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Bi-Stability Phenomenon in Constant On-Time Controlled Buck Converter With Small Output Capacitor ESR

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ABSTRACT This paper reports the finding of bi-stability phenomenon in a constant on-time (COT) controlled buck converter with a small output capacitor equivalent series resistance (ESR). In the previously published literatures, it has been reported that the COT controlled buck converter with a small output capacitor ESR operates in the reduced-frequency periodic oscillation state. However, it is newly validated by numerical simulations that for some specified initial states, such a buck converter with the same circuit parameters can operate in a chaotic oscillation state as well, reflecting the existence of bi-stable oscillation states. Based on the piecewise continuous model of COT controlled buck converter, the bi-stability phenomenon associated with two sets of initial states is revealed by numerical simulations and verified by PSIM circuit simulations. Consequently, when a small output capacitor ESR is used, the coexisting attractors' behavior of two bi-stable oscillation states appears in such a COT controlled buck converter, leading to the emergence of bi-stability phenomenon.

INDEX TERMS Constant on-time (COT) control, buck converter, equivalent series resistance (ESR), bi-stability.

I. INTRODUCTION

Multi-stability, the coexistence of topologically different and disconnected attractors related to different initial conditions, is an exciting phenomenon in a nonlinear dynamical system, which has been widely found in various scientific fields, such as engineering, physics, chemistry, biology, electronics, and so on [1]. For a determined set of system parameters, a nonlinear dynamical system with multi-stability has several finally stable operation states depending closely on its initial conditions [2]–[7].

A switching dc-dc converter is a typical nonlinear dynamical system, which plays a significant role in modern industrial power supply systems. The existence of multi-stability in the switching dc-dc converter has been previously reported [8]–[10]. In [8], some coexisting attractors are found in a voltage mode controlled buck converter with different initial conditions. In [9] and [10], when the compensation circuit of the control loop with high feedback gain is used, the multi-stability phenomenon can be observed in a switching dc-dc converter with multilevel control. However, compared with the voltage mode control and multilevel control, the constant-on-time (COT) control is a simple pulse frequency modulation (PFM) control technique without compensation circuit in the control loop, and it has recently attracted much attention due to its fast transient response and high light-load efficiency [11]. For a COT controlled buck converter, its dynamical behaviors are affected significantly by the output capacitor equivalent series resistance (ESR), and more concerns have been paid to the dynamical effects on the stability of circuit parameters [12]–[15]. Especially, when an output capacitor with a small ESR is utilized, the COT controlled buck converter operates in the reduced-frequency periodic oscillation state [13]. Interestingly, it is newly uncovered that with the same circuit parameters, the COT controlled buck converter can also operate in the chaotic oscillation state with the specified initial states, implying the occurrence of the bi-stability phenomenon [16].

When repeatedly powering on and off such a COT controlled buck converter, different initial values of the inductor current and capacitor voltage are randomly sensed, leading to the appearance of two types of uncertain operation states. To better understand the special nonlinear phenomenon, the time-domain waveforms, phase plane orbits and single-parameter bifurcation diagrams are used in this paper to exhibit the existence of bi-stability in a COT controlled buck converter with a small output capacitor ESR, which has not been previously reported in any published literature.

II. COT CONTROLLED BUCK CONVERTER

The circuit schematic of COT controlled buck converter and its main steady-state waveforms are depicted in Figs. 1(a) and 1(b), respectively. To the upper part of Fig. 1(a), the power stage circuit is shown, which contains the input voltage source V_{in}, switch S, diode D, inductor L, output capacitor C and its ESR r, and load resistor R. The control circuit to the lower part of Fig. 1(a) includes the comparator, RS trigger, and ON Timer. From Fig. 1(b), when the output voltage v_o decreases to the reference voltage V_{ref}, the switch S is turned on, and the inductor current i_L and output voltage v_o begin to increase. After the preset constant on-time interval T_{ON}, S is turned off by the ON Timer. When v_o decreases to V_{ref}, S is turned on again and a new switching cycle is initiated.

With respect to Fig. 1, it is known that the COT controlled buck converter is a simple second-order time-varying dynamical system with two state variables, the inductor current i_L and capacitor voltage v_C, denoted as $\mathbf{x} = [i_L, v_C]$. In this paper, we focus on the finding of the bi-stability phenomenon emerging from the COT controlled buck converter with a small output capacitor ESR.

III. COEXISTING DYNAMICAL BEHAVIORS

A. PIECEWISE MODEL AND ITS BI-STABILITY

The COT controlled buck converter is a structure-varying and piecewise-linear dynamical system. Referring to [13], the state equations for different switch states can be described as

$$\dot{x} = \mathbf{A}_j\mathbf{x}(t) + \mathbf{B}_j V_{in}(t_{j-1} \le t \le t_j \quad j = 1, 2, 3) \tag{1}$$

where j represents the j-th switch state, $[t_{j1}, t_j]$ is the time interval of the j-th switch state, and matrices $\mathbf{A}$'s and $\mathbf{B}$'s are expressed as

$$\mathbf{A}_1 = \mathbf{A}_2 = \begin{bmatrix} -\frac{\kappa r}{L} & -\frac{\kappa}{L} \\ \frac{\kappa}{C} & -\frac{\kappa}{RC} \end{bmatrix}, \quad \mathbf{A}_3 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{\kappa}{RC} \end{bmatrix},$$

$$\mathbf{B}_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}, \quad \mathbf{B}_2 = \mathbf{B}_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \kappa = \frac{R}{R+r}.$$

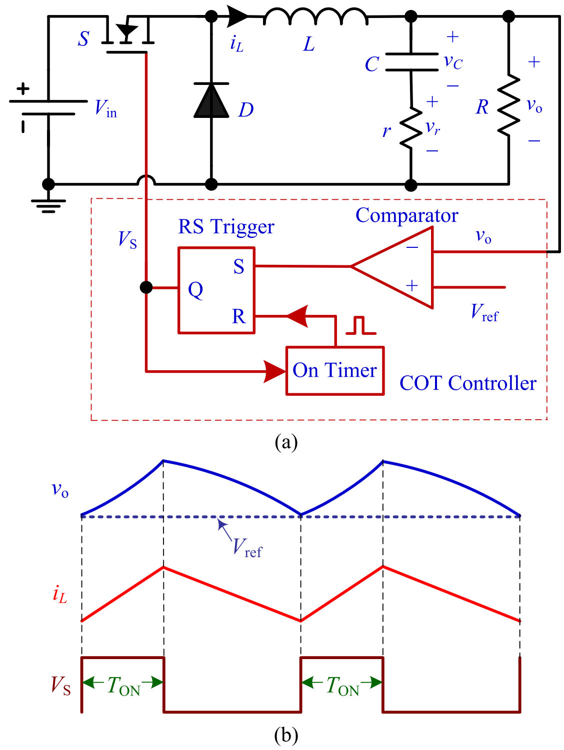

FIGURE 1. The COT controlled buck converter and its main steady-state waveforms. (a) Circuit schematic; (b) main steady-state waveforms.

TABLE 1. Circuit parameters for COT controlled buck converter.

Parameters	Significations	Values
V_{in}	Input voltage	15 V
V_{ref}	Reference voltage	5 V
L	Inductance	200 μH
C	Output capacitance	100 μF
r	Equivalent series resistance	8 mΩ
R	Load resistance	5 Ω
T_{ON}	Constant on-time	4 μs

Correspondingly, the two switched conditions of the COT controlled buck converter can be written as

$$v_o(t) = V_{ref} \quad \text{and } i_L(t) = 0 \tag{2}$$

Thus, the above equations (1) and (2) constitute the piecewise smooth continuous model of the COT controlled buck converter.

Define the initial states of the COT controlled buck converter as $\mathbf{x}_0 = [i_{L0}, v_{C0}]$. To demonstrate the bi-stability phenomenon, the circuit parameters of the COT controlled buck converter are given in Table 1 and kept unchanged. It should be stressed that the ESR of output capacitor we have chosen is relatively smaller because a multilayer ceramic capacitor

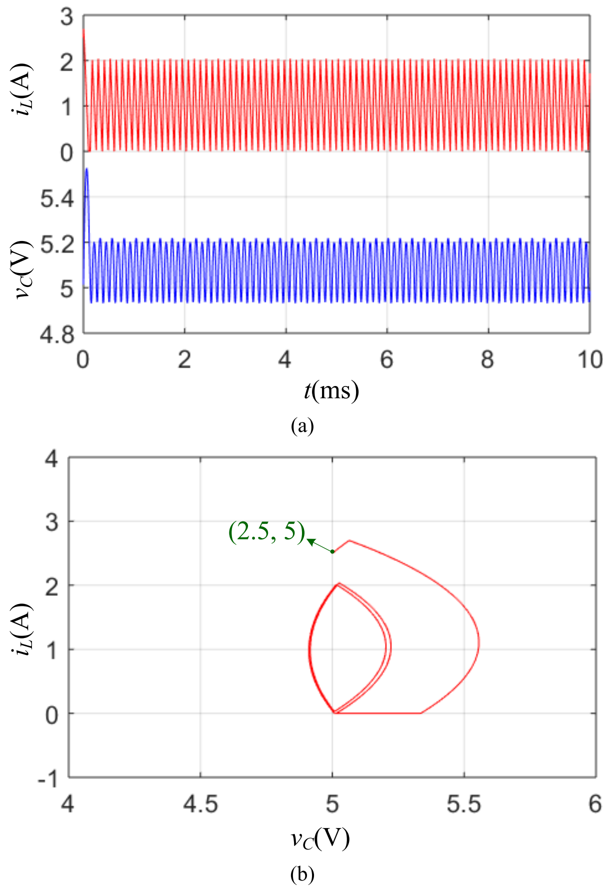

FIGURE 2. For $\mathbf{x}_0$ = [2.5A, 5V], MATLAB numerically simulated results of the COT controlled buck converter. (a) Periodic time-domain waveforms of i_L and v_C; (b) phase plane orbit of periodic limit cycle in the $v_C - i_L$ plane.

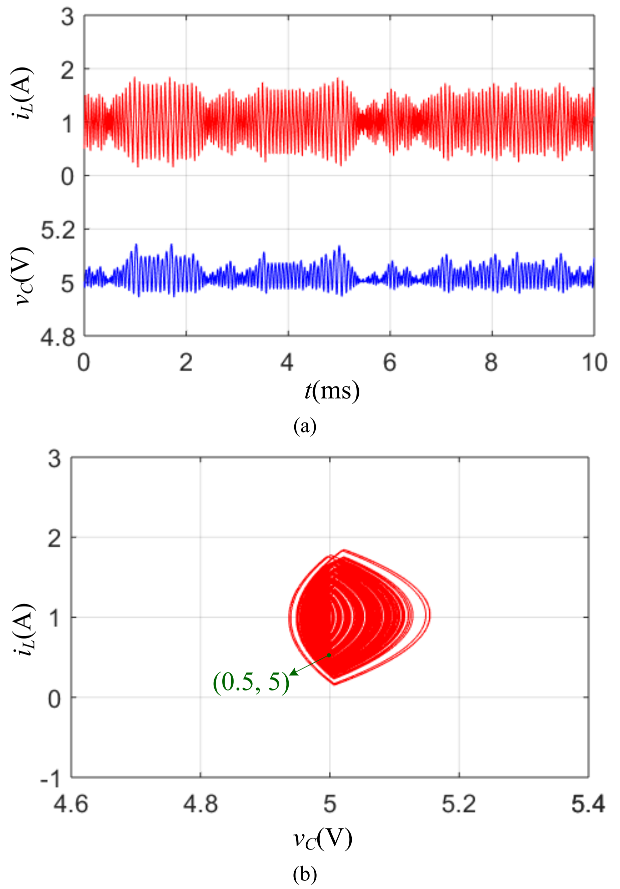

FIGURE 3. For $\mathbf{x}_0$ = [0.5A, 5V], MATLAB numerically simulated results of the COT controlled buck converter. (a) Chaotic time-domain waveforms of i_L and v_C; (b) phase plane orbit of chaotic attractor in the $v_C - i_L$ plane.

is employed. Based on the model given in (1) and (2), by using MATLAB software with Runge-Kutta ODE45 algorithm, the time-domain waveforms and the corresponding phase plane orbits of the COT controlled buck converter for two sets of the initial states are shown in Figs. 2 and 3, respectively, where the initial states in Fig. 2 are set to $\mathbf{x}_0$ = [2.5 A, 5 V] and the initial states in Fig. 3 are set to [0.5 A, 5 V].

Observed from Fig. 2, when starting from the initial states of $\mathbf{x}_0$ = [2.5 A, 5 V], the buck converter operates in the reduced- frequency periodic oscillation state with the inductor current discontinuous conduction mode (DCM). Whereas from Fig. 3, when starting from the initial states of $\mathbf{x}_0$ = [0.5A, 5V], the buck converter operates in the chaotic oscillation state with the inductor current continuous conduction mode (CCM). As a result, with the same circuit parameters, the COT controlled buck converter with a small output capacitor ESR can operate in completely different oscillation states for two sets of the initial states, which implies that an interesting and striking bi-stability phenomenon indeed exists in such a COT controlled buck converter with a small output capacitor ESR [16].

B. COEXISTING BIFURCATION BEHAVIORS

The initial output capacitor voltage is fixed as v_{C0} = 5 V and the initial inductor current i_{L0} is taken as a bifurcation parameter. Based on the mathematical model given in (1) and (2) and circuit parameters listed in Table 1, the single-parameter bifurcation diagram of the COT controlled buck converter with a small output capacitor ESR is shown in Fig. 4, where $i_{L,n}$ is the sampling value of the inductor current i_L at the beginning of the n-th constant on-time interval. It can be found from Fig. 4 that for the given circuit parameters, when the initial inductor current i_{L0} increases, the operation state of the COT controlled buck converter with a small output capacitor ESR is shifted from the DCM reduced-frequency periodic oscillation state to the CCM chaotic oscillation state at i_{L0} = 0.063 A, and then back to the DCM reduced-frequency periodic oscillation state at i_{L0} = 1.745 A. It should be pointed out that there is a periodic window of the DCM reduced-frequency periodic oscillation in [0.159 A, 0.167 A].

Furthermore, the single-parameter bifurcation diagrams of the COT controlled buck converter with the variation of output capacitor ESR under two sets of the initial states $\mathbf{x}_0$ = [2.5 A, 5 V] and [0.5 A, 5 V] are drawn

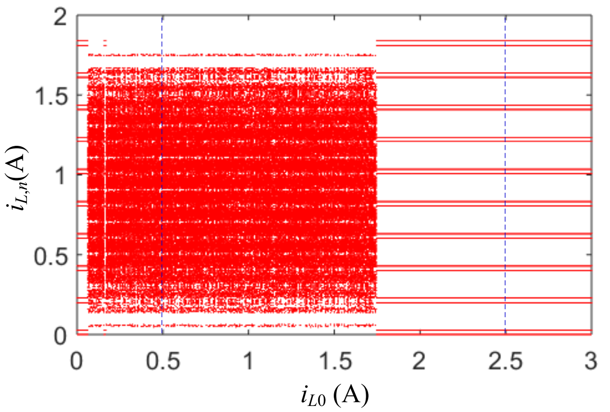

FIGURE 4. Single-parameter bifurcation diagram of the COT controlled buck converter with the variation of the initial inductor current i_{L0}, where the initial output capacitor voltage $v_{C0} = 5$ V.

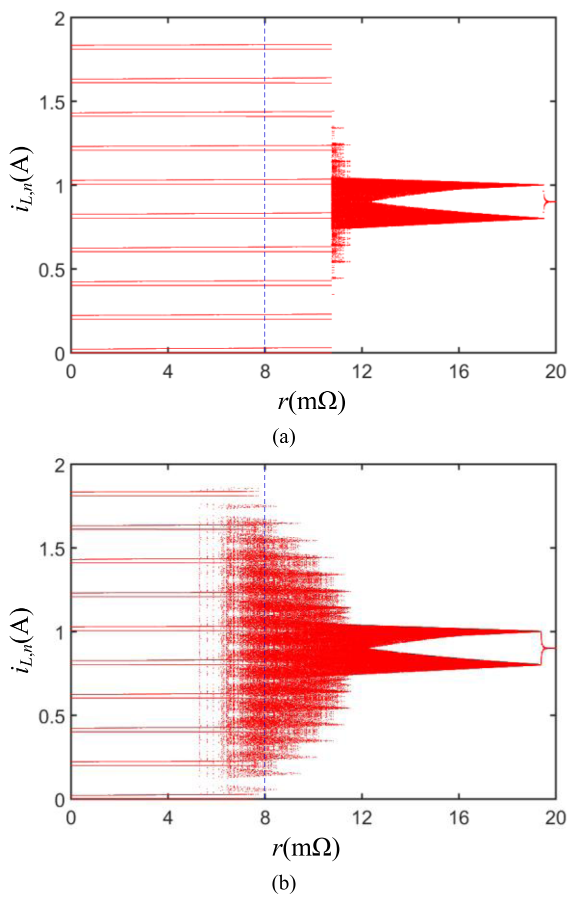

FIGURE 5. Single-parameter bifurcation diagrams of the COT controlled buck converter with the variation of output capacitor ESR for two sets of the initial states. (a) $\mathbf{x}_0$ = [2.5A, 5V]; (b) $\mathbf{x}_0$ = [0.5A, 5V].

in Figs. 5(a) and 5(b), respectively. As shown in Fig. 5(a), with the initial states $\mathbf{x}_0 = [2.5\text{A}, 5\text{V}]$, the buck converter operates in the DCM reduced- frequency periodic oscillation state when the output capacitor ESR locates in the neighboring range of $r = 8$ mΩ [13]. While as shown in Fig. 5(b), with the initial states of $\mathbf{x}_0 = [0.5\text{ A}, 5\text{ V}]$, the buck converter operates in the CCM chaotic oscillation state with the same range of output capacitor ESR.

From Figs. 4 and 5, it should be highly stressed that for the same given set of circuit parameters, the COT controlled buck converter with different sets of the initial states operates in two completely different oscillation states, resulting in the existence of bi-stability phenomenon, i.e., the coexistence of periodic and chaotic attractors, in the COT controlled buck converter with a small output capacitor ESR indeed.

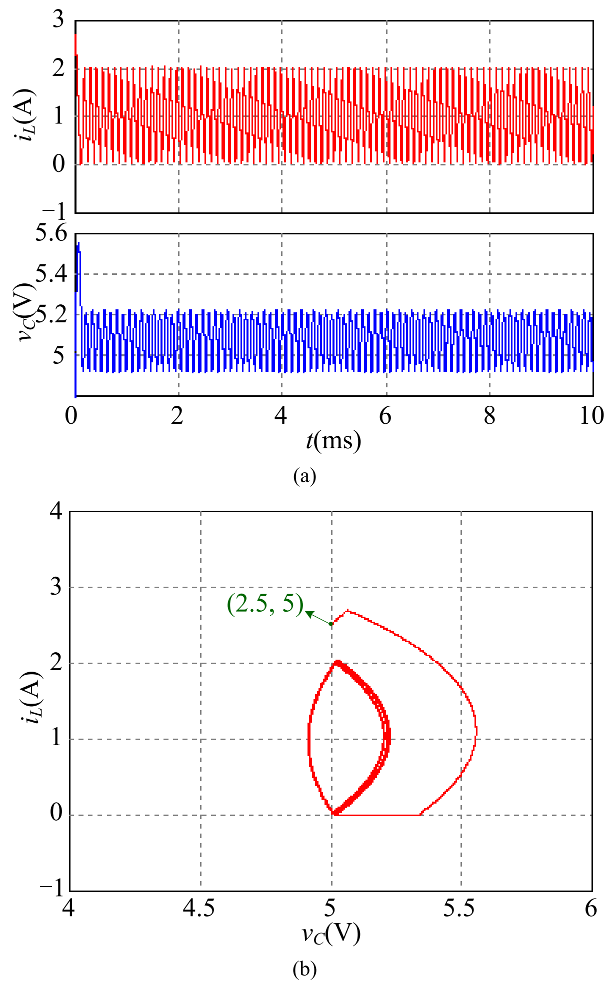

FIGURE 6. For $\mathbf{x}_0$ = [2.5A, 5V], PSIM circuit simulation results of the COT controlled buck converter. (a) Periodic time-domain waveforms of i_L and v_C; (b) phase plane orbit of periodic limit cycle in the $v_C - i_L$ plane.

IV. VERIFICATION BY PSIM CIRCUIT SIMULATIONS

By using PSIM (power simulation) circuit simulation software, a circuit simulation model of COT controlled buck converter with the circuit parameters listed in Table 1 is built. For two sets of the initial states of $\mathbf{x}_0$ used in Figs. 2 and 3, the corresponding PSIM simulation results of the time-domain waveforms and phase plane orbits are shown in Figs. 6 and 7, respectively.

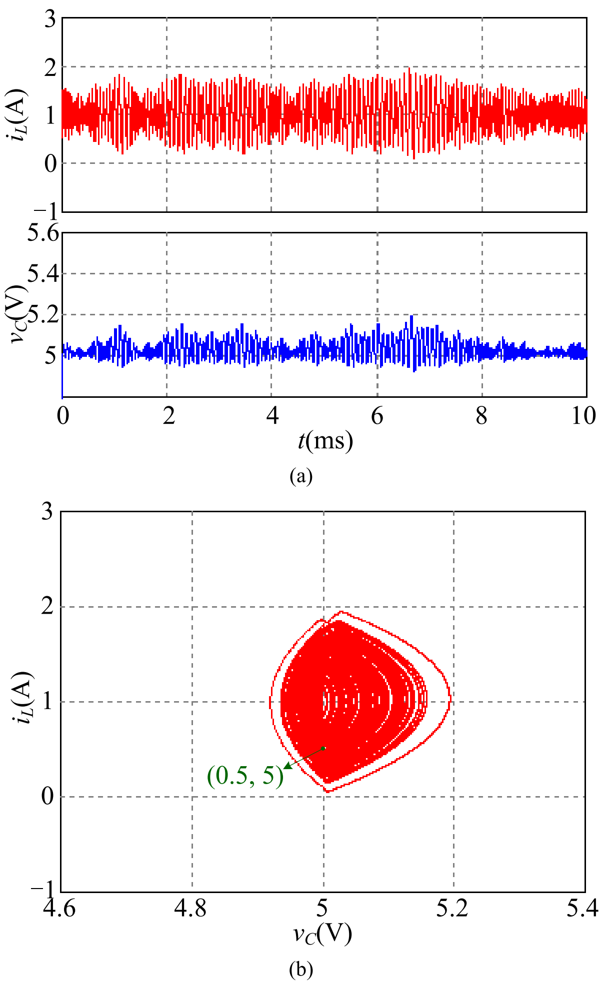

FIGURE 7. For $\mathbf{x}_0$ = [0.5A, 5V], PSIM circuit simulation results of the COT controlled buck converter. (a) Chaotic time-domain waveforms of i_L and v_C; (b) phase plane orbit of chaotic attractor in the $v_C - i_L$ plane.

When starting from the initial states $\mathbf{x}_0 = [2.5\text{ A}, 5\text{ V}]$, the COT controlled buck converter operates in the DCM reduced-frequency periodic oscillation state, as shown in Fig. 6. Whereas when starting from the initial states $\mathbf{x}_0 = [0.5\text{A}, 5\text{V}]$, the COT controlled buck converter operates in the CCM chaotic oscillation state, as shown in Fig. 7. Comparing Figs. 6 and 7 with Figs. 2 and 3, it can be seen that the PSIM circuit simulation results well verify the numerical ones.

It is remarkable that the parasitic parameters, e. g., the input inductor ESR and internal resistance of the input voltage, do not affect the bifurcation structures but they are found to shift the points at which bifurcation occurs when other major parameters are varied [17]. For example, for a current-mode-controlled boost converter reported in [17], when the load resistance is utilized as the bifurcation parameter, the period-doubling bifurcation occurs for a lower value if the inductor has a parasitic resistance. Therefore, if the power supply is a battery bank, the internal resistance of the battery bank can impact the system stability as well, but cannot cause the disappearance of the initial condition-dependent bi-stability phenomenon in such a COT controlled buck converter concerned in our paper, which has been verified in other PSIM circuit simulations omitted here.

V. CONCLUSION

The bi-stability phenomenon in the COT controlled buck converter with a small output capacitor ESR is newly found and faultlessly revealed in this paper. The new finding of the coexisting attractors' behavior has not been reported in the previously published literatures, which can contribute to a better understanding the complex dynamics existing in this kind of the switching dc-dc converter with the simple COT control. The study we have performed in this paper can also supply a new insight for the designer to analyze and design the COT controlled buck converter with a small output capacitor ESR. Further investigations on the dynamical mechanism of bi-stability phenomenon will be presented in our future research works.

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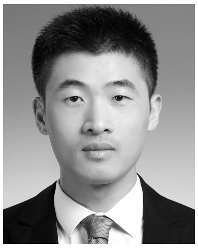

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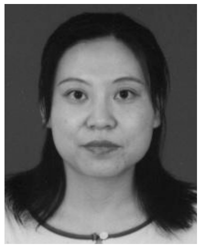

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