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# Nonlinear Disturbance Observer-Based Control for a Class of Port-Controlled Hamiltonian Disturbed Systems

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**ABSTRACT** In this paper, a composite control scheme is presented to asymptotically stabilize a class of port-controlled Hamiltonian systems under nonvanishing disturbances. First, based on the damping injection method and the nonlinear disturbance observer (NDOB) technique, the robust composite control strategy is designed. The NDOB, as an effective observation tool, is developed to estimate the disturbances, and furthermore, the disturbances can be feedforward compensated using the estimates of disturbances. Then, for the augmented system, an asymptotic stability theorem is proposed via an input-to-state stability technique and Lyapunov stability theorems. The proposed control approach exhibits not only good robustness and disturbance rejection performances but also the property of nominal performance recovery. Finally, a simulation example on a circuit system is given to show the feasibility and advantage of the composite control method.

**INDEX TERMS** Asymptotic stability, port-controlled Hamiltonian systems, damping injection, nonvanishing disturbances, nonlinear disturbance observer, input-to-state stability.

## I. INTRODUCTION

Since Maschke and Vander Schaft [1] and Vander Schaft and Maschke [2] presented Port-controlled Hamiltonian (PCH) systems, such systems have been extensively studied [3]-[5] until now. Since its structure represents clear physical meaning and the Hamilton function stands for the total energy which is the sum of potential energy and kinetic energy [6]–[8], the PCH system widely exists in the physical sciences, life sciences and engineering science fields, especially classical mechanics, aerospace science and biological engineering. The PCH system control theory has become a very important research direction in today's nonlinear science research. In recent years, many energy-based control approaches have been proposed for power system and mechanical system control [9]-[11]. Among these effective control approaches, the energy-shaping plus damping injection, first developed in [12], is a very important control technique for PCH systems. This provides a good idea for us to investigate the more difficult control issues of PCH systems.

As everyone knows, disturbances widely exist in practical engineering systems [13]-[15], which usually come from external environments or the system itself. Hence, the disturbances are the lumped disturbances of external disturbances, parameter perturbations and unmodeled dynamics, and always lead to the poor performance for these engineering systems. So far, there are two kinds of the advanced control methods to deal with control problems of PCH systems subject to disturbances. One kind of method is the robust control method which includes  $L_2$  disturbance rejection [13], [16],  $H_{\infty}$  control [17], [18], and so on. Although these control methods exhibit systems' robustness against uncertainties and disturbances rejection performance, the robustness is generally achieved at the expense of systems' nominal control performances. In other words, systems' nominal performance recovery is not guaranteed under these robust control methods mentioned. The integral control approach belongs to the second class of method. In [19]–[21], several effective control methods based on the integral action are given for PCH

systems to eliminate the effects caused by disturbances which are slowly time-varying. Although the integral action has a good disturbance rejection ability, the unexpected transient performance is always inevitable, such as larger overshoots and longer settling time.

In order to avoid the above methods' drawbacks and ameliorate the anti-disturbance performance of the controlled systems, an advanced control scheme has been developed, which is called the disturbance observer based control (DOBC) [22]–[24]. The DOBC, as an efficient disturbance estimation tool and compensation strategy, contains feedforward control and feedback control. The feedback control is used to control nominal systems. The feedforward control aims to compensate disturbances by estimating them effectively. As a result, the robustness against disturbances can be enhanced [25]–[30]. This inspires us to develop the DOBC approach to investigate the stabilization issues of PCH systems with disturbances.

The stabilization problem of PCH disturbed systems is solved by a composite control scheme in this paper. Firstly, since the positive definite of dissipative matrix of the PCH system is guaranteed by employing the damping injection technique [12], the asymptotic stability of the closed-loop system in the absence of disturbances is studied by designing a baseline feedback controller. Secondly, a nonlinear disturbance observer (NDOB) is employed and modified to estimate the disturbances. Furthermore, the compensation controller of disturbances is constructed based on the disturbances estimates. Finally, combining the baseline feedback controller and the disturbances compensation controller, a novel composite control strategy is presented. Under the composite controller, the asymptotic stability theorem of the PCH system is given by the input-to-state stability and Lyapunov stability theorems. A simulation example on circuit system is given to demonstrate the feasibility and advantage of the composite control scheme. The disturbance d is the nonvanishing disturbance in this paper, which means that  $\lim d(t) \neq 0$ . The main merits are in the following aspects.

1) The proposed control approach exhibits nice robustness and disturbance rejection performances.

2) The proposed control approach can still work for PCH systems without disturbances, i.e., the property of nominal performance recovery is guaranteed.

The remainder of the paper is listed below. In Section II, the problem statements and some preliminaries are proposed. The baseline feedback control law design, the NDOB design and the asymptotic stability analysis of the closed-loop augmented PCH system are developed in Section III, i.e., the main result. In Section IV, the feasibility and advantage of the control strategy is revealed by a circuit example. Finally, Section V presents the conclusion.

*Notation:*  $\mathbb{R}$  stands for the set of real numbers,  $\mathbb{R}^n$  denotes the set of *n*-dimensional vectors and  $\mathbb{R}^{n \times l}$  represents the set of  $n \times l$  real matrices. Positive semi-definite matrix A(x) is denoted by  $0 \leq A(x)$ , and positive definite matrix A(x)

is represented by 0 < A(x). For matrix B,  $\lambda(B)$  represents a eigenvalue,  $\lambda_{all}(B)$  denotes all eigenvalues, the minimum eigenvalue is denoted by  $\lambda_{\min}(B)$ , and  $\lambda_{nz\min}(B)$  means that the minimum eigenvalue does not equal zero.  $\nabla H(x)$  stands for  $\frac{\partial H(x)}{\partial x}$  and the matrix transposition is denoted by the superscript T.

### **II. PROBLEM FORMULATION AND PRELIMINARIES**

Under nonvanishing disturbances, a class of PCH system is given as follows

$$\dot{x} = [J(x) - R(x)]\nabla H(x) + g(x)u + \bar{g}(x)d(t),$$
(1)

where  $x \in \mathbb{R}^n$  and  $u \in \mathbb{R}^m$  are the state and the control input, respectively.  $J(x) = -J^{T}(x) \in \mathbb{R}^{n \times n}, \mathbb{R}^{n \times n} \ni R(x) \ge 0$ . As the Hamilton function, H(x) usually has a minimum point at x = 0. Gain matrices  $g(x) \in \mathbb{R}^{n \times m}$  and  $\overline{g}(x) \in \mathbb{R}^{n \times l}$  have full column rank,  $(R, g) \in \mathbb{R}^{n \times (n+m)}$  is a full row rank matrix, and the lumped disturbances  $d(t) = (d_1, d_2, \dots, d_l)^{T} \in \mathbb{R}^l$ are the nonvanishing disturbances that means  $\lim_{t \to \infty} d(t) \neq 0$ .

## A. PROBLEM FORMULATION

This paper aims to study the stabilization of Hamiltonian system (1) under nonvanishing disturbances. To achieve this goal, a composite controller is designed as  $u = u_1 + u_2$  by utilizing damping injection technique and NDOB technique, where  $u_1$  is a baseline feedback controller in the absence of disturbances, and  $u_2$  is a disturbance compensation controller with  $d(t) \neq 0$ .

## **B. PRELIMINARIES**

For the convenience of subsequent analysis, two lemmas need to be revisited again.

First, system

$$\dot{x} = f(x, u, t), \quad x \in \mathbb{R}^n, \ u \in \mathbb{R}^m,$$
(2)

is considered, where  $f : \mathbb{R}^n \times \mathbb{R}^m \times [0, \infty) \longrightarrow \mathbb{R}^n$  is locally Lipschitz in x and u and piecewise continuous in t.

*Lemma 1 [31]:* Consider system (2). A function V(t, x):  $[0, \infty) \times \mathbb{R}^n \longrightarrow \mathbb{R}$  is selected and its derivative is continuous, which satisfies the following conditions

$$\eta_1(\|x\|) \le V(t, x) \le \eta_2(\|x\|),\tag{3}$$

$$\dot{V}(t,x) \le -\iota_1(x), \quad \forall \|x\| \ge \kappa (\|u\|) > 0,$$
 (4)

where  $\iota_1(x)$  is a positive definite function which is continuous,  $\kappa$  is a class *K* function, and both  $\eta_1$  and  $\eta_2$  are class  $K_{\infty}$  functions. Then, system (2) is input-to-state stable (ISS).

*Lemma 2 [31]:* Suppose the following two conditions in system (2) hold,

1. system (2) is globally input-to-state stable,

2. as  $t \to \infty$ , u = 0,

then as  $t \to \infty$ , x = 0, i.e., system (2) is uniformly asymptotically stable.

# C. ASSUMPTIONS

For PCH disturbed system (1), the following preliminary assumptions are needed in this paper,.

Assumption 1: There exists a matrix Q(x) with appropriate dimension, such that  $g(x)Q(x) = \overline{g}(x)$ .

Assumption 2: Both the nonvanishing disturbances  $d(t) = (d_1, d_2, \dots, d_l)^T$  and their derivatives  $\dot{d}(t)$  are bounded, and the condition that as  $t \to \infty$ ,  $\dot{d}_i(t) = 0$ , is satisfied,  $i = 1, 2, \dots, l$ .

## **III. MAIN RESULTS**

The stabilization issue for system (1) is investigated in this part.

First, for the PCH system (1) with d(t) = 0, the baseline feedback control is designed. Then based on NDOB, the disturbance compensation part is developed. Finally, a composite control law is given which can make PCH disturbed system (1) stable.

## A. BASELINE FEEDBACK CONTROL DESIGN

Generally, choose  $H(x) = \sum_{i=1}^{n} x_i^2$  as the Hamilton function for PCH system (1). Disturbances are not considered in this part, i.e.,  $d_i = 0$ , and there are two cases to consider about the eigenvalues of matrix R(x).

*Case 1:* PCH system (1) is globally asymptotically stable if all the eigenvalues  $\lambda_{all}(R(x))$  lie in the right half plane. For this situation, we do not need to design the controller  $u_1$ , i.e., baseline feedback control law  $u_1 = 0$ .

*Case 2:* If there is at least one eigenvalue  $\lambda(R(x))$  which is not in the right half plane, a feedback controller  $u = u_1$ , called baseline feedback control law, needs to be designed such that  $\bar{R}(x) > 0$  and  $-\bar{J}(x) = \bar{J}^{T}(x)$  by employing the damping injection technique [12]. Furthermore, one gets the closed-loop system

$$\dot{x} = (\bar{J}(x) - \bar{R}(x))\nabla H(x).$$
(5)

According to the Hamilton function H(x), matrices  $\overline{R}(x) > 0$ and  $-\overline{J}(x) = \overline{J}^{T}(x)$ , it concludes that the global asymptotic stability of system (5) holds.

*Remark 1:* As the injected damping [12],  $R_1(x)$  is a part of the synthetic matrix  $\overline{R}(x)$ , i.e.,  $\overline{R}(x) := R(x) + R_1(x)$ , and the technique is called the "damping injection".  $J_1(x)$  is a part of the synthetic interconnection matrix  $\overline{J}(x) := J(x) + J_1(x)$ .

Since Hamilton function H(x) is ideal which does not need to be shaped, we only need to design the baseline feedback control law  $u_1$  such the  $J_1(x)$  and  $R_1(x)$  satisfy the following equation

$$g(x)u_1 = (J_1(x) - R_1(x))\nabla H(x),$$
(6)

where both skew-symmetric matrix  $J_1(x)$  and symmetric matrix  $R_1(x)$  are to be determined.

To get control law  $u_1$ , two situations are considered for full column rank matrix g(x).

If matrix g(x) is not invertible, the full row rank left annihilator  $g^{\perp}$  inspired by [32] is obtained, which satisfies

the equation

$$g^{\perp}(x) \cdot g(x) = 0. \tag{7}$$

Further, according to

$$g^{\perp}(x)(J_1(x) - R_1(x))\nabla H(x) = 0,$$
 (8)

and the help of the particular PCH system structure, one can get such a solution pair  $(J_1(x), R_1(x))$ . Finally, the baseline feedback control law

$$u_1 = [g^{\mathrm{T}}(x)g(x)]^{-1}g^{\mathrm{T}}(x)(J_1(x) - R_1(x))\nabla H(x)$$
(9)

is obtained according to (6), and system (1) is stabilized asymptotically under the controller (9).

If matrix g(x) is invertible, one can easily get the baseline feedback controller

$$u_1 = g(x)^{-1} (J_1(x) - R_1(x)) \nabla H(x), \tag{10}$$

which can stabilize system (1).

In fact, feedback control laws (9) and (10) are the same for invertible matrix g(x), so controller (9) is only used in the subsequent analysis.

## **B. NONLINEAR DISTURBANCE OBSERVER DESIGN**

Considering PCH system (1) in the presence of disturbances and substituting the baseline feedback control law (9) into PCH system (1), it yields the new expression of system (1)

$$\dot{x} = (\bar{J}(x) - \bar{R}(x))\nabla H(x) + g(x)u_2 + \bar{g}d(t),$$
 (11)

Because the disturbances d(t) exist in system (11), which need to be estimated and compensated in system control, a NDOB inspired by [25] is designed as follows:

$$\begin{cases} \dot{T} = -l(x)\bar{g}T - l(x)[\bar{g}p(x) \\ +(\bar{J}(x) - \bar{R}(x))\nabla H(x) + g(x)u_2], \\ \dot{d} = T + p(x), \end{cases}$$
(12)

where  $\hat{d}$  are the estimates of disturbances d(t), the NDOB gain l(x) is given as follows

$$l(x) = \frac{\partial p(x)}{\partial x},\tag{13}$$

the vector-valued function p(x) is to be designed, and T is the disturbance observer's state.

Define the disturbance estimation error as  $e_d = \hat{d} - d$ , and further one gets the disturbance estimation error system

$$\dot{e}_d = -l(x)\bar{g}e_d - \dot{d}.$$
(14)

Defining the new terms  $\overline{d} = \overline{g}d$ ,  $\overline{d} = \overline{g}d$  and  $M = \overline{g}e_d$ , it easily yields  $M = \overline{d} - \overline{d}$ . Furthermore, the error system is obtained as follows

$$\dot{M} = \bar{g}(-l\bar{g}e_d - \dot{d}) = -\bar{g}lM - \dot{\bar{d}}.$$
(15)

and PCH system (11) is reformulated as

$$\dot{x} = (\bar{J}(x) - \bar{R}(x))\nabla H(x) + g(x)u_2 + \bar{d}.$$
 (16)

Substituting error term  $M = \hat{d} - \bar{d}$  into PCH system (16), we obtain the following system

$$\dot{x} = (\bar{J}(x) - \bar{R}(x))\nabla H(x) + g(x)u_2 + \ddot{\bar{d}} - M.$$
 (17)

In system (17), the disturbance compensation controller  $u_2$  should be designed to compensate the disturbance estimates  $\hat{d}$ , which satisfies the following equation

$$g(x)u_2 = -\hat{\vec{d}}.$$
 (18)

Since g(x) is a full column rank matrix and Assumption 1 holds, we left multiply the equation (18) by  $[g^{T}(x)g(x)]^{-1}g^{T}(x)$ , and then obtain the disturbance compensation control law

$$u_2 = -[g^{\mathrm{T}}(x)g(x)]^{-1}g^{\mathrm{T}}(x)\hat{d}.$$
 (19)

## C. ASYMPTOTIC STABILITY ANALYSIS

This subsection will propose the asymptotic stability analysis of the closed-loop system.

Assumption 3: For matrix  $\overline{R}$ , the minimum eigenvalue satisfies the following condition

$$\lambda_{\min}(\bar{R}) > \frac{1}{4}.$$
 (20)

For matrix  $\bar{g}l$ , the following condition

$$\lambda_{nz\min}(\bar{g}l) > 1 \tag{21}$$

also holds for non-zero minimum eigenvalue  $\lambda_{n_z \min}(\bar{g}l)$ .

Based on the previous analysis, we have obtained the baseline feedback controller  $u_1$  and disturbances compensation control law  $u_2$ . Further, the robust composite control law uis developed for PCH system (1) based on  $u_1$  and  $u_2$ , which is expressed as

$$u = u_1 + u_2$$
  
=  $[g^{\mathrm{T}}(x)g(x)]^{-1}g^{\mathrm{T}}(x)$   
 $\cdot [(J_1(x) - R_1(x))\nabla H(x) - \hat{d}].$  (22)

In the following, a theorem of the PCH system (1) stabilization is presented.

Theorem 1: Consider PCH disturbed system (1). It is ensured that Assumptions 1-3 hold. Then the composite control law u (22) can asymptotically stabilize PCH system (1) and observer error system (14).

*Proof:* Since Subsections III-A and III-B present the related analysis, under the composite control law u (22), the PCH disturbed system (1) can be expressed as the following closed-loop Hamiltonian system

$$\dot{x} = (\bar{J}(x) - \bar{R}(x))\nabla H(x) - M.$$
(23)

A Lyapunov function is selected as

V

$$Y(x, M) = H(x) + \frac{1}{2}M^{\mathrm{T}}M.$$
 (24)



FIGURE 1. Circuit system.

Since  $-\overline{J}(x) = \overline{J}^{T}(x)$ , one gets  $\nabla^{T}H(x)\overline{J}(x)\nabla H(x) = 0$ . From  $H(x) = \sum_{i=1}^{n} x_{i}^{2}$ ,  $\nabla^{T}H(x)E = 2(x_{1}E_{1} + \dots + x_{n}E_{n})$  is obtained. Calculating the derivative of V(x, E), it yields

$$\dot{V}(x,M) = \nabla^{\mathrm{T}} H(x)(\bar{J}(x) - \bar{R}(x))\nabla H(x) - \nabla^{\mathrm{T}} H(x)M + M^{\mathrm{T}}[-\bar{g}lM - \dot{\bar{d}}] = -\nabla^{\mathrm{T}} H(x)\bar{R}(x)\nabla H(x) - 2(x_1M_1 + \dots + x_nM_n) + M^{\mathrm{T}}[-\bar{g}lM - \dot{\bar{d}}] \leq -4\lambda_{\min}(\bar{R})\|x\|^2 + \|x\|^2 + \|M\|^2 - \lambda_{nz\min}(\bar{g}l)\|M\|^2 + \|M\|\|\dot{\bar{d}}\| \leq -[4\lambda_{\min}(\bar{R}) - 1]\|x\|^2 - [\lambda_{nz\min}(\bar{g}l) - 1]\|M\|^2 + \|M\|\|\dot{\bar{d}}\| \leq -[4\lambda_{\min}(\bar{R}) - 1]\|x\|^2 - (1 - \theta)[\lambda_{nz\min}(\bar{g}l) - 1]\|M\|^2 + \|M\|\|\dot{\bar{d}}\|, \quad (25)$$

where  $0 < \theta < 1$ .

According to Assumption 3, we obtain that when  $||M|| \ge \frac{||\dot{d}||}{\theta[\lambda_{n_{z}\min}(\tilde{g}l)-1]},$ 

$$\dot{V}(x, M) \le -[4\lambda_{\min}(\bar{R}) - 1] \|x\|^2 - (1 - \theta) [\lambda_{n_z \min}(\bar{g}l) - 1] \|M\|^2.$$
(26)

According to Lemma 1, it yields that the closed-loop augmented systems consisting of systems (1) and (14) is global ISS. One gets  $\bar{g} \lim_{t \to \infty} \dot{d}_i(t) = 0$  and  $\lim_{t \to \infty} \dot{\bar{d}}_i(t) = 0$  based on Assumption 2. Furthermore, it concludes that the states M and x of the augmented system will converge to zero asymptotically according to the above analysis and Lemma 2, which means that  $\lim_{t \to \infty} e_d = [\bar{g}^T \bar{g}]^{-1} \bar{g}^T \lim_{t \to \infty} M = 0.$ 

Hence, it follows from the previous analysis that the augmented system is asymptotically stable which contains PCH disturbed system (1), robust composite control law (22) and error system (14).

*Remark 2:*  $\lambda_{\min}(\bar{R}) > \frac{1}{4}$  and  $\lambda_{nz\min}(\bar{g}l) > 1$  are reasonable in Assumption 3 since desired matrix  $\bar{R}$  and NDOB gain l(x) can be obtained by utilizing damping injection technique and designing appropriate vector-valued function p(x), respectively.



**FIGURE 2.** Response curves: (a) Electric charge q. (b) Magnetic flux  $\psi$ . (c) Current  $I_s$ . (d) Voltage  $U_s$ .

*Remark 3:* In PCH system (1), if d(t) = 0 and the initial state of the NDOB (12) is chosen as T(0) = -p, then the disturbance estimate satisfies  $\hat{d} \equiv 0$ . In this case, the system' composite control performance recovers to the system' performance under the baseline feedback control strategy. It implies that the system' nominal control performance can not be sacrificed under the robust composite control strategy in this paper.

*Remark 4:* The augmented system is deduced by the PCH system, the observer error system and the composite controller. Thus, it is complex, which makes it difficult to prove Theorem 1. The theory of input-to-state stability is applied to overcome the difficulty.

*Remark 5:* For port-controlled Hamiltonian systems with nonvanishing disturbances, the nonlinear disturbance observer-based control method is presented. Compared with robust control, the control method can improve the antidisturbance performance without sacrificing nominal control performances. Compared with integral control, the control method can avoid unexpected transient performances such as larger overshoots and longer settling time.

## **IV. SIMULATIONS**

This section presents a numerical example of circuit system and gives the simulation results, which reveals the feasibility and advantage of the composite control strategy.

*Example:* Figure 1 ([18]) shows the circuit system, where the electric charge q controls the capacitance, the magnetic flux  $\psi$  controls the inductance,  $U_1 = h_2(q)$  denotes the voltage,  $i_3 = h_1(\psi)$  stands for the current and  $i_w$  represents the current source disturbance.

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The circuit system can be written as follows based on Kirchhoff's law,

$$\begin{cases} \dot{q} = -I_s - h_1(\psi) - i_w, \\ \dot{\psi} = U_s + h_2(q) - R_3 h_1(\psi). \end{cases}$$
(27)

Let  $h_1(\psi) = 2\psi$ ,  $h_2(q) = 2q$  and  $R_3 = 3\Omega$ , and the control input is denoted by  $u = (I_s, U_s)^T$ . Further, the system (27) can be rewritten as the following PCH system

$$\begin{pmatrix} \dot{q} \\ \dot{\psi} \end{pmatrix} = \left[ \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix} \right] \cdot \begin{pmatrix} 2q \\ 2\psi \end{pmatrix}$$

$$+ \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} I_s \\ U_s \end{pmatrix}$$

$$+ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -i_w \\ 0 \end{pmatrix}$$

$$\coloneqq \left[ J(q, \psi) - R(q, \psi) \right] \nabla H(q, \psi)$$

$$+ g(q, \psi)u + \bar{g}d(t),$$

$$(28)$$

where  $H(q, \psi) = q^2 + \psi^2$ ,  $\bar{g} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  and  $d(t) = \begin{pmatrix} -i_w \\ 0 \end{pmatrix}$ . Since Subsection III-A presents the baseline feedback controller design process, by choosing  $J_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ,  $R_1 = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ , the baseline feedback control law is easily obtained

$$u_1 = \begin{pmatrix} 2aq + 2\psi \\ 2q - 2b\psi \end{pmatrix},\tag{29}$$

where  $a > \frac{1}{4}$  and  $b > -\frac{11}{4}$ .

The estimate  $z_1$  of disturbance  $-i_w$  is obtained according to the NDOB design, and further the disturbance compensation controller is developed as

$$u_2 = \begin{pmatrix} z_1 \\ 0 \end{pmatrix}. \tag{30}$$

Combining baseline feedback controller  $u_1$  with disturbance compensation controller  $u_2$  yields the robust composite control law u, i.e.,

$$u_{NDOB} = u_1 + u_2 = \begin{pmatrix} 2aq + 2\psi + z_1 \\ 2q - 2b\psi \end{pmatrix}.$$
 (31)

In order to illustrate the feasibility and superiority of the composite control scheme, a proportional integral (PI) control method is employed to compare the control performance. The PI control law is designed as follows

$$u_{PI} = \begin{pmatrix} k_{11}q + k_{12} \int_0^t q ds \\ -k_{21}\psi - k_{22} \int_0^t \psi ds \end{pmatrix}.$$
 (32)

In the simulation research, choose the controller parameters as a = 3, b = 3,  $k_{11} = 25$ ,  $k_{12} = 50$ ,  $k_{21} = 10$ ,  $k_{22} = 20$ . The vector-valued function is chosen as  $p(q, \psi) = (50 q, 0)^{T}$ in NDOB. The simulation with the initial states q(0) = -0.5and  $\psi(0) = 1.2$  is carried out. The current source disturbance  $-i_{w} = 1$  for  $2 s < t \le 6 s$  and  $-i_{w} = 1.5$  for  $t \ge 10 s$  is added to the system.



**FIGURE 3.** Response curves of the current source disturbance  $-i_W$  and its estimated value  $z_1$ .

Figure 2(*a*) is electric charge response of *q*, and Figure 2(*b*) is magnetic flux response of  $\psi$ . They show that the composite control strategy based on NDOB realizes a better control performance in the presence of disturbances in comparison with the PI control. Figure 2(*c*) and Figure 2(*d*) depict the current  $I_s$  and voltage  $U_s$ , respectively, which are the control signals. They show that each control signal has the same magnitude under NDOB and PI control strategy, which guarantees the fairness of comparison between them. From Figure 3, the disturbance estimate converges to the disturbance quickly. Thus, the NDOB can estimate the disturbance well.

### V. CONCLUSION

By a composite control strategy, this paper has studied the stabilization issue of a class of Port-controlled Hamiltonian systems in the presence of nonvanishing disturbances. The composite control law has been developed based on the disturbance compensation control law and the baseline feedback control law which are obtained by the nonlinear disturbance observer technique and the damping injection method, respectively. The asymptotic stability analysis has been given by means of input-to-state stability and Lyapunov stability theorems. Finally, a numerical example of circuit system with simulation results has been proposed to reveal the feasibility of the robust composite control strategy.

Many engineering systems can be modeled by PCH systems. For such practical systems, the composite control approach presented in the paper can be expected to realize better performances, and meanwhile put forward a challenge. The issue will be considered in the future.

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