

Social-Aware Incentive Mechanism for AP Based Mobile Data Offloading

FEN HOU¹ AND ZHANGYUAN XIE

Department of Electrical and Computer Engineering, University of Macau, Macao SAR, China

Corresponding author: Fen Hou (fenhou@umac.mo)

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ABSTRACT Without properly addressed, the dramatic increase in mobile data traffic may lead to severe traffic congestion problems in cellular networks. Mobile data offloading is a promising method to reduce the traffic congestion by redirecting the traffic of a cellular network to other types of access networks. In addition, with the popularity of online social platforms (e.g., Wechat and Facebook), the social relationship of mobile users (MUs) has become an important factor to impact the decision of data offloading. In this paper, we take the social relationship of MUs into the consideration of incentive mechanism design and design two social-aware incentive mechanisms to achieve efficient mobile data offloading, in which a PageRank-based algorithm is used to model the social relationship of MUs. The proposed incentive mechanisms satisfy the properties of individual rationality and truthfulness. Extensive simulation results demonstrate the nice performance of the proposed mechanisms by comparing with other counterparts.

INDEX TERMS Mobile data offloading, WiFi access point, incentive mechanism, social relationship.

I. INTRODUCTION

In recent years, we have been witnessing the explosive growth of mobile data traffic in cellular networks. In [1], it is stated that mobile data traffic will increase at a compound annual growth rate of 47% from 2016 to 2021, reaching 49 exabytes per month by 2021, which will result in the great pressure to cellular networks. Without properly addressed, the severe traffic congestion may degrade the quality of experience of mobile users (MUs), especially in crowded areas or rush hours. Some advanced algorithms [2], [3] and congestion control mechanisms [4] have been proposed to optimize the system performance. However, increasing the capacity of cellular networks, such as directly upgrading to the advanced technologies or building up more infrastructures, is time-consuming and efficient limited. Mobile data offloading is another way to alleviate the traffic congestion, which has been defined by 3GPP as a promising solution to handle the dramatic increase in mobile data traffic [5].

Mobile data offloading refers to redirecting the overloaded mobile data originally targeted to a cellular network to other networks, such as WiFi, femtocell or D2D networks [6]. In general, mobile data offloading can be classified into two categories: mobile-user initiated and cellular-network

initiated. Mobile-user initiated model assumes that MUs initiate the process of data offloading by choosing complimentary networks (e.g., femtocell, WiFi network, etc.) according to their own preference. While in a cellular-network initiated model, it is the cellular network operator that handles the process of mobile data offloading. In this paper, we focus on a cellular-network initiated offloading system composed of a base station (BS), multiple MUs, and several WiFi access points (APs). In specific, these APs are deployed by third-party companies. Due to the limited coverage area of a WiFi AP, an individual cellular network operator cannot provide the ubiquitous WiFi coverage. Therefore, some third-party companies can offer their WiFi APs to help a cellular operator to fulfill the efficient mobile data offloading.

However, serving the offloading data is resource-consuming (e.g., network capacity, energy consumption, etc.). Considering that the owner of each AP is rational and selfish, they will not take part in the data offloading without receiving the appropriate compensation (e.g., payment or reward). Therefore, how to design an incentive mechanism to motivate APs to participate in the mobile data offloading plays a key role in achieving the efficient mobile data offloading.

In addition, with the popularity of online social network, MUs are more likely to share the information with each other via the social platforms (e.g., Wechat, Facebook, etc.), thus an MU can get higher satisfaction when the other MUs having social connection with him can also get the nice service quality. For instance, MUs who are friends and playing online games together will get higher satisfaction if all of them can achieve high quality of service. Therefore, the social relationship of MUs can be utilized to improve the performance of a data offloading system.

In this paper, we jointly consider the social relationship of MUs into the incentive mechanism design for AP based mobile data offloading. In specific, we use a PageRank [7] based algorithm to model the social relationship of MUs, and design a social-aware incentive mechanism SRBA. The proposed incentive mechanism satisfies the properties of individual rationality and truthfulness with the objective of maximizing the system social welfare. Furthermore, we propose a greedy algorithm to reduce the computational complexity.

The main contributions of this paper include:

- *Novel social-aware problem formulation:* We consider the social relationship of MUs into the incentive mechanism design for AP based mobile data offloading.
- *Efficient incentive mechanism design:* By taking the social relationship of MUs into account, we design a social relationship based auction (SRBA), which maximizes the system social welfare and satisfies the properties of individual rationality and truthfulness.
- *Computational efficient algorithm design:* We propose a greedy algorithm to reduce the computation complexity of the AP selection and payment determination, which can be solved in polynomial time.
- *Extensive performance evaluation:* Extensive simulations have been conducted to demonstrate their nice properties of the proposed incentive mechanisms by comparing with other counterparts.

The remainder of the paper is organized as follows. The related work has been briefly reviewed in Section II. The system model and problem formulation are described in Section III. The proposed incentive mechanism SRBA is presented in Section IV, followed by the description of GSRBA in Section V. And the performance evaluation is given in Section VI. Finally we conclude the paper in Section VI.

II. RELATED WORK

A lot of works have been focused on the incentive mechanism design. Hua *et al.* [8] consider a mobile-user initiated model where MUs offer incentives to buy their wireless communication access time from the owner of each Femtocell. Iosifidis *et al.* [9] study the cellular-network initiated mobile data offloading, where BS pays for APs' offloading consumption to stimulate them to participate in the data offloading. Chen *et al.* [10] propose a reverse auction framework to motivate femtocell networks to participate into the two-tier hybrid access for improving the network

performance. Song and Zhao [11] propose a randomized reverse auction to maximize the system efficiency and reduce the computation complexity in D2D content dissemination.

Social relationship has been widely considered in spectrum access and D2D communication scenario. Chen *et al.* [12] propose a social group utility maximization model that considers both users' social link and physical interference to deal with a spectrum access problem. It shows that the popularity of social links among MUs plays an important role in filling the gaps between non-cooperative game and network utility maximization. Chen *et al.* [13] study cooperative D2D communications based on social trust and social reciprocity and formulate the relay selection problem as a coalitional game. Social relationship has also been considered in the data collection for mobile crowd sensing. Sun *et al.* [14] consider the social relationship in the incentive mechanism design to improve the social utility in the participatory sensing network.

Few of existing works in mobile data offloading consider social relationship of MUs. Wang *et al.* [15] focus on users' spreading impact in online social network and utilize it to guide the selection of relay users in opportunistic mobile data offloading. Zhang *et al.* [16] consider the social influence in users' utility when implementing D2D offloading via mobile participation. In this paper, we consider AP based mobile data offloading and explore how the social relationship of MUs impacts the decision of BS to achieve efficient mobile data offloading.

III. SYSTEM MODEL AND PROBLEM FORMULATION

A. SYSTEM MODEL

As shown in Fig. 1, we consider a mobile data offloading system consisting of a BS, several APs, and multiple MUs, where BS is deployed by a cellular network operator and the WiFi APs are deployed by different third-party companies. With the network congestion, BS will offload some mobile traffic to some of the APs. For MUs, we consider both physical domain and social domain in this paper. In the social domain, MUs have heterogeneous social relationship. In the physical domain, MUs have different channel qualities based on their physical locations. Let $\mathcal{N} = \{1, 2, \dots, n\}$ denote the set of MUs. Any user $i \in \mathcal{N}$ is associated with the following attributes.

- *Channel model:* For the channel model between each MU and BS, we consider both the path loss and short-term fading. For any MU $i \in \mathcal{N}$, the received signal noise ratio (SNR) and the corresponding data rate are given as

$$\begin{cases} \gamma_i = \frac{P_t}{N_0} \cdot \frac{1}{d_i^\alpha} \cdot h_i \\ r_{ib} = B \cdot \log_2(1 + \gamma_i) \end{cases} \quad (1)$$

where P_t is the transmit power of BS, N_0 is the channel noise, d_i is the distance between MU i and BS, α is the path-loss exponent, h_i denotes the small-scale channel fading which is exponentially distributed with unit mean, and B is the bandwidth of BS.

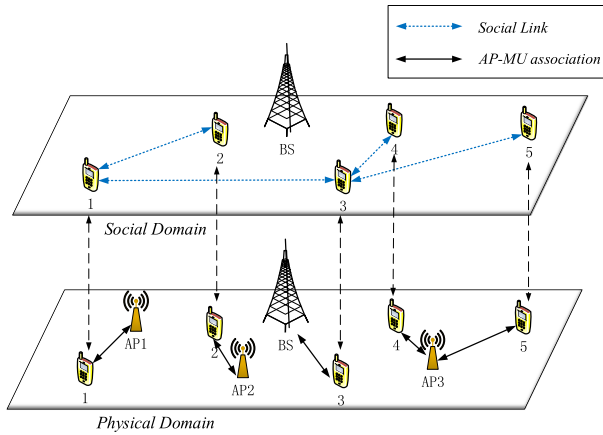


FIGURE 1. System model.

- *Social characteristic:* We use a parameter a_{ij} to denote the social influence that MU j exerts on i . If they are friends, then $a_{ij} \in (0, 1]$, otherwise $a_{ij} = 0$. The detailed discussion about how to calculate the value of a_{ij} is given in Subsection III.B.

Let $\mathcal{K} = \{1, 2, \dots, k\}$ denote the set of APs. Each AP $k \in \mathcal{K}$ is associated with the following attributes:

- *Available capacity c_k :* Since each AP, owned by a third-party company, has its own tasks, APs may have different spare capacities available for conducting the data offloading.
- *True value of consumption v_k :* It represents the true value of consumption caused by helping BS to serve the overloaded traffic. Note that v_k is the private information of AP k and cannot be known by any other APs or BS.
- *Ask price b_k :* It is the reward or payment that AP k wants to receive from BS to compensate its consumption caused by conducting the data offloading. Note that the ask price b_k may not be equal to the true value of consumption v_k if AP k can get a higher payment by asking another price.

B. MODELING THE SOCIAL RELATIONSHIP OF MUS

Based on the experiences in our daily life, the following features are usually observed for MUs.

- *Selfish:* MUs are always selfish. They mainly concern about the quality of their own services.
- *Selfless:* MUs will care for others to a certain extent. MUs will get extra satisfaction if their friends also get the high quality service. However, the level of concern for others will never too much exceed the level of concern for themselves.
- *The impact of fame:* An individual tends to care about famous people, and the famous people are more likely to have larger influence on the others.

Motivated by the aforementioned observation and inspired by [7], we assume MUs who have more social links with others are more popular and have higher influences on others,

which is fully in line with the reality. Let $rank_j$ denote the social rank of MU $j \in \mathcal{N}$, which reflects the popularity level of the user. We use PageRank algorithm to calculate the value of $rank_j$. PageRank is an algorithm used by Google Search to rank websites. It works by counting the number of links to a webpage to roughly estimate how important the website is. In this paper, we use the number of social links to an MU as a rough estimate about how popular an MU is. Based on the social relationship in Fig. 1, we give an example to elaborate how to derive the social rank of each MU. Let \mathcal{N}_i be the set of the MUs with a direct social link to MU i . Then, $|\mathcal{N}_i|$ is the number of social links directly connected to MU i . Base on the connection of MUs at the social domain in Fig. 1, we can get the connection matrix of MUs as follows.

$$M = [m_{i,j}]_{N \times N} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (2)$$

where N denotes the number of MUs, and the entity $m_{i,j}$ in the matrix M is give as

$$m_{i,j} = \begin{cases} \frac{1}{|\mathcal{N}_i|} & j \in \mathcal{N}_i \\ 0 & otherwise \end{cases}$$

For example, since MU 3 connects to 3 MUs (i.e., 1, 4, and 5), the entity $m_{3,1}, m_{3,4}$ and $m_{3,5}$ are $1/3$, respectively. Meanwhile, $m_{3,2} = 0$ since MU 3 doesn't directly connect with MU 2. Let $V = [rank_i]_{i \in \mathcal{N}}$ be the social rank vector. The social rank of MU i can be derived by solving the balance equations of the Markov process as follows.

$$\begin{cases} V = VM \\ \sum_{i \in \mathcal{N}} rank_i = 1 \end{cases}$$

That is, the social rank of MU $i \in \mathcal{N}$ is derived by solving the steady-state probability of a Markov process with the one-step transition matrix M in (2). In this example, we get the social rank of MU 2 and MU 3 is $rank_2 = 0.1250$ and $rank_3 = 0.3750$, respectively.

With the social rank of each MU, the influence of user j on i , denoted as a_{ij} , is defined as

$$a_{ij} = \frac{rank_j}{\sum_{n \in \mathcal{N}_i} rank_n} \quad (3)$$

where \mathcal{N}_i is the set of all the MUs that have social links with i .

For instance, with the social rank of MU 2 and MU 3 in the above example, we have $a_{12} = \frac{0.1250}{0.5000} = 0.2500$, and $a_{13} = \frac{0.3750}{0.5000} = 0.7500$.

TABLE 1. Notations.

Notations	Explanation
\mathcal{N}	The set of all MUs
\mathcal{K}	The set of all APs
a_{ij}	MU j 's social influence on i
r_{ik}	MU i 's data rate with AP k
r_{ib}	MU i 's data rate with BS
u_i	Utility of MU i
\mathcal{N}_i	The set of MUs that have social link with i
\mathcal{X}	The set of all feasible winner selection solutions
\mathcal{W}	The set of winning APs
\mathcal{F}	AP-MU association set
\mathcal{F}_k	AP-MU association set of AP k
$\mathcal{F}_{\mathcal{W}}$	Wining AP-MU association set
c_k	The available capacity of AP k
n_k	Number of MUs associated with AP k
s_i	Social utility of MU i
b_k	Ask price of AP k
v_k	AP k 's true value of consumption
p_k	BS operator's payment to AP k
$\mathcal{E}^{\mathcal{W}}$	System social welfare with the winning set \mathcal{W}
\mathcal{V}_k	AP k 's contribution
\mathcal{M}	Winner set contribution
\mathcal{M}_k	AP k 's marginal contribution

C. PROBLEM FORMULATION

In this paper, we use the auction theory to formulate the mobile data offloading problem, where the BS acts as the auctioneer who buys the capacity of APs while APs act as the sellers who offer their available capacity to serve the offloaded traffic. In specific, each AP reports its available capacity and ask price to BS while BS chooses some APs to participate in the data offloading based on the collected information (i.e., available capacity and ask price of all APs) and pay them the corresponding remuneration. The whole auction procedure includes three steps:

- APs submit their available capacities and ask prices to BS (i.e., c_k and $b_k, \forall k \in \mathcal{K}$).
- Each MU scans for the available WiFi connectivity and reports the AP with the strongest detected signal to BS. Based on the reported information, BS can generate an AP-MU association set $\mathcal{F} = \{\mathcal{F}_k\}_{k \in \mathcal{K}}$ which reflects the association between MUs and APs, where \mathcal{F}_k denotes the AP-MU association set of AP k . For example, as shown in Fig. 1, MU 4 and MU 5 are associated with AP 3. That is, AP-MU association set of AP 3 is $\mathcal{F}_3 = \{4, 5\}$.
- BS selects some APs to conduct the offloading task and decides the payment for each of the selected APs.

IV. SOCIAL-AWARE INCENTIVE MECHANISM DESIGN

In this section, we propose a social-aware incentive mechanism, called social relationship based auction (SRBA), to achieve efficient mobile data offloading, where BS will select the winning APs to participate in the data offloading and determine the corresponding payment for each AP. By jointly considering the social relationship of MUs, the objective of the proposed SRBA is to maximize the system social welfare. We firstly introduce some definitions, followed by the detailed elaboration about the AP selection rule and the payment rule of the proposed SRBA.

Definition 1 (Utility of MU): The utility of each MU reflects the data rate gain achieved by mobile data offloading. Therefore, the utility of an MU $i \in \mathcal{N}$ is defined as the difference between the achieved data rate by offloading its mobile traffic from BS to AP, which is given as

$$u_i = \begin{cases} r_{ik} - r_{ib} & i \in \mathcal{F}_{\mathcal{W}} \\ 0 & otherwise \end{cases} \quad (4)$$

where r_{ik} and r_{ib} are the data rate that MU i achieves through its associating AP k and directly from BS, respectively. $\mathcal{F}_{\mathcal{W}}$ denotes the set of MUs who are served by APs via the data offloading. For example, if AP 3 in Fig. 1 is selected to participate in the data offloading, MUs associated with AP 3 (i.e., MU 4 and MU 5) will be served by AP 3. Then, the utility of MU 4 and 5 is $u_4 = r_{43} - r_{4b}$ and $u_5 = r_{53} - r_{5b}$, respectively. Otherwise, if MU i is not covered by any AP or his associating AP is not selected to participate in the data offloading, his utility is zero because BS will not offload the traffic of this MU.

For the MUs served by an AP in the data offloading, we assume that they equally share the available capacity of the AP. That is

$$r_{ik} = \frac{c_k}{n_k} \quad (5)$$

where n_k is the number of MUs associated with AP k .

Definition 2 (Social Utility of MU): An MU's social utility is defined as his own utility plus the social contribution of other MUs. That is

$$S_i = \begin{cases} u_i + \sum_{j \in \mathcal{N}_i} a_{ij}u_j & i \in \mathcal{F}_{\mathcal{W}} \\ 0 & otherwise \end{cases} \quad (6)$$

where \mathcal{N}_i is the set of MUs that have social link with i , a_{ij} is the social relationship parameter between MU i and j , and $\mathcal{F}_{\mathcal{W}}$ is the set of MUs whose associating APs are selected to participate in the mobile data offloading.

Definition 3 (Utility of AP): The utility of AP $k \in \mathcal{K}$ is defined as the gain achieved by participating in the offloading, which is given as

$$u_k = \begin{cases} p_k - v_k & k \in \mathcal{W} \\ 0 & otherwise \end{cases} \quad (7)$$

where p_k is the payment that AP k obtains from BS, v_k is AP k 's true value of consumption caused by conducting the

offloading task. And \mathcal{W} is the set of winning APs (i.e., the set of APs selected to conduct the data offloading).

Definition 4 (System Social Welfare): System social welfare is defined as the sum of MUs' social utility achieved through the mobile data offloading minus the total cost of APs taking part in the data offloading, which is given as

$$\mathcal{E} = \sum_{i \in \mathcal{F}_{\mathcal{W}}} S_i - \sum_{k \in \mathcal{W}} b_k \quad (8)$$

where \mathcal{W} is the set of winning APs. With the truthful auction, the ask price b_k is equal to the true value of consumption caused by conducting the data offloading.

A. SOCIAL-AWARE INCENTIVE MECHANISM

In this subsection, we elaborate the proposed social-aware incentive mechanism SRBA in detail, which includes the AP selection rule and payment rule.

1) AP SELECTION RULE

Based on the collected information from all APs (e.g., ask price and the available capacity), the social relationship of MUs, channel condition of each MU, etc, BS selects a set of APs to participate in the data offloading with the objective of maximizing the system social welfare, which can be formulated as

$$\mathcal{W}^* = \arg \max_{\mathcal{W} \in \mathcal{X}} \left(\sum_{i \in \mathcal{F}_{\mathcal{W}}} S_i - \sum_{k \in \mathcal{W}} b_k \right) \quad (9)$$

$$s.t. |\mathcal{W}| = l \quad (10)$$

where \mathcal{X} is the set of all feasible solutions of AP selection. $|\mathcal{W}|$ denotes the number of APs in the set \mathcal{W} , and l is a parameter decided by BS operator. The pseudo code of the AP selection procedure is given in Algorithm 1.

2) PAYMENT RULE

BS determines the payment for each AP $k \in \mathcal{K}$. If AP k is not selected to conduct the data offloading, the payment $p_k = 0$; Otherwise, the payment is designed as

$$p_k = b_k + (\mathcal{E}^* - \mathcal{E}_{-k}^*) \quad (11)$$

where \mathcal{E}^* and \mathcal{E}_{-k}^* are the maximum value of the objective function in (9) with and without the participation of AP k , respectively, which are defined as follows.

$$\mathcal{E}^* = \max_{\mathcal{W} \in \mathcal{X}} \left(\sum_{i \in \mathcal{F}_{\mathcal{W}}} S_i - \sum_{k \in \mathcal{W}} b_k \right) \quad (12)$$

$$s.t. |\mathcal{W}| = l \quad (13)$$

$$\mathcal{E}_{-k}^* = \max_{\mathcal{W} \in \mathcal{X}_{-k}} \left(\sum_{i \in \mathcal{F}_{\mathcal{W}}} S_i - \sum_{k \in \mathcal{W}} b_k \right) \quad (14)$$

$$s.t. |\mathcal{W}| = l \quad (15)$$

Algorithm 1 AP Selection in SRBA

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1: Obtain the set of all feasible solutions  $\mathcal{X}$ 
2: for all  $\mathcal{W} \in \mathcal{X}$  do
3:   for all  $i \in \mathcal{F}_{\mathcal{W}}$  do
4:     for all  $k \in \mathcal{W}$  do
5:       if  $i \in \mathcal{F}_k$  then
6:          $r_{ik} = \frac{c_k}{n_k}$ ;
7:       end if
8:     end for
9:      $u_i = r_{ik} - r_{ib}$ ;
10:  end for
11:  for all  $i \in \mathcal{F}_{\mathcal{W}}$  do
12:     $s_i = u_i + \sum_{j \in \mathcal{N}_i} a_{ij}u_j$ ;
13:  end for
14:   $\mathcal{E}^{\mathcal{W}} = \sum_{i \in \mathcal{F}_{\mathcal{W}}} s_i - \sum_{k \in \mathcal{W}} b_k$ ;
15:   $\mathcal{E} \leftarrow \mathcal{E} \cup \{\mathcal{E}^{\mathcal{W}}\}$ ;
16: end for
17:  $\mathcal{W}^* = \arg \max_{\mathcal{W} \in \mathcal{X}} \mathcal{E}$ ;
18: return  $\mathcal{W}^*$ 

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where \mathcal{X}_{-k} denotes the set of all feasible solutions of AP selection without the participation of AP k .

B. PROOF OF PROPERTIES

In this subsection, we prove that the proposed incentive mechanism SRBA satisfies the properties of truthfulness and individual rationality.

The property of truthfulness means that the utility of any AP won't be improved by asking a price deviating from the true value of its consumption. Before presenting the proof of the truthfulness for SRBA, we first rewrite the utility of AP. As defined in (7), if an AP is not selected to perform the offloading, its utility is zero; otherwise, based on the payment given in (11), the utility of AP k defined in (7) can be rewritten as

$$\begin{aligned} u_k &= p_k - v_k \\ &= \mathcal{E}^* - \mathcal{E}_{-k}^* + b_k - v_k \end{aligned} \quad (16)$$

Lemma 1 (Truthfulness): Our proposed SRBA is truthful.

Proof: When AP k asks truthfully, we have $b_k = v_k$. When AP k does not ask truthfully (i.e., $b_k \neq v_k$), we have two possible cases.

Case 1 ($b_k > v_k$): In this case, we need to consider three subcases as follows.

Subcase 1.1: AP k is a winner when it asks truthfully, and AP k is still a winner when $b_k > v_k$.

In this subcase, the utility of AP k with the truthful ask price is $u_k = \mathcal{E}^* - \mathcal{E}_{-k}^*$, while the utility with a untruthful ask price is $\tilde{u}_k = \tilde{\mathcal{E}}^* - \tilde{\mathcal{E}}_{-k}^* + b_k - v_k$.

Since AP k is a winner with both truthful and untruthful ask prices, we have $\mathcal{E}_{-k}^* = \tilde{\mathcal{E}}_{-k}^*$. Meanwhile, the set of winning APs with the truthful ask price is the same

as that with a untruthful ask price. Thus, $u_k - \tilde{u}_k$ can be calculated as

$$\begin{aligned} u_k - \tilde{u}_k &= \mathcal{E}^* - \tilde{\mathcal{E}}^* - b_k + v_k \\ &= \left(\sum_{i \in \mathcal{F}_{\mathcal{W}}} S_i - \sum_{j \in \mathcal{W}} v_j \right) \\ &\quad - \left(\sum_{i \in \mathcal{F}_{\mathcal{W}}} S_i - \sum_{j \in \mathcal{W}, j \neq k} v_j - b_k \right) \\ &= \sum_{i \in \mathcal{F}_{\mathcal{W}}} S_i - \sum_{j \in \mathcal{W}} v_j - \sum_{i \in \mathcal{F}_{\mathcal{W}}} S_i + \sum_{j \in \mathcal{W}, j \neq k} v_j + v_k \\ &= \sum_{i \in \mathcal{F}_{\mathcal{W}}} S_i - \sum_{i \in \mathcal{F}_{\mathcal{W}}} S_i = 0 \end{aligned}$$

The utility gain of AP k is zero. So AP k cannot improve its utility in this subcase.

Subcase 1.2: AP k is a winner when it asks truthfully, and it is not a winner when $b_k > v_k$.

In this subcase, the utility with untruthful ask price is $\tilde{u}_k = 0$. However, the utility with the truthful ask price is

$$\begin{aligned} u_k &= p_k - v_k \\ &= \mathcal{E}^* - \mathcal{E}_{-k}^* + v_k - v_k \\ &= \mathcal{E}^* - \mathcal{E}_{-k}^* \\ &\geq 0 \end{aligned}$$

That is, we have $u_k \geq \tilde{u}_k$ in this subcase.

Subcase 1.3: AP k is not a winner when it asks truthfully, and it is still not a winner when $b_k > v_k$.

In this subcase, $u_k = \tilde{u}_k = 0$.

Case 2 ($b_k < v_k$): We need to consider three subcases as follows.

Subcase 2.1: AP k is a winner when it asks truthfully, and it is still a winner when $b_k < v_k$.

The proof for this subcase is similar to that in *Subcase 1.1*.

Subcase 2.2: AP k is not a winner when it asks truthfully, and it becomes a winner when $b_k < v_k$.

In this subcase, the utility $u_k = 0$, and \tilde{u}_k is given as

$$\begin{aligned} \tilde{u}_k &= \tilde{\mathcal{E}}^* - \tilde{\mathcal{E}}_{-k}^* + b_k - v_k \\ &= \tilde{\mathcal{E}}^* - \mathcal{E}^* + b_k - v_k \\ &= \sum_{i \in \mathcal{F}_{\mathcal{W}}} S_i - \sum_{j \in \mathcal{W}, j \neq k} v_j - b_k + b_k - v_k - \mathcal{E}^* \\ &= \sum_{i \in \mathcal{F}_{\mathcal{W}}} S_i - \sum_{j \in \mathcal{W}} v_j - \mathcal{E}^* \\ &\leq 0 \end{aligned}$$

where $\left(\sum_{i \in \mathcal{F}_{\mathcal{W}}} S_i - \sum_{j \in \mathcal{W}} v_j \right)$ is the system social welfare when the winner set contains AP k given that AP k asks truthfully. Obviously, it is smaller than \mathcal{E}^* , which is the optimal system social welfare given that AP k ask truthfully. So, \tilde{u}_k is not positive. That is, AP k cannot increase its utility in this subcase.

Subcase 2.3: AP k is not a winner when it asks truthfully, and it is still not a winner when $b_k < v_k$.

In this subcase, $u_k = \tilde{u}_k = 0$.

Based on above discussion, it is concluded that AP k cannot improve its utility by asking untruthfully. Hence our proposed SRBA is truthful. \square

Lemma 2 (Individual Rationality): Our proposed SRBA is individual rational.

Proof: The property of individual rational means that the utility of each AP is not a negative value. For the APs who are not selected to conduct the offloading, their utility is zero. In addition, based on the property of truthfulness, asking the true value is at least a weakly dominate strategy for each AP. So if AP k is selected to conduct the offloading, its utility is

$$u_k = p_k - v_k = \mathcal{E}^* - \mathcal{E}_{-k}^* \geq 0$$

\square

In summary, SRBA satisfies both truthfulness and individual rationality.

V. GREEDY ALGORITHM BASED INCENTIVE MECHANISM

Although SRBA has very nice properties such as the truthfulness and individual rationality, the optimal solution of AP selection in Algorithm 1 is obtained by comparing all feasible solutions in the set \mathcal{X} , which has exponential complexity and cannot adapt well with a large number of MUs or APs. In this section, we propose a greedy algorithm based incentive mechanism, named GSRBA, to reduce the computational complexity. Similar to SRBA, the objective of GSRBA is to maximize the system social welfare.

Firstly, we introduce some definitions. Then we elaborate the proposed GSRBA, including the AP selection rule and the payment rule.

Definition 5 (AP Contribution): The contribution of AP k is defined as the sum of achieved social utilities of all the MUs served by AP k , which is given as

$$v_k = \sum_{i \in \mathcal{F}_k} S_i \quad (17)$$

where \mathcal{F}_k denotes is the set of MUs served by AP k .

Definition 6 (Winner set contribution): Given a set of winning APs, denoted as \mathcal{W} , the contribution of the winner set \mathcal{W} is defined as the sum of the contribution of all APs in \mathcal{W} , which is given as

$$\mathcal{M}(\mathcal{W}) = \sum_{k \in \mathcal{W}} v_k. \quad (18)$$

Definition 7 (AP's Marginal Contribution): Given a set of winning APs, denoted as \mathcal{W} , the marginal contribution of AP $k \notin \mathcal{W}$ is defined as the increase in the winner set contribution caused by the winning AP k , which is given as

$$\mathcal{M}_k(\mathcal{W}) = \mathcal{M}(\mathcal{W} \cup \{k\}) - \mathcal{M}(\mathcal{W}). \quad (19)$$

The marginal contribution is a key factor in the AP selection rule for the proposed mechanism GSRBA, which will be discussed below.

A. GREEDY SOCIAL-AWARE INCENTIVE MECHANISM

To reduce the computational complexity, we further propose a greedy algorithm based incentive mechanism, named GSRBA. The AP selection rule and the payment determination rule are elaborated as follows.

1) AP SELECTION RULE

BS will select a set of APs to conduct the data offloading with the objective of maximizing the social welfare, which can be formulated as the following optimization problem.

$$\begin{aligned} \mathcal{W}^* &= \arg \max_{\mathcal{W}} \left(\sum_{i \in \mathcal{F}_{\mathcal{W}}} S_i - \sum_{k \in \mathcal{W}} b_k \right) \\ &= \arg \max_{\mathcal{W}} \left(\sum_{k \in \mathcal{W}} \mathcal{V}_k - \sum_{k \in \mathcal{W}} b_k \right) \\ &\text{s.t. } |\mathcal{W}| = l \end{aligned} \tag{20}$$

The pseudo code of selecting winning APs is given in Algorithm 2, which is based on the order of the marginal contribution of each AP minus its ask price (Lines 2-6).

The winning APs can be sorted as

$$\mathcal{M}_{\phi(1)} - b_{\phi(1)} \geq \dots \geq \mathcal{M}_{\phi(n)} - b_{\phi(n)} \geq \dots \tag{21}$$

where $\phi(n)$ denotes the index of the AP at the position n in the ordering. This descending ordering will be used in the proof for the property of truthfulness. In (21), we use $\mathcal{M}_{\phi(n)}$ instead of $\mathcal{M}_{\phi(n)}(\mathcal{W})$ to simplify the notation since the winner set \mathcal{W} will be updated in each while-loop during the selection of winning APs.

Algorithm 2 AP Selection in GSRBA

```

1:  $\mathcal{W} \leftarrow \emptyset, i = 0;$ 
2: while  $i < l$  do
3:    $k \leftarrow \arg \max_{h \in \mathcal{K} \setminus \mathcal{W}} (\mathcal{M}_h(\mathcal{W}) - b_h);$ 
4:    $\mathcal{W} \leftarrow \mathcal{W} \cup \{k\};$ 
5:    $i = i + 1;$ 
6: end while
7: return  $\mathcal{W}$ 

```

2) PAYMENT DETERMINATION RULE

The pseudo code of the payment determination rule is given in Algorithm 3. To compute the payment for AP $j \in \mathcal{W}$, we sort all APs in $\mathcal{K} \setminus j$ based on their marginal contribution minus ask price. At each repeat-loop, we find an AP with the maximum value of the marginal contribution minus ask price (line 7). Then, we update the payment p_j to be the maximum price that can make AP j become a winning AP by replacing at least one selected AP (Line 8).

After finishing the repeat-loop in Algorithm 3, the value of payment p_j is the maximum price that can make AP j become a winning AP by replacing at least one of the l selected APs in Γ . And we can sort the l selected APs in the descending

Algorithm 3 Payment Determination in GSRBA

```

1: for all  $h \in \mathcal{K}$  do
2:    $p_h \leftarrow 0;$ 
3: end for
4: for all  $j \in \mathcal{W}$  do
5:    $\mathcal{K}' \leftarrow \mathcal{K} \setminus \{j\}, \Gamma \leftarrow \emptyset, i = 0;$ 
6:   repeat
7:      $k \leftarrow \arg \max_{h \in \mathcal{K}' \setminus \Gamma} (\mathcal{M}_h(\Gamma) - b_h);$ 
8:      $p_j \leftarrow \max\{p_j, \min\{\mathcal{M}_j(\Gamma) - (\mathcal{M}_k(\Gamma) - b_k), \mathcal{M}_j(\Gamma)\}\};$ 
9:      $\Gamma \leftarrow \Gamma \cup \{k\};$ 
10:     $i = i + 1;$ 
11:   until  $i > l;$ 
12: end for
13: return  $p_h, \forall h \in \mathcal{K}$ 

```

order as follows

$$\tilde{\mathcal{M}}_{\xi(1)} - b_{\xi(1)} \geq \dots \geq \tilde{\mathcal{M}}_{\xi(m)} - b_{\xi(m)} \geq \dots \tag{22}$$

where $\xi(m)$ denotes the index of the AP at the position m of the sorting in (22), and $\tilde{\mathcal{M}}_{\xi(m)} = \mathcal{M}(\Gamma \cup \{\xi(m)\}) - \mathcal{M}(\Gamma)$.

B. PROOF OF PROPERTIES

In this subsection, we prove that the proposed mechanism GSRBA satisfies the properties of truthfulness, individual rationality, and computational efficiency. Before presenting the proof of the three properties, we introduce some notations and definitions to make the proof more clear. (i) $\phi(\cdot)$ and $\xi(\cdot)$ represent the function mapping the index of an AP and the position of the AP at the descending order in (21) and (22), respectively. For example, $\phi(3)$ represents the index of the AP at the 3rd position of the descending order in (21). (ii) $\mathcal{M}_{n(n)} \triangleq \mathcal{M}(\Gamma_{n-1} \cup \{\phi(n)\}) - \mathcal{M}(\Gamma_{n-1})$ denotes the marginal value of AP $\phi(n)$ when AP $\phi(n)$ substitutes AP $\xi(n)$ as the winning AP at the n^{th} position in (22).

Lemma 3 (Individual Rationality): The proposed incentive mechanism GSRBA is individual rational.

Proof: For the AP at the n^{th} position in (21) (i.e., $\phi(n)$), we have $\mathcal{M}_{n(n)} - b_{\phi(n)} \geq \tilde{\mathcal{M}}_{\xi(n)} - b_{\xi(n)}$, hence we get $\mathcal{M}_{n(n)} - (\tilde{\mathcal{M}}_{\xi(n)} - b_{\xi(n)}) \geq b_{\phi(n)}$. Meanwhile, by referring to line 8 in Algorithm 3, we have $p_{\phi(n)} \geq \mathcal{M}_{n(n)} - (\tilde{\mathcal{M}}_{\xi(n)} - b_{\xi(n)})$. Thus, we have $p_{\phi(n)} \geq b_{\phi(n)}$. Therefore, with the truthful asking price (i.e., $b_{\phi(n)} = v_{\phi(n)}$), we have $u_{\phi(n)} = p_{\phi(n)} - v_{\phi(n)} = p_{\phi(n)} - b_{\phi(n)} \geq 0$. That is, the utility of each AP is not a negative value. So our proposed incentive mechanism GSRBA is individual rational. Note that the property of truthfulness is presented in Lemma 4 as follows. \square

Lemma 4 (Truthfulness): The proposed mechanism GSRBA is truthful.

We prove the truthfulness property of GSRBA according to the theorem in [17], which states that an auction mechanism is truthful if and only if:

- The selection rule is monotone: If AP k wins the auction by asking b_k , it also wins by asking a price lower than b_k ;

- Each winner is paid the critical value: AP k would not win the auction if it asks a higher price than this value.

Therefore, the property of truthfulness can be proved by showing that the AP selection rule satisfies the monotonicity while the payment of each AP is the critical value in the proposed mechanism GSRBA.

(*Monotonicity*): The AP selection rule is monotone.

Proof: The monotonicity of the AP selection rule is obvious as asking a smaller bid price will lead to a larger value of marginal contribution minus ask price, which will push AP k backwards in the sorting and make AP k have a larger chance to be selected as a winning AP. That is, If AP k wins the auction by asking b_k , it also wins by asking a price lower than b_k . In other words, the proposed AP selection rule is monotonic. \square

(*The critical value of the payment*): The payment of each AP $k \in \mathcal{K}$ is the critical value for this AP.

Proof: We show that the payment p_k is the critical value for AP k in the sense that asking a price higher than p_k could prevent k from winning the auction.

Based on the payment determination rule, we have

$$p_{\phi(n)} = \max_{1 \leq m \leq l} \{\mathcal{M}_{n(m)} - (\widetilde{\mathcal{M}}_{\xi(m)} - b_{\xi(m)})\}.$$

If AP $\phi(n)$ asks a higher price $b_{\phi(n)} > p_{\phi(n)}$, we have $b_{\phi(n)} > \mathcal{M}_{n(m)} - (\widetilde{\mathcal{M}}_{\xi(m)} - b_{\xi(m)})$ for all $m \leq l$, which implies $\widetilde{\mathcal{M}}_{\xi(m)} - b_{\xi(m)} > \mathcal{M}_{n(m)} - b_{\phi(n)}$. So AP $\phi(n)$ will be sorted after the position l in the sequence of descending order of the marginal contribution minus ask price. Since the total number of winning APs is l , AP k , which is now at the position after l in the sorting sequence, will not be selected. That is, the payment $p_{\phi(n)}$ is the critical value of AP $\phi(n)$. \square

With the proof of monotonicity of the AP selection rule and the critical price of the payment, it is concluded that our proposed incentive mechanism GSRBA is truthful.

Lemma 5 (Computational Efficiency): The proposed GSRBA is computationally efficient.

Proof: The computational efficiency means that the AP selection and payment determination of the proposed mechanism GSRBA are solvable in polynomial time. Given that there are M MUs, K APs, and l winning APs. In the AP selection, the while-loop (lines 2-6 in Algorithm 2) executes l times. In each loop, finding an AP with the largest value of the marginal contribution minus ask price (line 3 in Algorithm 2) takes no more than $O(K)$ time, while calculating the marginal contribution of each AP takes no more than $O(lM)$ time. The computational complexity of AP selection is $O(l^2MK)$. In the payment determination process, the for-loop executes l times. In each for-loop, calculating the marginal contribution of each AP takes no more than $O(lM)$ time, and finding the AP with the largest marginal contribution minus ask price takes no more than $O(k)$ time. Therefore, the computational complexity of payment determination is $O(l^2MK)$. In summary, the computational complexity of the proposed mechanism GSRBA is $O(l^2MK)$, and the proposed GSRBA is computational efficient. \square

VI. PERFORMANCE EVALUATION

Extensive simulations have been conducted to evaluate the performance of the proposed mechanisms SRBA and GSRBA in terms of the property of truthfulness and the achieved system social welfare. We compare the proposed mechanisms with two counterparts: random selection and the incentive auction mechanism (denoted as IAM). With the random selection, BS randomly select a group of APs to join the mobile data offloading and the total number of selected APs is the same as that in the proposed mechanisms. With IAM, social influence of MUs is not considered, and BS selects APs only according to their own utilities.

In the simulation, the coverage of each AP is 150m, and the ask price of each AP is uniformly distributed over [180, 280]. $P_t/N_0 = 30dB$, $\alpha = 3$, $B = 20MHz$. In addition, the social link probability is 5%, and the number of selected APs is 5. In addition, we set the available capacity of each AP as 400 Mbps to simplify the system model.

A. TRUTHFULNESS

We verify the truthfulness property of SRBA and GSRBA by randomly choosing an AP and checking its utility with different bid prices.

Fig. 2 depicts the achieved utility of an AP with the true value of 201.34 versus different bid prices with SRBA. It is observed that the AP cannot achieve a higher utility by asking any price different from the true value, which demonstrates that the proposed SRBA satisfies the property of truthfulness.

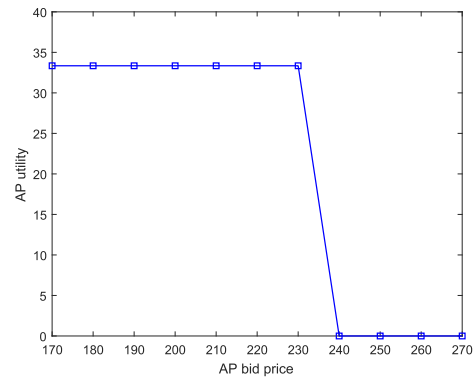


FIGURE 2. The utility of an AP with the true value $v_1 = 201.34$ with SRBA.

Fig. 3 depicts the achieved utility of an AP with the true value of 205.26 versus different bid prices with GSRBA. It is also observed that the AP cannot achieve a higher utility by asking a price different from its true value, which demonstrates that the proposed GSRBA satisfies the property of truthfulness.

B. SYSTEM SOCIAL WELFARE

Figs. 4-7 demonstrate the performance of the proposed mechanisms SRBA and GSRBA in terms of the achieved social welfare by comparing with IAM and randomly selection. Furthermore, considering that the collection of complete

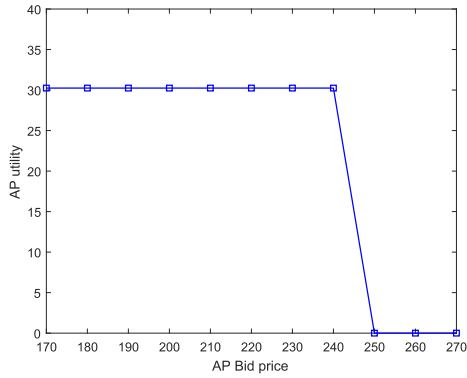


FIGURE 3. The utility of an AP with the true value $v_7 = 205.26$ with GSRBA.

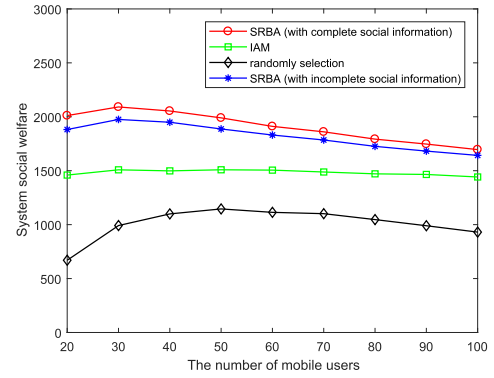


FIGURE 6. The achieved system social welfare with different number of MUs.

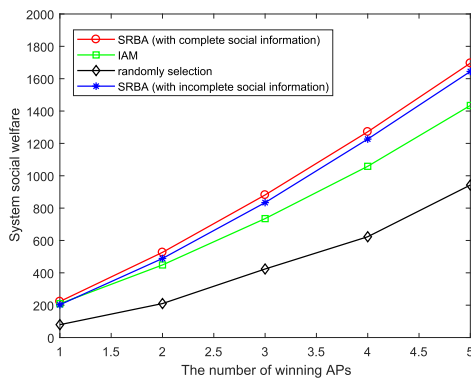


FIGURE 4. The system social welfare versus the number of winning APs.

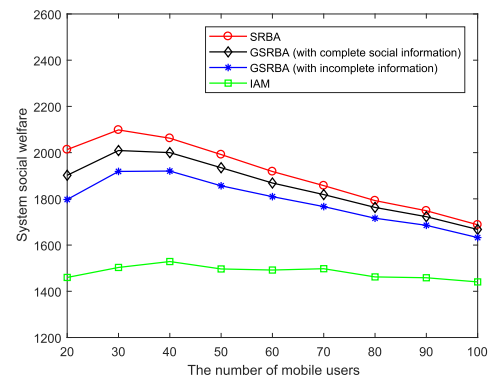


FIGURE 7. The achieved system social welfare with different number of MUs.

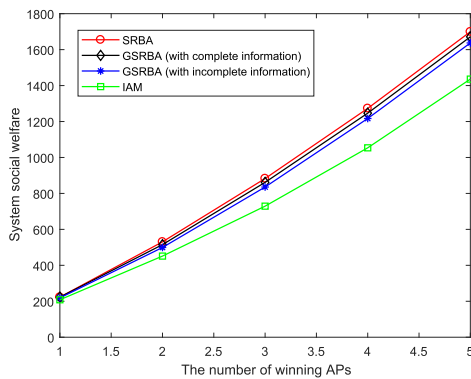


FIGURE 5. The system social welfare versus the number of winning APs.

information about the social relationship of MUs may be difficult due to various reasons. We further demonstrate the performance in the scenario of incomplete information. In specific, we assume 20% – 40% of information about the social relationship between MUs is missing. We compare the achieved social welfare with both complete information and incomplete information.

Figs. 4-5 depict the achieved system social welfare versus the number of winning APs. With more number of APs selected to participate in the mobile data offloading, a larger social welfare can be achieved. From Fig. 4, it is observed

that the proposed mechanism SRBA with both complete and incomplete information outperforms both IAM and the randomly selection. Meanwhile, from Fig. 5, it shows that the proposed GSRBA with both complete and incomplete information can achieve higher system social welfare compared with IAM.

Figs. 6-7 depict the achieved system social welfare versus the number of MUs. From Fig. 6, it is observed that the proposed mechanism SRBA with both complete and incomplete information outperforms both IAM and the randomly selection. Meanwhile, Fig. 7 shows that the proposed GSRBA with both complete and incomplete information can achieve larger social welfare compared with IAM.

From Figs. 6-7, it is also observed that the social welfare first increases with the increase of mobile users (e.g., 20-30), then slightly decrease with the increase of mobile users. When the number of MUs is small, each MU has few social links, and the level of their concern for a single friend is very high, then BS will try to select the set of APs such that both MUs and their friends can be served by APs. With the increase of MUs, each MU has more social links, and the level of their concern for a single friend is diluted. Meanwhile, an MU's friends may be more widely distributed and BS cannot make all of them be served by the selected APs, which makes the second term in (6) become small, and leads to the decrease

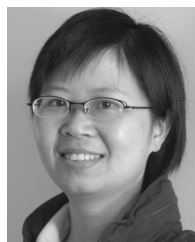
of the system social welfare. In addition, it is observed that the achieved system social welfare with GSRBA is very close to the optimal value achieved with SRBA.

VII. CONCLUSION

In this paper we have proposed two social-aware incentive mechanisms SRBA and GSRBA for achieving efficient mobile data offloading. By considering the social relationship of MUs, which is derived using a pageRank based algorithm, both SRBA and GSRBA can achieve higher system social welfare than the counterparts IAM and randomly selection. Meanwhile, GSRBA is computational efficiency with the performance approaching to SRBA. In addition, both SRBA and GSRBA satisfy the properties of individual rationality and truthfulness. Simulation results have been presented to demonstrate the nice performance of SRBA and GSRBA with both complete and incomplete information about the social relations of MUs.

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FEN HOU received the Ph.D. degree in electrical and computer engineering from the University of Waterloo, Waterloo, ON, Canada, in 2008. She was a Post-Doctoral Fellow with the Department of Electrical and Computer Engineering, University of Waterloo, and the Department of Information Engineering, The Chinese University of Hong Kong, from 2008 to 2009 and from 2009 to 2011, respectively. She is currently an Assistant Professor with the Department of Electrical and Computer Engineering, University of Macau. Her research interests include resource allocation and scheduling in broadband wireless networks, protocol design, and quality of service provisioning for multimedia communications in broadband wireless networks, mechanism design and optimal user behavior in mobile crowd sensing networks, and mobile data offloading. She was a recipient of the IEEE GLOBECOM Best Paper Award in 2010 and the Distinguished Service Award at the IEEE MMTC in 2011. She served as the Co-Chair for the ICCS 2014 Special Session on Economic Theory and Communication Networks, the INFOCOM 2014 Workshop on Green Cognitive Communications and Computing Networks, the IEEE GLOBECOM Workshop on Cloud Computing System, Networks, and Application from 2013 to 2014, the ICC 2015 Selected Topics in Communications Symposium, and the ICC 2016 Communication Software Services and Multimedia Application Symposium. She currently serves as the Vice-Chair (Asia) for the IEEE ComSoc Multimedia Communications Technical Committee. She serves as an Associate Editor for the *IET Communications*.



ZHANGYUAN XIE received the B.S. degree in electronic engineering from Tianjin University in 2014. He is currently pursuing the master's degree with the Department of Electrical and Computer Engineering, University of Macau. His research interests are in the area of mobile data offloading.

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