IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received July 5, 2018, accepted August 11, 2018, date of publication August 17, 2018, date of current version October 8, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2865933

Availability Analysis and Optimal Design of Multistate Weighted *k*-Out-of-*n* Systems With Component Performance Requirements

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This work was supported by the National Basic Research Program of China (973 Program) under Grant 2014CB744904.

ABSTRACT As one of the most typical multistate systems, weighted k-out-of-n system has been extensively studied in recent years. This paper presents a new kind of multistate weighted k-out-of-n system, which simultaneously considers performance requirements of each component and the system performance (cumulative performance threshold of all components). Then, we establish a new dynamic availability model by combining the Markov process and universal generating function. Moreover, an optimal design is proposed to achieve a tradeoff between system reliability/availability and cost. Non-dominated sorting genetic algorithm-II is used to optimize the probability and performance of components in different states. Finally, an example is illustrated to evaluate system availability and optimize component design. The optimization design can be referred as an optimal standard in system update.

INDEX TERMS Availability analysis, multistate system, k-out-of-n, optimal design, universal generating function, non-dominated sorting genetic algorithm-II (NSGA- II).

NOTATION

n	Number of system components.
m	Optimal operating state of the components.
$m+1$	Total number of states.
i	Index of component number in the system, $1 \leq i \leq n$.
j	Index of component state in the system, $1 \leq j \leq m$.
$\lambda_{j,l}^{i}$	State transition (failure) rate of component i from state j to state l ($j > l$).
$\mu_{j,l}^{i}$	State transition (repair) rate of component i from state j to state $l (j < l)$.
g_{ij}	Performance rate of component i in state j.
$G_i(t)$	Performance rate of component i at time t.
G_{Sj}	Performance rate of the system in state j.
$p_{ij}(t)$	State probability of component i in state j at time t.
$k_{ij}(t)$	Minimum performance required for each component to ensure that component i is in state j or above at time t.
$K_j(t)$	Minimum system performance (cumulative performance of all components) required to ensure that the system is in state j or above at time t.
$c_i(t)$	Total cost of component i at time t.
C_{sys}	Total cost of the system.
C_j	Cost of the system in a state below j.
$\tilde{A}_{sys}$	Minimum required probability for the system to remain in state j or above.
$\tilde{C}_{sys}$	Upper limit of the total cost for the system to remain in state j or above.
$c_i^p(t)$	Availability-associated cost of component i.
$c_i^u(t)$	Performance-associated cost of component i.

I. INTRODUCTION

Many practical systems comprise multistate components, which have various distinctive performance levels and failure modes with effects on the overall system performance. The system with different performance rates is called multistate system (MSS) [1]. Multistate weighted k-out-of-n systems have been widely investigated as typical MSSs. For example, some types of power systems, computer systems, pipeline transportation systems, and signal transmission systems are MSSs with performance characteristics of generator capacity, data processing speed, transport capacity, and channel capacity [2], [3], respectively. These systems usually have a high reliability, long operating cycle, and high cost of production and maintenance. Therefore, it is particularly crucial to conduct reliability/availability analysis and make optimal design of multistate weighted k-out-of-n systems.

Many studies have investigated the reliability and availability of MSSs, because of its significance and wide application. The earliest investigations of the reliability concept and theory of MSS were performed by Murchland [4], Barlow and Wu [5], and Ross [6] in the 1970s. Researchers have mainly considered the following three aspects in the availability/reliability analysis and optimal design of multistate weighted k-out-of-n systems:

A. MODELING METHODS

Researchers have mainly considered the following four methods for establishing the reliability/availability model of an MSS: 1) expanded Boolean models such as MSS fault trees [7], multistate paths and cut sets [8], [9], and multivalued decision diagrams [10], [11]. 2) simulations such as Monte Carlo simulations [8], [12], [13], and random Petri nets [14]. 3) random processes such as the homogeneous Markov process [15] and Quasi renewal process [16], [17]. 4) the universal generating function (UGF) proposed by Ushakov in 1987 [18], which is a discrete random variable operation tool. UGF is widely used in the reliability/availability analysis of MSSs because of its advantages, such as fast computing speed, easy programming, and numerical implementation [1], [19], [20].

B. MODEL ANALYSIS

In traditional multistate weighted k-out-of-n systems, every component in each possible state has a certain contribution to system performance. The output performance of weighted k-out-of-n systems depends on the sum of the weighted working components and is greater than the performance threshold. Wu and Chen [21] developed a recursive formula for analyzing the reliability of a binary weighted k-out-of-n system. Li and Zuo [22] extended the binary weighted k-out-of-n model to an MSS model and then analyzed its availability by combining recursive algorithms and the UGF. Ding *et al.* [23] developed a hierarchical weighted multistate k-out-of-n system, which can be resolved into different hierarchical levels, such as system, subsystem, and components. In this model, each level is represented as a multistate weighted k-out-of-n structure. Khorshidi *et al*. [24], [25] defined the multistate weighted k-out-of-n system from economic perspective, which demonstrated that the summation of components income (total performance of all components) in the system is equal to system income (system performance). Therefore, this definition agrees with the definition of Model I in reference [22]. Eryilmaz and Aksoy [26] studied the reliability of linear weighted k-out-of-n: G and F systems by using recursive formulas. Wang *et al.* [27] estimated the reliability of weighted k-out-of-n multistate systems based on component reliability data. An unbiased system reliability estimator and unbiased covariance estimator were obtained based on the propagation mechanism of uncertainty of component reliability data. Eryilmaz [28] developed the k-out-of-n system reliability model with random weights components by using recursive formula and Monte Carlo simulation through static and dynamic approaches. Eryilmaz and Bozbulut [29] considered the marginal and joint importance in weighted k-out-of-n systems through the UGF and probabilistic approaches. Amrutkar and Kamalja [30] developed a formula for evaluating the reliability and probability-based importance measures of weighted k-out-of-n: G systems. Pourkarim Guilania *et al.* [31] compared two methods of the Markov model, as well as the UGF and recursive algorithm for estimating the reliability of a nonrepairable three-state system. Results revealed that the Markov process method has a faster computing speed.

In the dynamic model, the probabilities and performance of components in different states can change with time. On the basis of multivariate copula, Eryilmaz [32] explored the reliability of a dynamic weighted k-out-of-n system with dependent component lifetime. Based on the capacity and weights as well as a cold standby component, Franko *et al.* [33] investigated the reliability of weighted k-out-of-n systems that included two types of components. Faghih-Roohia *et al*. [34] evaluated the dynamic availability of dynamic multistate weighted k-out-of-n systems based on the reliability model of Li and Zuo [22], and they developed the optimization model of Model I based on the nondynamic case to obtain the optimal design of dynamic components.

C. OPTIMAL DESIGN

Most reliability optimization and design problems of multistate weighted k-out-of-n systems are considered to be balancing system reliability/availability and cost. Metaheuristic has been considered as the most effective approach to solve complex reliability and cost optimization problems. Genetic algorithm (GA), which is the most well-known heuristic approach, has been widely used for solving optimization problems in weighted multistate weighted k-out-of-n systems [24], [35]–[40]. Li and Zuo [36] extended the component optimal design of the binary k-out-of-n system to multistate systems. Two single-objective optimization problems have been presented to solve the component optimal design. Khorshidi *et al*. [24] studied the optimization of the reliability-redundancy allocation problem for multistate weighted k-out-of-n systems to simultaneously maximize system reliability and minimize system cost. Khorshidi *et al.* [38] evaluated the dynamic unreliability of multistate weighted k-out-of-n: F systems and established a bi-objective optimization model for optimal maintenance strategies. Li and Peng [39] developed an availability and optimization model for analyzing dynamic multistate series-parallel systems. Li *et al.* [40] developed a multi-objective optimization model for multistate weighted series-parallel systems, which is used to maximize expected performance and reliability, and minimize system cost. An improved GA was also used for the optimization of multistate k-out-of-n systems. Ebrahimipur *et al*. [41] established a fuzzy optimal model for the reliability design of multistate k-out-of-n systems to minimize the system cost subject to the reliability constraint. By using the augmented Lagrangian genetic algorithm, Li *et al*. [42] achieved the optimal design parameters for the components of multistate weighted k-out-of-n systems.

The aforementioned literature indicates that most studies have focused on modeling and optimization. Moreover, most researchers have investigated multistate weighted k-out-of-n systems based on the two definitions proposed by Li and Zuo [22]. Li and Zuo [22] first extended the binary weighted k-out-of-n model to the MSS model and defined two types of systems named Model I and Model II. In addition, they [35] proposed two optimization problems for Model I and Model II.

Model I [22]: The system is in state j or above if the total performance of all components is greater than or equal to K_j. The performance of all components can contribute to the system.

Model II [22]: The system is in state j or above if the sum of the weights of only those components in state j or above is not less than K_j. The performance of components which are greater than or equal to K_j can contribute to the system.

According to the aforementioned discussion, Model I and Model II only require a single threshold for system or component performance. For Model I, only the cumulative performance threshold of all components is considered, whereas for Model II, each individual performance threshold of the components is considered. Therefore, we propose a new type of multistate weighted k-out-n: G system called Model III, in which performance of each component and cumulative performance of all components are considered simultaneously. This model extends the application areas of Models I and II. First, we establish an availability model for Model III by combining the Markov process and UGF. The performance and state probability of the system can also be obtained. Moreover, an optimization model is proposed by considering reduced costs and increased availability. The component costs comprise two parts: availability-associated cost and performance-associated cost. Non-dominated sorting genetic algorithm-II (NSGA-II) is selected for solving the optimization problem [43], [44]. Finally, a practical pumping system is considered as an example to illustrate the availability model and optimal component design. In addition, the influence of the output threshold for component and system performance on the system availability is investigated.

The rest of the paper is organized as follows. The system availability model is established in Section II based on the Markov process and UGF. The optimization model, which considers reduced costs and increased availability, is presented in Section III. A practical case is examined, and the system availability and component optimal design are analyzed in Section IV. The conclusions are presented in Section V.

II. SYSTEM AVAILABILITY

Typically, a multistate weighted k-out-of-n system has n multistate components and each component has $m+1$ performance states (i.e., $0, 1, \cdots, m$, where m implies the perfect performance state and 0 implies complete failure of the components). The components of repairable MSSs can gradually degenerate from a state of high performance to low performance; the component states can be repaired to improve the performance. Based on the discussion of Model III in Section I, the definition of Model III is given as follows:

Definition: The components whose performance is not less than the threshold $k_{ij}(t)$ can contribute to system performance. The system is in state j or above if the sum of component performance contributions is greater than or equal to K_j.

The performance of component i in state j is represented by the set $\mathbf{g}_i = \{g_{i0}, g_{i1}, \cdots, g_{im-1}, g_{im}\}$. The definition of Model III can be represented as

$$Pr\{\phi \geq j\} = Pr\left\{\sum_{i=1}^{n} g_{ij}\alpha(g_{ij} \geq k_{ij}(t)) \geq K_j\right\} \quad (1)$$

where $\alpha(\mathrm{g}_i \geq k_{ij})$ is an indicator function defined as

$$\alpha = \begin{cases} 1, & g_{ij} \geq k_{ij}(t) \\ 0, & g_{ij} < k_{ij}(t) \end{cases} \quad (j = 1, 2, 3 \ldots, m) \quad (2)$$

ϕ is the structure function of the system, which represents the state of system and the performance of all components. By comparing Model I and Model II, the definition of Model III simultaneously considers performance requirements of each component and system performance (cumulative performance threshold of all components). When $k_{ij}(t) = 0$ and $\sum_{i=1}^{n} g_{ij} \geq K_j$, the definition of Model III is the same as Model I; when $g_{ij}(t) \geq k_{ij}(t) = K_j$, components can make any contribution for system, and in this situation, the definition of Model II and III is the same. Therefore, Model III is a generalization and extension of traditional definition of multi-state weighted k-out-of-n system, and can be applied to a broader domain.

A. MODELING FOR MULTISTATE COMPONENT

The continuous-time discrete-state Markov process is used for establishing the multistate component model; the state probability distributions of the repaired components with perfect maintenance can be easily achieved. The multistate Markov state-transition model for repaired component i is illustrated in Fig. 1. The instantaneous state probabilities in state j at time t can be calculated as

$$p_{ij}(t) = Pr\left\{G_i(t) = g_{ij}\right\} \quad (3)$$

The state probabilities $p_{ij}(t)$ for component i can be computed by

$$\frac{dP}{dt} = P(t) * \Lambda_i \quad (4)$$

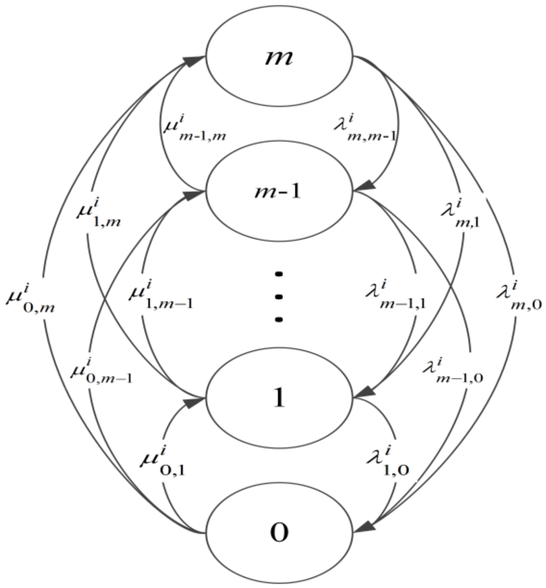

FIGURE 1. State-transition diagram for repairable components.

The state-transition rate matrix Λ_i of component i is presented as

$$\Lambda_i = \begin{pmatrix} \lambda^i_{m,m} & \lambda^i_{m,m-1} & \cdots & \lambda^i_{m,1} & \lambda^i_{m,0} \\ \mu^i_{m-1,m} & \lambda^i_{m-1,m-1} & \cdots & \lambda^i_{m-1,1} & \lambda^i_{m-1,0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mu^i_{0,m} & \mu^i_{0,m-1} & \cdots & \mu^i_{0,1} & \lambda^i_{0,0} \end{pmatrix} \tag{5}$$

The state probabilities of the components can be given as

$$\sum_{j=0}^{m} p_{ij}(t) = 1. \tag{6}$$

When the output performance of the components is less than the threshold $k_{ij}(t)$, the components cannot contribute to the system. This implies that $g_{ij} < k_{ij}(t), g_{ij} = g_{i0} = 0 (j = 0, 1, \cdots, w)$. When $g_{ij} \geq k_{ij}(t), g_{ij} = g_{ij} (j = w+1, \cdots, m)$, the contribution of components to the system remains unchanged. The indicator function α can be obtained by using (1), and the UGF $u_i(z, t)$ for each repairable component i with component output performance requirement $k_{ij}(t)$ is given by

$$u_i(z,t) = \sum_{j=0}^{m} p_{ij}(t)\, z^{g_{ij}\alpha(g_{ij} \geq k_{ij}(t))} \tag{7}$$

B. MODELING FOR MSS

The composition operator [45] should be used for the UGF of individual components with their thresholds and combinations. The UGF associated with the MSS output performance distribution at time t can be obtained as follows:

$$\begin{aligned} U_s(z,t) &= \Omega\left(u_1(z,t), u_2(z,t), \ldots, u_{n-1}(z,t), u_n(z,t)\right) \\ &= \sum\nolimits_{j_1=0}^{m} \sum\nolimits_{j_2=0}^{m} \cdots \sum\nolimits_{j_{n-1}=0}^{m} \sum\nolimits_{j_n=0}^{m} \\ &\quad \left(\prod_{i=1}^{n} p_{ij_i}(t) z^{\phi(g_{1,j_1}, g_{2,j_2}, \ldots, g_{n,j_n})} \right) \\ &= \sum\nolimits_{j=0}^{m} p_{Sj}(t) z^{g_{sj}} \end{aligned} \tag{8}$$

Ω is a composition operator and it is also designated as $\underset{\phi}{\otimes}$. $\phi()$ is structure function for the system, and the calculation approach is determined according to the structure of the system.

$$\phi\left(g_{1,j_1}, g_{2,j_2}, \ldots, g_{n,j_n}\right) = \sum\nolimits_{i=1}^{n} g_{i,j_i} \tag{9}$$

Thus, the performance and probability of the system can be achieved under the perfect maintenance. At time t, the availability $A(t)$ of the multistate weighted k-out-of-n system with system demand $K_j(t)$ is as follows:

$$\begin{aligned} A(t) &= Pr\left\{\phi\left(g_{1,j_1}, g_{2,j_2}, \ldots, g_{n,j_n}\right) - K_j(t) \geq 0\right\} \\ &= \sum_{j=0}^{M} p_{sj}(t)\, \alpha\left(G_{Sj} - K_j(t)\right) \end{aligned} \tag{10}$$

where $\alpha\left(G_{Sj} - K_j(t)\right)$ is an indicator function defined as

$$\alpha = \begin{cases} 1, & G_{Sj} \geq K_j(t) \\ 0, & G_{Sj} < K_j(t) \end{cases} \quad (j = 1, 2, 3 \ldots, m) \tag{11}$$

III. OPTIMIZATION MODEL

Most reliability optimization and design problems are a trade-off between system reliability/availability and cost. In this study, the optimal MSS design is essentially a bi-objective optimization problem, in which a Pareto-optimal set of design plans is created by maximizing system availability and minimizing system cost.

Based on the optimization model in [34] and [36], we establish the optimization model for Model III with dynamic design. The optimization problem for Model III is formulated as follows:

Problem 1:

$$\text{Minimize: } C_{sys} = \sum_{i=1}^{n} c_i(t) + (1 - A(t)) C_j$$

$$\text{Subject to: } A(t) \geq \tilde{A}_{sys} \tag{12}$$

$$\begin{aligned} &\sum_{j=0}^{m} p_{ij}(t) = 1, \quad 0 \leq p_{ij}(t) \leq 1 \\ &(i = 1, 2, \cdots, n; j = 1, 2, \cdots, m) \\ &u_{i0}(t) = 0, \quad u_{ij}(t) \geq 0 \\ &(i = 1, 2, \cdots, n; j = 1, 2, \cdots, m) \end{aligned} \tag{13}$$

Problem 2:

$$\text{Maximize: } A(t)$$
$$\text{Subject to: } C_{sys} = \sum_{i=1}^{n} c_i(t) + (1 - A(t)), \quad C_j \le \tilde{C}_{sys}$$
$$\sum_{j=0}^{m} p_{ij}(t) = 1, \quad 0 \le p_{ij}(t) \le 1$$
$$(i = 1, 2, \cdots, n; j = 1, 2, \cdots, m)$$
$$u_{i0}(t) = 0, \quad u_{ij}(t) \ge 0$$
$$(i = 1, 2, \cdots, n; j = 1, 2, \cdots, m) \tag{14}$$

where the system availability $A(t)$ can be obtained by using (10). Li and Zuo [36] assumed that the cost of component $c_i(t)$ comprises two parts: performance-associated cost and availability-associated cost. Mettas [46] and Li and Zuo [36] proposed a component cost function, which considers the reliability/availability, performance, and cost of components.

In the current paper, $\sum_{j=1}^{m} p_{ij}(t)\, u_{ij}(t)\, \alpha$ is the expected performance of multistate component i, which contributes to the system. The value of α is obtained by using (2). If the performance of component i at state j reaches the threshold $k_{ij}(t)$, α is 1; otherwise, it is 0. The performance-associated cost of component i can be calculated as

$$c_i^u(t) = g_i \exp\left[\sum_{j=1}^{m} p_{ij}(t)\, u_{ij}(t)\, \alpha - u_{i_{min}}(t)\right] \tag{15}$$

where g_i is the feasibility of increasing performance of component i, and $u_{i_{min}}(t)$ is the minimum performance of component i in the time interval from 0 to t.

The availability-associated cost of component i can be calculated as follows:

$$c_i^p(t) = \exp\left[(1 - f_i)\frac{\sum_{j=1}^{m} p_{ij}(t) - p_{i_{min}}(t)}{p_{i_{max}}(t) - \sum_{j=1}^{m} p_{ij}(t)}\right] \tag{16}$$

where f_i is the feasibility of increasing availability of component i; $p_{i_{min}}(t)$ and $p_{i_{max}}(t)$ are respectively the minimum and maximum availability of component i in the time interval from 0 to t.

The total cost of the component i can be calculated by:

$$c_i(t) = c_i^u(t) + c_i^p(t) \tag{17}$$

The search heuristic algorithm has been widely used to solve the complex reliability and cost optimization problems. In this paper, NSGA-II is used to solve optimization problems. The NSGA-II [47] offers a greater improvement in computational efficiency than traditional GA; moreover, it incorporates unique dominated sorting and elitism without sharing parameters [48]. It can also prevent the loss of favorable solutions once they are determined.

IV. CASE STUDY

A. AVAILABILITY ANALYSIS

In this section, we considered a pumping system as an example for analyzing and optimizing multistate weighted k-out-of-n systems with component performance requirements (Model III). The pumping system has three pumps, through which water is pumped from a river to a main pipeline and then transported from the main pipeline to a higher reservoir. The three pumps have different pump heads, and each pump has four performance states. The distance from the river to main pipeline is different; therefore, the requirement of pump head (performance) for each pump is different. The minimum performance requirements of pumps 1, 2, and 3 are 20, 15, and 10 m, respectively. The lift of demand for transporting the water from the main pipeline to the reservoir is 35 m, which indicates that the threshold of system $K_j(t)$ is constant at 35 m. Therefore, the pumping system is a typical weighted k-out-of-3 system with various $k_{ij}(t)$ values. The different state levels 3, 2, 1, and 0 represent perfect, high, low, and zero performance states, respectively. State 3 represents the perfect operation state and state 0 is regarded as a complete failure state. We assume that the pumps must be repaired only when they are in state 0. Therefore, the pumping system is analyzed by using Model III, where the state probabilities of the pumps are assumed to change exponentially with time. Model III requires not only the performance of the system but also that of the components. The component performance requirement is $k_{ij}(t)$ and the system performance requirement is $K_j(t)$. The state transition and performance of three pumps are shown in Table 1.

TABLE 1. Performance and transition rates of pumps.

Pumps Types	States Levels	Transition rates (1/yr) S_3	S_2	S_1	S_0	Pump Head (m)
1	S_3	0	1.8	1.4	0.8	35
	S_2	0	0	1.1	0.6	26
	S_1	0	0	0	0.3	18
	S_0	4.1	0	0	0	0
2	S_3	0	1.5	0.9	0.6	31
	S_2	0	0	0.7	0.3	23
	S_1	0	0	0	0.1	15
	S_0	5	0	0	0	0
3	S_3	0	1.3	1.0	0.7	27
	S_2	0	0	0.8	0.5	17
	S_1	0	0	0	0.2	10
	S_0	3.5	0	0	0	0

On the basis of (4), we achieved the following equations:

$$\frac{dP_{i3}(t)}{dt} = -\left(\lambda_{3,2}^i + \lambda_{3,1}^i + \lambda_{3,0}^i\right) P_{i3}(t) + \mu_{0,3}^i P_{io}(t)$$
$$\frac{dP_{i2}(t)}{dt} = \lambda_{3,2}^i P_{i3}(t) - \left(\lambda_{2,1}^i + \lambda_{2,0}^i\right) P_{i2}(t)$$
$$\frac{dP_{i1}(t)}{dt} = \lambda_{3,1}^i P_{i3}(t) + \lambda_{2,1}^i P_{i3}(t) - \lambda_{1,0}^i P_{i1}(t)$$
$$\frac{dP_{i0}(t)}{dt} = -\mu_{0,3}^i P_{io}(t) + \lambda_{3,0}^i P_{i3}(t) + \lambda_{2,0}^i P_{i2}(t) + \lambda_{1,0}^i P_{i1}(t).$$

The initial state of the pump is fully operational; therefore, the initial state probability of the pumps is given by

$$P_{i3}(0) = 1, \quad P_{i2}(0) = 0,\ P_{i1}(0) = 0, P_{i0}(0) = 0 \quad \text{and} \quad P_{i3}(t) + P_{i2}(t) + P_{i1}(t) + P_{i0}(t) = 1.$$

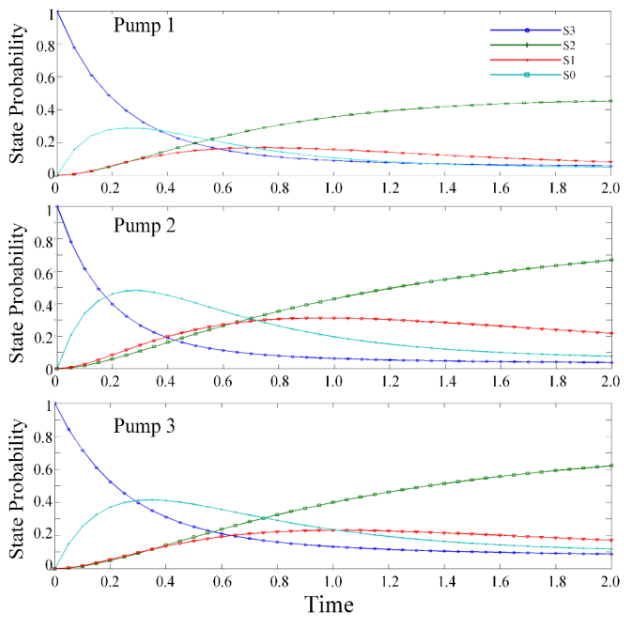

FIGURE 2. State probability distributions of pumps.

The state probability distributions of the three pumps for the dynamic system are achieved by using the ODE45 method, as shown in Fig. 2.

On the basis of (7), the performance UGF of each pump can be represented as follows:

$$u_1(z,t) = P_{13}(t)z^{35\alpha} + P_{12}(t)z^{26\alpha} + P_{11}(t)z^{18\alpha} + P_{10}(t),$$
$$u_2(z,t) = P_{23}(t)z^{31\alpha} + P_{22}(t)z^{23\alpha} + P_{21}(t)z^{15\alpha} + P_{20}(t),$$
$$u_3(z,t) = P_{33}(t)z^{27\alpha} + P_{32}(t)z^{17\alpha} + P_{31}(t)z^{10\alpha} + P_{30}(t).$$

From (8), the UGF of the pumping system for the three pumps can be obtained as

$$U_s(z,t) = \underset{\phi}{\otimes}\left(u_1(z,t), u_2(z,t), u_3(z,t)\right) = \underset{\phi}{\otimes}\begin{pmatrix} P_{13}(t)z^{35\alpha} + P_{12}(t)z^{26\alpha} + P_{11}(t)z^{18\alpha} + P_{10}(t) \\ P_{23}(t)z^{31\alpha} + P_{22}(t)z^{23\alpha} + P_{21}(t)z^{15\alpha} + P_{20}(t) \\ P_{33}(t)z^{27\alpha} + P_{32}(t)z^{17\alpha} + P_{31}(t)z^{10\alpha} + P_{30}(t) \end{pmatrix}.$$

In this paper, we consider the thresholds $k_{ij}(t)$ and $K_j(t)$ as constants, and threshold changes over time are not analyzed. First, the real system is studied, and the minimum demand performance $k_{ij}(t)$ of component i ($i = 1, 2, 3$) is 20, 15, and 10 m, respectively. The minimum system performance $K_j(t)$ is constant at 35 m. The instantaneous availability of the real pumping system is shown in Fig. 3.

Three hypothetical situations are considered for analyzing the effect of model threshold on availability.

Situation 1: The minimum demand performance $k_{ij}(t)$ of each component is 18 m and $K_j(t)$ is 35 m. The instantaneous availability in situation 1 is depicted in Fig. 4, which indicates that the availability in the situation 1 is initially similar to that

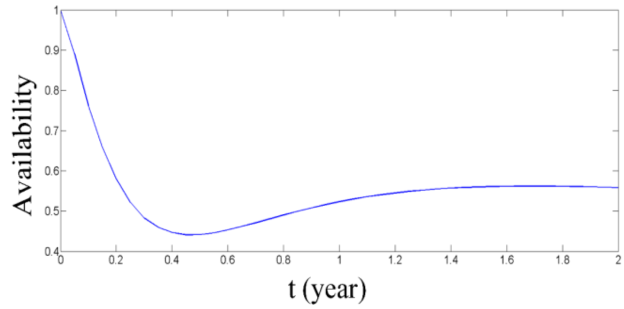

FIGURE 3. The instantaneous availability of the real pumping system.

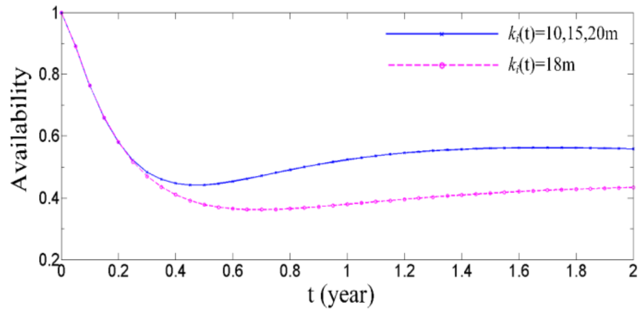

FIGURE 4. Comparison on instantaneous availability in situation 1 and reality.

of actual situation. However, the availability in situation 1 is lower than that in the actual situation after 0.25 years.

Situation 2: Comparison of the availability in Model III with that in Models I and II, which were established by Li and Zuo [22]. Different performance thresholds are adopted in the three types of Models. Model I: $k_{ij}(t) = 35$ m, $K_j(t) = 0$ m; Model II: $k_{ij}(t) = 0$ m, $K_j(t) = 35$ m; and Model III: $k_{ij}(t) = 18$ m, $K_j(t) = 35$ m. The instantaneous availabilities in Models I, II, and III are presented in Fig. 5.

Situation 3: the performance threshold of the five pumping systems is the same ($K_j(t) = 35$ m); however, the component threshold of each system is different. The component thresholds of systems 1, 2, 3, 4, and 5 are 0, 12, 16, 18, and 20 m, respectively. The availability of five pumping systems is shown in Fig. 6. The threshold $K_j(t)$ of the systems remains constant. When the component threshold is increased, the system availability is reduced.

B. OPTIMAL DESIGN

In this section, we assume that the pumping system needs to be upgraded to four-state weighted k-out-of-n system. However, in optimal design, the performance and availability distribution of pumps in different states are unknown. Therefore, we need to evaluate different systems and find the optimal design (availability and performance distributions) of the pumps.

We use NSGA-II to solve the optimization problem. Expressing the optimization problem as penalty function facilitates the calculation.

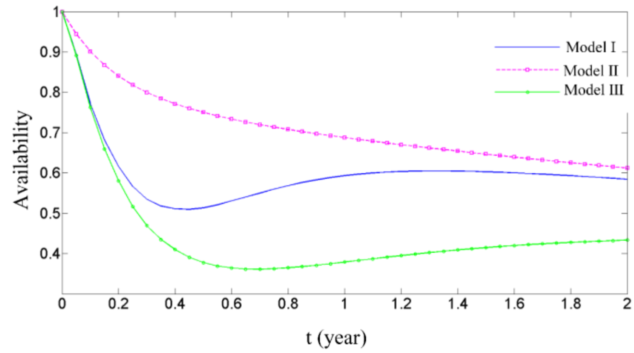

FIGURE 5. Comparison on instantaneous availability in situation 2 and reality.

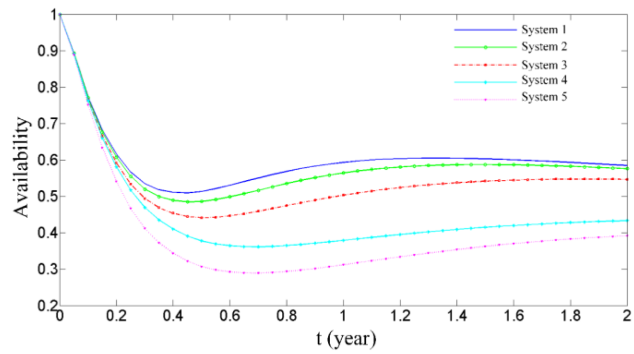

FIGURE 6. Comparison on instantaneous availability in situation 3 and reality.

Problem:

Minimize:

$$C_{sys} = \sum_{i=1}^{n} c_i(t) + (1 - A(t))C_j + \max\left(\tilde{A}_{sys} - A(t), 0\right)\eta - A(t) + \max\left(\sum_{i=1}^{n} c_i(t) + (1 - A(t))C_j - \tilde{C}_{sys}, 0\right)\eta$$

where η is a large number.

Subject to:

$$\sum_{j=0}^{m} p_{ij}(t) = 1, \quad 0 \le p_{ij}(t) \le 1 \quad (i = 1, 2, \cdots, n; j = 1, 2, \cdots, m)$$

$$u_{i0}(t) = 0, \quad u_{ij}(t) \ge 0 \quad (i = 1, 2, \cdots, n; j = 1, 2, \cdots, m)$$

The instantaneous availability $A(t)$ can be obtained by using Model III as described in Section 2. The other parameter values of the objective function are shown in Table 2.

In this paper, NSGA-II is used to find the Pareto-optimal solution of optimization problems; the solving process can be divided into the following six steps:

Step 1 (Population Initialization): Genes are decision variables for the component design, which are the state probability and performance of components. When the components

TABLE 2. Parameter values of the Optimization function.

f_i	g_i	$u_{i_{min}}$	$p_{i_{min}}$	$p_{i_{max}}$
0.99	1	2000	0.93	0.9999
$\tilde{A}_{sys}$	C_j	$\tilde{C}_{sys}$	η	
0.9	10	9.5	99 999	

are in state 0 ($j = 0$), the performance of the components is 0. This study contains 21 decision variables. The population size is set at 100 chromosomes. Then, the initial population is generated randomly based on the problem range and the constraints of the design variables. The first generation of the chromosomes is shown in Fig. 7.

Step 2 (Non-Dominated Sorting): The initialization population is sorted based on non-domination [44], [49].

Step 3 (Crowding Distance): The NSGA-II incorporates elitism without sharing parameters, and the calculation of crowding distance is the key to ensuring the diversity of the population.

Step 4 (Selection): Once the individuals are sorted in steps 3 and 4, the selection process performed by using a crowded comparison operator ($\prec$). Individuals with favorable genetic operations are adopted through selection process, thus improving the convergence of the algorithm.

Step 5 (Genetic Operators): Genetic operators mainly include crossover and mutation, which can be used to avoid local optimal and improve the convergence performance.

There are many types of crossover types, such as one-point crossover, multipoint crossover, and uniform crossover. The second gene is used as a crossover object in the parent chromosomes, and they are exchanged to generate two new chromosomes.

Mutation mainly changes the value of the gene in the chromosome from the initial state to a new value. Through a two-point mutation, the parent chromosome generates a new offspring chromosome.

Step 6 (Recombination and Selection): This step combines the current population and a temporary population to generate a new population. The new generation, which is limited by the population size, is selected based on the results of Step 2.

In this NSGA-II program, the other parameters of the NSGA-II are set as follows: the number of individuals in a population is 6; the maximum generation number is 200; the proportion of crossover is 0.8; the proportion of mutation is 0.3; and the probability of mutation is 0.7. In this study, we ran the NSGA-II program in MATLAB R2014a by using a computer with 3.40-GHz Intel Core(TM) CPU and 8 GB of RAM with the Windows 10 Professional operating system. We selected time $t = 2$s for solving the optimization model. The Pareto-optimal solution was obtained based on the NSGA-II in approximately 11 s. The optimization result is presented in Table 3. The optimal design result is certainly not the only favorable solution. In the optimization model, the number of decision variables rapidly increases with the number of components and performance state.

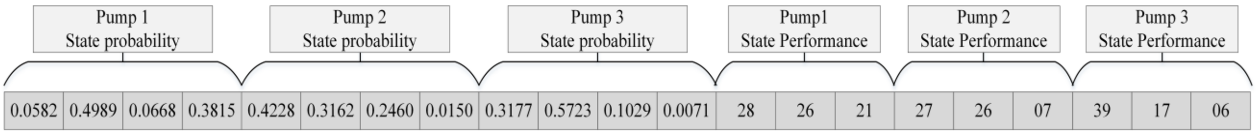

FIGURE 7. First generation of chromosomes based on the problem range and the constraints of the design variables.

TABLE 3. Optimal solution ($t = 2$ s).

Reliability=0.9625		k=18		K=35	
Cost=3.3481		j=3	j=2	j=1	j=0
U	i=1	16	3	2	0
	i=2	28	8	6	0
	i=3	34	29	19	0
P	i=1	0.0133	0.1575	0.0866	0.7424
	i=2	0.4041	0.0832	0.2728	0.2397
	i=3	0.4151	0.2145	0.1818	0.1883

For example, there are 90 decision variables in 6 components with 8 states, and 372 decision variables in 12 components with 16 states. The NSGA-II offers rapid convergence and can be used to solve the optimization problem easily and quickly.

V. CONCLUSION

In this paper, a dynamic availability model for weighted k-out-of-n systems, with component output performance requirements, is established by combining the Markov process and UGF method. Then, we propose an optimization method which considers availability maximization and cost minimization. Finally, a practical case is presented to illustrate how to evaluate system availability and optimize component design.

Therefore, the probabilities and performance of system and its components in different states are obtained over time. Secondly, we achieve the availability of Model III with consideration of performance threshold of each component and cumulative performance of all components. Then, according to comparison on availability in hypotheses and the reality, system availability decreases faster with the increase of component threshold $k_{ij}(t)$; availability of Model III declines more quickly compared to Models I and II. Forth, state probability and performance of components are optimized by choosing NSGA-II, which offers high accuracy and speed.

This paper mainly focuses on evaluating the availability of Model III and optimizing component design. In the future, system availability analysis needs to be studied under imperfect maintenance; component importance or sensitivity analysis also should be discussed. Moreover, optimization problem of fuzzy constraints and uncertainties need to be further studied.

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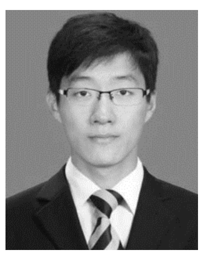

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