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Piecewise Affine Identification of Tire Longitudinal Properties for Autonomous Driving Control Based on Data-Driven

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ABSTRACT The tire longitudinal properties exhibit highly nonlinear dynamical behaviors influenced by the tire slip coefficient, the road longitudinal adhesion coefficient, and the tire vertical load. This paper presents a system identification approach to approximate such a nonlinear dynamic for autonomous driving controller design. The tire longitudinal properties tests are conducted using a flat-plate test bench. On the basis of the experimental data, the piecewise affine (PWA) identification of the tire longitudinal properties involves the classification of the cluster data and the parameter estimation of the affine submodels. Using the least-square algorithm, the parameter vectors of the affine submodels are estimated, and the problem of region partition is solved via heuristic approach. The simulation results of the identified PWA model match the experimental data accurately, which demonstrates the effectiveness of the proposed identification approach for the tire longitudinal properties.

INDEX TERMS Tire longitudinal properties, autonomous driving, piecewise affine systems, system identification, data-driven.

I. INTRODUCTION

The tire longitudinal properties are mainly used to predict the longitudinal (acceleration/deceleration) forces acted on the tire, which are crucial to the design and dynamical performance evaluation of the vehicles [1]–[4]. Since the influencing factors of the tire longitudinal properties change rapidly under different driving conditions, the tire longitudinal properties exhibit a highly nonlinear dynamic [5]. In the past research of vehicle longitudinal dynamics control, the nonlinearities in the tire longitudinal properties are often ignored and linearized [6]–[8]. This treatment may have little influence on the vehicle control performance for driving on the dry asphalt road, which is assumed that no slip occurs at the tire-road interface. However, with the rapid development of intelligent transportation systems, autonomous vehicles, which need to drive on a wide range of road conditions, are likely to play a major role in the future [9]–[11]. The vehicle driving on the icy and slippery roads will cause behavioral changes and variations of the tire longitudinal properties due to slip occurrence. Therefore, the nonlinearities in the tire longitudinal properties should be considered carefully for longitudinal motion controller design of the autonomous vehicles [12].

Modeling the tire longitudinal properties has been regarded as an important aspect over the last decade. In this respect, several tire models, e.g. the unified semi-empirical model [13], the magic formula model [14], the Dugoff's model [15] and the Kiencke's model [16], have been developed with quite different properties. To meet the accuracy requirement, these models involve multiple aspects relevant to the tire characteristics. Although these existing tire force models are widely used to reflect the tire longitudinal dynamics, most of them are complicated and difficult to fit the parameters. To achieve desired control performance for the autonomous vehicles, the system needs to be controlled appropriately based on accurate dynamics model. However, for control law synthesis purpose, the most suitable system model should present a good compromise between accuracy and simplicity [17], [18]. Too complicated models may cause difficulties in controller design. From these viewpoints, the existing tire models are not sufficient in the analysis of the tire longitudinal dynamics and especially the autonomous driving controller design.

Implementing an ideal control performance for autonomous vehicle longitudinal dynamics requires sufficient information about the tire longitudinal properties by either mathematical modeling or system identification. One of the effective ways to meet those requirements is to approximate the nonlinear tire longitudinal properties with piecewise affine (PWA) system. The PWA systems are a special class of nonlinear systems, established by partitioning the state-input domain into a limited number of polyhedral regions and obtaining the affine submodels in each region [19]–[22]. Since the PWA models have universal approximation capability, arbitrarily nonlinear systems, which are sufficiently smooth, can be approximated well by a PWA function [23], [24]. In addition, among different frameworks of hybrid systems, which are proposed to describe a class of systems which can switch between many operating modes where each mode is governed by its own characteristic dynamical laws, PWA systems have been also suitably used for hybrid controller design due to their equivalencies to other classes of hybrid systems [25]–[27], which also makes it outstanding to the others tire modeling methods. Thus, in this paper, for autonomous driving controller design, the tire longitudinal properties are considered to be approximated by the PWA model.

The approach for approximating a nonlinear system through the PWA model will consist of the classification of the partitions of the regressor domain and the estimation of the parameters of the affine submodels. The main intricacy is that the identification of each submodel cannot be decoupled directly from the data classification problem [28]–[30]. Several methods to identify PWA models, such as the Bayesian procedure [31], the mixed-integer quadratic programming approach [32] and the greedy bounded-error approach [33], have been proposed. These identification methods are often applicable only to the different identification objects, which have different input-output characteristics, such as the Single-input Multiple-output system or the Multi-input Single-output system. In addition, the data form of the system inputs and outputs, such as the random data and the deterministic data, also has great influence on the selection of the identification method. In this paper, considering the computation complexity of the identification algorithm and the actual characteristics of the tire longitudinal properties, the least-square algorithm is used to estimate the parameter vectors of the affine submodels, and the region partition is conducted by using the heuristic approach [34].

The originality of the present paper is that the PWA model of the tire longitudinal properties is identified through the experimental data. That is, the study includes two major contributions. The first is that the accurate tests data of the tire longitudinal properties is provided. To accurately reflect the nonlinear mapping relation between the tire longitudinal force and its influencing factors, the tire longitudinal properties tests are conducted using a flat-plate tire test bench for different vertical loads, slip coefficients and longitudinal adhesion coefficients. The other contribution is the identification of the PWA model of the tire longitudinal properties, which not only provides a novel modeling approach for the tire dynamics, but also presents the best model accuracy/simplicity compromise for the control design.

The paper is organized as follows. The tire longitudinal properties and the major influence factors are discussed in Section II. The tire longitudinal properties tests and the tests results analyses are given in Section III. Section IV is on the PWA model identification. Finally, accuracy validations and results analyses are presented.

II. TIRE LONGITUDINAL PROPERTIES

The tire model is used to describe the mathematical relationships between the force- and moment- generating properties and the wheel motion parameters, i.e. the input-output characteristics of the tire under specific driving conditions, whose main inputs and outputs are shown in Fig. 1 [35], [36].

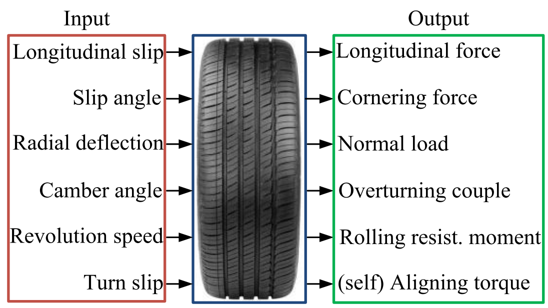

FIGURE 1. Input-output characteristics of the tire.

According to different research objectives of the vehicle dynamics, the tire model can be further divided into three kinds, i.e. the tire cornering properties, the tire vertical vibration properties and the tire longitudinal properties. In this paper, only the tire longitudinal properties, which reflect the non-linear relationship between the tire longitudinal force and its influencing factors, is researched. For vehicle driving normally on the dry asphalt road, which is assumed that no slip occurs at the tire-road interface, the longitudinal forces generated by the tire have a nearly linear relationship with their influencing factors. However, for vehicle driving on the icy and slippery roads or starting and braking conditions, the tire longitudinal properties show unsteady state and nonlinear characteristics. Thus, the modeling of the tire longitudinal properties has great significance for the vehicle longitudinal dynamics analysis and control.

The main contribution of this paper is just to establish the tire longitudinal properties model through the PWA identification method based on data-driven. The longitudinal forces generated by the tire during vehicle driving and braking conditions are mainly influenced by three factors, among of which, the longitudinal slip is defined as [37]:

$$\kappa = -\frac{V_x - r_e\Omega}{V_x} = \frac{r_e\Omega}{V_x} - 1 \tag{1}$$

where κ is the longitudinal slip coefficient, V_x is the vehicle forward speed, Ω is the wheel angular velocity, r_e is the effective rolling radius of the tire. The sign is taken such that a positive longitudinal force F_x arises for a positive κ. In that case, i.e. for vehicle driving, the product of Ω and r_e is larger than V_x. For vehicle braking, the fore-and-aft slip becomes negative, thus a negative longitudinal force F_x arises, that is, a brake force. The reader is referred to [37] for further details about the tire free body diagram.

The other two influencing factors of the tire longitudinal forces are the longitudinal adhesion coefficient and the vertical load, which depend on the road condition and the vehicle weight respectively. In this paper, the experimental results of the tire longitudinal properties are firstly provided. By analyzing the influence of each factor on the tire longitudinal forces, the model form of the tire longitudinal properties is further clarified. On this basis, the PWA identification of the nonlinear tire longitudinal properties is conducted. To verify the accuracy of the identification model, the simulation results of the PWA model are compared with the experimental results finally. The overall research procedure of the PWA identification of the tire longitudinal properties based on data-driven is shown in Fig. 2.

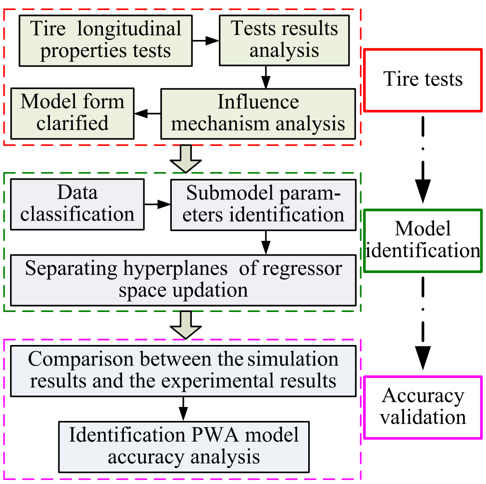

FIGURE 2. Research procedure of the PWA identification of the tire longitudinal properties.

III. TIRE LONGITUDINAL PROPERTIES TESTS

To obtain the accurate test data about the tire longitudinal properties, the tire tests are conducted using a flat-plate tire test bench, which is assisted by KH Automotive Technologies (Guangzhou). The experimental setup of the tire longitudinal properties is shown in Fig.3. During the test procedure, since the variation of the tire pressure is negligible, thus the tire pressure is assumed to be constant, and the tire slip angle is assumed to be zero. These assumptions are also the research premises of this paper.

FIGURE 3. Experimental setup of the tire longitudinal properties.

For specific tire load, tire pressure and longitudinal adhesion coefficient, the tire longitudinal forces are measured by starting the sliding table to drive the tire to move at a constant speed. In order to analyze the influence of each factor on the tire longitudinal forces, five different tire loads and two different longitudinal adhesion coefficients are provided by the test bench. Modeling the different road conditions is just one part of these experimental settings, the other part is making them feasible and economical for the tests. It is noted that the two different longitudinal adhesion coefficients are achieved by changing two different rolling plates. However, to solve the guiding problem of the rolling plate, the required technical cost is high. Therefore, in this study, only two different longitudinal adhesion coefficients are provided for the test. The range of the longitudinal slip provided by the test bench is $[-1, 0.5]$, which means that, at wheel lock, obviously, $\kappa = -1$, while at driving on slippery roads, the maximum longitudinal slip is 0.5. The specific parameter settings of the tire tests based on the flat-plate bench are given in Table 1, among of which, the two different longitudinal adhesion coefficients are estimated according to the materials of the rolling plates.

TABLE 1. List of parameter settings.

Parameter	Setting
Tire pressure (kPa)	880
Vertical load (N)	3124, 6530 8036, 9468, 11760
Slip angle (rad)	0
Longitudinal slip	-1~0.5
Adhesion coefficient (peak value)	0.34 (low) 0.77 (high)

During the test procedure, five different vertical loads are respectively applied to the tire for each test, and the rolling

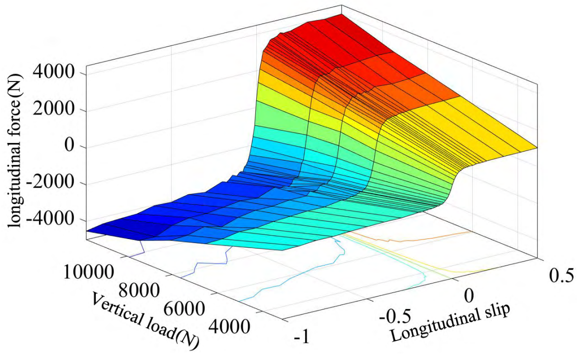

FIGURE 4. Nonlinear relationship between the tire longitudinal force and its influence factors for low adhesion coefficient.

plate is started to drive the tire at a constant speed. Then, the tire longitudinal forces are measured for each slip coefficient. The tire longitudinal properties test results, i.e. the nonlinear relationship between the tire longitudinal force and its influence factors, i.e. the vertical load and the longitudinal slip, for the two different longitudinal adhesion coefficients are shown in Fig. 4 and Fig. 5 respectively.

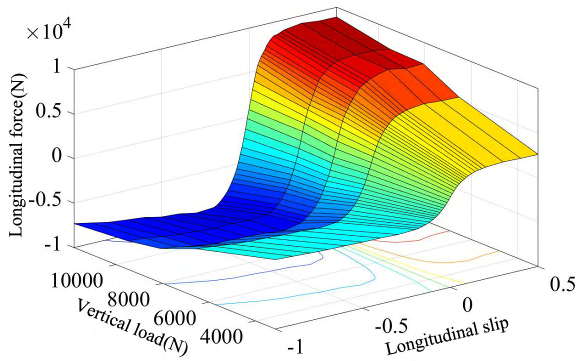

FIGURE 5. Nonlinear relationship between the tire longitudinal force and its influence factors for high adhesion coefficient.

As it can be observed from the tests results, the relationship between the tire longitudinal force and its influence factors is manifested as an irregular curved surface, thus the tire longitudinal properties indeed show highly nonlinear dynamical behaviors. If this curved surface can be decomposed into several flat surfaces for different operating regions, the nonlinear tire longitudinal properties can be linearized accurately. This idea is just the research object of this study for the PWA identification of the tire longitudinal properties. For a class of nonlinear systems like the tire longitudinal properties with a wild range of operating regions, the multiple affine models are an effective way to describe the whole system dynamics. The technique used in this paper is to identify a local affine submodel for each desired operating region. Then, the PWA system can be developed to cover the entire operating conditions of the tire longitudinal properties. Each identified affine submodel will have an effective region, in which the system generates minimum deviation from the original plant. However, the model's performance will be reduced if out of this region, hence another affine model with shifted operating region is needed. The switching behaviors of the PWA system with several affine submodels can be shown in Fig. 6, where t is the time step.

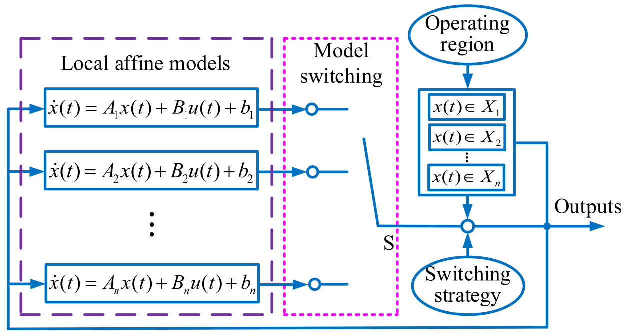

FIGURE 6. Switching behaviors of the PWA system.

Based on the experimental data, it can also be found that the longitudinal adhesion coefficient has great influence on the tire longitudinal force. The variation of the road longitudinal adhesion coefficient affects the tire-road interface so that the tire longitudinal force changes correspondingly. By further comparing the tire longitudinal forces in the same operating regions for the two different longitudinal adhesion coefficients, it can be seen that the relationship between the tire longitudinal force and the road longitudinal adhesion coefficient is linear approximately, i.e. in the same operating region, the ratio between the longitudinal forces is approximately equal to the ratio between the longitudinal adhesion coefficients. Therefore, the road longitudinal adhesion coefficient is not considered as a nonlinear impact factor of the tire longitudinal properties in this study. That is to say, if the nonlinear relationship between the tire longitudinal force and its influence factors, i.e. the vertical load and the longitudinal slip, for a specific road adhesion coefficient is determined, the corresponding relationships for the other road adhesion coefficients will also be proportionally obtained. This simplification is not only consistent with the actual situation, but also reduces the difficulty of the PWA identification of the tire longitudinal properties. In addition, several research works have been devoted to estimate the road longitudinal adhesion coefficient [38]–[40], thus this factor can be regarded as a known condition for the tire longitudinal properties modeling.

According to the aforementioned simplifications and analyses, the PWA identification of the tire longitudinal properties based on experimental data is finally determined as constructing several affine models to approximate the nonlinear relationship between the tire longitudinal force and its influence factors, i.e. the vertical load and the longitudinal slip, for a specific road adhesion coefficient. The number of the local affine submodels of the PWA system highly affects the stability of the modeling and control as well as the complexity of computations. This variable mainly depends on the identification precision requirements.

IV. PIECEWISE AFFINE SYSTEM IDENTIFICATION

PWA systems are those whose operating space can be partitioned into a finite number of nonoverlapping convex polyhedral regions and whose individual subsystem in each region is affine [41]. If there is no limit on the total number of the subsystem, the PWA system can then be used to approximate arbitrarily nonlinear dynamical systems by switching among those subsystems based on the operating regions. Therefore, the nonlinear tire longitudinal properties can be described effectively by the identified PWA systems. The PWA identification of the tire longitudinal properties mainly involves the classification of the cluster data, which reflects the relationship between the tire longitudinal force and its influence factors, and the estimation of the parameter vectors for the affine submodels.

A. PWA IDENTIFICATION PROBLEM STATEMENT

A PWA model of the dynamical system is defined as:

$$y(t) = \begin{cases} \theta_1^{\mathrm{T}} \begin{bmatrix} \varphi(t) \\ 1 \end{bmatrix} + \varepsilon(t), & \textit{if } \varphi(t) \in \chi_1 \\ \vdots \\ \theta_c^{\mathrm{T}} \begin{bmatrix} \varphi(t) \\ 1 \end{bmatrix} + \varepsilon(t), & \textit{if } \varphi(t) \in \chi_c \end{cases} \tag{2}$$

where $y(t)$ is the PWA model output, θ_i $(i = 1, \ldots, c)$ are the parameter vectors defining each submodel, c is the number of the subsystems, $\varphi(t)$ represents the regression vector. The regression vector $\varphi(t)$ studied in this paper, which consists of the system past inputs and outputs, is formed as:

$$\varphi(t) = [y(t-1), \cdots, y(t-n_y), u(t-1), \cdots, u(t-n_u)]^{\mathrm{T}} \tag{3}$$

where n_y and n_u are the PWA model orders, $u(t)$ is the input to the system. χ_i $(i = 1, \ldots, c)$ represents the whole polyhedral region of the affine submodels, and each region χ_i is a convex polyhedron represented in the following form:

$$\chi_i = \{F_i\varphi(t) + g_i \leq 0\} \tag{4}$$

where F_i and g_i are the corresponding coefficient matrices. By letting $H_{\mathrm{i}} = [F_i g_i]$, $(i = 1, \ldots, c)$, the convex polyhedron region χ_i can be rewritten as:

$$\chi_i = \left\{H_i\, [\varphi(t)\ 1]^T \leq 0\right\} \tag{5}$$

The PWA system defined by (2), (3) and (5) can be regarded as a collection of affine subsystems connected by dynamical switches which depend on the partition of the polyhedral region [42]. Since the PWA model is the simplest extension of linear models but can approximate the system nonlinearities with arbitrary accuracy, this model form is attractive for identification purposes. The solution of the system PWA identification problem is just to obtain a PWA model for the target system according to a specified fitting criterion based on the experimental data. It mainly includes the following two steps [43]:

1) SEGMENTATION

At the first step, the number of the affine submodels is determined and the coefficient matrices of hyperplanes H_i which define the partition of the operating region are got.

2) REGRESSION

At the second step, the order of affine submodels should be determined and the parameter vectors of each submodel θ_i are identified from the experimental data.

This identification problem underlies the classification of each data point associated to one polyhedral region and to the affine submodel. The simultaneous solution of the entire above-mentioned problem is hard and difficult to compute. One of the main difficulties is the determination of the number of affine submodels. From the viewpoint of fitting precision, if one data point corresponds to one submodel, the fit is perfect but clearly an inappropriate solution. Thus, constraints on the number of affine submodels must be introduced to keep a good compromise between the fitting accuracy and the number of the affine submodels. Heuristic and suboptimal approaches, which have been proposed in the related research, are applicable to solve this problem [44]. To improve the fitting precision, most of these methods either assume a fixed number of affine submodels or adjust it iteratively (e.g., by adding one submodel at a time).

B. SEGMENTATION

To find a good balance between the number of the affine model partitions and the overall identification accuracy, several approaches for partitioning are proposed, such as average [45], z-score [46], and k-means [47]. These methods have their own applicable occasions and advantages. The average method considers all observed data in one region, and thus identifies the affine submodel for the entire system. This approach is the base for a dynamical system. The z-score method divides the observed data into two partitions based on the empirical likelihood of the observation. Finally, k-means clustering aims to partition n observations into k clusters in which each observation belongs to the cluster with the nearest mean value. For convenience and considering the actual characteristics of the PWA identification problem for the tire longitudinal properties, heuristic approach is used in this study according to the system steady-state response surfaces, which have been shown in Fig. 4 and Fig. 5. It can be seen from the two figures that several affine submodels can well approximate the nonlinear relationship between the tire longitudinal force and its influence factors. Since the relations between the tire longitudinal properties with different road longitudinal adhesion coefficients are assumed to be linearly dependent in this paper, thus only the nonlinear relationship shown in Fig. 4 is chosen to be PWA approximated.

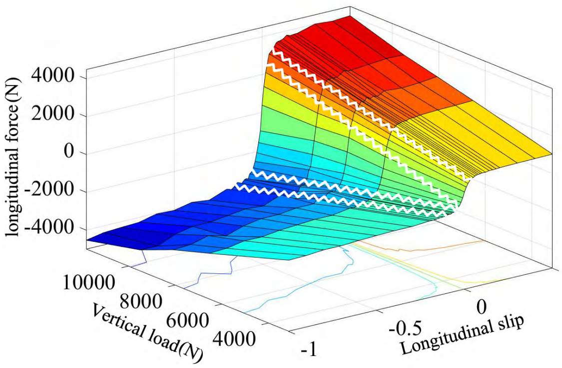

FIGURE 7. All five affine submodels of the system derived coarsely from steady-state response.

By further observing the shape of the irregular curved surface, it can be concluded that five affine submodels can reasonably approximate this curved surface, which determines the number of the affine submodels as five. Fig. 7 coarsely shows the five affine submodels.

Then, the next step is segmentation, i.e. determining the coefficient matrices of hyperplanes H_i $(i = 1, \ldots, c)$, which defines the partition of the operating region. The curved surface shown in Fig. 7 embodies several fractures, which can reflect the partition of regressor set and be described by several space lines based on data points. To distinguish between these five submodels and derive the space line equations, the following operating points are listed as:

$$\begin{cases} A(-0.19, 11800, -3450); \\ B(-0.04, 11800, -2505); \\ C(0.03, 11800, 2815); \\ D(0.07, 11800, 3400); \end{cases} \quad \begin{cases} E(0.036, 3124, 799.6); \\ F(-0.02, 3124, -425.2); \\ G(-0.03, 3468, -607.6); \\ L(-0.03, 3124, -529.2); \\ I(-0.07, 3124, -805.6); \end{cases}$$

According to the irregular curved surface shown in Fig. 7, it seems that eight data points are enough to distinguish these five submodels, i.e. four straight lines are formatted. However, to better fit the nonlinear tire longitudinal properties, some flat surfaces need to be distinguished by broken lines after repeated analyses and comparisons, thus the aforementioned nine data points are finally provided to distinguish these five submodels.

Projecting these nine data points on the xy plane results in H_i coefficients, which are described as the following five linear equations:

$$\begin{cases} F_z = -69636\kappa - 1751.7; \\ F_z = -867600\kappa - 22904; \\ F_z = -34797\kappa + 2410.5; \\ F_z = 137960\kappa + 7662.2; \\ F_z = 260520\kappa - 6441.4; \end{cases} \tag{6}$$

where F_z denotes the tire vertical load. These five linear equations define the partition of the operating region.

C. REGRESSION

For linear systems, the order of the affine submodels can be derived according to the system transfer function. However, since the relationship between the tire longitudinal force and its influence factors is nonlinear, the order of the submodels can only be specified by trial and error. It is also obvious that the submodel order should present a good compromise between the fitting accuracy and the identification simplicity. After repeated analyses and comparisons, the order of the affine submodels is finally determined as: $n_y = 2$ and $n_u = 1$.

Once the operating region is partitioned and the order of the submodels is determined, the parameter vectors of the affine submodels can then be estimated by using the least-square algorithm. On the basis of the aforementioned segmentation, the data points have been classified into several clusters, thus the regression aim is to estimate an affine model for each cluster. If N data points are provided for a fixed number of the affine submodels, the considered regression problem can be formulated as follows [48]–[50]:

$$\lambda_{ki} = \begin{cases} 1 & if\ \varphi(k) \in \chi_i \\ 0 & otherwise \end{cases} \quad k = 1, \cdots, N,\ i = 1, \cdots, c$$

$$\min_{\theta_i} \frac{1}{N} \sum_{k=1}^{N} \sum_{i=1}^{c} (y_k - \theta_i^T \begin{bmatrix} \varphi(t) \\ 1 \end{bmatrix})^2 \lambda_{ki} \tag{7}$$

By solving the problem shown in Eq. (7), the five affine submodels with parameters can be obtained as follows:

$$\begin{cases} F_x(k) = -0.156F_x(k-1) + 0.183F_x(k-2) \\ \qquad + 461.72\kappa(k-1) - 0.312F_z(k-1) + 202.44 \\ \qquad if\ F_z \leq -69636\kappa - 1751.7 \end{cases}$$

$$\begin{cases} F_x(k) = -0.284F_x(k-1) + 0.267F_x(k-2) \\ \qquad + 6910\kappa(k-1) - 0.22F_z(k-1) + 364.65 \\ \qquad if\ F_z > -69636\kappa - 1751.7\& \\ \qquad (F_z \leq -867600\kappa - 22904 | F_z \\ \qquad \leq -34797\kappa + 2410.5) \end{cases}$$

$$\begin{cases} F_x(k) = 1.304F_x(k-1) - 1.218F_x(k-2) \\ \qquad + 71757.1\kappa(k-1) - 0.145F_z(k-1) + 2076.57 \\ \qquad if\ F_z > 867600\kappa - 22904\&F_z \\ \qquad \geq 137960\kappa + 7662.2 \end{cases}$$

$$\begin{cases} F_x(k) = -0.424F_x(k-1) + 0.367F_x(k-2) \\ \qquad + 22050\kappa(k-1) + 0.215F_z(k-1) - 684.12 \\ \qquad if\ F_z < 137960\kappa + 7662.2\&F_z > 34797\kappa \\ \qquad + 2410.5\&F_z \geq 260520\kappa - 6441.4 \end{cases}$$

$$\begin{cases} F_x(k) = -0.168F_x(k-1) + 0.157F_x(k-2) \\ \qquad + 505\kappa(k-1) + 0.298F_z(k-1) - 151.42 \\ \qquad if\ F_z < 260520\kappa - 6441.4 \end{cases}$$

V. VALIDATION AND ANALYSIS

Obtaining the aforementioned affine submodels of the nonlinear dynamic and their region of operation, a PWA model with

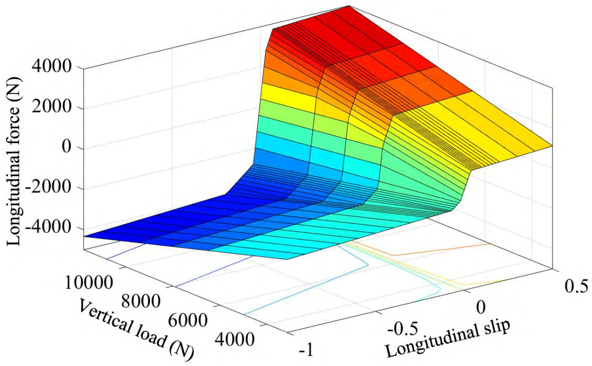

FIGURE 8. Simulation result of the PWA identified model for low adhesion coefficient.

switching rule is implemented. Since the PWA identification of the tire longitudinal properties is conducted based on the experimental data, thus to verify the actual performance of the identified model, its simulation results are compared with that of the experiments. The PWA model of the relationship between the tire longitudinal force and its influence factors for low adhesion coefficient is shown in Fig. 8. As it can be observed, the surface shown in Fig. 8 is very similar to that shown in Fig. 4, including the amplitude and variation tendency of the system output.

To further validate the effectiveness of the proposed PWA identification approach for the tire longitudinal properties, the fitting error, i.e. the tire longitudinal force difference between the PWA model and the experimental data for low adhesion coefficient, is shown in Fig. 9. It can be seen from Fig. 9 that the distribution of the longitudinal force error is concentrated near zero, and its amplitude range is very small compared with that of the actual tire longitudinal force, which indicates that the PWA model can effective approximate the dynamical behaviors of the tire longitudinal properties.

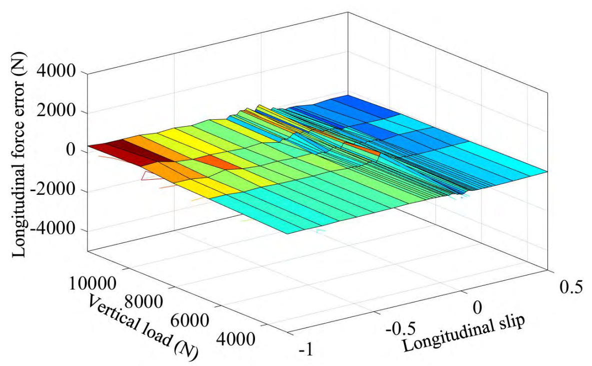

FIGURE 9. Fitting error between the PWA model and the experimental data for low adhesion coefficient.

Since only the relationship between the tire longitudinal force and its influence factors for low adhesion coefficient is conducted PWA identification in this study, and the tire longitudinal properties with different road longitudinal adhesion coefficients are assumed to be linearly dependent.

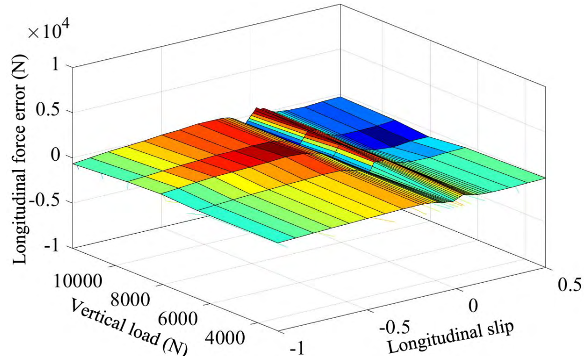

FIGURE 10. Fitting error between the PWA model and the experimental data for high adhesion coefficient.

Thus, the fitting error between the PWA model and the experimental data for high adhesion coefficient can be further shown in Fig. 10. As it can be demonstrated from the figure, similar conclusion can be obtained as the previous scenario, i.e. the output of the PWA model matches the experimental results accurately, which further validates the effectiveness of the PWA identification method.

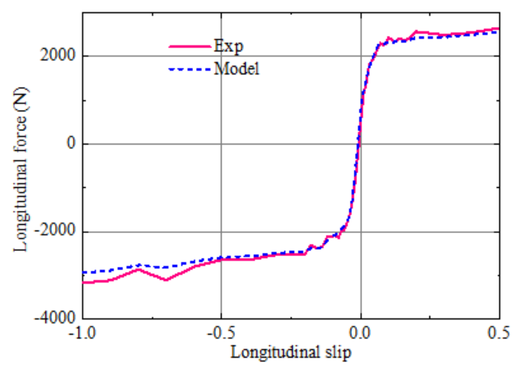

FIGURE 11. Comparison of the tire longitudinal force between the PWA model and the experimental data for low adhesion coefficient (8036 N).

Based on the identified PWA model of the tire longitudinal properties, the relationship between the tire longitudinal force and the longitudinal slip for specific tire load can be further obtained. Fig. 11 shows the comparison result between the longitudinal force output of the PWA model and the experimental data for low adhesion coefficient under tire load 8036 N. Fig. 12 shows the comparison result between the longitudinal force output of the PWA model and the experimental data for high adhesion coefficient under tire load 11760 N. Output comparisons of the the identified PWA model and the experimental results show a close match with an accuracy of 90%. Therefore, the PWA system can be used to describe a highly nonlinear dynamic of the tire longitudinal properties, which also demonstrates sufficiently that the proposed PWA approach can be generalized to the tire longitudinal properties modeling.

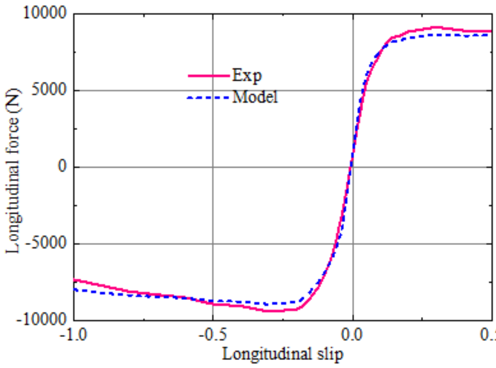

FIGURE 12. Comparison of the tire longitudinal force between the PWA model and the experimental data for high adhesion coefficient (11760 N).

VI. CONCLUSION

This paper proposed a novel PWA identification approach to approximate the tire longitudinal properties. By conducting the tire tests using a flat-plate tire test bench, the accurate test data about the tire longitudinal properties was obtained. On this basis, the system PWA identification problem is determined as constructing several affine submodels to approximate the nonlinear relationship between the tire longitudinal force and its influence factors, and the development of the tire PWA model mainly includes two steps, i.e. segmentation and regression. The region partition problem was solved via heuristic approach and the parameter vectors of the affine submodels were estimated using the least square algorithm. Comparisons between the PWA model output and the experimental data demonstrated the effectiveness of the proposed identification approach for the modeling of the tire longitudinal properties. Thus, the longitudinal motion controller design of the autonomous vehicles can be conducted based on the PWA model in the near future. In addition, the research idea of this paper, i.e. identifying the tire longitudinal properties as a PWA system based on experimental data, can provide a certain reference for modeling other types of the tire models.

REFERENCES

[1] H. Yamashita, Y. Matsutani, and H. Sugiyama, "Longitudinal tire dynamics model for transient braking analysis: ANCF-LuGre tire model," *J. Comput. Nonlinear Dyn.*, vol. 10, no. 3, pp. 1–11, May 2015.

[2] T. Chen, R. Liu, and Z. Yang, "Estimate method of tire-road friction coefficient in accident reconstruction," *J. Jiangsu Univ. Nat. Sci.*, vol. 36, no. 4, pp. 386–391, Nov. 2015.

[3] E. Velenis, P. Tsiotras, C. Canudas-de-Wit, and M. Sorine, "Dynamic tyre friction models for combined longitudinal and lateral vehicle motion," *Vehicle Syst. Dyn.*, vol. 43, no. 1, pp. 3–29, 2005.

[4] G. Wang, M. Yin, and C. Liang, "Measurement system of tire footprint geometric parameters based on image processing," *J. Jiangsu Univ. Nat. Sci.*, vol. 38, no. 2, pp. 139–143, Mar. 2017.

[5] F. Cheli, F. Braghin, M. Brusarosco, F. Mancosu, and E. Sabbioni, "Design and testing of an innovative measurement device for tyre–road contact forces," *Mech. Syst. Signal. Process.*, vol. 25, no. 6, pp. 1956–1972, Aug. 2011.

[6] S. Thanok and M. Parnichkun, "Longitudinal control of an intelligent vehicle using particle swarm optimization based sliding mode control," *Adv. Robot.*, vol. 29, no. 8, pp. 525–543, 2015.

[7] H. A. Hamersma and P. S. Els, "Longitudinal vehicle dynamics control for improved vehicle safety," *J. Terramech.*, vol. 54, pp. 19–36, Aug. 2014.

[8] X. Xu, T. Chen, L. Chen, and W. Wang, "Longitudinal force estimation for motorized wheels driving electric vehicle based on improved closed-loop subspace identification," *J. Jiangsu Univ. Nat. Sci.*, vol. 37, no. 6, pp. 650–656, Nov. 2016.

[9] S. Huang and R. Wei, "Autonomous intelligent vehicle and its performance in automated traffic systems," *Int. J. Control.*, vol. 72, no. 18, pp. 1665–1688, 1999.

[10] D. Cerotti, S. Distefano, G. Merlino, and A. Puliafito, "A crowd-cooperative approach for intelligent transportation systems," *IEEE Trans. Intell. Transp. Syst.*, vol. 18, no. 6, pp. 1529–1539, Jun. 2017.

[11] H. Jiang, C. Li, S. Ma, S. H. Ding, and C. Zhang, "Path tracking control of automatic parking for intelligent vehicle based on non-smooth control strategy," *J. Jiangsu Univ. Nat. Sci.*, vol. 38, no. 5, pp. 497–502, 2017.

[12] K. EI Majdoub, F. Giri, H. Ouadi, L. Dugard, and F. Z. Chaoui, "Vehicle longitudinal motion modeling for nonlinear control," *Control. Eng. Pract.*, vol. 20, no. 1, pp. 69–81, Jan. 2012.

[13] K. H. Guo, Z. C. Yuan, and D. Lu, "A study on the prediction capability of UniTire model for combined slips mode," *Automot Eng.*, vol. 28, no. 6, pp. 565–568, 2006.

[14] H. Pacejka, *Tire and Vehicle Dynamics*. London, U.K.: Butterworth, 2002.

[15] P. Dugoff and L. Segel, "An analysis of tire traction properties and their influence on vehicle dynamic performance," *SAE Trans.*, vol. 79, pp. 1219–1243, Jan. 1970.

[16] E. Sabbioni, R. Bao, F. Cheli, and D. Tarsitano, "A particle filter approach for identifying tire model parameters from full-scale experimental tests," *J. Mech. Des.*, vol. 139, no. 2, p. 021403, 2017.

[17] Y. Ding, C. Zhu, and H. Zhang, "A type of distribution control method for nonlinear stochastic systems," *J. Drainage Irrigation Mach. Eng.*, vol. 35, no. 9, pp. 774–779, Sep. 2017.

[18] X. Sun, Y. Cai, C. Yuan, S. Wang, and L. Chen, "Vehicle height and leveling control of electronically controlled air suspension using mixed logical dynamical approach," *Sci. China. Technol. Sci.*, vol. 59, no. 12, pp. 1814–1824, Dec. 2016.

[19] G. Pola and M. D. Di Benedetto, "Symbolic models and control of discrete-time piecewise affine systems: An approximate simulation approach," *IEEE Trans. Autom. Control*, vol. 59, no. 1, pp. 175–180, Jan. 2014.

[20] H. Molla-Ahmadian, F. Tahami, A. Karimpour, and N. Pariz, "Hybrid control of DC–DC series resonant converters: The direct piecewise affine approach," *IEEE Trans. Power Electron.*, vol. 30, no. 3, pp. 1714–1723, Mar. 2015.

[21] Y. Zhang, Z. Wei, X. Zhu, Y. Hu, and L. Li, "Control strategy for precision water-fertilizer irrigation system and its verification," *J. Drain. Irrig. Mach. Eng.*, vol. 35, no. 12, pp. 1088–1095, Dec. 2017.

[22] Y. Zhao, H. Fu, F. Lin, and Y. Li, "Advancement of non-pneumatic wheels and mechanical characteristics," *J. Jiangsu Univ. Nat. Sci.*, vol. 37, no. 6, pp. 621–627, 2016.

[23] E. D. Sontag, "Nonlinear regulation: The piecewise linear approach," *IEEE Trans. Autom. Control*, vol. AC-26, no. 2, pp. 346–358, Apr. 1981.

[24] C. Y. Lai, C. Xiang, and T. H. Lee, "Data-based identification and control of nonlinear systems via piecewise affine approximation," *IEEE Trans. Neural Netw.*, vol. 22, no. 12, pp. 2189–2200, Dec. 2011.

[25] X. Zhang, D. Zhang, and Y. Liu, "Hydraulic turbine governing system based on fractional-order fuzzy PID control," *J. Drain. Irrig. Mach. Eng.*, vol. 34, no. 6, pp. 504–510, Jun. 2016.

[26] M. di Bernardo, U. Montanaro, R. Ortega, and S. Santini, "Extended hybrid model reference adaptive control of piecewise affine systems," *Nonlinear Anal., Hybrid Syst.*, vol. 21, pp. 11–21, Aug. 2016.

[27] W. P. M. H. Heemels, B. De Schutter, and A. Bemporad, "Equivalence of hybrid dynamical models," *Automatica*, vol. 37, no. 7, pp. 1085–1091, 2001.

[28] N. Groot, B. De Schutter, and H. Hellendoorn, "Integrated model predictive traffic and emission control using a piecewise-affine approach," *IEEE Trans. Intell. Transp.*, vol. 14, no. 2, pp. 587–598, Jun. 2013.

[29] G. Andrikopoulos, G. Nikolakopoulos, I. Arvanitakis, and S. Manesis, "Piecewise affine modeling and constrained optimal control for a pneumatic artificial muscle," *IEEE Trans. Ind. Electron.*, vol. 61, no. 2, pp. 904–916, Feb. 2014.

[30] M. Wang, L. Dai, H. Dai, J. Mao, and L. Liang, "Support vector regression based model for predicting water level of Dongting Lake," *J. Drain. Irrig. Mach. Eng.*, vol. 35, no. 11, pp. 954–961, Nov. 2017.

[31] A. L. Juloski, S. Weiland, and W. P. M. H. Heemels, "A Bayesian approach to identification of hybrid systems," in *Proc. 43rd IEEE Conf. Decision Control*, Nassau, Bahamas, Dec. 2004, pp. 13–19.
[32] J. Roll, A. Bemporad, and L. Ljung, "Identification of piecewise affine systems via mixed-integer programming," *Automatica*, vol. 40, no. 1, pp. 37–50, Jan. 2004.
[33] A. Bemporad, A. Garulli, S. Paoletti, and A. Vicino, "A bounded-error approach to piecewise affine system identification," *IEEE Trans. Autom. Control*, vol. 50, no. 10, pp. 1567–1580, Oct. 2005.
[34] M. Vaezi and A. Izadian, "Piecewise affine system identification of a hydraulic wind power transfer system," *IEEE Trans. Control Syst. Technol.*, vol. 23, no. 6, pp. 2077–2086, Nov. 2015.
[35] S.-L. Koo and H.-S. Tan, "Tire dynamic deflection and its impact on vehicle longitudinal dynamics and control," *IEEE/ASME Trans. Mechatronics*, vol. 12, no. 6, pp. 623–631, Dec. 2007.
[36] T. R. Botha and P. S. Els, "Digital image correlation techniques for measuring tyre–road interface parameters: Part 2—Longitudinal tyre slip ratio measurement," *J. Terramechanics*, vol. 61, pp. 101–112, Oct. 2015.
[37] E. Lee, J. Lee, and S. B. Choi, "String tyre model for evaluating steering agility performance using tyre cornering force and lateral static characteristics," *Vehicle Syst. Dyn.*, vol. 55, no. 2, pp. 231–243, 2017.
[38] B. Ma, C. Lv, Y. Liu, M. Zheng, Y. Yang, and X. Ji, "Estimation of road adhesion coefficient based on tire aligning torque distribution," *J. Dyn. Syst. Meas., Control*, vol. 140, no. 5, pp. 1–17, Dec. 2017.
[39] L. Chen, Y. Luo, M. Bian, Z. Qin, J. Luo, and K. Li, "Estimation of tire-road friction coefficient based on frequency domain data fusion," *Mech. Syst. Signal. Process.*, vol. 85, pp. 177–192, Feb. 2017.
[40] J.-H. Yoon, S. E. Li, and C. Ahn, "Estimation of vehicle sideslip angle and tire-road friction coefficient based on magnetometer with GPS," *Int. J. Automot. Technol.*, vol. 17, no. 3, pp. 427–435, Jun. 2016.
[41] L. Fabien, "On the complexity of piecewise affine system identification," *Automatica*, vol. 62, pp. 148–153, Dec. 2015.
[42] S. Kersting and M. Buss, "Direct and indirect model reference adaptive control for multivariable piecewise affine systems," *IEEE Trans. Autom. Control*, vol. 62, no. 11, pp. 5634–5649, Nov. 2017.
[43] J. Sun, M. Liu, Y. Li, Z. Su, and Y. Kang, "Analysis on steady fluid dynamics of cylindrical gas seal by CFD," *J. Drain. Irrig. Mach. Eng.*, vol. 35, no. 11, pp. 968–974, Nov. 2017.
[44] J. Liu, S. Liu, and H. Fu, "Analysis of AFS based on car navigation electronic map," *J. Jiangsu Univ. Nat. Sci.*, vol. 39, no. 1, pp. 1–6, Jan. 2018.
[45] B. Zhang, M. Hsu, and U. Dayal, "Harmonic average based clustering method and system," U.S. Patent 6 584 433 B1, Jun. 24, 2003.
[46] V. Kathiresan and P. Sumathi, "An efficient clustering algorithm based on Z-score ranking method," in *Proc. Int. Conf. Comput. Commun. Inform. (ICCCI)*, Coimbatore, India, Jan. 2012, pp. 1–4.
[47] M. Yedla, S. R. Pathakota, and T. M. Srinivasa, "Enhancing K-means clustering algorithm with improved initial center," *Int. J. Comput. Sci. Inf. Technol.*, vol. 1, no. 2, pp. 121–125, 2010.
[48] S. Paoletti, "Identification of piecewise affine models," Ph.D. dissertation, Dept. Inf. Eng., Siena Univ., Siena, Italy, 2004.
[49] C. Yin, J. Zhang, S. Wang, X. Sun, and X. Xu, "Influence of damper indicator diagram area on vehicle ride comfort," *J. Jiangsu Univ. Nat. Sci.*, vol. 38, no. 6, pp. 645–651, Nov. 2017.
[50] M. Momeni, H. M. Kelk, and H. Talebi, "Rotating switching surface control of series-resonant converter based on a piecewise affine model," *IEEE Trans. Power Electron.*, vol. 30, no. 3, pp. 1762–1772, Mar. 2015.

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