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# Impact of Social Interaction on the Capacity of Hybrid Wireless Networks

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**ABSTRACT** In this paper, we jointly consider the optimal max-$L$-hops routing policy and social features for the throughput capacity of a hybrid wireless network, which is different from the existing works that ignored the network traffic model. For the social feature, each node's social group is determined by a probability related to the distance from the source node. To embody the features of social behavior, we propose a traffic model within a social group. Under the max-$L$-hops routing policy in the hybrid wireless network, we analyze the effects of physical distance, clustering property, and a number of base station on throughput capacity. We also study the influence of different social group sizes on the throughput capacity. Our results demonstrate that: 1) the optimal $L$ is not only related to social group size, but also the selection of destination nodes; 2) when the social contact factors $\alpha$ and $\beta$ increase, the throughput capacity trends to be independent of the number of base station and routing parameter $L$. Particularly, our results show that the wireless network is scalable with the number of node, if the social contact factor is large enough; and 3) the results demonstrate that the base station and social interaction improve the throughput capacity of hybrid wireless networks.

**INDEX TERMS** Throughput capacity, social group, paw-law distribution, hybrid wireless network.

## I. INTRODUCTION

Wireless ad hoc network is a decentralized type of wireless network which consists of a set of nodes. Each node communicates with each other over a wireless channel using multi-hops transmission. Due to the distributed control and potential mobility of nodes, it causes a lot of problems in the network design. The main problem is that of network capacity. Network capacity has been a hot topic in the past few years. Gupta and Kumar [1] started a groundbreaking work on network capacity. They analyzed the network capacity from the perspective of scaling law. Specifically, for a network that contains $n$ nodes, an average per-node throughput capacity was $\Theta\left(\frac{1}{\sqrt{n\log n}}\right)$[1] as $n \to \infty$, when each node followed independent identical distribution (i.i.d) and the destination was selected randomly and independently. The results indicated that per-node throughput would be vanished as $n \to \infty$. The reason of the pessimistic result is that each node not only needs to relay other traffics, but also interfered by simultaneous transmission in the network. To alleviate the relay load, Liu *et al.* [2] and Li *et al.* [3] assigned some base stations into the network, which was named as hybrid wireless network (HWN). In the HWN, the base stations were connected by an infinite capacity wired network and did not consume wireless bandwidth resource. Each node could utilize ad hoc mode or cellular mode to deliver packets. If the destination was located far from the source node, the packets would be delivered by cellular mode. It not only mitigates the relay load of each node, but also alleviates the interference, such that per-node throughput could be increased in some extent. There are two major routing policies employed in the HWN: same cell routing policy [2] and Max-$L$-hops routing policy [3]. For first policy, the data is delivered by ad hoc mode if the destination node is served by a same base station. Otherwise, it will be served by cellular mode using one hop transmission. As for the Max-$L$-hops routing policy, the data is delivered by ad hoc mode if the destination node is located within $L$ hops from the source node. Otherwise, it would be served by cellular mode.

Most works on the two policies focused on the destination was randomly and uniformly selected, such that the traffic flows were uniform. However, it can not reflect the realistic

[1]Given two functions $x(n)$ and $y(n)$: $x(n) = O(y(n))$ indicates $\lim_{n\to\infty} x(n)/y(n) = c < \infty$; $x(n) = o(y(n))$ indicates $\lim_{n\to\infty} x(n)/y(n) = 0$; if $y(n) = O(x(n))$, $x(n) = \Omega(y(n))$ w.h.p.; if both $x(n) = \Omega(y(n))$ and $x(n) = O(y(n))$, $x(n) = \Theta(y(n))$; $x(n) = \widetilde{\Theta}(y(n))$ indicates $x(n) = \Theta(y(n))$ when logarithmic terms are ignored.

node/user's behavior, where each user may appear social behaviors. Studies on wireless social networks in conformity with abundant datasets of realistic networks had received widely concern. In [4], Palla *et al.* demonstrated that actual networks were made of highly overlapping cohesive groups of nodes, and in [5], Mislove *et al.* studied the characteristics of large scale online social network, the result showed that social network could be interpreted as a power-law, small-world, and scale-free networks. Later, Viswanath *et al.* [6] extended the work in [5] and addressed an evolution model between users in the online social network. For the wireless social networks, each node belongs to a social group and only communicates to its social group nodes. In particular, each source chooses its destination according to a given priority. In [7], Barabási and Albert showed that the social group had a power-law distribution and the connection between source and destination was related to the distance between them. Milgram [8] showed that the social network was essential a small-world network, which was proposed by Kleinberg [9]. They investigated a planar grid network and found the social connection of each node could be divided into local contact and long range contact. The local contact was defined as the four possible directions within one hop transmission. While the long range contact denoted the connection of a destination far away from the source. The probability of long range contact was inverse proportion to the distance between source and destination. Based on the Kleinberg's model, which is a fundamental mode of social network, Li *et al.* [10] investigated the effect of power-law distribution traffic flows on the capacity of wireless network. However, they did not consider the influence of social group. Later, Azimdoost *et al.* [11], [12] and Garcia-Luna-Aceves [13] jointly considered the social network and wireless ad hoc network to derive the throughput capacity. Specifically, they assumed there were four neighboring contacts and one long range contact of each node. Nevertheless, the assumption of only one long range contact was too limited. In [14], they extended the number of long range contact node to exactly $q$. The selection of $q$ nodes followed a power-law distribution related to their location. In particular, for a node $v$, the probability of node $v$ was selected as a long range contact of node $s$ was proportional to $d^{-\alpha}(s, v)$, where $\alpha > 0$ and $d^{-\alpha}(s, v)$ was the lattice distance from $s$ to $v$. Then the source node uniformly and randomly selected a destination from its $q$ size social group, i.e., for any social group node $k$, the probability of node $k$ was selected as the destination was $\frac{1}{q}$. Varying $q = \Theta(1)$ to $q = \Theta(n)$, in [12], they studied the effect of social group size on the throughput capacity of wireless network. The results revealed that the throughput capacity was not only impacted by social group selection factor $\alpha$, but also influenced by social group size $q$. Since the social group would be evolutional, Fu *et al.* [16] studied the capacity of wireless network based on an evolutional social network, where each node followed a power law distribution. While Wang [17] analyzed the throughput capacity under a population-based formation social model, in which contained three-layered structure and each layer was controlled by a factor. As for the mobile wireless network, Lu *et al.* [18] analyzed capacity and delay in the vehicular network with social-proximity. While in [19], Ren *et al.* focused on the multicast capacity by employing directional antenna to decrease the interference. After that, plenty of works had been done on the throughput capacity of wireless social network. Following the social network model in [16], Zheng *et al.* [20] analyzed the throughput capacity of inhomogeneous wireless network. While Wei *et al.* [21] studied the throughput capacity of three-dimensional wireless social network. Due to the broadcast nature of wireless channel [22], Zheng *et al.* [23] further considered the secrecy capacity of inhomogeneous wireless social network. Since directional antenna can improve the capacity to some extent, Qin *et al.* [24] improved the throughput capacity by assuming that each node was equipped with directional antenna. Besides, Jia *et al.* [25] combined the social network with cognitive network and derived the throughput capacity. However, all the works mentioned above only concerned pure ad hoc network. As for the hybrid wireless network, Hou *et al.* [26] assigned base stations to the wireless network. According to the social contact model proposed in [14], they addressed the impact of social interaction on the throughout capacity of HWN, where each node had exactly $q$ social group nodes. Nevertheless, they only considered a simple social traffic model, which can not reflect the realistic flow of social network.

In this work, we extend our previous work [27] to hybrid wireless network. Specifically, in order to better reflect the data flows of social network, each node chooses nodes as its social group according to a power-law distribution with factor $\alpha$. The selection of destination node within the social group also follows a power-law distribution with factor $\beta$. By using the Max-$L$-hops routing policy, we investigate the impact of base stations, routing policy, social group factors $\alpha$ and destination selection factor $\beta$ on the throughput capacity. The results show that, under the optimal routing parameter $L$, social interaction can increase the throughput capacity of hybrid wireless network. Moreover, we also find the optimal number of base station to maximize the throughput capacity.

The main contributions in this work are concluded as follows:

- We proposed a more realistic social network model. Different from that of [26] where the selection of destination node within social group is uniform, we consider an evolution social interaction model, where the selection of destination node follows a power-law distribution with factor $\beta$.
- Considering the impact of factor $\beta$ on the selection of destination node, we find an optimal Max-$L$-hops routing policy in HWN, as well as the optimal number of base station.
- We also jointly analyze the effect of social feature factors $\alpha$ and $\beta$, optimal hop length $L$ and number of base station on the throughput capacity.

The rest of the paper is organized as follows. In section II, we introduce the system model with social features. The procedure of capacity analysis is presented in section III. In section IV, we derive the throughput capacity of ad hoc layer and combine it with cellular layer in section V. We give a numerical simulation and comparison with the previous works in Section VI. Finally, the paper is concluded in section VII.

## II. SYSTEM MODELS

We consider a hybrid wireless network (HWN) with $n$ ad hoc nodes and $m$ base stations in an unit disk. As shown in Fig. 1, there are $n$ nodes randomly and uniformly distributed in the network, while $m$ base stations are regularly deployed. The HWN can be divided into two layers: ad hoc layer and cellular layer. The cellular layer is composed by $m$ cells which is divided by $m$ base stations, such that the coverage area of each cell is $\frac{1}{m}$. The base stations are connected by an infinite bandwidth wired network. In order to guarantee the connectivity of network, we assume the transmission range of each node is $r(n)$, where $r(n) = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$ [28]. Since there exists interference caused by simultaneous transmission, we employ protocol model [1] to define a successful transmission. Let $X_i$, $X_j$ and $X_k$ be the locations of node $v_i$, $v_j$ and $v_k$, respectively. Node $v_i$ at location $X_i$ can successfully transmit information to node $v_j$ at location $X_j$, if for any nodes $v_k$ at location $X_k$, $v_k \neq v_i$, that transmit information simultaneously with $v_i$, then node $v_i$ can transmit information successfully to node $v_j$ if $|X_i - X_j| \leq r(n)$ and $|X_k - X_j| \geq (1 + \Delta)r(n)$, where $\Delta > 0$ is a constant of guard zone factor.

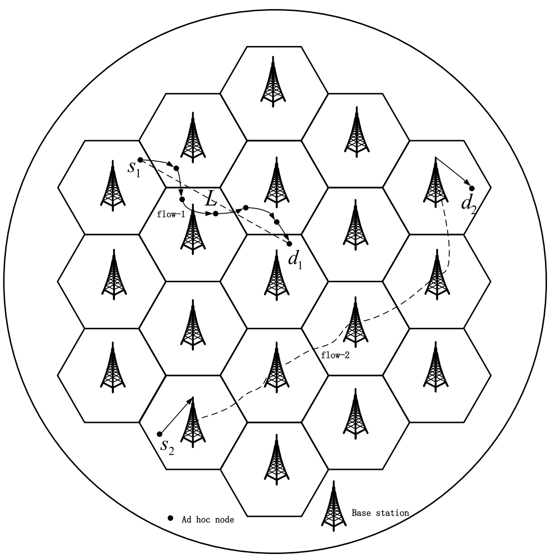


**FIGURE 1.** Hybrid network mode. If flow 1 (source $s_1$ and destination $d_1$ line) is within $L$ hops, the transmission uses multihops ad hoc model. Otherwise uses cellular mode, such as flow 2 (source $s_2$ and destination $d_2$ line).

Without loss of generality, we employ the time division multiple access (TDMA) medium access strategy to schedule the transmission. In particular, the system is time-slots and the network area is tesselated into squares with side-length $C_1 r(n)$ $(C_1 < \frac{1}{4})$. Each square in a big square grouped by $K^2$ squares takes turns to transmit, where $K \geq (2+\Delta)$. As shown in Fig. 2, the squares with cross sign are the simultaneous transmitting squares. The packet of source node can be transmitted by multi-hops transmission (ad hoc model), also can be directly delivered to the base station (cellular mode). Let $W_a$ and $W_c$ denote the bandwidth resources allocated to the ad hoc model and cellular mode, respectively. The total bandwidth is $W = W_a + W_c$. We assume each source node predicts the location of its local contacts and long range contacts. Thus, the packet is firstly transmitted to the local contact which is nearest to the destination, then continued to deliver to the destination.

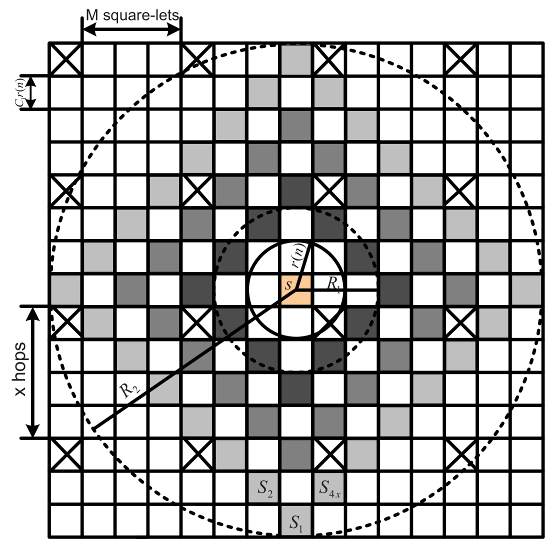


**FIGURE 2.** The network is divided into squares with side length $C_1 r(n)$. For a given source node $s$, the innermost circle denotes the transmission range. Gradual gray level from inner to outside indicates the probability that a node is selected as a member of social group is decreasing. $R1(R2)$ is the distance from the source $s$ to its destination. Squares $(\{s_1, s_2, \ldots s_{4x}\})$ contain the nodes with probability $P(h_{vt} = x)$. Squares with cross are the squares that can transmit simultaneously and $M$ (16 squares in this figure) squares form a group to take turns to transmit.

Since the network contains ad hoc layer and cellular layer, before transmitting the packet to the destination, the source should decide which mode to transmit the packet, i.e., ad hoc mode and cellular mode. As shown in Fig. 1, we apply the Max-$L$-hops policy to determine the transmission mode. That is, if the location of destination is within $L$ hops far from the source node, the packet would be delivered by ad hoc model, where $L$ is variable and impacts the performance of network. Otherwise, the packet would be transmitted by one hop cellular model. In the ad hoc model, the packet is delivered by multi-hops transmission which is relayed by the intermediate nodes. While for the cellular model, source node sends the packet to its nearest base station using one hop transmission, then the packet is delivered to destination from the base station near to the destination.

To evaluate the influence of social interaction on the throughput capacity, we adopt the social model proposed in our previous work [27] to describe the node's social behaviors. Specifically, for each source node, there exists one local contact within its four adjacent cells. As shown in Fig.2, each source actually has four local contacts. However, as we are investigating the asymptotic result, the communication of local contact does not impact the order of throughput capacity results. Thus, we ignore the influence of local contact, only consider the long range contacts. Additionally, we assume the number of social group node for each node is $q$, where $q$ can vary from a constant to $n$. Define $G$ as the social group of node $s$, then, for a node $j$ located at a distance $d_j$ away from $s$, the probability that $j$ belongs to $G$ is proportional to $d_j^{-\alpha}$. That is, for nodes $n_1, n_2, \ldots, n_q$, the probability that these nodes belong to $G$ is

$$Pr(G = \{n_1, n_2, \ldots, n_q\}) = \frac{d_{n_1}^{-\alpha} d_{n_2}^{-\alpha} \ldots d_{n_q}^{-\alpha}}{\sum_{1 \le i_1 < i_2 \ldots < i_q \le n} d_{i_1}^{-\alpha} \ldots d_{i_q}^{-\alpha}}. \tag{1}$$

Particularly, the probability of a particular node $v_k$ as a member of $G$ is

$$\begin{aligned} Pr(v_k \in G) &= \frac{\sum_{1 \le i_1 < i_2 \ldots < i_{q-1} \le n, i_j \ne k} d_k^{-\alpha} d_{i_1} \ldots d_{i_{q-1}}}{\sum_{1 \le i_1 < i_2 \ldots < i_q \le n} d_{i_1}^{-\alpha} \ldots d_{i_q}^{-\alpha}} \\ &= \frac{d_k^{-\alpha} \sigma_{q-1,n-1}(\mathbf{d}_\mathbf{n}^{\bar{\mathbf{k}}})}{\sigma_{q,n}(\mathbf{d}_\mathbf{n})}. \end{aligned} \tag{2}$$

where $\mathbf{d_n} \triangleq (d_{i_1}^{-\alpha}, \ldots, d_{i_n}^{-\alpha})$.

To simplify the formula, we utilize some notions in **elementary symmetric polynomials** [29]. For an $n$ variables $\mathbf{x} = \{x_1, \ldots, x_n\}$, an elementary symmetric polynomials with $q$-th degree of these variables is defined as $\sigma_{q,n}(\mathbf{x}) = \sigma_{q,n}(x_1, \ldots, x_n) = \sum_{1 \le i_1 < i_2 \ldots < i_q \le n} x_{i_1} \ldots x_{i_q}$. Similarly, the elementary symmetric polynomials of the set where $x_k$ is absent from the former set of variables as $\sigma_{q,n-1}^{\bar{k}}(\mathbf{x}) = \sigma_{q,n-1}(x_1, \ldots, x_{k-1}, x_{k+1}, \ldots, x_n)$.

For each source node, the destination node is selected within its social group. Given a destination $v_t$ and node $v_k$, we have $Pr(v_t = v_k | v_k \notin G) = 0$, and

$$Pr(v_t = v_k) = Pr(v_t = v_k | v_k \in G) Pr(v_k \in G). \tag{3}$$

Different from [26] in which the authors considered the source uniformly selected destination from its social group, we consider that, in each social group, source node constructs its social nodes related to the distance from source node which follows a power law distribution with factor $\alpha$. Within the social group, the probability that a node is selected as the destination is related to the distance from its source, which is also followed a power law distribution [9], [30] with factor $\beta$. For a social group, let $\mathbf{d_q} = \{d_{i_1}^{-\beta}, d_{i_2}^{-\beta}, \ldots, d_{i_q}^{-\beta}\}$, we have

$$Pr(v_t = v_k | v_k \in G) = \frac{d_k^{-\beta}}{\sum_{j=1}^{q} d_j^{-\beta}} = \frac{d_k^{-\beta}}{\sigma_{1,q}(\mathbf{d_q})}. \tag{4}$$

Combining (2) and (4), (3) can be rewritten as

$$\begin{aligned} Pr(v_t = v_k) &= Pr(v_t = v_k | v_k \in G) Pr(v_k \in G) \\ &= \frac{d_k^{-\alpha-\beta} \sigma_{q-1,n-1}(\mathbf{d_n}^{\bar{k}})}{\sigma_{1,q}(\mathbf{d_q}) \sigma_{q,n}(\mathbf{d_n})}. \end{aligned} \tag{5}$$

In [26], Hou *et al.* considered that nodes in a social group enjoy an equal probability to be selected as a destination. Actually, nodes may exhibit different intimating degree, even they are in the same social group, such that we introduce a power law distribution on the selection of destination. Although it is just a factor difference on destination selection, the derivation of throughput capacity would be more complicated than previous work [26]. The result shows that the intimating degree can influence the throughput capacity significantly.

## III. CAPACITY ANALYSIS

Based on the proposed social traffic model, we investigate the impact of social interaction on the throughput capacity of HWN. Varying social group size $q$ and maximum hop length $L$ and social group size factor $\alpha$, social interaction factor $\beta$, the throughput capacity is derived.

### A. DEFINITION AND NOTIONS

The definition of throughput capacity is the average bits or packets transmitted per unit time. Each node transmits $\Lambda$ bits or packets per second is feasible if there exists a spatial and temporal scheme to schedule the transmission. Correspondingly, the network capacity is $\Lambda' = n\Lambda$. Let $\Theta(f(n))$ bits or packets per second define the throughput capacity. For constants $c' > 0$ and $c'' < \infty$, the throughput capacity is achievable with high probability, and

$$\lim_{n \to \infty} P\{\Lambda = c' f(n) \text{ is feasible}\} = 1, \tag{6}$$

$$\lim_{n \to \infty} P\{\Lambda = c'' f(n) \text{ is feasible}\} < 1. \tag{7}$$

In Table 1, we list all the parameters that will be used in later analysis, proofs and discussions.

**TABLE 1. Notations.**

| Notation | Definition |
|---|---|
| $n$ | Number of nodes in the network. |
| $\alpha$ | Social factor represents a node in the social group. |
| $\beta$ | Social factor that a node is selected as a destination. |
| $L$ | Maximum hop counts of ad hoc flows. |
| $d$ | Distance from source to destination. |
| $m$ | Number of base station. |
| $q$ | Social group size. |
| $W_a$ | Bandwidth for ad hoc model. |
| $W_c$ | Bandwidth for cellular mode. |
| $r(n)$ | Transmission range of ad hoc node. |
| $\Lambda_c$ | Capacity contributed by cellular layer. |
| $\Lambda_a$ | Capacity contributed by ad hoc layer. |
| $\Lambda$ | Per-node throughput capacity. |

### B. DERIVATION OF THROUGHPUT CAPACITY

We firstly analyze the capacity contributed by the base stations. In the cellular layer, if we allocate $W_c$ bandwidth resource to the cellular transmission, then throughput capacity of each cell is upper bound by $W_c$ and lower bounded by $\frac{W_c}{\Delta_c}$, where $\Delta_c$ is an interfering coefficient [3] and it is independent of $n$ and $m$. Thus, for each cell, the capacity contributed by a base station is $\Theta(W_c)$. Let $\Lambda_c$ denote the capacity contributed by cellular layer, we can conclude that $\Lambda_c = \Theta(mW_c)$ since the network contains $m$ base stations.

As for the ad hoc layer, the case is much more complicated. Firstly, we present the baseline to derive the capacity contributed by ad hoc layer.

Similar to the work of [11] and [14], as shown in Fig. 2, the network is divided into squares with side length of $C_1 r(n)$, where $r(n)$ is the transmission range of each node. Such that each node can transmit packets to any nodes located in the neighboring squares. Note that a cell of cellular layer may consist several squares. Since each source may select ad hoc mode or cellular mode to transmit, let $P_a$ denote the probability that the source transmits data using ad hoc mode, which is determined by the distance from source node to the destination. Then the average number of ad hoc flows in the entire network is $N_a = nP_a$. Correspondingly, the expected number of hops for each ad hoc flow is calculated according to the social traffic model. Let $E[h]$ denote the average hop count of each ad hoc flow. Then, the total number of hops in ad hoc layer is $H = N_a E[h]$.

Since we are considering an uniform network model and there are $1/r^2(n)$ squares in the network, the average number of ad hoc flows intersected a certain square is $E[I] = Hr^2(n)$. It indicates that the rate at which each square needs to transmit is less than $c_0 nE[I]$ with high probability, where $c_0$ is a constant. Without loss of generality, we adopt TDMA to schedule the transmission. In particular, the number of interfering squares of a given square is at most $c = O((2 + \Delta)^2)$ [1]. As shown in Fig. 2, only one square (crossed square) in each grouped squares (with group size $c = 16$) can transmit simultaneously, where $c$ is independent of $n$ and $m$. Since the bandwidth allocated to ad hoc layer is $W_a$, the rate at which each square gets to transmit is $W_a/c$ bits per second. This rate can be achieved by all squares if it is less than the rate available, i.e., $c_0 nE[I] \le W_a/c$. Considering the perspective of scaling law, the average throughput capacity of each ad hoc flow is $\Lambda_a^0 = \Theta(\frac{W_a}{E[I]})$. Correspondingly, the network capacity contributed by ad hoc layer is $\Lambda_a = N_a \Lambda_a^0$.

As illustrated in Fig. 2, under the ad hoc model, the maximum hop count from source node to destination node is $L$. If a destination located at $x$ hops from the source node, there exist $4x$ squares that contain such destination. Ignoring the effect of edge in a surface torus and defining $P(h_{v_t} = x)$ as the probability that the destination located at $x$ hops, we have

$$P(h_{v_t} = x) = \sum_{l=1}^{4x} \sum_{v_k \in A_l} P(v_t = v_k), \tag{8}$$

where $A_l$ is a square at a distance of $l$ hops away from the source square.

Correspondingly, on the basis of Max-$L$-hops resource allocation strategy, we now calculate the total number of hops that using ad hoc transmission mode.

$$N_a = nP_a = n \sum_{x=1}^{L} P(h_{v_t} = x) = n \sum_{x=1}^{L} \sum_{l=1}^{4x} \sum_{v_k \in A_l} P(v_t = v_k). \tag{9}$$

Combining the Max-$L$-hops routing policy [3], we get Theorem 1 as follows,

*Theorem 1: Under the Max-$L$-hops policy, the average number of ad hoc flows cross each square is $nr^2(n)E[h]$, where $E[h] = \sum_{x=1}^{L} xP(h_{v_t} = x)$.*

*Proof:* Let $h_i$ denote the number of hops of ad hoc flow $i$. Since there are $N_a$ ad hoc flows in the ad hoc layer, the average total number of hops $H$ generated by ad hoc flows is

$$E[H] = E\left[\sum_{i=1}^{N_a} h_i\right] = \sum_{i=1}^{N_a} E[h]. \tag{10}$$

Let $I_f^g = 1$ represent that flow $f$ passes square $g$, otherwise, $I_f^g = 0$. According to the Law of Large Numbers (LLN), we have $I_f^g = r^2(n)$. Thus, the average number of flows passing a given square can be denoted as:

$$\begin{aligned} E[I] &= E_H[E[I|H]] = E_H[HE[I_f^g]] \\ &= E[H] \cdot I[I_g^f] = N_a \cdot E[h] \cdot r^2(n).s \end{aligned} \tag{11}$$

Additionally, we have $N_a = nP_a$ and

$$\begin{aligned} E[h] &= \sum_{x=1}^{L} xP(h_{v_t} = x|\text{ad hoc flow}) \\ &= \sum_{x=1}^{L} x \frac{P(h_{v_t} = x, \text{ad hoc flow})}{P(\text{ad hoc flow})} \\ &= \frac{1}{P_a} \sum_{x=1}^{L} xP(h_{v_t} = x). \end{aligned} \tag{12}$$

□

According to (8), we also have

$$\sum_{x=1}^{L} xP(h_{v_t} = x) = \sum_{x=1}^{L} x \sum_{l=1}^{4x} \sum_{v_k \in s_l} P(v_t = v_k). \tag{13}$$

Combining (12) and (13), we get the average number of ad hoc flows crossing each square is

$$E[I] = nr^2(n) \sum_{x=1}^{L} x \sum_{l=1}^{4x} \sum_{v_k \in s_l} P(v_t = v_k). \tag{14}$$

To derive the capacity, the key point is $N_a$ and $E[I]$, which is influenced by the size of social group. To analyze the effect of social interaction on $N_a$ and $E[I]$, we will discuss the size of social group with $q = \Theta(n)$ and $q = \Theta(1)$, respectively.

### C. WHEN $q = \Theta(n)$

Before we analyze the case $q = \Theta(n)$, we present the following Lemma firstly.

*Lemma 1: If $q = \Theta(n)$, then $\frac{d_k^{-\alpha}\sigma_{q-1,n-1}^{\bar{k}}(\mathbf{d_n}^{-\alpha})}{\sigma_{q,n}(\mathbf{d_n}^{-\alpha})} = \frac{q}{n} = \Theta(1)$.*

*Proof:* According to definition of elementary symmetric polynomials, the probability that a particular node $v_k$ is a member of $G$ is

$$Pr(v_k \in G) = \frac{d_k^{-\alpha}\sigma_{q-1,n-1}(\mathbf{d_n}^{\bar{k}})}{\sigma_{q,n}(\mathbf{d_n})} = \frac{\sum_{1\le i_1<\ldots<i_q\le n, \exists h: i_h=k}\prod_{i=1}^{q} d_{i_j}^{-\alpha}}{\sum_{1\le i_1<\ldots<i_q\le n}\prod_{i=1}^{q} d_{i_j}^{-\alpha}}. \quad (15)$$

Assume that $d_{ij}^{-\alpha} = Y_{i_j} = \exp(Z_{ij})$ for $1 \le i_j \le n$,

$$\frac{d_k^{-\alpha}\sigma_{q-1,n-1}(\mathbf{d_n}^{\bar{k}})}{\sigma_{q,n}(\mathbf{d_n})} = \frac{\sum_{1\le i_1<\ldots<i_q\le n, \exists h: i_h=k}\prod_{i=1}^{q} Y_{i_j}}{\sum_{1\le i_1<\ldots<i_q\le n}\prod_{i=1}^{q} Y_{i_j}} = \frac{\sum_{1\le i_1<\ldots<i_q\le n, \exists h: i_h=k}\exp(\sum_{j=1}^{q} Z_{i_j})}{\sum_{1\le i_1<\ldots<i_q\le n}\exp(\sum_{j=1}^{q} Z_{i_j})}. \quad (16)$$

Due to $q$ ranges from $q_0 + 1$ to $n - 1$, we can exploit the LLN for $q > q_0$. For any small $\varepsilon > 0$, there exists a small $\delta(\varepsilon)$ such that $\lim_{(\text{Large q})} \frac{1}{q}\sum_{i=1}^{q} Z_i = \bar{Z} + \varepsilon$, with probability $1 - \delta(\varepsilon) \to 1$. where $\bar{Z}$ is the mathematical expectation of random variable $Z_i$.

$$\frac{d_k^{-\alpha}\sigma_{q-1,n-1}(\mathbf{d_n}^{\bar{k}})}{\sigma_{q,n}(\mathbf{d_n})} = \frac{\sum_{1\le i_1<\ldots<i_q\le n, \exists h: i_h=k}\exp(q(\bar{Z}+\varepsilon))}{\sum_{1\le i_1<\ldots<i_q\le n}\exp(q(\bar{Z}+\varepsilon))} = \frac{C_{n-1}^{q-1}}{C_n^q} = \frac{q}{n}. \quad (17)$$

□

*Lemma 2: when $q = \Theta(n)$, we have*

$$E[I] = \begin{cases} \Theta\left(n\left(\sqrt{\frac{\log n}{n}}\right)^{4-\beta} L^{3-\beta}\right), & 0 \le \beta \le 2 \\ \Theta(\log nL^{3-\beta}), & 2 \le \beta \le 3 \\ \Theta(\log n), & 3 \le \beta \end{cases} \quad (18)$$

*Proof:* According to Lemma 1, (14) can be simplified as

$$E[I] = nr^2(n)\sum_{x=1}^{L} x \sum_{l=1}^{4x}\sum_{v_k\in s_l} P(v_t = v_k) = nr^2(n)\sum_{x=1}^{L} x \sum_{l=1}^{4x}\sum_{v_k\in s_l} \frac{d_k^{-\beta}}{\sigma_{1,q}(\mathbf{d_q}^{-\beta})}. \quad (19)$$

This summation can be further simplified as follow:

$$\sum_{x=1}^{L} x \sum_{l=1}^{4x}\sum_{v_k\in s_l} d_k^{-\beta} \equiv \sum_{x=1}^{L}\sum_{l=1}^{4x}\sum_{v_k\in s_l}(c_k x^2 r(n))^{-\beta} \equiv nr(n)^{2-\beta}\sum_{x=1}^{L} x^{2-\beta}$$

(Approximated by integral for sufficient large $L$)

$$\equiv nr(n)^{2-\beta}\int_{x=1}^{L} u^{2-\beta}du \equiv \begin{cases} \Theta(nr(n)^{2-\beta}L^{3-\beta}), & 0 \le \beta \le 3 \\ \Theta(nr(n)^{2-\beta}), & 3 \le \beta \end{cases} \quad (20)$$

By the same approximation method, we also have

$$\sigma_{1,q}(\mathbf{d_q}^{-\beta}) \equiv \sigma_{1,\Theta(n)}(\mathbf{d}_{(\Theta(\mathbf{n}))}{}^{-\beta}) \equiv \sum_{k=1}^{\Theta(n)} d_k^{-\beta} \equiv \sum_{x=\lceil \frac{1}{C_1}+1\rceil}^{\lceil \frac{2}{C_1 r(n)}\rceil}\sum_{l=1}^{4x}\sum_{v_k\in s_l}(c_k x r(n))^{-\beta} \equiv nr(n)^{2-\beta}\sum_{x=\lceil \frac{1}{C_1}+1\rceil}^{\lceil \frac{2}{C_1 r(n)}\rceil} x^{1-\beta} \equiv nr(n)^{2-\beta}\int_{x=\lceil \frac{1}{C_1}+1\rceil}^{\lceil \frac{2}{C_1 r(n)}\rceil} u^{1-\beta}du \equiv \begin{cases} \Theta(n), & 0 \le \beta \le 2 \\ \Theta(nr(n)^{2-\beta}), & 2 \le \beta \end{cases} \quad (21)$$

Let $r(n) = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$ substitute to (19), (20) and (21). Then combining (20) and (21), we finish the proof. □

Since

$$N_a = nP_a = n\sum_{x=1}^{L} P(h_{v_t} = x) = n\sum_{x=1}^{L}\sum_{l=1}^{4x}\sum_{v_k\in A_l} P(v_t = v_k). \quad (22)$$

Removing $x$ and $r^2(n)$ in (19) and using the same process as (20) and (21), we obtain the following Lemma without proof.

*Lemma 3: when $q = \Theta(n)$, we have*

$$N_a = nP_a = \begin{cases} \Theta\left(n\left(\sqrt{\frac{\log n}{n}}\right)^{2-\beta} L^{2-\beta}\right), & 0 \le \beta \le 2 \\ \Theta(n), & 2 \le \beta \end{cases} \quad (23)$$

### D. WHEN $q = \Theta(1)$

The case of $q = \Theta(1)$ is more complicated than that of $q = \Theta(n)$, since it is hard to directly calculate $E[I]$. To derive $E[I]$, we analyze the upper bound and lower bound of $E[I]$, respectively. Before we calculate $E[I]$ and $N_a$, we refer the following Lemma from [29].

*Lemma 4: Let $\Psi = \{\psi_1, \ldots, \psi_n\}$ denote a set of nonnegative real numbers, where $n \ge 2$. Then we have*

$$\frac{\sigma_{1,n}(\Psi)\sigma_{p,n}(\Psi)}{(p+1)\sigma_{p+1,n}(\Psi)} = \Theta\left(\frac{n}{n-p}\right), \quad (24)$$

*where $p$ is finite and $n$ goes to infinite [29].*

*Lemma 5: When $q = \Theta(1)$, we have*

$$E[I] = \begin{cases} \Theta\left(n\left(\sqrt{\frac{\log n}{n}}\right)^{4-\alpha} L^{3-\alpha-\beta}\right), & 0 \le \alpha \le 2, \; 0 \le \alpha+\beta \le 3 \\ \Theta(\log n L^{3-\alpha-\beta}), & 2 \le \alpha, \; 0 \le \alpha+\beta \le 3 \\ \Theta(\log n), & else \end{cases} \tag{25}$$

*Proof:* To derive the upper bound and low bound of average number of ad hoc flow crossing each square, we refer the following two inequalities:

$$\sigma_{q-1,n}(\mathbf{d_n}^{-\alpha}) - d_k^{-\alpha}\sigma_{q-2,n}(\mathbf{d_n}^{-\alpha}) \le \sigma^{\bar{k}}_{q-1,n-1}(\mathbf{d_n}^{-\alpha}), \tag{26}$$

and

$$\sigma^{\bar{k}}_{q-1,n-1}(\mathbf{d_n}^{-\alpha}) \le \sigma_{q-1,n}(\mathbf{d_n}^{-\alpha}). \tag{27}$$

First, using the upper bound in (27), we have

$$\begin{aligned} \overline{E[I]} &= nr^2(n)\sum_{x=1}^{L} x \sum_{l=1}^{4x}\sum_{v_k\in s_l} \frac{d_k^{-\alpha-\beta}\sigma_{q-1,n}(\mathbf{d_n}^{-\alpha})}{\sigma_{1,q}(\mathbf{d_q}^{-\beta})\sigma_{q,n}(\mathbf{d_n}^{-\alpha})} \\ &= nr^2(n)\frac{\sigma_{q-1,n}(\mathbf{d_n}^{-\alpha})}{\sigma_{1,q}(\mathbf{d_q}^{-\beta})\sigma_{q,n}(\mathbf{d_n}^{-\alpha})}\sum_{x=1}^{L} x \sum_{l=1}^{4x}\sum_{v_k\in s_l} d_k^{-\alpha-\beta}. \end{aligned} \tag{28}$$

Using (20), we have

$$\sum_{x=1}^{L} x \sum_{l=1}^{4x}\sum_{v_k\in s_l} d_k^{-\alpha-\beta} \equiv \begin{cases} \Theta(nr(n)^{2-\alpha-\beta}L^{3-\alpha-\beta}), & 0 \le \alpha+\beta \le 3 \\ \Theta(nr(n)^{2-\alpha-\beta}), & 3 \le \alpha+\beta \end{cases} \tag{29}$$

According to the lemma 4, we have

$$\begin{aligned} \frac{\sigma_{q-1,n}(\mathbf{d_n}^{-\alpha})}{\sigma_{q,n}(\mathbf{d_n}^{-\alpha})} &\equiv \frac{1}{\sigma_{1,n}(\mathbf{d_n}^{-\alpha})}\Theta\left(\frac{nq}{n-q+1}\right) \\ &\equiv \Theta\left(\frac{1}{\sigma_{1,n}(\mathbf{d_n}^{-\alpha})}\right). \end{aligned} \tag{30}$$

Substituting factor $\beta$ for $\alpha$ in (21), we have

$$\Theta\left(\sigma_{1,n}(\mathbf{d_n}^{-\alpha})\right) \equiv \begin{cases} \Theta(n), & 0 \le \alpha \le 2 \\ \Theta(nr(n)^{2-\alpha}), & 2 \le \alpha \end{cases} \tag{31}$$

Next, we need to find the order of $\sigma_{1,q}(\mathbf{d_q}^{-\beta})$

Let $x_{q_i}$ be the hop length of the $i^{th}$ member of long range social group from the source node, then we have

$$\begin{aligned} \sigma_{1,q}(\mathbf{d_q}^{-\beta}) = \sum_{i=1}^{q} d_{q_i}^{-\beta} &= \sum_{i=1}^{q}(C_i r(n)x_{q_i})^{-\beta} \\ &= r^{-\beta}(n)\sum_{i=1}^{q}(C_i x_{q_i})^{-\beta}. \end{aligned} \tag{32}$$

It holds with probability one as $n \to \infty$. It means that, within a distance of $\Theta(1)$, we can find at least one long range contact node, which is the dominant term in (32). Thus the summation of (32) is $\Theta(1)$, and

$$\sigma_{1,q}(\mathbf{d_q}^{-\beta}) = \Theta(r^{-\beta}(n)). \tag{33}$$

According to (31) and (33), we have

$$\frac{\sigma_{q-1,n}(\mathbf{d_n}^{-\alpha})}{\sigma_{1,q}(\mathbf{d_q}^{-\beta})\sigma_{q,n}(\mathbf{d_n}^{-\alpha})} = \begin{cases} \Theta(\frac{r(n)^{\beta}}{n}), & 0 \le \alpha \le 2 \\ \Theta(\frac{r(n)^{\alpha+\beta-2}}{n}), & 2 \le \alpha \end{cases} \tag{34}$$

Combining (29) and (34), we have

$$\begin{aligned} &\sum_{x=1}^{L} x \sum_{l=1}^{4x}\sum_{v_k\in s_l} \frac{d_k^{-\alpha-\beta}\sigma_{q-1,n}(\mathbf{d_n}^{-\alpha})}{\sigma_{1,q}(\mathbf{d_q}^{-\beta})\sigma_{q,n}(\mathbf{d_n}^{-\alpha})} \\ &\equiv \begin{cases} \Theta(r(n)^{2-\alpha}L^{3-\alpha-\beta}), & 0 \le \alpha \le 2, \; 0 \le \alpha+\beta \le 3 \\ \Theta(r(n)^{2-\alpha}), & 0 \le \alpha \le 2, \; 3 \le \alpha+\beta \\ \Theta(L^{3-\alpha-\beta}), & 2 \le \alpha, \; 0 \le \alpha+\beta \le 3 \\ \Theta(1), & 2 \le \alpha, \; 3 \le \alpha+\beta \end{cases} \end{aligned} \tag{35}$$

Next, we compute the lower bound on $E[I]$. Using (26), we have

$$\begin{aligned} \underline{E[I]} = nr^2(n)\Big(&\sum_{x=1}^{L} x \sum_{l=1}^{4x}\sum_{v_k\in s_l} \frac{d_k^{-\alpha-\beta}\sigma_{q-1,n}(\mathbf{d_n}^{-\alpha})}{\sigma_{1,q}(\mathbf{d_q}^{-\beta})\sigma_{q,n}(\mathbf{d_n}^{-\alpha})} \\ &- \sum_{x=1}^{L} x \sum_{l=1}^{4x}\sum_{v_k\in s_l} \frac{d_k^{-2\alpha-\beta}\sigma_{q-2,n}(\mathbf{d_n}^{-\alpha})}{\sigma_{1,q}(\mathbf{d_q}^{-\beta})\sigma_{q,n}(\mathbf{d_n}^{-\alpha})}\Big). \end{aligned} \tag{36}$$

The first term in (36) has been derived in (35) and the second term in (36) can be computed as

$$\begin{aligned} &\sum_{x=1}^{L} x \sum_{l=1}^{4x}\sum_{v_k\in s_l} \frac{d_k^{-2\alpha-\beta}\sigma_{q-2,n}(\mathbf{d_n}^{-\alpha})}{\sigma_{1,q}(\mathbf{d_q}^{-\beta})\sigma_{q,n}(\mathbf{d_n}^{-\alpha})} \\ &= \frac{\sigma_{q-2,n}(\mathbf{d_n}^{-\alpha})}{\sigma_{1,q}(\mathbf{d_q}^{-\beta})\sigma_{q,n}(\mathbf{d_n}^{-\alpha})}\sum_{x=1}^{L} x \sum_{l=1}^{4x}\sum_{v_k\in s_l} d_k^{-2\alpha-\beta}. \end{aligned} \tag{37}$$

Following Lemma 4, we get

$$\frac{\sigma_{q-2,n}(\mathbf{d_n}^{-\alpha})}{\sigma_{q-1,n}(\mathbf{d_n}^{-\alpha})} \equiv \frac{1}{\sigma_{1,n}(\mathbf{d_n}^{-\alpha})}\Theta\left(\frac{n(q-1)}{n-q+2}\right) \equiv \frac{1}{\sigma_{1,n}(\mathbf{d_n}^{-\alpha})}. \tag{38}$$

Thus we have

$$\begin{aligned} \frac{\sigma_{q-2,n}(\mathbf{d_n}^{-\alpha})}{\sigma_{q,n}(\mathbf{d_n}^{-\alpha})} &\equiv \left(\frac{1}{\sigma_{1,n}(\mathbf{d_n}^{-\alpha})}\right)^2 \\ &\equiv \begin{cases} \Theta((\frac{1}{n})^2), & 0 \le \alpha \le 2 \\ \Theta((\frac{1}{nr(n)^{2-\beta}})^2), & 2 \le \alpha \end{cases} \end{aligned} \tag{39}$$

Based on (29), we also have

$$\sum_{x=1}^{L} x \sum_{l=1}^{4x} \sum_{v_k \in s_l} d_k^{-2\alpha-\beta} \equiv \begin{cases} \Theta(nr(n)^{2-2\alpha-\beta}L^{3-2\alpha-\beta}), & 0 \le 2\alpha+\beta \le 3 \\ \Theta(nr(n)^{2-2\alpha-\beta}), & 3 \le 2\alpha+\beta \end{cases} \quad (40)$$

Combining (33), (39) and (40), we have

$$\sum_{x=1}^{L} x \sum_{l=1}^{4x} \sum_{v_k \in s_l} \frac{d_k^{-2\alpha-\beta}\sigma_{q-2,n}(\mathbf{d_n}^{-\alpha})}{\sigma_{1,q}(\mathbf{d_q}^{-\beta})\sigma_{q,n}(\mathbf{d_n}^{-\alpha})} \equiv 1 \begin{cases} \Theta(\frac{1}{n}r(n)^{2-\alpha}L^{3-\alpha-\beta}), & 0 \le \alpha \le 2,\ 0 \le 2\alpha+\beta \le 3 \\ \Theta(\frac{1}{n}r(n)^{2-\alpha}), & 0 \le \alpha \le 2,\ 3 \le 2\alpha+\beta \\ \Theta(\frac{1}{nr(n)^{2-\beta}}L^{3-\alpha-\beta}), & 2 \le \alpha, 0 \le 2\alpha+\beta \le 3 \\ \Theta(1), & 2 \le \alpha,\ 3 \le 2\alpha+\beta \end{cases} \quad (41)$$

Comparing the upper bound and lower bound, we find the order of lower bound is much less than that of upper bound. Due to the number of average hop needs to large than 1, the average number of hop, when $0 \le \alpha \le 2, 3 \le 2\alpha+\beta$, is modified to $\Theta(1)$. Thus, on the basis of (35) and (41), and substituting $r(n) = \Theta(\sqrt{\frac{\log n}{n}})$, we have the Lemma 5. □

Similarly, when $q = \Theta(1)$, according to (9), the total number of ad hoc flows $N_a$ can be derived.

*Lemma 6: when $q = \Theta(1)$, we have*

$$N_a = \begin{cases} \Theta\left(n\left(\sqrt{\frac{\log n}{n}}\right)^{2-\alpha} L^{2-\alpha-\beta}\right), & 0 \le \alpha \le 2,\ 0 \le \alpha+\beta \le 2 \\ \Theta\left(n\left(\sqrt{\frac{\log n}{n}}\right)^{2-\alpha}\right), & 0 \le \alpha \le 2,\ 2 \le \alpha+\beta \\ \Theta(n), & 2 < \alpha,\ 2 \le \alpha+\beta \end{cases} \quad (42)$$

## IV. AD HOC NETWORK CAPACITY ANALYSIS

The throughput capacity of ad hoc layer can be calculated by

$$\Lambda_a^0 = \Theta\left(\frac{W_a}{E[I]}\right), \quad \text{and } \Lambda_a = N_a\Lambda_a^0. \quad (43)$$

### A. CASE I: WHEN $q = \Theta(n)$

Substituting (18) to (43), we have

*Lemma 7: In the case $q = \Theta(n)$, we have*

$$\Lambda_a^0 = \Theta\left(\frac{W_a}{E[I]}\right) = \begin{cases} \Theta\left(\frac{W_a}{n\left(\sqrt{\frac{\log n}{n}}\right)^{4-\beta} L^{3-\beta}}\right), & 0 \le \beta \le 2,\ L = \Omega\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\beta}}\right) \\ \Theta\left(\frac{W_a}{\log n}\right), & 0 \le \beta \le 2,\ L = O\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\beta}}\right) \\ \Theta\left(\frac{W_a}{L^{3-\beta}\log n}\right), & 2 \le \beta \le 3 \\ \Theta\left(\frac{W_a}{\log n}\right), & 3 \le \beta \end{cases} \quad (44)$$

*Proof:* To prove the above Lemma, we need discuss the impact of parameter $L$ on the throughout capacity.

(a) Case $0 \le \beta \le 2$: according to (14) and $\sum_{x=1}^{L} x \sum_{l=1}^{4x} \sum_{v_k \in s_l} P(v_t = v_k) \ge 1$, i.e., the average number of hops needs to larger than 1, such that the throughput capacity will be discussed under the cases of $L = \Omega\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\beta}}\right)$ and $L = O\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\beta}}\right)$, respectively. When $L = \Omega\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\beta}}\right)$, there is a deterministic constant $c > 0$ not depending on $n$, $W_a$, such that we have

$$\Lambda_a^0 = \begin{cases} \Theta\left(\frac{W_a}{n\left(\sqrt{\frac{\log n}{n}}\right)^{4-\beta} L^{3-\beta}}\right), & L = \Omega\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\beta}}\right) \\ \Theta\left(\frac{W_a}{\log n}\right), & L = O\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\beta}}\right) \end{cases} \quad (45)$$

(b) When $2 \le \beta \le 3$: since $L \ge 1$, we have

$$\Lambda_a^0 = \Theta\left(\frac{W_a}{L^{3-\beta}\log n}\right). \quad (46)$$

(c) When $3 \le \beta$, the destination is near to the source with high probability and located at a constant hop count away. Therefore, the flow would be transmitted using a constant hops in the ad hoc layer. Such that the per-node throughput capacity is $\Lambda_a^0 = \Theta(\frac{W_a}{\log n})$, which is independent of routing scheme parameter $L$. □

As for the network capacity contributed by ad hoc layer, we have

*Lemma 8: If $q = \Theta(n)$, we have*

$$\Lambda_a = \Theta\left(\frac{N_a W_a}{E[Z]}\right) = \begin{cases} \Theta\left(\frac{nW_a}{L\log n}\right), & 0 \le \beta \le 2, \; L = \Omega\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\beta}}\right) \\ \Theta\left(W_a\left(\sqrt{\frac{n}{\log n}}\right)^{\beta} L^{2-\beta}\right), & 0 \le \beta \le 2, \; L = O\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\beta}}\right) \\ \Theta\left(\frac{nW_a}{L^{3-\beta}\log n}\right), & 2 \le \beta \le 3 \\ \Theta\left(\frac{nW_a}{\log n}\right), & 3 \le \beta \end{cases} \tag{47}$$

### B. CASE II: $q = \Theta(1)$

Similarly, substituting (25) to (43), the following Lemma is obtained.

*Lemma 9: If $q = \Theta(1)$, we have (48), as shown at the bottom of this page.*

Similarly, we have

*Lemma 10: If $q = \Theta(1)$, we have (49), as shown at the bottom of this page.*

## V. THROUGHPUT CAPACITY ANALYSIS OF THE NETWORK

The network capacity is composed by the capacity of cellular layer and that of ad hoc layer, i.e.,

$$\Lambda' = \Lambda_a + \Lambda_m. \tag{50}$$

We will discuss the impact of number of base station on the throughput capacity, **we firstly take the following case as an example, then get the other cases similar to the example case. In addition, we summarize all the cases in the table 2.**

Considering the case, $\Lambda_a = \Theta\left(\frac{nW_a}{L\log n}\right)$, where $q = \Theta(n)$, $0 \le \beta \le 2$, and $L = \Omega\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\beta}}\right)$, we have

$$\Lambda' = \Lambda_a + \Lambda_m = \Theta\left(\frac{nW_a}{L\log n}\right) + \Theta(mW_c). \tag{51}$$

If $m = \Omega\left(\frac{n}{L\log n}\right)$, then we can have higher throughput when the traffic flows are transmitted through cellular layer and all bandwidth can be allocated to cellular communication, i.e., $W_a = 0$ and $W_c = W/2$. Correspondingly, the network capacity is

$$\Lambda'_{\max} = \Theta(mW). \tag{52}$$

and per-node throughput capacity is

$$\Lambda_{\max} = \begin{cases} \Theta(W), & m = \Omega(n) \\ \Theta(\frac{mW}{n}), & m = o(n) \end{cases} \tag{53}$$

If $m = o\left(\frac{n}{L\log n}\right)$, then we can have higher throughput when the traffic flows are transmitted through ad hoc layer and all bandwidth can be allocated to ad hoc communication, i.e., $W_c = 0$ and $W_a = W$. Correspondingly, the network capacity is

$$\Lambda'_{\max} = \Theta\left(\frac{nW}{L\log n}\right). \tag{54}$$

$$\Lambda_a^0 = \Theta\left(\frac{W_a}{E[Z]}\right) = \begin{cases} \Theta\left(\frac{W_a}{n\left(\sqrt{\frac{\log n}{n}}\right)^{4-\alpha} L^{3-\alpha-\beta}}\right), & 0 \le \alpha \le 2, 0 \le \alpha+\beta \le 2, L = \Omega\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\alpha-\beta}}\right) \\ \Theta\left(\frac{W_a}{\log n}\right), & 0 \le \alpha \le 2, 0 \le \alpha+\beta \le 2, L = O\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\alpha-\beta}}\right) \\ \Theta\left(\frac{W_a}{L^{3-\alpha-\beta}\log n}\right), & 2 \le \alpha+\beta \le 3 \\ \Theta\left(\frac{W_a}{\log n}\right), & \text{else} \end{cases} \tag{48}$$

$$\Lambda_a = N_a\Lambda_a^0 = \begin{cases} \Theta\left(\frac{nW_a}{L\log n}\right), & 0 \le \alpha \le 2, 0 \le \alpha+\beta \le 2, L = \Omega\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\alpha-\beta}}\right) \\ \Theta\left(W_a\left(\sqrt{\frac{n}{\log n}}\right)^{\alpha} L^{2-\alpha-\beta}\right), & 0 \le \alpha \le 2, 0 \le \alpha+\beta \le 2, L = O\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\alpha-\beta}}\right) \\ \Theta\left(\frac{nW_a}{L^{3-\alpha-\beta}\log n}\right), & 2 \le \alpha+\beta \le 3 \\ \Theta\left(\frac{nW_a}{\log n}\right), & \text{else} \end{cases} \tag{49}$$

**TABLE 2.** The throughput capacity results of wireless social hybrid network.

| q | $\alpha$ and $\beta$ | L | m | Per-node Throughput Capacity |
|---|---|---|---|---|
| $\Theta(n)$ | $0 \leq \beta \leq 2$ | $L = \Omega\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\beta}}\right)$ | $m = \Omega(n)$ | $\Theta(W)$ |
| | | | $m \in \left(\Omega\left(\frac{n}{L\log n}\right), o(n)\right)$ | $\Theta\left(\frac{mW}{n}\right)$ |
| | | | $m = o\left(\frac{n}{L\log n}\right)$ | $\Theta\left(\frac{W}{L\log n}\right)$ |
| | | $L = O\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\beta}}\right)$ | $m = \Omega(n)$ | $\Theta(W)$ |
| | | | $m \in \left(\Omega\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\beta} L^{2-\beta}\right), o(n)\right)$ | $\Theta\left(\frac{mW}{n}\right)$ |
| | | | $m = o\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\beta} L^{2-\beta}\right)$ | $\Theta\left(\frac{W}{n}\left(\sqrt{\frac{n}{\log n}}\right)^{\beta} L^{2-\beta}\right)$ |
| | $2 \leq \beta \leq 3$ | $L \geq 1$ | $m = \Omega(n)$ | $\Theta(W)$ |
| | | | $m \in \left(\Omega\left(\frac{n}{L^{3-\beta}\log n}\right), o(n)\right)$ | $\Theta\left(\frac{mW}{n}\right)$ |
| | | | $m = o\left(\frac{n}{L^{3-\beta}\log n}\right)$ | $\Theta\left(\frac{W}{L^{3-\beta}\log n}\right)$ |
| | $3 \leq \beta$ | not impacted by $L$ | $m = \Omega(n)$ | $\Theta(W)$ |
| | | | $m \in \left(\Omega\left(\frac{n}{\log n}\right), o(n)\right)$ | $\Theta\left(\frac{mW}{n}\right)$ |
| | | | $m = o\left(\frac{n}{\log n}\right)$ | $\Theta\left(\frac{W}{\log n}\right)$ |
| $\Theta(1)$ | $0 \leq \alpha \leq 2$ and $0 \leq \alpha+\beta \leq 2$ | $L = \Omega\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\alpha-\beta}}\right)$ | $m = \Omega(n)$ | $\Theta(W)$ |
| | | | $m \in \left(\Omega\left(\frac{n}{L\log n}\right), o(n)\right)$ | $\Theta\left(\frac{mW}{n}\right)$ |
| | | | $m = o\left(\frac{n}{L\log n}\right)$ | $\Theta\left(\frac{W}{L\log n}\right)$ |
| | | $L = O\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\alpha-\beta}}\right)$ | $m = \Omega(n)$ | $\Theta(W)$ |
| | | | $m \in \left(\Omega\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\alpha} L^{2-\alpha-\beta}\right), o(n)\right)$ | $\Theta\left(\frac{mW}{n}\right)$ |
| | | | $m = o\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\alpha} L^{2-\alpha-\beta}\right)$ | $\Theta\left(\frac{W}{n}\left(\sqrt{\frac{n}{\log n}}\right)^{\alpha} L^{2-\alpha-\beta}\right)$ |
| | $2 \leq \alpha+\beta \leq 3$ | $L \geq 1$ | $m = \Omega(n)$ | $\Theta(W)$ |
| | | | $m \in \left(\Omega\left(\frac{n}{L^{3-\alpha-\beta}\log n}\right), o(n)\right)$ | $\Theta\left(\frac{mW}{n}\right)$ |
| | | | $m = o\left(\frac{n}{L^{3-\alpha-\beta}\log n}\right)$ | $\Theta\left(\frac{W}{L^{3-\alpha-\beta}\log n}\right)$ |
| | else | not impacted by $L$ | $m = \Omega(n)$ | $\Theta(W)$ |
| | | | $m \in \left(\Omega\left(\frac{n}{\log n}\right), o(n)\right)$ | $\Theta\left(\frac{mW}{n}\right)$ |
| | | | $m = o\left(\frac{n}{\log n}\right)$ | $\Theta\left(\frac{W}{\log n}\right)$ |

and per-node throughput capacity is

$$\Lambda_{\max} = \Theta\left(\frac{W}{L\log n}\right). \tag{55}$$

## VI. NUMERICAL RESULTS AND DISCUSSIONS

Table 2 lists the throughput capacity results with different social interaction extent and number of base station, as well as optimal routing hops $L$. Fig. 3 illustrates the throughput capacity versus the number of base stations. Due to the diversity of results, we only consider two cases, i.e., $0 < \beta < 2$, $L = \Omega\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\beta}}\right)$, $q = \Theta(n)$ and $0 < \alpha < 2$, $0 < \alpha+\beta < 2$, $L = O\left(\left(\sqrt{\frac{n}{\log n}}\right)^{\frac{2-\beta}{3-\alpha-\beta}}\right)$, $q = \Theta(1)$. While for other cases, they are similar. As shown in Fig. 3, unsurprisingly, the throughput capacity is increased with the number of base stations. For these two cases, if the number of base station is up to $\Omega(n)$, the throughput capacity attains its maximum order $\Theta(\frac{W}{\log n})$. The reason is that, with the number of base station increasing, more traffic flow would prefer to the cellular mode.

In this paper, the results are obtained by mathematical proofs and expressed in terms of scaling laws. To validate the theoretical results with simulations, the number of nodes $n$ needs to be set very large. It is impossible to conduct a practical simulation since the computational complexity is non-polynomial. In this way, we will validate our results by numerical simulation with appropriate social factor $\alpha$ and $\beta$. Since our work employs the Max-$L$-hops routing policy, we also compare our results with those in [3] and [26]. Without loss of generality, as shown in Table 2, if the size of social group $q = \Theta(n)$ and the destination is selected uniformly $\beta = 0$, i.e., uniform traffic mode, then our results are the same with those in [3]. The main influence of social interaction on the throughput capacity of HWN can be summarized as follows:

- The results demonstrate that, when $q = \Theta(n)$ and the destination is chosen according to its distance from the source, the throughput capacity is independent of social group size factor $\alpha$. The reason is that, as the social group size is proportional to $n$, the construction of social group with factor $\alpha$ does not make much different, since most of nodes are selected as to a social group. Fig. 4 shows the maximum throughput capacity with different $\beta$ when applying the optimal $L$ and $q = \Theta(n)$, as well as comparison with that in [3] and [26]. The results reveal that increasing the value of $\beta$, the throughput is increased dramatically. It indicates that the selection of destination would dominate the throughput.

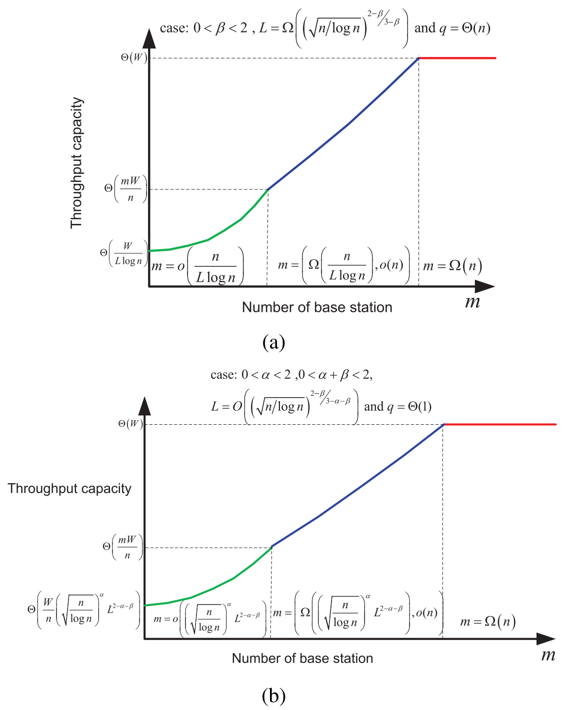


**FIGURE 3.** The bounds of throughput capacity vs the number of base station: 3(a) when $q = \Theta(n)$, 3(b) when $q = \Theta(1)$.

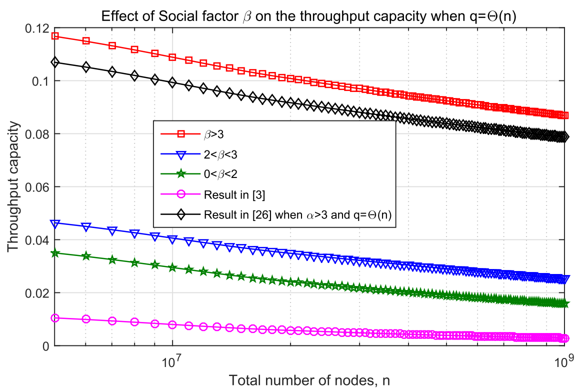


**FIGURE 4.** Effect of factor $\beta$ on the throughput capacity when $q = \Theta(n)$.

However, if the social factor $\alpha$ in [26] is larger than 3, our result shows that the capacity order is less than that of [26] when $\beta$ is less than 3. The reason is that, for large values of $\alpha$, social groups are localized, transmission distance from sources to destinations involves only $\Theta(1)$ hops.

- While $q = \Theta(1)$, the case is much different. As shown in Fig. 5, it shows that the factor $\alpha$ will play a role and the nodes appear more selective in the construction of their social groups. In particular, with increasing the value of $\alpha$, the throughput capacity increases and the influence of cellular layer decreases. When $\alpha + \beta > 3$, social interaction becomes dominant factor on the throughput capacity and the influence of cellular layer can be ignored. The reason is that within a constant hop counts, there exists a destination with high probability.

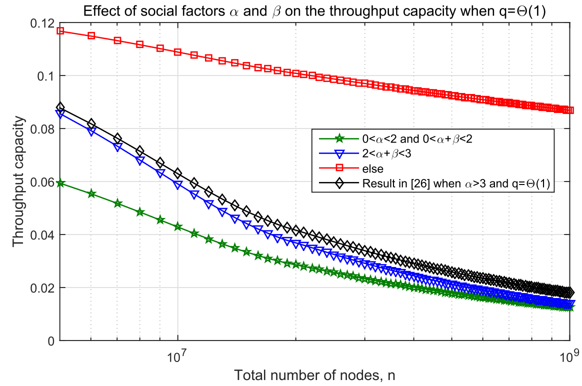


**FIGURE 5.** Effect of factors $\alpha$ and $\beta$ on the throughput capacity when $q = \Theta(1)$.

Such that the average hop count is proportional to $\Theta(1)$ which means all the traffic flows would transmit by ad hoc layer.

- Intuitively, we observe that the throughput capacity increases as $\alpha$ and $\beta$ increase. Nevertheless, when $\alpha$ increases, the probability of selecting a node far from the the source node as a member of social group reduces. Comparing with [26], we observe that the throughput capacity is not only influence by $\alpha$, but also impact by $\beta$. That is, even $\alpha$ equals 0, which means each node has a equal probability to be select to a social group independent of its distance from the source, the throughput capacity is still relevant to its distance from the source, since bigger $\beta$ indicates that the source node would communicate with the near social group nodes more frequently. Thus, the average hop count of ad hoc flows would be reduced. As a result, more ad hoc resources would be allocated for each flow, and the ad hoc layer capacity would be increased. Particularly, when $\beta > 3$, the throughput capacity is independent of $\alpha$.
- In the Max-$L$-hops routing scheme, we observe that optimal $L$ is determined by both $\alpha$ and $\beta$, but not $q$. Intuitively, the optimal $L$ is set to derive an appropriate average hop count. Particularly, if $\beta > 3$ when $q = \Theta(n)$ or $\alpha + \beta > 3$ when $q = \Theta(1)$, the throughput capacity is not impacted by $L$. The reason is that the destination node is located at a constant hops from the source with high probability.

## VII. CONCLUSION

This paper investigate the impact of an improved social traffic model on the throughput capacity of HWN. Two cases of social group size have been addressed as well as the selection of destination node. According to the double power-law distribution on social characteristic, we derive the throughput of HWN under the Max-$L$-hops routing scheme. The results show that the throughput capacity is associated with group size factor $\alpha$, destination selection factor $\beta$, number of nodes $n$, number of station $m$ and routing policy parameter $L$.

It is worth to emphasize that the effects of variable social group of each node is not involved, Thus, a more solid work, should be considered in the future work. Furthermore, the evolution and mobility of social group is also ignored, to make a comprehensive understand the impact of social interaction on the throughput capacity, all of these characteristics should be integrated in the future work.

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