IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received July 10, 2018, accepted August 15, 2018, date of publication August 28, 2018, date of current version September 21, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2867558

Hybrid Precoding Design for MIMO System With One-Bit ADC Receivers

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This work was supported in part by the National Natural Science Foundation of China under Grant 61771345 and Grant 61571330, in part by the Fundamental Research Funds for the Central Universities, and in part by the Fund of the State Key Laboratory of Integrated Services Networks, Xidian University, under Project ISN19-01.

ABSTRACT When wireless transmission is performed over the bandwidth in the order of a gigahertz, high-resolution analog-to-digital converters (ADCs), and the large number of radio frequency chains significantly increase the power consumption. To address this issue, one promising technique is to use low-resolution, even one-bit ADCs. Another promising technique is to apply a hybrid precoding architecture to reduce the number of RF chains. In this paper, we propose to combine those techniques to reduce the hardware costs in multi-input multi-output system. Our objective is to optimize the hybrid precoder with the aim of increasing the achievable rate. To this end, we first derive an expression for the achievable rate in flat fading channels based on the Bussgang theorem, which is able to reformulate the nonlinear quantitative process as a linear function with identical first- and second-order statistics. To solve the non-convex hybrid precoding design problem, we treat the hybrid precoding design as a matrix factorization problem, which can be solved with an efficient alternating minimization algorithm. That is, we solve the digital precoder and the analog precoder in an alternative way in two separate subproblems. To find the optimal precoder in the first subproblem, we first prove the optimal structure of the digital precoding matrix. With it, we transfer the digital precoding design to a power allocation problem, the closed-form solution of which is then optimally found by using Karush–Kuhn–Tucker conditions. In the second subproblem, due to the non-convex modulus-norm constraint, it is challenging to directly solve the analog precoder. To resolve this problem, we propose to optimize the phases in the analog precoding matrix and adopt the subgradient algorithm to find the local optimal solution. Our simulation results show that the proposed hybrid precoding design effectively improves the achievable rates.

INDEX TERMS Multi-input multi-output, analog-to-digital converter, one-bit quantization, hybrid precoding, alternating minimization.

I. INTRODUCTION

The capacity of wireless communication system has to exponentially increase to meet the explosive demands for high-data-rate multimedia access [1]. Multi-input multi-output (MIMO) system plays a significant role in 5G networks as an efficient way in improving the system spectral efficiency and transmission reliability [2], [3]. Based on the knowledge of information theory, the larger the number of antennas is, the more significant improvement on spectral efficiency and transmission reliability it brings. Unfortunately, the large number of antennas brings a great challenge for hardware costs and power consumption, which makes the high-resolution analog-to-digital converters (ADCs) and the fully-digital precoding solution allocating a radio frequency (RF) chain per antenna difficult to implement in practical applications [4], [5]. New architectures relaxing the requirement of associating an RF chain per antenna need to be developed to overcome this challenge. The promising solutions for hybrid digital/analog precoding architectures [6]–[10] and one-bit ADCs [11]–[14] have gained much interest recently. However, these two solutions are on behalf of two extreme cases of decreasing either the number of ADC bits or the number of RF chains, which cannot save hardware costs or power consumption effectively. The objective of this paper is to explore the combination of the two architectures.

According to [8], the classical full-digital precoding architecture can achieve the optimal antenna gain. However, this kind of architecture requires the same number of RF chains as that of antennas, which is costly for MIMO systems [10]. As a result, the hybrid digital/analog precoding architectures have received much attention recently since they only require a much smaller number of RF chains connecting a low-dimensional digital precoder with a high-dimensional analog precoder [15]. It is noted that the high-dimensional analog precoder is still impractical to be implemented in the RF domain due to the power-hungry variable voltage amplifiers [16]. A solution for this is to realize analog precoders with low-cost phase shifters by sacrificing the magnitude changing ability of the RF signals. In accordance with the mapping relationship between RF chains and antennas, which determines the number of the needed phase shifters, the hybrid precoding architectures can be categorized into fully-connected and partially-connected structures [7], [9]. In [7], for the fully-connected structure, an alternating minimization algorithm based on manifold optimization is proposed to design the hybrid precoding architecture. In [9], for the partially-connected structure, the transmitter design problem is transformed into a single-stream optimal transmitter beamforming design problem with per-antenna power constraint, and the optimal design is achieved using the water-filling algorithm. In this paper, in order to guarantee the system performance, we adopt the fully-connected hybrid precoding architecture.

An alternative to high-resolution ADCs is to employ ultra low-resolution ADCs, i.e. 1-3 bits. As the power cost exponentially increases with the number of antennas, using low-resolution ADCs is able to highly reduce the power penalty. As is proven in [13], the power consumption caused by the one-bit quantitative process is approximately equal to $\frac{2}{\pi}$ (1.96dB) in the low signal-to-noise ratio (SNR) region. On the other hand, at high SNRs one-bit quantitative process can produce a large capacity loss [14], but there is a reason to believe that for improved energy efficiency and exploiting array gain, MIMO systems will operate at relatively low SNRs to overcome the capacity loss at high SNRs [12]. Using low-resolution ADCs can greatly modify the physical signs and the corresponding design in the low-resolution scenario is generally more challenging. For example, the optimal input distribution has not been found a case with more than two-bit ADCs. To make an effort to the progress in the direction, in this paper, we focus on one-bit ADCs consisting of a simple comparator, which consumes negligible power. Since one-bit ADCs require no automatic gain control, the cost and power consumption for the corresponding RF chains can be very low. The exact capacity of quantized MIMO scenario is generally unknown at present, except for some simple channel models, like multiple-input single-output (MISO) channel and some special cases [11], [12], [17]. In [12], the lower bound of the achievable rate is obtained under the assumption of a maximum-ratio combining (MRC) receiver. In [17] and [18], the achievable rate for hybrid architectures with few-bit ADCs is derived with the transmission methods based on channel inversion (CI) and singular value decomposition (SVD). The analog precoding matrix is assumed to be composed of columns from DFT matrices, but the optimality of the precoders is not proven.

This paper considers an architecture of MIMO system with hybrid analog/digital precoding and one-bit ADCs. The hybrid analog/digital precoding architecture is composed of a digital precoder and an analog precoder. In our system, two one-bit quantizers separately quantize the real and imaginary part of each received signal at each receive antenna.

In current work, there is still much room for the improvement in the achievable rate of the MIMO system with hybrid analog/digital precoding and one-bit ADCs. According to the existing literature, the studied hybrid architecture and one-bit ADC receiver architecture represent two extremes, namely the hybrid precoding architecture employed a small number of RF chains but with high-resolution ADCs and one-bit ADC receivers with the assumption of the number of RF chains equal to that of antennas. In this paper, we investigate the hybrid precoding architecture with one-bit ADC receivers and design the hybrid precoding with an aim to maximize the achievable rate of the system. Moreover, since the analog precoding is implemented with analog phase shifters, the optimization problem is with a unit-norm constraint. The existing methods for solving the unit-norm constraint optimization problem are mainly based on a Matlab toolbox Manopt [7], [19], [20], which is not suitable for some complex objective functions. Other literature deals with the unit-norm optimization problem based on a DFT matrix assumption [17], [18], but the optimality of the precoders is not proven. In this paper, we propose a new method to solve the unit-norm constraint optimization problem suitable for both simple and complex objective functions. In our work, the contributions are listed as follows.

- We derive an expression of the achievable rate in flat fading channels according to the Bussgang theorem [21]. The advantage of the Bussgang theorem is that it is able to approximate the nonlinear quantitative process as a linear function with identical first- and second-order statistics.
- To solve the non-convex hybrid precoding design problem, we treat the hybrid precoding design as a matrix factorization problem, which can be solved with an efficient alternating minimization algorithm. That is, we solve the digital precoder and the analog precoder in an alternative way in two separate subproblems.
- In the process of optimizing the digital precoding matrix, with the analog precoding matrix fixed, we first prove the optimal structure of the digital precoding matrix using the SVD method. With it, we treat the digital precoding design as a power allocation problem, which is then optimally solved by Karush-Kuhn-Tucker (KKT) conditions.

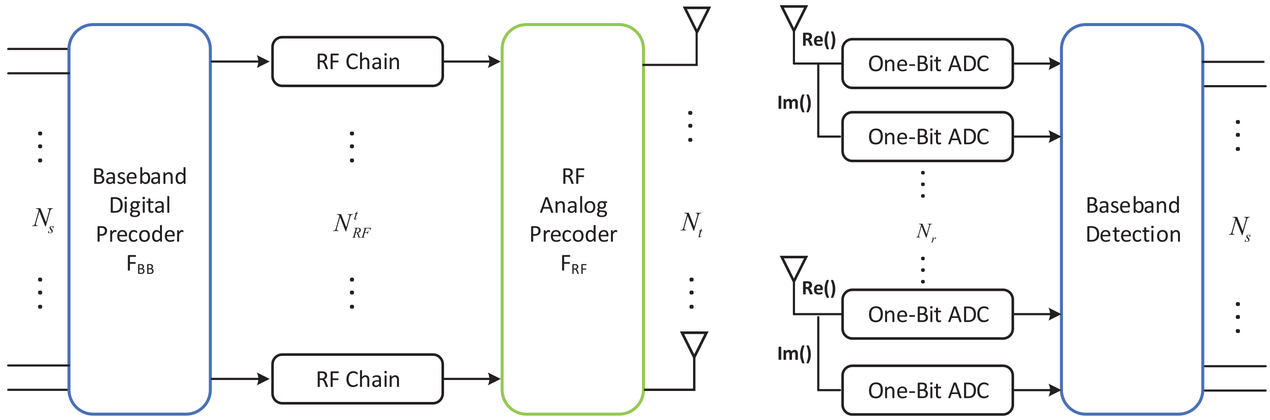

FIGURE 1. System model.

- In the second subproblem, due to the non-convex modulus-norm constraint, directly solving the analog precoder is challenging. To resolve this problem, we propose to optimize the phases in the analog precoding matrix and adopt the subgradient algorithm to find the local optimal solution.
- Simulation results reveal that the performance of the channel can be significantly improved compared with the existing works.

Notations: The notations below are used throughout this paper. a, $\mathbf{a}$ and $\mathbf{A}$ respectively stand for a scalar, a vector and a matrix. The set of complex numbers is denoted as $\mathbb{C}$. The conjugate, transpose and Hermitian transpose of $\mathbf{A}$ are denoted as $\mathbf{A}^*$, $\mathbf{A}^T$ and $\mathbf{A}^H$, respectively. The Frobenius norm and the ℓ_2-norm are denoted as $\|\cdot\|_F$ and $\|\cdot\|_2$, respectively. A diagonal matrix containing only the diagonal elements of $\mathbf{A}$ is denoted as diag($\mathbf{A}$). $\mathbf{a} \sim \mathcal{CN}(\mathbf{m}, \mathbf{N})$ indicates that $\mathbf{a}$ is a complex Gaussian vector with mean $\mathbf{m}$ and convariance matrix $\mathbf{N}$. $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary part of a complex variable. The expected value of a random variable is denoted as $\mathbb{E}(\cdot)$. We define $(x)^\dagger \triangleq \max(0, x)$.

The rest of this paper is organized as follows. In Section II, we present the system model of the hybrid precoding architecture with one-bit ADC receivers in MIMO system. The expression for the achievable rate is proposed in Section III. In Section IV, we analyze the hybrid precoding design by optimizing the achievable rate of the system. Simulation results are shown in Section V, followed by the conclusion in Section VI.

II. SYSTEM MODEL

In this section, we present the system model of the considered point-to-point MIMO system. We assume that the low-resolution ADCs are one-bit ADCs, and the hybrid analog/digital precoding is assumed as shown in Fig. 1. In this paper, the transmitter and receiver are respectively equipped with N_t and N_r antennas. With the hybrid precoding architecture, the transmitter is assumed to have N_{RF}^t RF chains, which satisfy $N_{RF}^t \leq N_t$. N_s data streams are sent and collected by transmit and receive antennas, satisfying $N_s \leq N_{RF}^t$.

In terms of different signal mapping strategies between RF chains and antennas, the transmitter architecture can be categorized into the fully-connected and partially-connected hybrid precoding structures. The output signal of each RF chain is sent to all the antennas through phase shifters for the fully-connected structure, while for the partially-connected structure, only partial antennas are connected to each RF chain. In this paper, in order to guarantee the system performance, we adopt the fully-connected hybrid precoding architecture, enjoying full beamforming gains by connecting each RF chain with all antennas.

Assuming a flat fading channel and perfect synchronization, the signal at the receive antenna is

$$\mathbf{y} = \mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{y} \in \mathbb{C}^{N_s \times 1}$ is the received signal before one-bit quantization, $\mathbf{s} \in \mathbb{C}^{N_s \times 1}$ is the digital baseband signal with the covariance $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \frac{P_t}{N_s}\mathbf{I}_{N_s}$, and P_t is the transmission power, $\mathbf{n} \sim \mathcal{CN}(0, \mathbf{I}_{N_r})$ is the white Gaussian noise, $\mathbf{H} \in \mathbb{C}^{N_s \times N_t}$ is the flat fading channel. The hybrid precoding architecture consists of an analog RF precoder $\mathbf{F}_{RF} \in \mathbb{C}^{N_t \times N_{RF}^t}$ and a digital baseband precoder $\mathbf{F}_{BB} \in \mathbb{C}^{N_{RF}^t \times N_s}$, satisfying the power constraint $\|\mathbf{F}_{RF}\mathbf{F}_{BB}\|_F^2 \leq P_t$.

Moreover, the analog precoders are implemented with phase shifters, which can only adjust the phases of the signals. As a result, all the elements of $\mathbf{F}_{RF}$ should satisfy the unit-modulus constraint, that is $|[\mathbf{F}_{RF}]_{m,n}| = 1$.

The received signal after the one-bit quantization can be expressed as

$$\begin{aligned} \mathbf{r} &= \mathcal{Q}(\mathbf{y}) \\ &= \mathcal{Q}(\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{s} + \mathbf{n}), \end{aligned} \tag{2}$$

where $\mathcal{Q}(\cdot)$ is the one-bit quantization function which applies to component-wise and separately to the real and imaginary parts as $\mathcal{Q} = \frac{1}{\sqrt{2}}(\text{sign}(\Re(\cdot)) + j\text{sign}(\Im(\cdot)))$. The outcome of the one-bit quantization lies in the set $\mathcal{R} = \frac{1}{\sqrt{2}}\{1 + 1j, 1 - 1j, -1 + 1j, -1 - 1j\}$, which includes without loss of generality a scaling factor so that the power of each quantized signal is one.

III. ACHIEVABLE RATE IN ONE-BIT MASSIVE MIMO SYSTEMS

In this section, we derive an expression of the achievable rate in flat fading channels based on the Bussgang theorem, reformulating the nonlinear quantitative process as a linear function with identical first- and second-order statistics.

A. BUSSGANG DECOMPOSITION

Although the one-bit quantitative process is a nonlinear operation, we can transfer it into an equivalent linear operation using the Bussgang theorem, which obtains a statistically equivalent linear operator for any nonlinear function of a Gaussian signal. Particularly, for the one-bit quantizer in (2), the Bussgang theorem can be written as

$$\begin{aligned}\mathbf{r} &= \mathbf{A}\mathbf{y}+\mathbf{q}\\ &= \mathbf{A}(\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{s}+\mathbf{n})+\mathbf{q}\\ &= \mathbf{A}\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{s}+\tilde{\mathbf{n}}, \end{aligned} \tag{3}$$

where $\mathbf{A}$ is the linear operator and $\mathbf{q}$ is the statistically equivalent quantizer noise, and the total noise $\tilde{\mathbf{n}} = \mathbf{A}\mathbf{n}+\mathbf{q}$ is composed of Gaussian noise and quantizer noise.

According to the total noise $\tilde{\mathbf{n}} = \mathbf{A}\mathbf{n}+\mathbf{q}$, we can observe that the quantizer noise $\mathbf{q}$ is related to the matrix $\mathbf{A}$. A particularly meaningful choice for $\mathbf{A}$ is the one minimizing the power of the quantizer noise $\mathbf{q}$, or equivalently, yielding $\mathbf{q}$ uncorrelated with $\mathbf{y}$. This value of $\mathbf{A}$ is the result of

$$\arg\min_{\mathbf{A}} \mathbf{E}\{\|\mathbf{q}\|_2^2\} = \arg\min_{\mathbf{A}} \mathbf{E}\{\|\mathbf{r}-\mathbf{A}\mathbf{y}\|_2^2\}, \tag{4}$$

whose solution is given by

$$\mathbf{A} = \mathbf{R}_{\mathbf{yr}}^H\mathbf{R}_{\mathbf{y}}^{-1}, \tag{5}$$

where $\mathbf{R}_{\mathbf{yr}}$ denotes the cross-correlation matrix between the received signal after the hybrid precoding architecture $\mathbf{y}$ and the quantized signal $\mathbf{r}$, and $\mathbf{R}_{\mathbf{y}}$ denotes the auto-correlation matrix of the received signal after the hybrid precoding $\mathbf{y}$. With one-bit quantization and Gaussian signals, $\mathbf{R}_{\mathbf{yr}}$ is given by

$$\mathbf{R}_{\mathbf{yr}} = \sqrt{\frac{2}{\pi}}\mathbf{R}_{\mathbf{y}}\mathrm{diag}(\mathbf{R}_{\mathbf{y}})^{-\frac{1}{2}} \tag{6}$$

according to [12].

Substituting (6) into (5), we can get the expression of $\mathbf{A}$, whose value is only related to that of $\mathbf{R}_{\mathbf{y}}$

$$\mathbf{A} = \sqrt{\frac{2}{\pi}}\mathrm{diag}(\mathbf{R}_{\mathbf{y}})^{-\frac{1}{2}}. \tag{7}$$

According to (7), in order to apply the Bussgang theorem, the covariance matrix $\mathbf{R}_{\mathbf{y}}$ of the quantizer input must be known at the base station. However, we can use the same technique proposed in [22] in practice to reconstruct the covariance matrix $\mathbf{R}_{\mathbf{y}}$ using the measurements of the quantizer input. Additionally, relying on the channel hardening in MIMO systems and for independent identically distributed unit-variance channel coefficients [12], [23], [24], the matrix $\mathbf{A}$ can be approximated as

$$\mathbf{A} \cong \sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{1+P_t}}\mathbf{I}_{N_r} = \alpha\mathbf{I}_{N_r}, \tag{8}$$

where $\alpha = \sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{1+P_t}}$. It is noted this approximation does not require perfect channel state information (CSI).

B. ACHIEVABLE RATE

To obtain the expression of achievable rate, which is related to the auto-correlation of the total noise $\tilde{\boldsymbol{n}}$, we need to derive the auto-correlation of the quantizer noise $\mathbf{R}_{\mathbf{q}}$ correlated with the auto-correlation of the received signal after one-bit quantization $\mathbf{R}_{\mathbf{r}}$.

As shown in [25], for one-bit ADCs, the arcsine law can be used to obtain the auto-correlation expression of $\mathbf{r}$, which is given by

$$\begin{aligned}\mathbf{R}_{\mathbf{r}} = \frac{2}{\pi}&(\arcsin(\mathrm{diag}(\mathbf{R}_{\mathbf{y}})^{-\frac{1}{2}}\Re(\mathbf{R}_{\mathbf{y}})\mathrm{diag}(\mathbf{R}_{\mathbf{y}})^{-\frac{1}{2}})\\ &+j\arcsin(\mathrm{diag}(\mathbf{R}_{\mathbf{y}})^{-\frac{1}{2}}\Im(\mathbf{R}_{\mathbf{y}})\mathrm{diag}(\mathbf{R}_{\mathbf{y}})^{-\frac{1}{2}})).\end{aligned} \tag{9}$$

Although the quantizer noise is not Gaussian distributed in general, since the Gaussian case corresponds to the worst case as additive noise that minimizes the input-output mutual information, we can obtain the achievable rate by modeling the quantizer noise as Gaussian distributed. Thus the achievable rate can be obtained by modeling the quantizer noise $\mathbf{q}$ as white Gaussian noise with the covariance matrix:

$$\begin{aligned}\mathbf{R}_{\mathbf{q}} &= \mathbf{R}_{\mathbf{r}} - \mathbf{A}\mathbf{R}_{\mathbf{y}}\mathbf{A}^H\\ &= \frac{2}{\pi}(\arcsin(\mathbf{X})+j\arcsin(\mathbf{Y})) - \frac{2}{\pi}(\mathbf{X}+j\mathbf{Y}),\end{aligned} \tag{10}$$

where we define

$$\mathbf{X} = \mathrm{diag}(\mathbf{R}_{\mathbf{y}})^{-\frac{1}{2}}\Re(\mathbf{R}_{\mathbf{y}})\mathrm{diag}(\mathbf{R}_{\mathbf{y}})^{-\frac{1}{2}}, \tag{11}$$

$$\mathbf{Y} = \mathrm{diag}(\mathbf{R}_{\mathbf{y}})^{-\frac{1}{2}}\Im(\mathbf{R}_{\mathbf{y}})\mathrm{diag}(\mathbf{R}_{\mathbf{y}})^{-\frac{1}{2}}. \tag{12}$$

As shown in (10) that $\mathbf{R}_{\mathbf{q}}$ is not a diagonal matrix in general, implying that there is correlation between the quantizer noise on each antenna. However, $\mathbf{R}_{\mathbf{y}}$ is diagonally dominant under the assumption of this paper [23] and we can obtain the following approximation by applying the arcsine law:

$$\frac{2}{\pi}\arcsin(a) = \begin{cases}1, & a=1\\ \frac{2a}{\pi}, & a<1.\end{cases} \tag{13}$$

The non-diagonal elements of $\mathbf{X}$ and $\mathbf{Y}$ are much smaller than 1, thus we can approximate (10) as

$$\mathbf{R}_{\mathbf{q}} \cong (1-\frac{2}{\pi})\mathbf{I}_{N_r}, \tag{14}$$

implying that the quantizer noise can be approximated as uncorrelated noise with a variance of $1-\frac{2}{\pi}$.

Again, based on the Bussgang theorem and the assumption of a Gaussian input, we can get the auto-correlation of $\mathbf{y}$:

$$\mathbf{R}_{\mathbf{y}} = \frac{P_t}{N_s}\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^H\mathbf{F}_{RF}^H\mathbf{H}^H + \mathbf{I}_{N_r}. \tag{15}$$

Moreover, the quantizer noise $\mathbf{q}$ is brought by the quantization process, while the white Gaussian noise $\mathbf{n}$ is introduced by the channel environment. Thus, $\mathbf{q}$ is uncorrelated with $\mathbf{n}$. Therefore, using $\mathbf{A}$ as in (8) according to the Bussgang theorem, we can get the covariance matrix of $\tilde{\boldsymbol{n}}$ by

$$\begin{aligned}\mathbf{R}_{\tilde{n}} &= \mathbf{A}\mathbf{A}^H + \mathbf{R}_{\mathbf{q}}\\ &\cong (\alpha^2 + 1 - \frac{2}{\pi})\mathbf{I}_{N_r}.\end{aligned} \tag{16}$$

Thus the expression of the achievable rate for the considered single-user MIMO system with hybrid analog/digital precoding architecture deploying one-bit ADCs can be obtained as the form of

$$C = \log_2|\mathbf{I} + \frac{P_t}{N_s}\mathbf{R}_{\tilde{\boldsymbol{n}}}^{-1}\mathbf{A}\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^H\mathbf{F}_{RF}^H\mathbf{H}^H\mathbf{A}^H|. \tag{17}$$

After substituting (8), (15) and (16) into (17), the achievable rate can be obtained as

$$C = \log_2|\mathbf{I} + \frac{P_t}{N_s}\frac{\pi\alpha^2}{\pi\alpha^2 + \pi - 2}\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^H\mathbf{F}_{RF}^H\mathbf{H}^H|. \tag{18}$$

IV. PROBLEM FORMULATION

In this section, with the aim of increasing the achievable rate, we formulate the hybrid precoder design optimization problem to design the digital baseband precoder $\mathbf{F}_{BB}$ and the analog precoder $\mathbf{F}_{RF}$.

Since the analog precoders are implemented with analog phase shifters, which only adjust the phases of the signals, each element of the analog precoding matrix $\mathbf{F}_{RF}$ is limited to have the same norm. Without loss of generality, we assume that the elements of analog precoding matrix have unit norm, i.e., $|[\mathbf{F}_{RF}]_{m,n}| = 1$. Moreover, the hybrid architecture must satisfy the power constraint, which can be expressed as $\|\mathbf{F}_{RF}\mathbf{F}_{BB}\|_F^2 \leq P_t$.

The optimization problem is to optimize the digital precoding matrix $\mathbf{F}_{BB}$ and the analog precoding matrix $\mathbf{F}_{RF}$ maximizing the achievable rate as follows

$$\begin{aligned}\max_{\mathbf{F}_{BB},\mathbf{F}_{RF}} \quad & C\\ \text{subject to } & |[\mathbf{F}_{RF}]_{m,n}| = 1,\\ & \|\mathbf{F}_{RF}\mathbf{F}_{BB}\|_F^2 \leq P_t.\end{aligned} \tag{19}$$

It is noted that the optimization problem given in (19) is non-convex. The reason is twofold. The first one is that the objective function is non-convex as it includes the product of two matrix variables $\mathbf{F}_{BB}$ and $\mathbf{F}_{RF}$. The second one is the non-convex unit-modulus constraint. The element-wise unit-modulus constraints of $\mathbf{F}_{RF}$ provide a great challenge for optimizing the two matrix variables jointly. In order to develop a feasible algorithm to find the solution, we propose to decompose the optimization problem into two subproblems. In the first subproblem, we optimize the digital baseband precoder $\mathbf{F}_{BB}$ with given analog precoder $\mathbf{F}_{RF}$. While in the second subproblem, we optimize the analog precoder $\mathbf{F}_{RF}$. Moreover, we can treat the optimization problem as a matrix factorization problem involving two matrix variables $\mathbf{F}_{BB}$ and $\mathbf{F}_{RF}$, for which the alternating minimization can be adopted as an efficient method. Alternating minimization method plays a significant role in the optimization problems regarding to different subsets of variables by decoupling the optimization problem of these two variables. With the principle of alternating minimization algorithm, the $\mathbf{F}_{BB}$ and $\mathbf{F}_{RF}$ matrix will be optimized alternatively while fixing the other, which is the essential idea throughout this paper.

A. DIGITAL BASEBAND PRECODER DESIGN

We first consider to design the digital baseband precoding matrix $\mathbf{F}_{BB}$ with the analog precoding matrix $\mathbf{F}_{RF}$ fixed. Thus, the optimization problem concerning $\mathbf{F}_{BB}$ can be reformulated as

$$\begin{aligned}&\max_{\mathbf{F}_{BB}} \ C\\ &\text{subject to } \operatorname{trace}\{\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^H\mathbf{F}_{RF}^H\} \leq P_t.\end{aligned} \tag{20}$$

Here, we define $\mathbf{M} = \mathbf{F}_{RF}^H\mathbf{H}^H\mathbf{H}\mathbf{F}_{RF}$. To reduce the optimization complexity, we first derive the optimal structure of $\mathbf{F}_{BB}$. It is shown that with the obtained structure, the precoding design problem can be transferred into a simple power allocation problem.

Lemma 1: The optimal structure of the baseband digital precoding matrix $\mathbf{F}_{BB}$ *is* $\mathbf{F}_{BB} = \mathbf{M}^{-\frac{1}{2}}\mathbf{U}\boldsymbol{\Sigma}^{\frac{1}{2}}$, *where* $\mathbf{U}$ *is a para-unitary matrix which can be obtained by the singular value decomposition of* $\mathbf{M}^{-\frac{1}{2}}\mathbf{F}_{RF}^H\mathbf{F}_{RF}\mathbf{M}^{-\frac{1}{2}}$ *and* $\boldsymbol{\Sigma}$ *is a diagonal matrix.*

Proof: For (20), without loss of generality [26], we make the assumption that the optimal $\mathbf{F}_{BB}$ makes the term of $\mathbf{F}_{BB}^H\mathbf{M}\mathbf{F}_{BB}$ diagonal. Otherwise, we can also multiply $\mathbf{F}_{BB}$ with a unitary matrix, i.e. the eigenvector matrix of $\mathbf{F}_{BB}^H\mathbf{M}\mathbf{F}_{BB}$ at the right side, to make $\mathbf{F}_{BB}^H\mathbf{M}\mathbf{F}_{BB}$ diagonal, which does not affect the objective function value or the power constraint. Thus, by assuming $\mathbf{F}_{BB}^H\mathbf{M}\mathbf{F}_{BB} = \boldsymbol{\Sigma}$, where $\boldsymbol{\Sigma}$ is a diagonal matrix with the diagonal elements arranged in increasing order. Thus we can obtain $\mathbf{M}^{\frac{1}{2}}\mathbf{F}_{BB} = \mathbf{U}\boldsymbol{\Sigma}^{\frac{1}{2}}$, where $\mathbf{U} \in \mathbb{C}^{N_{RF}^t \times N_s}$ is a para-unitary matrix satisfying $\mathbf{U}^H\mathbf{U} = \mathbf{I}_{N_s}$. Then, we can obtain the optimal structure of the digital precoding matrix $\mathbf{F}_{BB}$ as follows

$$\mathbf{F}_{BB} = \mathbf{M}^{-\frac{1}{2}}\mathbf{U}\boldsymbol{\Sigma}^{\frac{1}{2}}. \tag{21}$$

Substituting the optimal $\mathbf{F}_{BB}$ expression (21) into the power constraint (20) yields

$$\begin{aligned}&\operatorname{trace}\{\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^H\mathbf{F}_{RF}^H\}\\ &= \operatorname{trace}\{\mathbf{F}_{BB}^H\mathbf{F}_{RF}^H\mathbf{F}_{RF}\mathbf{F}_{BB}\}\\ &= \operatorname{trace}\{\boldsymbol{\Sigma}^{\frac{1}{2}}\mathbf{U}^H\mathbf{M}^{-\frac{1}{2}}\mathbf{F}_{RF}^H\mathbf{F}_{RF}\mathbf{M}^{-\frac{1}{2}}\mathbf{U}\boldsymbol{\Sigma}^{\frac{1}{2}}\}\\ &\geq \operatorname{trace}\{\boldsymbol{\Sigma}\tilde{\boldsymbol{\Sigma}}'\},\end{aligned} \tag{22}$$

where the SVD of matrix $\mathbf{M}^{-\frac{1}{2}}\mathbf{F}_{RF}^{H}\mathbf{F}_{RF}\mathbf{M}^{-\frac{1}{2}}$ can be denoted as the form of $\mathbf{M}^{-\frac{1}{2}}\mathbf{F}_{RF}^{H}\ \mathbf{F}_{RF}\mathbf{M}^{-\frac{1}{2}} = \tilde{\boldsymbol{U}}\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{U}}^{H}$. $\tilde{\boldsymbol{\Sigma}}$ is a diagonal singular value matrix with the diagonal elements being the singular values arranged in increasing order. $\tilde{\boldsymbol{U}}$ denotes the unitary singular matrix with the columns being the singular vectors of matrix $\mathbf{M}^{-\frac{1}{2}}\mathbf{F}_{RF}^{H}\mathbf{F}_{RF}\mathbf{M}^{-\frac{1}{2}}$. We choose the N_s largest singular values arranged in decreasing order as the diagonal elements of the diagonal matrix $\tilde{\boldsymbol{\Sigma}}' \in \mathbb{C}^{N_s\times N_s}$. We define $\tilde{\boldsymbol{\Sigma}}'$ as

$$\tilde{\boldsymbol{\Sigma}}' = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_{N_s} \end{bmatrix} \tag{23}$$

The inequality is obtained by using the fact that trace$(\mathbf{AB}) \geq$ trace$(\Lambda_{\mathbf{A}}\Lambda_{\mathbf{B}})$, where $\Lambda_{\mathbf{A}}$ is a diagonal matrix with the singular values of $\mathbf{A}$ as its diagonal matrix arranged in increasing order, and $\Lambda_{\mathbf{B}}$ is a diagonal matrix with the singular values of $\mathbf{B}$ as its diagonal elements arranged in decreasing order. It is noted that the equality in (22) holds when $\mathbf{U} = \tilde{\boldsymbol{U}}'$, where $\tilde{\boldsymbol{U}}'$ is the matrix corresponding with the singular values in $\tilde{\boldsymbol{\Sigma}}'$. Thus, we conclude that the optimal precoding matrix for $\mathbf{F}_{BB}$ is $\mathbf{M}^{-\frac{1}{2}}\tilde{\boldsymbol{U}}'\boldsymbol{\Sigma}^{\frac{1}{2}}$. ■

To solve $\boldsymbol{\Sigma}$, we can formulate another optimization problem. First, we define $\boldsymbol{\Sigma}$ as

$$\boldsymbol{\Sigma} = \begin{bmatrix} \lambda_{q,1} & & & \\ & \lambda_{q,2} & & \\ & & \ddots & \\ & & & \lambda_{q,N_s} \end{bmatrix} \tag{24}$$

Thus, the optimization problem regarding variables $\lambda_{q,i}$, $\forall i$ can be expressed as

$$\begin{aligned} \min_{\lambda_{q,i}} \quad & -\sum_{i=1}^{N_s}\log_2(1+\gamma\lambda_{q,i}) \\ \text{subject to } & \lambda_{q,i} \geq 0, \\ & \sum_{i=1}^{N_s}\lambda_{q,i}\lambda_i \leq P_t. \end{aligned} \tag{25}$$

Lemma 2: Optimization problem (25) is convex. The i-th diagonal element is denoted as $\lambda_{q,i}$, thus the optimal $\lambda_{q,i}$ is given by

$$\lambda_{q,i} = [\frac{1}{\lambda_i\beta\ln 2} - \frac{1}{\gamma}]^{\dagger}, \tag{26}$$

where $\gamma = \frac{\pi\alpha^2}{\pi\alpha^2+\pi-2}$ and β is the Lagrangian multiplier.

Proof: In (25), it is seen that the constraint is linear with respect to the λ_i, $\forall i$, which is convex. Thus, to prove the convexity of the problem (25), we only need to prove that the objective function is concave. By defining $\gamma = \frac{\pi\alpha^2}{\pi\alpha^2+\pi-2}$, the objective function can be expressed as compositional function given as $r = f(g(\lambda_{q,i}))$ with $f(x) = \log_2(x)$ and $g(\lambda_{q,i}) = 1+\gamma\lambda_{q,i}$. Since $f(x)$ is a non-decreasing concave function and $g(\lambda_{q,i})$ is a concave function with respect to $\lambda_{q,i}$, we can prove that the compositional function $f(g(\lambda_{q,i}))$ is concave [27].

Due to the convexity of problem (25), the optimal solution can be found by using KKT conditions. Here, we first formulate the Lagrangian function of (25), which is given by

$$\mathcal{L} = -\sum_{i=1}^{N_s}\log_2(1+\gamma\lambda_{q,i}) + \beta(\sum_{i=1}^{N_s}\lambda_{q,i}\lambda_i - P_t) - \sum_{i=1}^{N_s}\beta_i\lambda_{q,i}, \tag{27}$$

where β and β_i are Lagrangian multipliers. Then, the resultant set of KKT conditions can be obtained as

$$\frac{\partial\mathcal{L}}{\partial\lambda_{q,i}} = -\frac{\gamma}{1+\gamma\lambda_{q,i}}\cdot\frac{1}{\ln 2} + \beta\lambda_i - \beta_i = 0 \tag{28a}$$

$$\beta(\sum_{i=1}^{N_s}\lambda_{q,i}\lambda_i - P_t) = 0 \tag{28b}$$

$$\beta_i\lambda_{q,i} = 0 \tag{28c}$$

$$\beta \geq 0 \tag{28d}$$

$$\beta_i \geq 0, \quad \forall i. \tag{28e}$$

Multiplying (28a) by $\lambda_{q,i}$, we have

$$\lambda_{q,i}(\lambda_i\beta - \frac{1}{\ln 2}\frac{\gamma}{1+\gamma\lambda_{q,i}}) = \lambda_{q,i}\beta_i = 0. \tag{29}$$

To satisfy (29), we consider the following two cases:
Case 1: If $\beta > \frac{\gamma}{\lambda_i\ln 2}$, we must have $\lambda_{q,i} = 0$.
Case 2: Else, if $\beta \leq \frac{\gamma}{\lambda_i\ln 2}$, as $\beta_i \geq 0$, we must have $\lambda_{q,i} > 0$. Furthermore, we have $\beta = \frac{1}{\lambda_i\ln 2}\frac{\gamma}{1+\gamma\lambda_{q,i}}$. In Case 2, the optimal $\lambda_{q,i}$ can be expressed by

$$\lambda_{q,i} = \frac{1}{\lambda_i\beta\ln 2} - \frac{1}{\gamma}. \tag{30}$$

By combining Case 1 and Case 2, we can obtain the final solution of $\lambda_{q,i}$ by

$$\lambda_{q,i} = [\frac{1}{\lambda_i\beta\ln 2} - \frac{1}{\gamma}]^{\dagger}. \tag{31}$$

■

Then, we must solve the value of the Lagrangian multiplier β. This should be solved by the bisection method and we should get the lower and upper bound of β. It is the constraint in (28d) that β must be larger than zero, so we choose zero as the lower bound of β. Moreover, β must be chosen to satisfy $\sum_{i=1}^{N_s}\lambda_{q,i}\lambda_i = P_t$. Then, we get the constraint of β respect to λ_i from

$$\sum_{i=1}^{Ns}[\frac{1}{\beta\ln 2} - \frac{\lambda_i}{\gamma}]^{\dagger} = P_t. \tag{32}$$

Since λ_{N_s} is the minimum value in the set of λ_i $(1 \leq i \leq N_s)$, we get $\frac{1}{\beta\ln 2} - \frac{\lambda_i}{\gamma} \leq \frac{1}{\beta\ln 2} - \frac{\lambda_{N_s}}{\gamma}$. The upper bound of β can be solved from the following inequality

$$\sum_{i=1}^{Ns}\frac{1}{\beta\ln 2} - \frac{\lambda_{N_s}}{\gamma} \geq P_t, \tag{33}$$

then the upper bound of the Lagrangian multiplier β is

$$\beta_{max} = \frac{1}{\ln 2} \frac{\gamma N_s}{\gamma P_t^2 + N_s \lambda_{N_s}}. \tag{34}$$

B. ANALOG RF PRECODER DESIGN

For the fully-connected hybrid precoding structure, the constraint regarding to the analog precoder can be specified by $|[\mathbf{F}_{RF}]_{m,n}| = 1$, since each RF chain is connected to all the antennas. After optimizing the digital baseband precoding matrix $\mathbf{F}_{BB}$, we fix $\mathbf{F}_{BB}$ to seek an optimal analog precoder for the following problem

$$\begin{aligned} \min_{\mathbf{F}_{RF}} \quad & -C \\ \text{subject to } & |[\mathbf{F}_{RF}]_{m,n}| = 1, \\ & \|\mathbf{F}_{RF}\mathbf{F}_{BB}\|_F^2 \leq P_t. \end{aligned} \tag{35}$$

The structure of hybrid precoding constricts every element in the analog precoding matrix to be unit-modulus, and this non-convex element-wise constraint makes the precoder design problem intractable. To make the optimization problem more tractable, we propose to use an exponential expression for the analog precoding matrix $\mathbf{F}_{RF}$ given by

$$\mathbf{F}_{RF} = \begin{bmatrix} e^{j\varphi_{1,1}} & e^{j\varphi_{1,2}} & \cdots & e^{j\varphi_{1,N_{RF}^t}} \\ e^{j\varphi_{2,1}} & e^{j\varphi_{2,2}} & \cdots & e^{j\varphi_{2,N_{RF}^t}} \\ \vdots & \cdots & \ddots & \vdots \\ e^{j\varphi_{N_t,1}} & e^{j\varphi_{N_t,2}} & \cdots & e^{j\varphi_{N_t,N_{RF}^t}} \end{bmatrix} \tag{36}$$

As a result, the optimization problem regarding to $\varphi_{i,j}$ can be reformulated as

$$\begin{aligned} \min_{\varphi_{i,j}} \quad & -C \\ \text{subject to } & \|\mathbf{F}_{RF}(\varphi_{i,j})\mathbf{F}_{BB}\|_F^2 \leq P_t. \end{aligned} \tag{37}$$

Subgradient algorithm [28] is an iterative algorithm. In every iteration, to minimize the objective function value, it removes the power constraint and chooses primal variables. Since the objective and power constraint functions are separable, the optimization problem (37) can be decomposed into parallel smaller problems. Thus, the subgradient method can be developed to design the analog precoding matrix $\mathbf{F}_{RF}$.

We first construct $\mathbf{F}_{RF}$ with random phases as the initial matrix value of the analog precoding matrix, and we optimize $\mathbf{F}_{RF}$ by updating each $\varphi_{i,j}$. $c > 0$ is a constant step size and $\lambda(0) = 0$ is a given constant vector. The subgradient method can be treated as a subgradient method which is applied to the Lagrangian dual function. The Lagrangian dual function can be defined as

$$\mathcal{L} = -C + \lambda(t)[\text{trace}\{\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^H\mathbf{F}_{RF}^H\} - P_t]. \tag{38}$$

In each iteration t of subgradient method, to get the value of $\varphi_{i,j}$ that minimizes Lagrangian dual function value, we optimize the element of $\varphi_{i,j}$ for $\mathbf{F}_{RF}$ using the steepest descent algorithm to satisfy

$$\varphi_{i,j}(t) = \arg\min_{\varphi_{i,j}} \mathcal{L}(\varphi_{i,j}), \tag{39}$$

where we define $\mathbf{F}_{RF}^{i,j}$ as the analog precoding matrix with the element in ith row, jth column to be zero, thus the analog precoding matrix can be expressed as $\mathbf{F}_{RF} = \mathbf{F}_{RF}^{i,j} + e^{j\varphi_{i,j}}\mathbf{e}_i\mathbf{e}_j^H$. $\mathbf{e}_i \in \mathbb{C}^{N_t \times 1}$ is a column vector with the ith element 1 and others 0 and $\mathbf{e}_j \in \mathbb{C}^{N_{RF}^t \times 1}$ is a column vector with the jth element 1 and others 0. For example,

$$\mathbf{F}_{RF}^{2,1} = \begin{bmatrix} e^{j\varphi_{1,1}} & e^{j\varphi_{1,2}} & \cdots & e^{j\varphi_{1,N_{RF}^t}} \\ 0 & e^{j\varphi_{2,2}} & \cdots & e^{j\varphi_{2,N_{RF}^t}} \\ \vdots & \cdots & \ddots & \vdots \\ e^{j\varphi_{N_t,1}} & e^{j\varphi_{N_t,2}} & \cdots & e^{j\varphi_{N_t,N_{RF}^t}} \end{bmatrix} \tag{40}$$

Thus, $\mathbf{F}_{RF} = \mathbf{F}_{RF}^{i,j} + e^{j\varphi_{i,j}}\mathbf{e}_i\mathbf{e}_j^H$.

We then update $\lambda(t+1)$ as

$$\lambda(t+1) = \max\{\lambda(t) + c[\text{trace}\{\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^H\mathbf{F}_{RF}^H\} - P_t]\}. \tag{41}$$

The iteration of $\varphi_{i,j}$ stops when a stopping criterion triggers. The overall algorithm based on the subgradient algorithm is summarized as follows.

Algorithm 1 Subgradient Algorithm for Analog RF Precoder Design

Input: $P_t, N_s, \alpha, \mathbf{H}, \mathbf{F}_{BB}$
1: Construct $\mathbf{F}_{RF}$ with random phases;
2: $c > 0$, $\lambda(0) = 0$, $i = 1$ and $j = 1$;
repeat
repeat
3: Optimize the element of $\varphi_{i,j}$ for $\mathbf{F}_{RF}$ using Algorithm 2:
$\varphi_{i,j}(t) = \arg\min_{\varphi_{i,j}} \mathcal{L}(\varphi_{i,j})$;
4: Update the analog precoding matrix $\mathbf{F}_{RF}$:
$\mathbf{F}_{RF} = \mathbf{F}_{RF}^{i,j} + e^{j\varphi_{i,j}}\mathbf{e}_i\mathbf{e}_j^H$;
5: Update the value of λ:
$\lambda(t+1) =$
$\max\{\lambda(t) + c[\text{trace}\{\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^H\mathbf{F}_{RF}^H\} - P_t]\}$;
6: $t \leftarrow t + 1$;
until *a stopping criterion triggers*;
until *all the elements of* $\mathbf{F}_{RF}$ *have been optimized*;

In the process of obtaining the value of $\varphi_{i,j}$ that minimizes Lagrangian dual function value, the Armijo backtracking line search step θ_k guarantees the objective function to be non-increasing in each iteration. Moreover, the gradient $\text{grad}(\varphi_{i,j}(k))$ on $\varphi_{i,j}(k)$ can be calculated by

$$\begin{aligned} &\text{grad}(\varphi_{i,j}(k)) \\ &= -\frac{1}{\ln 2}\text{trace}\{(\mathbf{I} + \gamma\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^H\mathbf{F}_{RF}^H\mathbf{H}^H)^{-1} \\ &\quad \times (je^{j\varphi_{i,j}}\gamma\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^H\mathbf{F}_{RF}^H\mathbf{H}^H)\} \\ &\quad + \lambda(t)\text{trace}\{-je^{-j\varphi_{i,j}}\mathbf{e}_j\mathbf{e}_i^H\mathbf{F}_{RF}^{i,j}\mathbf{F}_{BB}\mathbf{F}_{BB}^H \\ &\quad + je^{j\varphi_{i,j}}\mathbf{F}_{RF}^{i,j}\mathbf{e}_j\mathbf{e}_i^H\mathbf{F}_{BB}\mathbf{F}_{BB}^H\}. \end{aligned} \tag{42}$$

The steepest descent direction is the opposite of the gradient of $\varphi_{i,j}$. Before the next iteration, we update the value of $\varphi_{i,j}$ by $\varphi_{i,j}(k+1) = \varphi_{i,j}(k) + \theta_k d(\varphi_{i,j}(k))$.

Based on the analysis above, the algorithm regarding to the steepest descent method is given as:

Algorithm 2 Steepest Descent Algorithm for Lagrangian Dual Function in Algorithm 1

Input: $P_t, N_s, \alpha, \mathbf{H}, \mathbf{F}_{BB}, \mathbf{F}_{RF}, \mathbf{F}_{RF}^{i,j}, \varphi_{i,j}$

1: $k = 0$;

repeat

2: Choose Armijo backtracking line search step size θ_k;

3: Calculate the gradient grad$(\varphi_{i,j}(k))$ using (42);

4: Determine the steepest descent direction: $d(\varphi_{i,j}(k)) = -\text{grad}(\varphi_{i,j}(k))$;

5: Update the value of $\varphi_{i,j}(k+1)$: $\varphi_{i,j}(k+1) = \varphi_{i,j}(k) + \theta_k d(\varphi_{i,j}(k))$;

6: $k \leftarrow k+1$;

until *a stopping criterion triggers*;

Algorithm 1 together with Algorithm 2 guarantees the convergence to a critical point, achieving the optimized analog precoder design $\mathbf{F}_{RF}$ under the fixed digital precoder design $\mathbf{F}_{BB}$.

C. HYBRID PRECODER DESIGN

With the methods for optimizing the digital precoding matrix $\mathbf{F}_{BB}$ and the analog precoding matrix $\mathbf{F}_{RF}$, the hybrid precoding matrix design using the alternating minimization algorithm is described in Algorithm 3 by solving the optimization problems (20) and (35) iteratively.

Algorithm 3 Alternating Minimization Algorithm for Hybrid Precoding Design

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Input: $P_t, N_s, \alpha, \mathbf{H}$

1: Construct $\mathbf{F}_{RF}^{(0)}$ with random phases and set $k = 0$;

repeat

2: Fix $\mathbf{F}_{RF}^{(k)}$ and $\mathbf{F}_{BB} = \mathbf{M}^{-\frac{1}{2}}\mathbf{U}\Sigma^{\frac{1}{2}}$ according to Digital Baseband Precoding Design;

3: Optimize $\mathbf{F}_{RF}^{(k+1)}$ using Algorithm 1 and Algorithm 2 when $\mathbf{F}_{BB}$ is fixed;

4: $k \leftarrow k+1$;

until *a stopping criterion triggers*;

Since the value of the objective function in (19) is maximized at Step 2 and 3, in each iteration, the objective function value will never decrease. Moreover, the value of the objective function is nonnegative. These two properties jointly guarantee that the alternating minimization algorithm can converge to a local optimal solution.

V. SIMULATION RESULTS

In this section, we numerically evaluate the performance of our proposed algorithms using MATLAB.

We present simulation results of the proposed optimization method convergence behaviors. The relationship between the achievable rates and the transmit/receive antennas and SNR will also be shown. In this paper, due to the normalized power of the noise, we can define SNR as SNR $= P_t$. In different SNRs, we compare the achievable rates obtained by the alternating minimization algorithm with that obtained by optimizing the digital precoding matrix $\mathbf{F}_{BB}$ or the analog precoding matrix $\mathbf{F}_{RF}$ only and by optimizing with the assumption of the analog precoding matrix composed of columns of DFT matrices, which is adopted in many previous works such as [17] and [18]. Moreover, we also compare with the performance of other specific precoding systems, i.e. the digital architecture with one-bit ADCs, the hybrid precoding architecture with ∞-bit ADCs and the digital architecture with ∞-bit ADCs.

Without loss of generality, in this paper, the number of RF chains is assumed to be equal to that of the data streams, i.e. $N_{RF}^t = N_s$. For the proposed alternating minimization algorithm, the initial phases of the analog precoding matrix follow a uniform distribution over the range of $[0, 2\pi)$.

Firstly, we investigate the convergence behavior of subgradient algorithm at $N_t = 6$, $N_r = 4$, $N_s = 2$ and SNR $=$ 30dB. The subgradient algorithm behavior for the first three elements of the analog precoding matrix $\mathbf{F}_{RF}$ is plotted in Fig. 2. It can be seen that with the number of iterations increasing, the absolute value of the objective function $\mathcal{L} = -C + \lambda(t)[\text{trace}\{\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^H\mathbf{F}_{RF}^H\} - P_t]$ is becoming larger, which implies that the achievable rate is becoming larger.

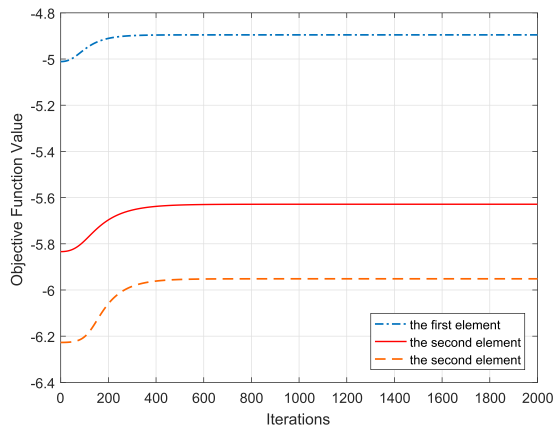

FIGURE 2. The convergence behavior of subgradient algorithm.

Fig. 3 shows the performance of the alternating minimization algorithm at $N_t = 6$, $N_r = 4$, $N_s = 2, 3, 4$ and SNR $=$ 10dB. In Fig. 3, the convergence behavior of the alternating minimization algorithm is illustrated by plotting a point after each iteration of optimizing the digital precoding matrix $\mathbf{F}_{BB}$ and the analog precoding matrix $\mathbf{F}_{RF}$, and the improvement of the achievable rate can also be observed. This can prove that the alternating minimization algorithm is an effective method that guarantees the achievable rate to be non-decreasing in each iteration of optimizing the digital precoding matrix $\mathbf{F}_{BB}$ and the analog precoding matrix $\mathbf{F}_{RF}$ alternatively. As the number of the data streams N_s increases, the achievable rates after optimized also increase.

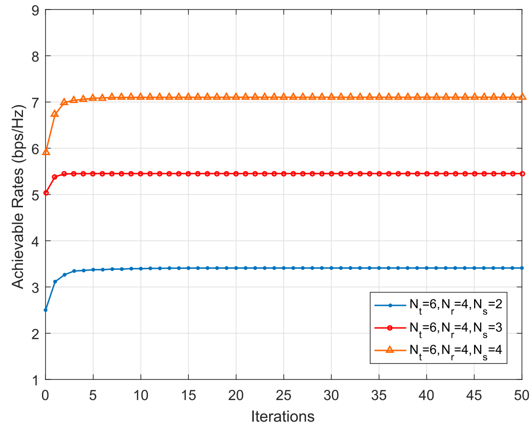

FIGURE 3. The performance of alternating minimization algorithm.

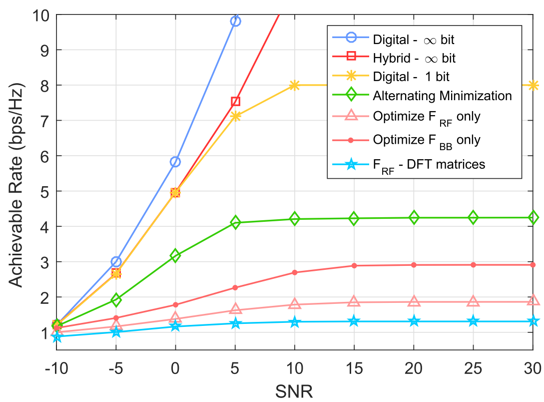

FIGURE 4. The achievable rates versus SNR.

In Fig. 4, we evaluate the performance of the proposed algorithm in different SNRs at $N_t = 6$, $N_r = 4$ and $N_s = 2$. We compare the performances of proposed algorithm with the scheme only optimizing the digital precoder $\mathbf{F}_{BB}$ and the scheme only optimizing the analog precoder $\mathbf{F}_{RF}$. Furthermore, we compare with the achievable rate obtained with DFT analog precoding matrix. In DFT analog precoding matrix, the analog precoding matrix $\mathbf{F}_{RF}$ consists of columns from the DFT matrices, which are used in many previous works, such as [17] and [18]. We can observe from Fig. 4 that the achievable rates of the proposed algorithm in this paper are superior to the ones with the DFT analog precoding matrix in all SNRs, which implies that the DFT analog precoding matrix limits the improvement of the achievable rates. Moreover, the achievable rates obtained by optimizing the digital precoding matrix $\mathbf{F}_{BB}$ only is larger than that obtained by optimizing the analog precoding matrix $\mathbf{F}_{RF}$ only. This is because the optimization of the digital precoding matrix $\mathbf{F}_{BB}$ can change the amplitudes of the hybrid precoding architecture, while the analog precoding matrix $\mathbf{F}_{RF}$ can only adjust the phases, which impedes the performance improvement. By optimizing the digital precoding matrix $\mathbf{F}_{RF}$ and the digital precoding matrix $\mathbf{F}_{BB}$ jointly, the achievable rate can be significantly improved. Furthermore, with SNR increasing gradually, the quantization process of one-bit ADCs at the receiver will impede the increasing trend of the achievable rate. Further, with SNR increasing gradually, the quantization process of one-bit ADCs at the receiver will impede the increasing trend of the achievable rate. Thus, the achievable rate will finally become converged and get saturated even if SNR increases.

Moreover, we compare with other specific precoding systems, including the digital architecture with one-bit ADCs, the hybrid precoding architecture with ∞-bit ADCs and the digital architecture with ∞-bit ADCs. With the receive antenna N_r at the receiver, there are at most 2^{2N_r} possible quantization outputs, which implies that $2N_r$ bps/Hz is a simple upper bound on the channel capacity for systems with one-bit ADCs. Thus in Fig. 4, the upper bound of the achievable rate of digital architecture with one-bit ADCs is 8 bps/Hz. It is also observed that the performance of the digital architecture with ∞-bit ADCs is better than that of the hybrid architecture with ∞-bit ADCs, because the hybrid architecture utilizes less RF chains than the digital architecture.

We then analyze the achievable rates on different numbers of transmit and receive antennas.

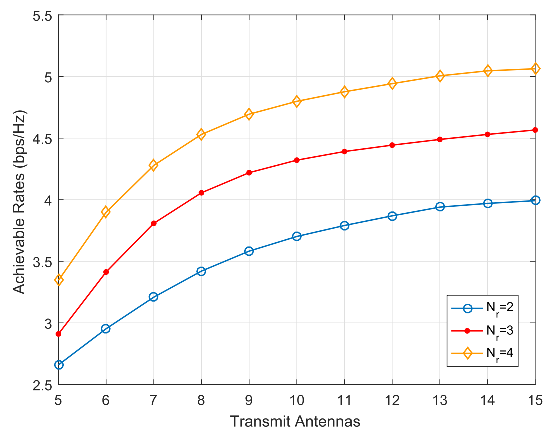

FIGURE 5. The achievable rates versus transmit antennas.

Fig. 5 is plotted at SNR = 10dB. As it shows, the achievable rates increase as the number of transmit antennas increases. However, it cannot achieve the upper bound of the achievable rate for systems with one-bit ADCs due to the cost- and energy-saving hybrid precoding systems. Meanwhile, the increasing rate is getting slower as the number of the transmit antennas increases, which is because for given number of the receive antennas, the one-bit quantization process at the receiver impedes the increasing trend of the achievable rate. This also indicates that the numbers of transmit and receive antennas should be chosen properly to realize the economical efficiency in practice. It can also be observed that the achievable rates increase as the number of the receive antennas increases, which will further be illustrated in Fig. 6.

Fig. 6 is plotted at SNR = 10dB. With the number of the receive antennas increasing, the achievable rates increase. The increasing rate also slightly decreases as the number of the receive antennas increases because one-bit ADCs at the

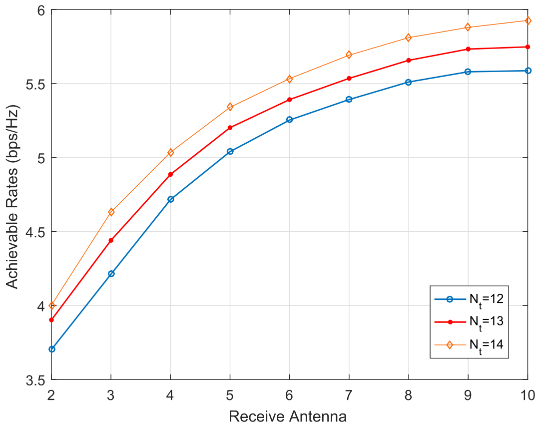

FIGURE 6. **The achievable rates versus receive antennas.**

receiver cause a bottleneck for the given number of transmit antennas, which leads us to consider the choice of the number of the transmit and receive antennas to realize the economical efficiency in reality. The conclusion of Fig. 5 is proven again here, that is, as the number of the transmit antennas increases, the achievable rates increase.

VI. CONCLUSION

In this paper, an architecture of single-user MIMO system with the hybrid analog/digital precoding architecture and one-bit ADCs is investigated. We use the Bussgang Theorem to derive the expression of the achievable rates for the proposed architecture and formulate the optimization problem for the achievable rates with the power constraint of the hybrid architecture and the unit-modulus constraint of the analog precoder. Based on the alternating minimization algorithm, we solve the digital precoder and the analog precoder in an alternative way in two separate subproblems. In the subproblem for optimizing the digital precoding matrix, we use SVD to obtain the optimal structure of the digital precoding matrix. We simplify the digital precoding design as a power allocation problem which is then optimally solved by using KKT conditions. We then determine the parameters using the bisection method. In the subproblem of optimizing the analog precoding matrix, we express the analog precoding matrix with an exponential form satisfying the unit-modulus constraints and solve the optimization problem with the subgradient algorithm and the steepest descent algorithm. Finally, by comparing the performance of the proposed method with the existing methods and systems, our numerical results show that designing the hybrid precoding architecture with our proposed method can improve the achievable rates effectively. The results also illustrate that the achievable rates increase with the number of transmit and receive antennas increasing, but there is a bottleneck due to the one-bit quantization process at the receiver.

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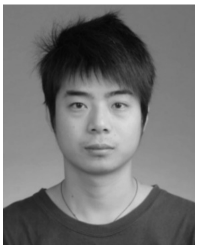

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