(\mathbf{z})$ represents the intensity of the clutter.

## III. STUDENT'S T MIXTURE CBMeMBer FILTER

The CBMeMBer filter is effective for multi-target tracking for the scenarios with low clutter density [17]. By representing the multi-target density with a multi-Bernoulli set form, the CBMeMBer filter can achieve multi-target estimation by recursively calculating the parameters of the multi-Bernoulli set. The Gaussian Mixture CBMeMBer (GM-CBMeMBer) filter is one realization of the CBMeMBer filter, which can achieve a promising result for linear system with Gaussian noise model. However, the performance of the GM-CBMeMBer filter may be degraded when outliers occur in process noise and measurement noise due to the lightweight tails of Gaussian distribution.

In this section, we propose a novel CBMeMBer realization, named Student's t mixture CBMeMBer (STM-CBMeMBer) filter, to deal with the multi-target tracking problem when outliers occur in process noise and measurement noise. In the STM-CBMeMBer filter, the joint density of the process noise and the state is modelled as a joint Student's t distribution. Similarly, the joint density of measurement noise and state is also assumed to be a Student's t distribution. Then the outliers occurring in process noise and measurement noise can be well handled due to the heavy tailed Student's t distribution. Similar to the derivation of the GM-CBMeMBer filter, the following assumptions are adopted to facilitate the derivation of the STM-CBMeMBer filter.

*Assumption 1:* Each target follows the evolution and measurement models according to (1) and (2), i.e.,

$$f_{k|k-1}(\mathbf{x}|\xi) = St(\mathbf{x}; \mathbf{F}_{k-1}\xi, \mathbf{Q}_{k-1}, \upsilon_1), \tag{25}$$

$$g_k(\mathbf{z}|\mathbf{x}) = St(\mathbf{z}; \mathbf{H}_k\mathbf{x}, \mathbf{R}_k, \upsilon_2), \tag{26}$$

where $f_{k|k-1}(\mathbf{x}|\xi)$ and $g_k(\mathbf{z}|\mathbf{x})$ are transition probability density and likelihood function, respectively. Note that Assumption 1 is obtained according to the linear transformation property of the Student's distribution.

*Assumption 2:* The joint pdf $p(\mathbf{x}_{k-1}, \mathbf{w}_{k-1}|\mathbf{Z}_{k-1})$ of the process noise and the state $\mathbf{x}_{k-1}$ at time $k-1$ follows Student's t distribution given by

$$p(\mathbf{x}_{k-1}, \mathbf{w}_{k-1}|\mathbf{Z}_{k-1}) = St\left(\begin{bmatrix}\mathbf{x}_{k-1}\\ \mathbf{w}_{k-1}\end{bmatrix}; \begin{bmatrix}\hat{\mathbf{x}}_{k-1|k-1}\\ \mathbf{0}\end{bmatrix}, \begin{bmatrix}\mathbf{P}_{k-1|k-1} & 0\\ 0 & \mathbf{Q}_{k-1}\end{bmatrix}, \upsilon_{k-1}\right), \tag{27}$$

*Assumption 3:* The joint pdf $p(\mathbf{x}_k, \mathbf{v}_k|\mathbf{Z}_{k-1})$ of the measurement noise and the predicted state vector at time $k$ follows Student's t distribution given by

$$p(\mathbf{x}_k, \mathbf{v}_k|\mathbf{Z}_{k-1}) = St\left(\begin{bmatrix}\mathbf{x}_k\\ \mathbf{v}_k\end{bmatrix}; \begin{bmatrix}\hat{\mathbf{x}}_{k|k-1}\\ \mathbf{0}\end{bmatrix}, \begin{bmatrix}\mathbf{P}_{k|k-1} & 0\\ 0 & \mathbf{R}_k\end{bmatrix}, \upsilon_{k-1}\right), \tag{28}$$

*Assumption 4:* Both the survival probability and the detection probability are assumed to be independent with state, i.e.,

$$p_{S,k}(\mathbf{x}) = p_{S,k} \tag{29}$$

$$p_{D,k}(\mathbf{x}) = p_{D,k} \tag{30}$$

*Assumption 5:* The birth targets are modelled by a multi-Bernoulli form $\{(r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)})\}_{i=1}^{M_{\Gamma,k}}$ with existence probability $r_{\Gamma,k}^{(i)}$ and probability density $p_{\Gamma,k}^{(i)}$, and $p_{\Gamma,k}^{(i)}$ is supposed to be a Student's t mixture described as

$$p_{\Gamma,k}^{(i)}(\mathbf{x}) = \sum_{j=1}^{J_{\Gamma,k}^{(i)}} w_{\Gamma,k}^{(i,j)} St(\mathbf{x}; \mathbf{m}_{\Gamma,k}^{(i,j)}, \mathbf{P}_{\Gamma,k}^{(i,j)}, \upsilon_{\Gamma,k}^{(i,j)}), \tag{31}$$

where $w_{\Gamma,k}^{(i,j)}$, $\mathbf{m}_{\Gamma,k}^{(i,j)}$, $\mathbf{P}_{\Gamma,k}^{(i,j)}$ and $\upsilon_{\Gamma,k}^{(i,j)}$ denote the weight, mean, scale matrix and dof of the $j$th Student's t component, respectively.

Next, we present the STM-CBMeMBer filter in detail.

### A. PREDICTION

Suppose that Assumptions 1-5 hold, the multi-target posterior density of the target RFS is given by a multi-Bernoulli expression as (14) at time $k-1$. Moreover, the probability density $p_{k-1}^{(i)}$ is comprised of Student's t mixtures, i.e.,

$$p_{k-1}^{(i)}(\mathbf{x}) = \sum_{j=1}^{J_{k-1}^{(i)}} w_{k-1}^{(i,j)} St(\mathbf{x}; \mathbf{m}_{k-1}^{(i,j)}, \mathbf{P}_{k-1}^{(i,j)}, \upsilon_{k-1}^{(i,j)}). \tag{32}$$

Then the multi-target predicted density of the survival target RFS in (15) can be obtained as

$$r_{P,k|k-1}^{(i)} = r_{k-1}^{(i)} p_{S,k}, \tag{33}$$

$$p_{P,k|k-1}^{(i)}(\mathbf{x}) = \sum_{j=1}^{J_{k-1}^{(i)}} w_{k-1}^{(i,j)} St(\mathbf{x}; \mathbf{m}_{P,k|k-1}^{(i,j)}, \mathbf{P}_{P,k|k-1}^{(i,j)}, \upsilon_{P,k|k-1}^{(i,j)}), \tag{34}$$

where

$$\upsilon_{P,k|k-1}^{(i,j)} = \upsilon_{k-1}^{(i,j)}, \quad (35)$$

$$\mathbf{m}_{P,k|k-1}^{(i,j)} = \mathbf{F}_{k-1}\mathbf{m}_{k-1}^{(i,j)}, \quad (36)$$

$$\mathbf{P}_{P,k|k-1}^{(i,j)} = \mathbf{F}_{k-1}\mathbf{P}_{k-1}^{(i,j)}\mathbf{F}_{k-1}^{T} + \mathbf{Q}_{k-1}. \quad (37)$$

The birth multi-Bernoulli parameters $r_{\Gamma,k}^{(i)}$ and $p_{\Gamma,k}^{(i)}$ are given by the birth model in (31).

### B. UPDATE

Suppose that the multi-target predicted density of the target RFS at time $k$ is given by (18). Moreover, each probability density of the predicted multi-Bernoulli is expressed by a Student's t mixture, i.e.,

$$p_{k|k-1}^{(i)}(\mathbf{x}) = \sum_{j=1}^{J_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} St(\mathbf{x}; \mathbf{m}_{k|k-1}^{(i,j)}, \mathbf{P}_{k|k-1}^{(i,j)}, \upsilon_{k|k-1}^{(i,j)}). \quad (38)$$

Then the multi-target posterior density can be obtained by (19). The multi-Bernoulli parameters of the undetected targets are calculated similar to that of the GM-CBMeMBer filter, i.e.,

$$r_{L,k}^{(i)} = r_{k|k-1}^{(i)} \frac{1 - p_{D,k}}{1 - r_{k|k-1}^{(i)} p_{D,k}}, \quad (39)$$

$$p_{L,k}^{(i)}(\mathbf{x}) = p_{k|k-1}^{(i)}(\mathbf{x}). \quad (40)$$

The multi-Bernoulli parameters of the detected targets can be calculated as

$$r_{U,k}(\mathbf{z}) = \frac{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}(1-r_{k|k-1}^{(i)})\rho_{U,k}^{(i)}(\mathbf{z})}{(1-r_{k|k-1}^{(i)}p_{D,k})^2}}{\kappa_k(\mathbf{z}) + \sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}\rho_{U,k}^{(i)}(\mathbf{z})}{1-r_{k|k-1}^{(i)}p_{D,k}}}, \quad (41)$$

$$p_{U,k}(x;\mathbf{z}) = \frac{\sum_{i=1}^{M_{k|k-1}} \sum_{j=1}^{J_{k|k-1}^{(i)}} w_{U,k}^{(i,j)}(\mathbf{z}) St(\mathbf{x}; \mathbf{m}_{U,k}^{(i,j)}, \mathbf{P}_{U,k}^{(i,j)}, \upsilon_{U,k}^{(i,j)})}{\sum_{i=1}^{M_{k|k-1}} \sum_{j=1}^{J_{k|k-1}^{(i)}} w_{U,k}^{(i,j)}(\mathbf{z})}, \quad (42)$$

where

$$\upsilon_{U,k|k-1}^{(i,j)} = \upsilon_{k|k-1}^{(i,j)} + d_{\mathbf{z}}, \quad (43)$$

$$\rho_{U,k}^{(i)}(\mathbf{z}) = p_{D,k} \sum_{j=1}^{J_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} q_k^{(i,j)}(\mathbf{z}), \quad (44)$$

$$q_k^{(i,j)}(\mathbf{z}) = St(\mathbf{z}; \mathbf{H}_k\mathbf{m}_{k|k-1}^{(i,j)}, \mathbf{S}_k^{(i,j)}, \upsilon_{k|k-1}^{(i,j)}), \quad (45)$$

$$\mathbf{S}_k^{(i,j)} = \mathbf{H}_k\mathbf{P}_{k|k-1}^{(i,j)}\mathbf{H}_k^T + \mathbf{R}_k, \quad (46)$$

$$w_{U,k}^{(i,j)}(\mathbf{z}) = \frac{r_{k|k-1}^{(i)}}{1 - r_{k|k-1}^{(i)}} p_{D,k} w_{k|k-1}^{(i,j)} q_k^{(i,j)}(\mathbf{z}), \quad (47)$$

$$\mathbf{m}_{U,k}^{(i,j)} = \mathbf{m}_{k|k-1}^{(i,j)} + \mathbf{K}_{U,k}^{(i,j)}(\mathbf{z}_k - \mathbf{H}_k\mathbf{m}_{k|k-1}^{(i,j)}), \quad (48)$$

$$\mathbf{P}_{U,k}^{(i,j)} = \frac{\upsilon_{k|k-1}^{(i,j)} + (\Delta_{\mathbf{z},k}^{(i,j)})^2}{\upsilon_{U,k|k-1}^{(i,j)}}[\mathbf{I} - \mathbf{K}_{U,k}^{(i,j)}\mathbf{H}_k]\mathbf{P}_{k|k-1}^{(i,j)}, \quad (49)$$

$$\mathbf{K}_{U,k}^{(i,j)} = \mathbf{P}_{k|k-1}^{(i,j)}\mathbf{H}_k^T(\mathbf{S}_k^{(i,j)})^{-1}, \quad (50)$$

$$(\Delta_{\mathbf{z},k}^{(i,j)})^2 = (\mathbf{z}_k - \mathbf{H}_k\mathbf{m}_{k|k-1}^{(i,j)})^T(\mathbf{S}_k^{(i,j)})^{-1}(\mathbf{z}_k - \mathbf{H}_k\mathbf{m}_{k|k-1}^{(i,j)}). \quad (51)$$

An intuitive inspection of (41) and (42) reveals that the formulation for calculating the multi-Bernoulli parameters are similar to that of the GM-CBMeMBer filter. However, there is a significant difference in calculating the mean $\mathbf{m}_{U,k}^{(i,j)}$, scale matrix $\mathbf{P}_{U,k}^{(i,j)}$ and likelihood function $q_k^{(i,j)}(\mathbf{z})$ due to the Student's t assumption.

The closed-form recursion of the CBMeMBer based Student's t approximation can be finished based on the following lemmas.

*Lemma 1:* Given Assumption 2, the following equation will hold if $\mathbf{P}$ and $\mathbf{Q}$ are positive definite,

$$\int St(\mathbf{x}; \mathbf{F}\xi, \mathbf{Q}, \upsilon_1) St(\xi; \mathbf{m}, \mathbf{P}, \upsilon_3) d\xi = St(\mathbf{x}; \mathbf{Fm}, \mathbf{FPF}^T + \mathbf{Q}, \upsilon_3). \quad (52)$$

*Lemma 2:* Given Assumption 2, the following equation will hold if $\mathbf{P}$ and $\mathbf{R}$ are positive definite,

$$St(\mathbf{z}; \mathbf{Hx}, \mathbf{R}, \upsilon_2) St(\mathbf{x}; \mathbf{m}, \mathbf{P}, \upsilon_3) = q(\mathbf{z}) St(\mathbf{x}; \tilde{\mathbf{m}}, \tilde{\mathbf{P}}, \tilde{\upsilon}_3) \quad (53)$$

where

$$q(\mathbf{z}) = St(\mathbf{z}; \mathbf{Hm}, \mathbf{S}, \upsilon_3), \quad (54)$$

$$\mathbf{S} = \mathbf{R} + \mathbf{HPH}^T, \quad (55)$$

$$\tilde{\mathbf{m}} = \mathbf{m} + \mathbf{PH}^T\mathbf{S}^{-1}(\mathbf{z} - \mathbf{Hm}), \quad (56)$$

$$\tilde{\mathbf{P}} = \frac{\upsilon_3 + \Delta_{\mathbf{z}}^2}{\tilde{\upsilon}_3}(\mathbf{P} - \mathbf{PH}^T\mathbf{S}^{-1}\mathbf{HP}), \quad (57)$$

$$\tilde{\upsilon}_3 = \upsilon_3 + d_{\mathbf{z}}, \quad (58)$$

$$\Delta_{\mathbf{z}}^2 = (\mathbf{z} - \mathbf{Hm})^T\mathbf{S}^{-1}(\mathbf{z} - \mathbf{Hm}). \quad (59)$$

Lemma 1 and Lemma 2 can be proved using the properties of Student's t distribution as shown in section II-A, readers may refer to [28] and [29] for more details.

### C. IMPLEMENTATION ISSUES

#### 1) PRUNE AND MERGE

Note that the amount of the multi-Bernoulli components increases without limit as recursion goes ahead, which is caused by the target birth in the prediction procedure as well as the hypothesized tracks in the measurement update procedure. Therefore, a prune operation is performed to the multi-Bernoulli components through eliminating the components, whose existence probabilities are below a predetermined threshold $P$. Moreover, the number of the Student's t components to represent each multi-Bernoulli component increases with recursion similar to the GM-PHD filter [8]. Therefore, the Student's t components whose weights are below a threshold $T$ are eliminated to reduce the number of the Student's t. Furthermore, a merge operation is performed to combine the Student's t components within a distance $U$. Note that the maximum number of the Student's t mixture for each multi-Bernoulli component is set as $J_{\max}$. These operations are similar to that of the GM-PHD and GM-CBMeMBer [8], [17].

### 2) MOMENT MATCH

A close inspection of (43) reveals that the dof parameter $\upsilon_{U,k|k-1}^{(i,j)}$ will approach infinity with recursion proceeding. As a result, the Student's t mixture

$$p_{U,k}^{(i)}(\mathbf{x}) = \sum_{j=1}^{J_{U,k}^{(i)}} w_{U,k}^{(i,j)} St(\mathbf{x};\, \mathbf{m}_{U,k}^{(i,j)}, \mathbf{P}_{U,k}^{(i,j)}, \upsilon_{U,k|k-1}^{(i,j)}) \quad (60)$$

will converge to a Gaussian mixture. This may lose the heavy tailed property so that the proposed filter may fail to deal with the outliers in process and measurement noises. In this paper, we perform a moment matching [28], [29] operation to solve this problem. We match the first two moments similar to [28] and [29], i.e.,

$$\mathbf{m}_{U,k}^{*(i,j)} = \mathbf{m}_{U,k}^{(i,j)}, \quad (61)$$

$$\frac{\upsilon_{k|k-1}^{(i,j)}}{\upsilon_{k|k-1}^{(i,j)} - 2}\mathbf{P}_{U,k}^{*(i,j)} = \frac{\upsilon_{U,k|k-1}^{(i,j)}}{\upsilon_{U,k|k-1}^{(i,j)} - 2}\mathbf{P}_{U,k}^{(i,j)}. \quad (62)$$

Then we can obtain the Student's t mixture presentation of the posterior multi-Bernoulli parameter $p_{U,k}^{(i)}(\mathbf{x})$ updated by measurement

$$p_{U,k}^{(i)}(\mathbf{x}) = \sum_{j=1}^{J_{U,k}^{(i)}} w_{U,k}^{(i,j)} St(\mathbf{x};\, \mathbf{m}_{U,k}^{*(i,j)}, \mathbf{P}_{U,k}^{*(i,j)}, \upsilon_{k|k-1}^{(i,j)}), \quad (63)$$

where

$$\mathbf{P}_{U,k}^{*(i,j)} = \frac{(\upsilon_{k|k-1}^{(i,j)} - 2)\upsilon_{U,k|k-1}^{(i,j)}}{(\upsilon_{U,k|k-1}^{(i,j)} - 2)\upsilon_{k|k-1}^{(i,j)}}\mathbf{P}_{U,k}^{(i,j)}. \quad (64)$$

### 3) STATE EXTRACTION

Similar to the GM-CBMeMBer filter, the mean cardinality of the posterior multi-target density $\hat{N}_k = \sum_{i=1}^{M_{k|k}} r_k^{(i)}$ is regarded as the target number estimates. Then we extract $\hat{N}_k$ individual state estimates by calculating the mean of the Student's t components for the $\hat{N}_k$ hypothesized tracks with highest existence probabilities.

### 4) COMPARISONS WITH THE GM-CBMEMBER FILTER

In [17], the GM-CBMeMBer filter is designed for linear systems with Gaussian process and measurement noises. As mentioned above, the Student's t distribution will converge to a Gaussian distribution as the dof approaches infinity. Hence, the GM-CBMeMBer filter can be regarded as a special case of the proposed STM-CBMeMBer filter, which is proved as follows.

*Proof:* The predicted multi-target multi-Bernoulli density can be expressed by a Gaussian mixture when the dof approaches infinity, i.e.,

$$\lim_{\upsilon_{\Gamma,k}^{(i,j)}\to+\infty} p_{\Gamma,k}^{(i)}(\mathbf{x}) = \sum_{j=1}^{J_{\Gamma,k}^{(i)}} w_{\Gamma,k}^{(i,j)}\mathcal{N}(\mathbf{x};\, \mathbf{m}_{\Gamma,k}^{(i,j)}, \mathbf{P}_{\Gamma,k}^{(i,j)}), \quad (65)$$

$$\lim_{\upsilon_{P,k|k-1}^{(i,j)}\to+\infty} p_{P,k|k-1}^{(i)}(\mathbf{x}) = \sum_{j=1}^{J_{k-1}^{(i)}} w_{k-1}^{(i,j)}\mathcal{N}(\mathbf{x};\mathbf{m}_{P,k|k-1}^{(i,j)},\mathbf{P}_{P,k|k-1}^{(i,j)}), \quad (66)$$

the mean and covariance of the Gaussian components can be calculated in a similar way as (37). Then the multi-Bernoulli parameter $p_{U,k}(x;\,\mathbf{z})$ becomes

$$\lim_{\upsilon_{U,k}^{(i,j)}\to+\infty} p_{U,k}(x;\,\mathbf{z}) = \frac{\sum_{i=1}^{M_{k|k-1}}\sum_{j=1}^{J_{k|k-1}^{(i)}} w_{U,k}^{(i,j)}(\mathbf{z})\mathcal{N}(\mathbf{x};\, \mathbf{m}_{U,k}^{(i,j)}, \mathbf{P}_{U,k}^{(i,j)})}{\sum_{i=1}^{M_{k|k-1}}\sum_{j=1}^{J_{k|k-1}^{(i)}} w_{U,k}^{(i,j)}(\mathbf{z})}. \quad (67)$$

The likelihood function $q_k^{(i,j)}(\mathbf{z})$ will also converge to a Gaussian expression

$$\lim_{\upsilon_{k|k-1}^{(i,j)}\to+\infty} q_k^{(i,j)}(\mathbf{z}) = \mathcal{N}(\mathbf{z};\, \mathbf{H}_k\mathbf{m}_{k|k-1}^{(i,j)}, \mathbf{S}_k^{(i,j)}). \quad (68)$$

When $\upsilon_{k|k-1}^{(i,j)} \to +\infty$, we have

$$\frac{\upsilon_{k|k-1}^{(i,j)} + (\Delta_{\mathbf{z},k}^{(i,j)})^2}{\upsilon_{U,k|k-1}^{(i,j)}} = \frac{\upsilon_{k|k-1}^{(i,j)} + (\Delta_{\mathbf{z},k}^{(i,j)})^2}{\upsilon_{k|k-1}^{(i,j)} + d_{\mathbf{z}}} \to 1, \quad (69)$$

then (49) can be rewritten as

$$\lim_{\upsilon_{k|k-1}^{(i,j)}\to+\infty} \mathbf{P}_{U,k}^{(i,j)} = [\mathbf{I} - \mathbf{K}_{U,k}^{(i,j)}\mathbf{H}_k]\mathbf{P}_{k|k-1}^{(i,j)}. \quad (70)$$

□

This completes the proof.

### 5) EXTENSION TO NONLINEAR MODELS

Consider the following nonlinear evolution and measurement equations

$$\mathbf{x}_k = f_k(\mathbf{x}_{k-1}) + \mathbf{w}_{k-1} \quad (71)$$

$$\mathbf{z}_k = h_k(\mathbf{x}_k) + \mathbf{v}_k \quad (72)$$

where $f_k(\cdot)$ and $h_k(\cdot)$ are nonlinear functions, $\mathbf{w}_{k-1}$ and $\mathbf{v}_k$ are additional heavy tailed process and measurement respectively. Due to the nonlinearity of $f_k(\cdot)$ and $h_k(\cdot)$, the multi-target multi-Bernoulli density can not be approximated as Student's t mixture.

Similar to the GM-CBMeMBer filter [17], the proposed STM-CBMeMBer filter for linear models can be extended to nonlinear models. However, the key issue of STM-CBMeMBer filter for nonlinear models is how to compute the Student's t integrals. In single-target filtering, the unscented transform [29] is used to compute the Student's t weighted integrals. In this paper, we utilize the unscented transform to extend the proposed filter to nonlinear models according to [17] and [29]. The extension of the proposed filter is conceptually straightforward, therefore, we only give a brief description of the basic approach for the approximate recursions. Readers may refer to [17] and [29] for more details.

## IV. SIMULATION RESULTS

In this section, we design various multi-target tracking experiments and compare the proposed filter with the GM-CBMeMBer filter in order to verify the effectiveness of the proposed STM-CBMeMBer filter. The Optimal Sub-Pattern Assignment (OSPA) distance [35] is used as the performance metrics in our experiments due to its capability to capture the differences in both cardinality and individual state between two multi-target RFSs.

### A. SITUATIONS WITHOUT OUTLIERS IN PROCESS AND MEASUREMENT NOISES

We apply the STM-CBMeMBer filter and the GM-CBMeMBer filter to deal with multi-target tracking problem in models (1) and (2) in absence of outliers in neither process nor measurement noise. The multi-target scenario is similar to that of [17] as shown in Fig. 1. The state transition matrix $\mathbf{F}_k$ and measurement matrix $\mathbf{H}_k$ are set to

$$\mathbf{F} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{73}$$

Let the sample interval be $T = 1$. The state vector at time $k$ is denoted by $\mathbf{x}_k = [p_k^x, v_k^x, p_k^y, v_k^y]^T$, which is comprised of the position components $[p_k^x, p_k^y]^T$ and velocity components $[v_k^x, v_k^y]^T$ on the $x$ and $y$ axes. The process and measurement noises are modeled as

$$\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \sigma_w^2 I), \tag{74}$$
$$\mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \sigma_v^2 I), \tag{75}$$

where $\sigma_w = 1m$, $\sigma_v = 2m$. The surviving probability and detection probability of individual target are assumed to be $p_{S,k} = 0.99$ and $p_{D,k} = 0.98$. The clutter intensity $\kappa_k(z) = \lambda_c/V$ and clutter rate $\lambda_c = 10$ denotes the average number of clutter per scan, $V = 4 \times 10^6 m^2$ is the area of the surveillance region $[-1000, 1000]m \times [-1000, 1000]m$. The multi-Bernoulli density of the spontaneous birth of the STM-CBMeMBer filter is denoted by $\pi_{S\Gamma} = \{(r_{S\Gamma,k}, p_{S\Gamma,k}^{(i)})\}_{i=1}^{M_{\Gamma,k}}$, while $\pi_{G\Gamma} = \{(r_{G\Gamma,k}, p_{G\Gamma,k}^{(i)})\}_{i=1}^{M_{\Gamma,k}}$ denotes the multi-Bernoulli density of the spontaneous birth of the GM-CBMeMBer filter. The parameters are set to $r_{S\Gamma,k} = r_{G\Gamma,k} = 0.03$,

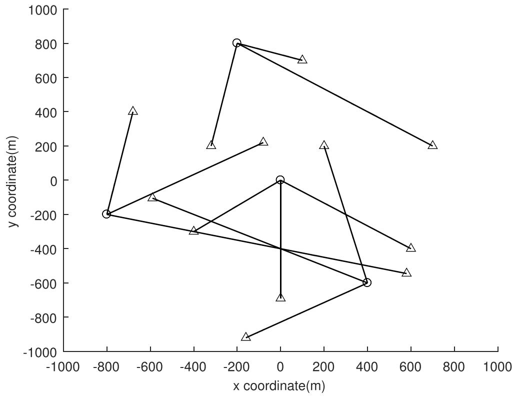



**FIGURE 1.** Target trajectories with start and stop positions denoted as circles and triangles, respectively.

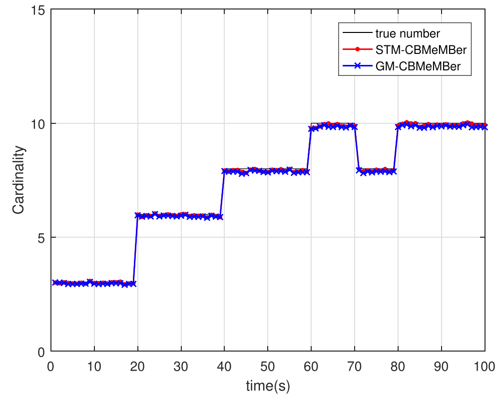



**FIGURE 2.** Cardinality estimations of the two filters in absence of process and measurement outliers.

$p_{S\Gamma,k}^{(i)}(\mathbf{x}) = St(\mathbf{x}; \mathbf{m}_{\Gamma,k}^i, \mathbf{P}_{\Gamma,k}, \upsilon_{\Gamma,k})$, $p_{G\Gamma,k}^{(i)}(\mathbf{x}) = \mathcal{N}(\mathbf{x}; \mathbf{m}_{\Gamma,k}^i, \mathbf{P}_{\Gamma,k})$, where $\mathbf{m}_{\Gamma,k}^1 = [400, 0, -600, 0]^T$, $\mathbf{m}_{\Gamma,k}^2 = [0, 0, 0, 0]^T$, $\mathbf{m}_{\Gamma,k}^3 = [-800, 0, -200, 0]^T$, $\mathbf{m}_{\Gamma,k}^4 = [-200, 0, 800, 0]$. The dof of the STM-CBMeMBer filter is set to be $\upsilon_{\Gamma,k} = 8$. The pruning threshold for existence probability is set to be $P = 10^{-3}$. Meawhile, the pruning threshold is $T = 10^{-3}$, merging threshold is $U = 4$ and the maximum number is $J_{\max} = 100$ for each hypothesized track. The order $p$ and cut-off parameter $c$ of the OSPA are set to be $p = 1$ and $c = 200$. To verify the performance, 100 independent Monte Carlo (MC) simulations are performed.

Fig. 2 shows the average of the target number estimates of the proposed STM-CBMeMBer and GM-CBMeMBer filters. The average OSPA distances of the two filters are shown in Fig. 3. It can be observed that the GM-CBMeMBer filter can achieve a good performance in linear state space whitout outlier in process and measurements. We can see from Figs. 2 and 3, the proposed STM-CBMeMBer filter can achieve comparable performance with the GM-CBMeMBer filter. The simulation results reveal that the proposed STM-CBMeMBer filter can handle multi-target tracking problem.

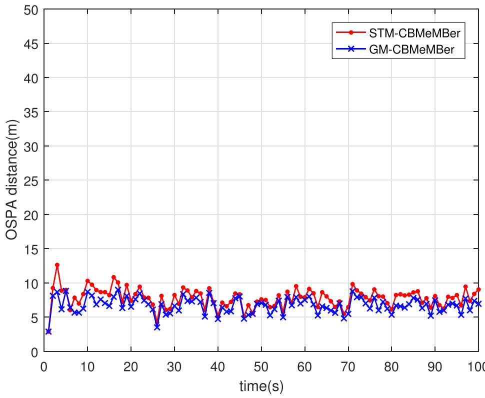



**FIGURE 3. OSPA distances of the two filers in absence of process and measurement outliers.**

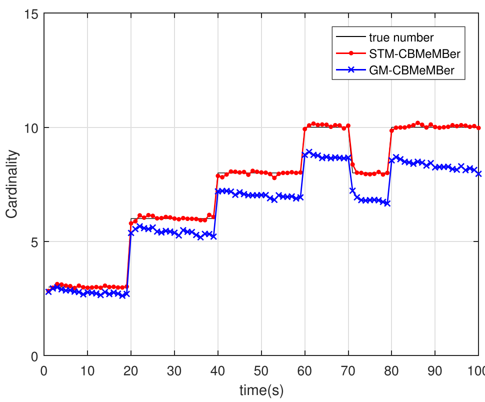



**FIGURE 4. Cardinalities estimation of the two filters with measurement outlier probability $p_{mo} = 0.1$.**

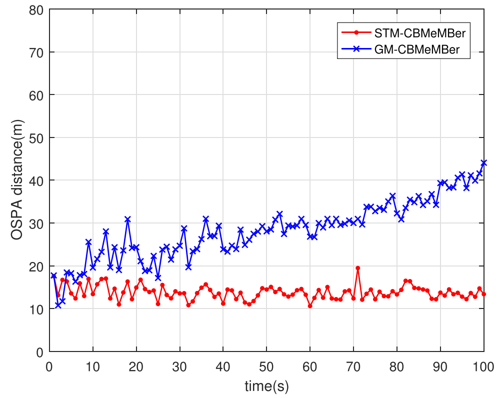



**FIGURE 5. Comparison of OSPA of the two filters with measurement outlier probability $p_{mo} = 0.1$.**

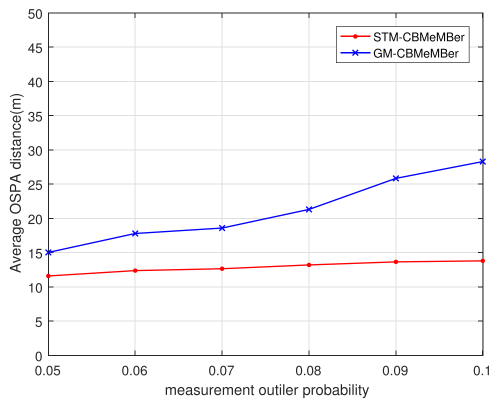



**FIGURE 6. Average OSPA of the two filters with different probabilities of measurement outlier under a fixed clutter rate $\lambda_c = 10$.**

### *B. SITUATIONS WITH ONLY MEASUREMENT OUTLIERS*

In order to verify the multi-target tracking capability in the presence of measurement outliers, an experiment with only measurement outliers is designed. In this simulation, the measurement noise decomposed by outlier is modeled similar to [28] and [29] with

$$\mathbf{v}_k \sim \begin{cases} \mathcal{N}(\mathbf{0}, \sigma_v^2\mathbf{I}), & \text{w.p. } 1 - p_{mo}, \\ \mathcal{N}(\mathbf{0}, 100\sigma_v^2\mathbf{I}), & \text{w.p. } p_{mo}. \end{cases} \tag{76}$$

where w.p. represents "with probability", $p_{mo} = 0.1$ denotes the probability of measurement noise outlier. In this experiment, the dof of the STM-CBMeMBer filter is set to be $\upsilon_{\Gamma,k} = 5$ . Other parameters are set to be the same as that of simulation 1.

Figs. 4 and 5 show the average target number estimates and OSPA distance, respectively. As can be seen from Figs. 4 and 5, the performance of the GM-CBMeMBer filter degrades severely by the measurement outliers. This is because the weight of the Gaussian tends to be a small value or even zero in some cases due to its lightweight tail property when outliers occur in measurement. As a result, the GM-CBMeMBer filter may obtain an underestimate of the target number, which will further worsen the tracking performance, i.e., increasing the OSPA distance. It can be observed from Figs. 4 and 5 that our proposed STM-CBMeMBer filter outperforms the GM-CBMeMBer filter because of the heavy tail of the Student's t distribution. When the outliers occur, the heavy tailed Student's t distribution can obtain a non-negligible weight which is helpful to track the targets without missing. The results above imply that the proposed filter can deal with the multi-target tracking problem with measurement outliers.

In order to investigate the impact of the measurement outlier probability on the two filters, the time averaged OSPA distances under different measurement outlier probability are evaluated. Fig. 6 shows the average OSPA distances with different measurement outlier probabilities of the proposed STM-CBMeMBer filter and the GM-CBMeMBer filter with a fixed clutter rate $\lambda_c = 10$. Fig. 6 illustrates that the average OSPA distances of both filters increase with the increase of the measurement outlier probability. Meanwhile, it is clear that the OSPA distance of the GM-CBMeMBer filter rises faster with the increase of the outlier probability. This is because the larger the measurement outlier probability is, the higher probability the target will be missed tracking, resulting in a degraded performance of the GM-CBMeMBer filter. On the contrary, the OSPA distance of the proposed filter rises slower than that of the GM-CBMeMBer filter. This is because the Student's distribution with heavy tail can capture the outliers and give a non-negligible weight to the target. It can be concluded that the proposed STM-CBMeMBer filter can achieve a more stable performance than the GM-CBMeMBer filter for heavy tailed measurement outliers, especially when the outlier probability is large.

### C. SITUATIONS WITH BOTH PROCESS AND MEASUREMENT OUTLIERS

In this simulation, experiments are designed to evaluate the multi-target tracking capability of the proposed STM-CBMeMBer filter with outliers in both process and measurement noises. Similar to [28] and [29] the measurement noise outlier can be generated according to (76), the process noise with heavy tails corrupted by outliers are modeled as

$$\mathbf{w}_k \sim \begin{cases} \mathcal{N}(\mathbf{0}, \sigma_w^2 \mathbf{I}), & \text{w.p. } 1 - p_{po}, \\ \mathcal{N}(\mathbf{0}, 25\sigma_w^2 \mathbf{I}), & \text{w.p. } p_{po}. \end{cases} \quad (77)$$

where $p_{po}$ denotes the probability of process noise outlier. Other parameters are set to be the same as that in simulation 2.

Figs. 7 and 8 show the number estimates and OSPA distances of the two filters under different outliers occurrence probabilities in the process and measurement noises. It can seen from Figs. 7 and 8 that, when the measurement noise outlier probabilities are fixed, the performance of the GM-CBMeMBer filter degrades with the appearance of the process noise outlier. It reveals that the GM-CBMeMBer filter is very sensitive to the process noise outlier. This is because the process noise outlier may cause target maneuver, and the GM-CBMeMBer filter cannot capture the target due to the lightweight tail of Gaussian distribution. We can also observe that the process noise outlier has little impact on the performance of our proposed STM-CBMeMBer filter due to the heavy tailed Student's distribution. It is demonstrated that our proposed filter can handle maneuvering target tracking problem. The results above reveal that our proposed STM-CBMeMBer filter can realize reliable multi-target tracking with outliers in both process and measurement noises.

Fig. 9 shows the average OSPA distances with different probabilities of measurement noise outlier of the two filters under a fixed process noise outlier probability $p_{po} = 0.1$. We can see from Fig. 9, the average OSPA distances of the GM-CBMeMBer filter increases with the increase of the measurement outlier probability, which implies larger measurement outlier probability will worsen the performance of the GM-CBMeMBer filter. Fig. 10 shows the relationship between the average OSPA distances and the probability of process outlier with a fixed measurement outlier probability $p_{mo} = 0.1$ . As can be seen from Fig. 10, the larger the process outlier probability is, the worse performance of the GM-CBMeMBer filter will achieve. This is

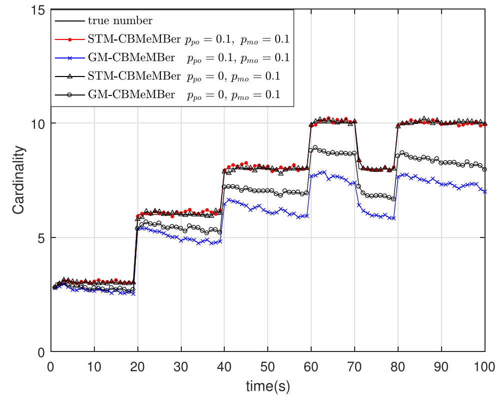



**FIGURE 7.** Cardinality estimations of the two filters with different probabilities of process and measurement outliers.

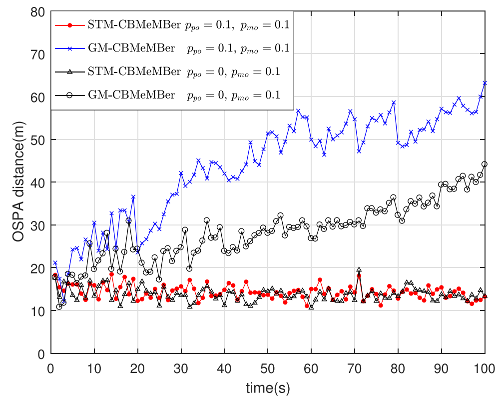



**FIGURE 8.** OSPA distances of the two filters with different probabilities of process and measurement outliers.

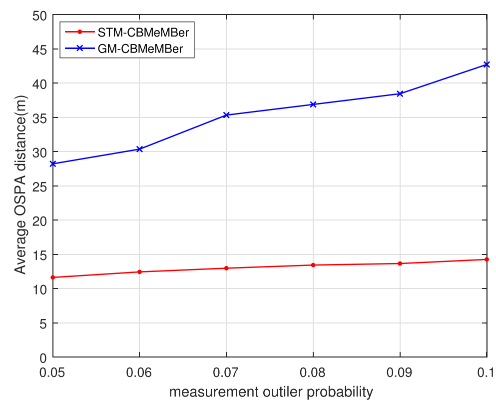



**FIGURE 9.** Average OSPA of the two filters with different probabilities of measurement outlier under a fixed process outlier probability $p_{po} = 0.1$.

because large probability of outliers in process noise will cause more maneuver of targets, which makes it harder for the GM-CBMeMBer filter to capture. We can observe from Figs. 9 and 10 that our proposed can achieve a relatively stable performance for different process and measurement outlier probabilities. Therefore, we can conclude that our proposed STM-CBMeMBer filter outperforms the GM-CBMeMBer

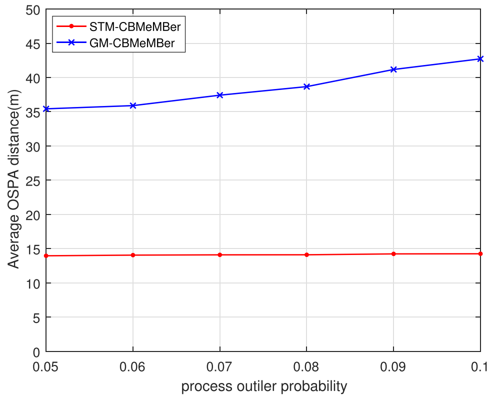



**FIGURE 10.** Average OSPA of the two filters with different probabilities of process outlier under a fixed measurement outlier probability $p_{mo} = 0.1$.

filter in multi-target scenarios where outliers occur in process and measurement noises.

Note that, although the noise models (76) and (77) are not consistent with the assumptions set in section III. Our proposed filer still can achieve a good tracking performance, which implies that our proposed STM-CBMeMBer filter is robust to process and measurement noises modelling uncertain.

### D. SIMULATIONS FOR NONLINEAR SYSTEMS

In this section, we compare the performance of our proposed Unscented Kalman(UK) STM-CBMeMBer approximation with the UK GM-CBMeMBer approximation for nonlinear systems. The multi-target scenario is similar to that of [17] as shown in Fig. 11. The state vector at time $k$ is denoted by $\mathbf{x}_k = [\tilde{\mathbf{x}}_k^T, \omega_k]^T$, where $\tilde{\mathbf{x}}_k^T$ represents the position and velocity $\tilde{\mathbf{x}}_k^T = [p_k^x, v_k^x, p_k^y, v_k^y]$ and $\omega_k$ represents the turn rate. The state dynamic equation is

$$\tilde{\mathbf{x}}_k^T = F(\omega_{k-1})\tilde{\mathbf{x}}_{k-1}^T + G\mathbf{w}_{k-1}, \tag{78}$$

$$\omega_k = \omega_{k-1} + T u_{k-1}. \tag{79}$$

where

$$F(\omega) = \begin{bmatrix} 1 & \dfrac{\sin\omega T}{\omega} & 0 & -\dfrac{1-\cos\omega T}{\omega} \\ 0 & \cos\omega T & 0 & -\sin\omega T \\ 0 & \dfrac{1-\cos\omega T}{\omega} & 1 & \dfrac{\sin\omega T}{\omega} \\ 0 & \sin\omega T & 0 & \cos\omega T \end{bmatrix},$$

$$G = \begin{bmatrix} \dfrac{T^2}{2} & 0 \\ T & 0 \\ 0 & \dfrac{T^2}{2} \\ 0 & T \end{bmatrix}. \tag{80}$$

Assume the noisy measurement is composed of bearing and range vector given by

$$\mathbf{z}_k = \begin{bmatrix} \arctan(p_k^x/p_k^y) \\ \sqrt{(p_k^x)^2 + (p_k^y)^2} \end{bmatrix} + \mathbf{v}_k. \tag{81}$$

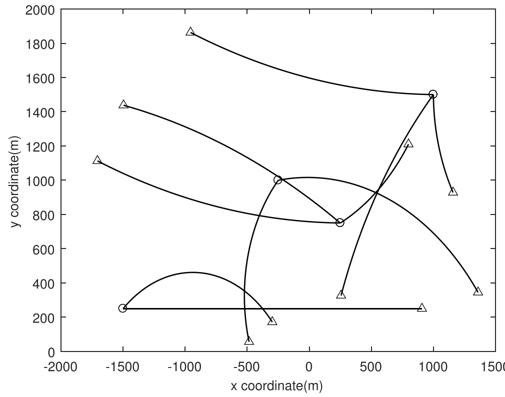



**FIGURE 11.** Target trajectories of nonlinear systems with start and stop positions denoted as circles and triangles, respectively.

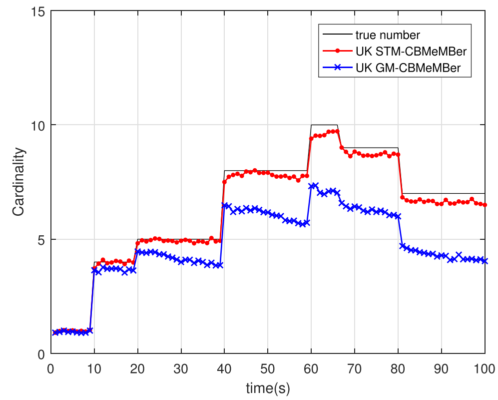



**FIGURE 12.** Cardinalities estimation of the two filters with outlier probabilities $p_{po} = 0.05$ and $p_{mo} = 0.05$.

Similar to the linear models, the process and measurement noises with heavy tails corrupted by outliers are modeled by

$$\mathbf{w}_k \sim \begin{cases} \mathcal{N}(\mathbf{0}, \mathbf{Q}), & \text{w.p. } 1 - p_{po}, \\ \mathcal{N}(\mathbf{0}, 25\mathbf{Q}), & \text{w.p. } p_{po}. \end{cases} \tag{82}$$

$$\mathbf{v}_k \sim \begin{cases} \mathcal{N}(\mathbf{0}, \mathbf{R}), & \text{w.p. } 1 - p_{mo}, \\ \mathcal{N}(\mathbf{0}, 100\mathbf{R}), & \text{w.p. } p_{mo}. \end{cases} \tag{83}$$

with $\mathbf{Q} = \mathrm{diag}([2, 2, \pi/180]^T)^2$ and $\mathbf{R} = \mathrm{diag}([\pi/180, 5]^T)^2$. The clutter rate is set to $\lambda_c = 10$ over the surveillance region $[-\frac{\pi}{2}, \frac{\pi}{2}]rad \times [0, 2000]m$. $\pi_{S\Gamma} = \{(r_{S\Gamma,k}, p_{S\Gamma,k}^{(i)})\}_{i=1}^{M_{\Gamma,k}}$ and $\pi_{S\Gamma} = \{(r_{G\Gamma,k}, p_{G\Gamma,k}^{(i)})\}_{i=1}^{M_{\Gamma,k}}$ represent the spontaneous birth multi-Bernoulli densities of the STM-CBMeMBer and GM-CBMeMBer filter respectively, where $r_{S\Gamma,k} = r_{G\Gamma,k} = 0.03$, $p_{S\Gamma,k}^{(i)}(\mathbf{x}) = St(\mathbf{x}; \mathbf{m}_{\Gamma,k}^i, \mathbf{P}_{\Gamma,k}, \upsilon_{\Gamma,k})$, $p_{G\Gamma,k}^{(i)}(\mathbf{x}) = \mathcal{N}(\mathbf{x}; \mathbf{m}_{\Gamma,k}^i, \mathbf{P}_{\Gamma,k})$, $\mathbf{P}_{\Gamma,k} = \mathrm{diag}([10, 10, 10, 10, 6(\pi/180)]^T)^2$, $\mathbf{m}_{\Gamma,k}^1 = [-250, 0, 1000, 0, 0]^T$, $\mathbf{m}_{\Gamma,k}^2 = [-1500, 0, 250, 0, 0]^T$, $\mathbf{m}_{\Gamma,k}^3 = [1000, 0, 1500, 0, 0]^T$, $\mathbf{m}_{\Gamma,k}^4 = [250, 0, 750, 0, 0]^T$. The dof of the STM-CBMeMBer filter is set to be $\upsilon_{\Gamma,k} = 5$, the other parameters are set to be the same as that of linear systems.

To demonstrate the performance of the UK STM-CBMeMBer filter, 100 independent Monte Carlo (MC)

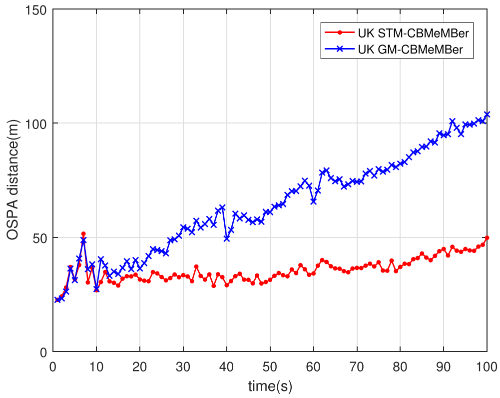



**FIGURE 13.** Comparison of OSPA of the two filters with outlier probabilities $p_{po} = 0.05$ and $p_{mo} = 0.05$.

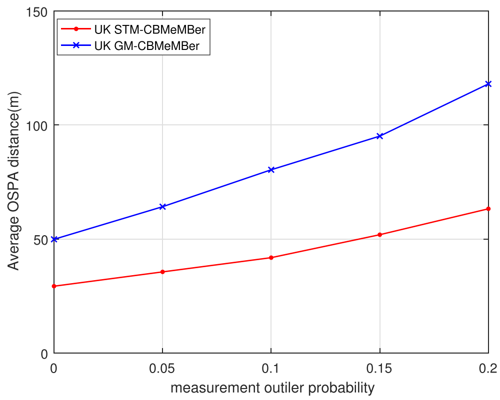



**FIGURE 14.** Average OSPA of the two filters with different probabilities of measurement outlier under a fixed process outlier probability $p_{po} = 0.05$.

simulations are performed. Figs. 12 and 13 show the average target number estimates and OSPA distance with $p_{po} = 0.05$ and $p_{mo} = 0.05$. It is observed from Fig. 12 that the UK GM-CBMeMBer filter may obtain an underestimate of the target number. This is due to the Gaussian distribution can not capture the heavy tailed process and measurement outliers. As a result, the UK GM-CBMeMBer filter may achieve worse tracking performance with a higher OSPA distance as shown in Fig. 13. It can be seen from Figs. 12 and 13 that our proposed UK STM-CBMeMBer filter outperforms the UK GM-CBMeMBer filter.

Fig. 14 shows the average OSPA distances with different probabilities of measurement noise outlier of the two filters under a fixed process noise outlier probability $p_{po} = 0.05$. Fig. 15 shows the relationship between the average OSPA distances and the probability of process outlier with a fixed measurement outlier probability $p_{mo} = 0.05$ . As can be seen from Figs. 14 and 15, the averaged OSPA distances of the two filters increase with increase of the process and measurement outlier probabilities. However, the proposed UK STM-CBMeMBer filter outperforms the UK GM-CBMeMBer filter overall. In addition, the gaps of the OSPA curves between the two filters get wider as the probabilities of the outliers increase. The results indicate that the

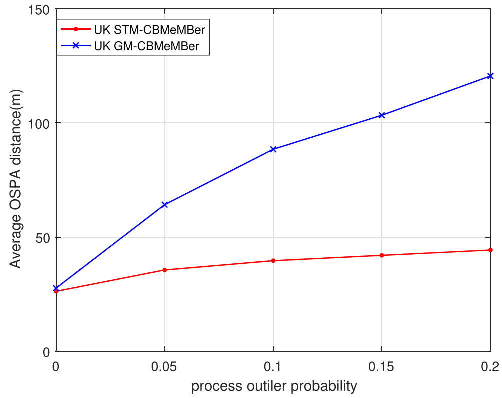



**FIGURE 15.** Average OSPA of the two filters with different probabilities of process outlier under a fixed measurement outlier probability $p_{mo} = 0.05$.

proposed UK STM-CBMeMBer filter can achieve a relatively stable performance for process and measurement outlier probabilities, especially for high oultier probabilities.

## V. CONCLUSION

In this paper, we have proposed a novel filter named STM-CBMeMBer filter to achieve reliable multi-target tracking when there are heavy-tailed process and measurement noises. By approximating the joint pdf of the process noise and state and the joint pdf of the state and measurement noise with Student's t distributions, we have derived a closed-form solution of the CBMeMBer recursion. Simulation results have shown that the proposed filter is robust to outliers in both process and measurement noises in multi-target tracking, which greatly outperforms the conventional GM-CBMeMBer filter.

## REFERENCES

[1] Y. Bar-Shalom, *Tracking and Data Association*. San Diego, CA, USA: Academic, 1987.
[2] D. Musicki and R. Evans, "Joint integrated probabilistic data association: JIPDA," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 40, no. 3, pp. 1093–1099, Jul. 2004.
[3] S. S. Blackman and R. Popoli, *Design and Analysis of Modern Tracking Systems*. Norwood, MA, USA: Artech House, 1999.
[4] S. S. Blackman, "Multiple hypothesis tracking for multiple target tracking," *IEEE Aerosp. Electron. Syst. Mag.*, vol. 19, no. 1, pp. 5–18, Jan. 2004.
[5] R. P. S. Mahler, *Statistical Multisource-Multitarget Information Fusion*. Norwood, MA, USA: Artech House, 2007.
[6] R. P. S. Mahler, "Multitarget Bayes filtering via first-order multitarget moments," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 39, no. 4, pp. 1152–1178, Oct. 2003.
[7] R. Mahler, "PHD filters of higher order in target number," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 43, no. 4, pp. 1523–1543, Oct. 2007.
[8] B. N. Vo and W. K. Ma, "The Gaussian mixture probability hypothesis density filter," *IEEE Trans. Signal Process.*, vol. 54, no. 11, pp. 4091–4104, Nov. 2006.
[9] B. T. Vo, B. N. Vo, and A. Cantoni, "Analytic implementations of the cardinalized probability hypothesis density filter," *IEEE Trans. Signal Process.*, vol. 55, no. 7, pp. 3553–3567, Jul. 2007.
[10] B.-N. Vo, S. Singh, and A. Doucet, "Sequential Monte Carlo methods for multitarget filtering with random finite sets," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 41, no. 4, pp. 1224–1245, Oct. 2005.
[11] T. Zajic and R. P. S. Mahler, "Particle-systems implementation of the PHD multitarget-tracking filter," *Proc. SPIE*, vol. 5096, pp. 291–299, Aug. 2003.

[12] B. Ristic, D. Clark, B.-N. Vo, and B.-T. Vo, "Adaptive target birth intensity for PHD and CPHD filters," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 48, no. 2, pp. 1656–1668, Apr. 2012.
[13] A. Masnadi-Shirazi and B. D. Rao, "A covariance-based superpositional CPHD filter for multisource DOA tracking," *IEEE Trans. Signal Process.*, vol. 66, no. 2, pp. 309–323, Jan. 2018.
[14] J. Yang and H. Ge, "Adaptive probability hypothesis density filter based on variational Bayesian approximation for multi-target tracking," *IET Radar, Sonar Navigat.*, vol. 7, no. 9, pp. 959–967, Dec. 2013.
[15] A. Daniyan, Y. Gong, S. Lambotharan, P. Feng, and J. Chambers, "Kalman-gain aided particle PHD filter for multi-target tracking," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 53, no. 5, pp. 2251–2265, Oct. 2017.
[16] Z. Fu, P. Feng, F. Angelini, J. Chambers, and S. M. Naqvi, "Particle PHD filter based multiple human tracking using online group-structured dictionary learning," *IEEE Access*, vol. 6, pp. 14764–14778, Mar. 2018.
[17] B.-T. Vo, B.-N. Vo, and A. Cantoni, "The cardinality balanced multi-target multi-Bernoulli filter and its implementations," *IEEE Trans. Signal Process.*, vol. 57, no. 2, pp. 409–423, Feb. 2009.
[18] Y. Zhang, H. Ji, X. Gao, and Q. Hu, "An ellipse extended target CBMeMBer filter using gamma and box-particle implementation," *Signal Process.*, vol. 149, pp. 88–102, Aug. 2018.
[19] J.-L. Yang and H.-W. Ge, "An improved multi-target tracking algorithm based on CBMeMBer filter and variational Bayesian approximation," *Signal Process.*, vol. 93, no. 9, pp. 2510–2515, Sep. 2013.
[20] X. Hu, H. Ji, and M. Wang, "Cardinality balanced multi-target multi-Bernoulli filter with adaptive target birth intensity," *IET Signal Process.*, to be published, doi: 10.1049/iet-spr.2017.0567.
[21] B.-T. Vo, B.-N. Vo, and H. G. Hoang, "An efficient implementation of the generalized labeled multi-Bernoulli filter," *IEEE Trans. Signal Process.*, vol. 65, no. 8, pp. 1975–1987, Apr. 2017.
[22] V. Barnett and T. Lewis, *Outliers in Statistical Data*. Chichester, U.K.: Wiley, 1994.
[23] G. Agamennoni, J. I. Nieto, and E. M. Nebot, "Approximate inference in state-space models with heavy-tailed noise," *IEEE Trans. Signal Process.*, vol. 60, no. 10, pp. 5024–5037, Oct. 2012.
[24] Y. Huang, Y. Zhang, N. Li, Z. Wu, and J. A. Chambers, "A novel robust Student's t-based Kalman filter," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 53, no. 3, pp. 1545–1554, Jun. 2017.
[25] Y. Huang, Y. Zhang, N. Li, and J. Chambers, "A robust Gaussian approximate fixed-interval smoother for nonlinear systems with heavy-tailed process and measurement noises," *IEEE Signal Process. Lett.*, vol. 23, no. 4, pp. 468–472, Apr. 2016.
[26] Y. Huang, Y. Zhang, B. Xu, Z. Wu, and J. Chambers, "A new outlier-robust Student's t based Gaussian approximate filter for cooperative localization," *IEEE/ASME Trans. Mechatronics*, vol. 22, no. 5, pp. 2380–2386, Oct. 2017.
[27] Y. Huang, Y. Zhang, P. Shi, Z. Wu, J. Qian, and J. A. Chambers, "Robust Kalman filters based on Gaussian scale mixture distributions with application to target tracking," *IEEE Trans. Syst., Man, Cybern. Syst.*, to be published.
[28] M. Roth, E. Özkan, and F. Gustafsson, "A Student's t filter for heavy tailed process and measurement noise," in *Proc. IEEE Int. Conf. Acoust., Speech Signal Process. (ICASSP)*, Vancouver, BC, Canada, May 2013, pp. 5770–5774.
[29] Y. Huang, Y. Zhang, N. Li, and J. Chambers, "Robust Student's t based nonlinear filter and smoother," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 52, no. 5, pp. 2586–2596, Oct. 2016.
[30] Y. Huang and Y. Zhang, "Robust student's t-based stochastic cubature filter for nonlinear systems with heavy-tailed process and measurement noises," *IEEE Access*, vol. 5, pp. 7964–7974, May 2017.
[31] O. Straka and J. Duník, "Stochastic integration student's-t filter," in *Proc. 20th Int. Conf. Inf. Fusion (FUSION)*, Xi'an, China, Jul. 2017, pp. 1–8.
[32] Y. Huang, Y. Zhang, N. Li, S. M. Naqvi, and J. Chambers, "A robust Student's t based cubature filter," in *Proc. 19th Int. Conf. Inf. Fusion (FUSION)*, Heidelberg, Germany, Jul. 2016, pp. 9–16.
[33] Y. Huang and Y. Zhang, "A new process uncertainty robust Student's t based Kalman filter for SINS/GPS integration," *IEEE Access*, vol. 5, pp. 14391–14404, Jul. 2017.
[34] M. Roth, "On the multivariate t distribution," Division Autom. Control, Linköping Univ. Norrköping, Sweden, Tech. Rep. LiTH-ISY-R-3059, Apr. 2013.
[35] D. Schuhmacher, B.-T. Vo, and B.-N. Vo, "A consistent metric for performance evaluation of multi-object filters," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 3447–3457, Aug. 2008.

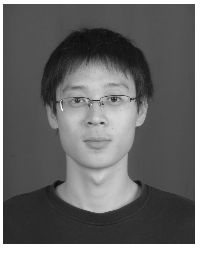
**MINGJIE WANG** was born in Chifeng, China, in 1988. He received the B.Eng. degree in information countermeasure technology from Xidian University, China, in 2011, where he is currently pursuing the Ph.D. degree in pattern recognition and intelligent systems. His research interests include multi-sensor information fusion and target tracking.

**HONGBING JI** (M'96–SM'07) graduated from the Northern West Telecommunications Engineering College (now Xidian University), Xi'an, China, and received the B.S. degree in radar engineering in 1983, and the M.S. degree in circuit, signals and systems and the Ph.D. degree in signal and information processing from Xidian University in 1989 and 1999, respectively. Since 1989, he has been with the School of Electronic Engineering, Xidian University.

He is currently a Professor and an Advisor for Ph.D. students. His primary areas of research have been radar signal processing, automatic targets recognition, multi-sensor information fusion, and target tracking.

**YONGQUAN ZHANG** was born in Gansu, China, in 1985. He received B.S. degree in computer science and technology and the M.S. degree in computer applications technology from the Lanzhou University of Technology 2007 and 2010, respectively.

He is currently a Lecturer with the School of Electronic Engineering, Xidian University. His research interests include machine learning, signal processing, target tracking, and data fusion.



**XIAOLONG HU** was born in Xian, China, in 1989. He received the B.Eng. degree in information countermeasure technology from Xidian University, China, in 2011, where he is currently pursuing the Ph.D. degree in pattern recognition and intelligent systems. His research interests include target tracking and radar signal processing.