IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received May 30, 2018, accepted July 19, 2018, date of publication August 16, 2018, date of current version September 7, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2865625

ADMM-Based Distributed Auction Mechanism for Energy Hub Scheduling in Smart Buildings

WEIFENG ZHONG[1], CHAO YANG[1], KAN XIE[1], SHENGLI XIE[1], (Senior Member, IEEE), AND YAN ZHANG[2], (Senior Member, IEEE)

[1]Guangdong Key Laboratory of IoT Information Technology, School of Automation, Guangdong University of Technology, Guangzhou 510006, China
[2]Department of Informatics, University of Oslo, 0315 Oslo, Norway

Corresponding author: Shengli Xie (shlxie@gdut.edu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grants 61333013, 61727810, 61703113, 61603099, 61773126, and 61501127, in part by the Natural Science Foundation of Guangdong under Grant 2016A030313705, in part by the Science and Technology Plan Project of Guangdong under Grants 2014B090907010, 2015B010131014, and 2017B010125002, and in part by the Pearl River S&T Nova Program of Guangzhou under Grant 201806010176.

ABSTRACT Energy hub integrates various energy conversion and storage technologies, which can yield complementarity among multiple energy and provide consumers with stable energy services, such as electricity, heating, and cooling. This enables energy hub to be an ideal energy system design for smart and green buildings. This paper proposes a distributed auction mechanism for multi-energy scheduling of an energy hub that serves numbers of building energy users. In the auction, users first submit their demand data to the hub manager. Then, the hub manager allocates energy to users via optimization of energy scheduling based on the users' data. The auction mechanism is designed to be incentive compatible, meaning that users are incentivized to truthfully submit their demand data. Next, to mitigate the computational burden of the hub manager, a distributed implementation of the auction is developed, in which an algorithm based on alternating direction method of multipliers (ADMM) is adopted to offload auction computation onto the users. Distributed computation offloading may bring in new chances for users to manipulate the auction outcome since the users participate part of the auction computation. It is proven that the proposed distributed auction mechanism can achieve incentive compatibility in a Nash equilibrium, which indicates that rational users will faithfully report demand data and complete the assigned computation as well. Finally, simulation results based on a household energy consumption dataset are presented to evaluate the energy scheduling performance and to verify the incentive compatibility of the auction mechanism.

INDEX TERMS Alternating direction method of multipliers, auction, energy hub, energy scheduling.

I. INTRODUCTION

The building sector, including residential and commercial buildings, has been always one of the biggest sectors in the world energy consumption. As analyzed in [1], the building sector accounts for 19.58% of the world delivered energy consumption in 2015, which will increase to 21.21% in 2050; total world energy consumption in buildings is projected to increase by 47.25% between 2015 and 2050. With the constantly growing energy demand of buildings, advanced energy system designs are significantly needed to reach further energy saving and enhanced energy security in buildings. Under such circumstance, energy hub has been considered as a promising energy system design that can serve multiple types of building loads (e.g., electricity, heating, and cooling loads) all together [2]–[4]. An energy hub represents a combination of energy technologies, where multiple types of energy can be converted, conditioned, and stored, and thus it allows energy demand and supply to shift among different energy carriers and different time scales [5]. Through optimized multi-energy management, the inherent flexibility of energy hubs can greatly improve the system efficiency and security during energy production and consumption. In recent years, the researches on energy hubs have attracted considerable attention, and they can be categorized into single-hub [6]–[12] and multi-hub [13]–[17] researches.

Generally, an energy hub acquires raw energy (e.g., solar radiation, high-voltage electricity, and natural gas) at its input ports and provides refined energy services that consumers need (e.g., low-voltage electricity, heating, and cooling) at its output ports [5]. An energy hub consists of various energy conversion and storage devices, and different configurations of the devices can lead to different benefits in economy and environment [7]. Energy management for a single energy hub has been wildly studied. In [8], a probabilistic scheduling model for an energy hub was proposed, in which a demand response program was developed to reallocate the hub's responsive loads based on energy market prices. A model was developed in [9] to analyze the capacity of an energy hub to participate in power system demand response without affecting the hub's loads. In [10], robust optimization theory was employed in energy hub management to produce robust solutions in the presence of system parameter uncertainties. In [11], energy hub management was formulated as a stochastic bi-level problem, where the hub manager determines energy prices to clients with uncertain demands. By vehicle-to-grid technology [18]–[20], massive electric vehicles can be controlled to provide energy systems with storage service. In [12], market behavior of electric vehicles was modeled in an energy hub, where vehicles' batteries were aggregated to serve as a bulk storage.

In the multi-hub scenarios, energy hubs are interconnected via energy transmission networks, forming a macro multi-energy system. In [13], a heuristic algorithm was proposed to solve the optimal power flow problem of a multi-hub system with non-constant efficiency of hub devices. A long-term expansion planning approach was provided in [14] to estimate appropriate investment candidates for a multi-energy system, including power generation units, transmission lines, and hub devices. In [15], a multi-energy demand response problem was formulated as a game, where energy hubs are players and their energy purchase strategies are responsive to dynamic prices. The methodology of alternating direction method of multipliers (ADMM) was employed in [16] to allow energy hubs to collectively solve a global system optimization problem in a distributed fashion. In [17], energy flows among building-level energy hubs were optimized to lower the costs of both hubs and distribution networks.

To enable operational optimization of energy hubs, energy consumers at the hubs' output ports are usually asked to report their load information, such as controllable load constraints and utility/cost functions. However, most of the existing energy hub studies assume that the load information is reported truthfully. In fact, consumers are very likely to misreport load information if they can gain more benefits by doing so. The energy system efficiency and security will be badly impacted when the hub manager cannot distinguish the misreporting. In addition, most of the previous works consider that an energy hub carries out the computation centrally. In such case, the hub's computational complexity increases as the number of loads increases. To mitigate the computational burden, distributed optimization methods are more beneficial in the case where an energy hub has massive loads.

To address the aforementioned issues, this paper proposes a distributed auction mechanism for optimal scheduling of an energy hub. We first design an auction mechanism which is able to achieve truth-telling of energy consumers. Then, we develop a distributed implementation of the auction, in which an ADMM-based algorithm is employed to solve the auction' optimization problem, offloading computation onto consumers. Specifically, we consider that an energy hub serves multiple building energy users. The users submit demand information to the hub manager at the beginning of the auction. Given the reported demand information, the hub manager, as an auctioneer, determines an energy allocation to users and payments that users should make. The payment rule is designed based on Vickrey-Clarke-Groves (VCG) pricing method [21] so that the proposed auction is incentive compatible, which means that a user' utility is maximized only when it truthfully reports its demand information. Next, we develop a distributed version of the auction, where a distributed algorithm based on dual consensus ADMM [22], [23] is proposed to assign part of the auction computation to users. A distributed auction implementation may introduce new chances for users to manipulate the auction outcome since users are allowed to participate in the auction computation. The proposed distributed mechanism is designed precisely to prevent the manipulation of users and implement an incentive compatible outcome as well. The contributions of this paper are as follows:

- An auction mechanism is designed for multi-energy scheduling of an energy hub that serves numbers of building energy users. In the auction, users are incentivized to truthfully submit their demand information.
- Employing dual consensus ADMM, a distributed implementation of the auction is developed to offload auction computation onto the users.
- It is proven that the distributed auction mechanism can avoid user's manipulation and ensure the incentive compatibility in a Nash equilibrium.

Household energy consumption data and solar data are used in the simulation, where we evaluate the energy hub scheduling performance and verify the incentive compatibility of the auction mechanism.

The rest of this paper is organized as follows: In Section II, the model of an energy hub is presented. In Section III, the centralized auction mechanism is proposed for energy hub scheduling, and the auction properties are discussed. In Section IV, the distributed implementation of the auction is developed, and the properties of the distributed auction are discussed. Section V presents and analyzes simulation results. Finally, conclusion is drawn in Section VI.

II. SYSTEM MODEL

A. ENERGY HUB FOR BUILDINGS

Fig. 1 shows the schematic model of an energy hub studied in this paper. Energy inputs of the energy hub include solar radiation, electricity from distribution networks, and natural gas. Output ports of the energy hub are connected to multiple building users, providing them with electricity, heating energy, and cooling energy. If it is a building-level energy hub, the energy users can be apartments in a large residential building or offices in a commercial building. If the energy hub serves a community, the users can be individual buildings, e.g., houses. The energy hub is comprised

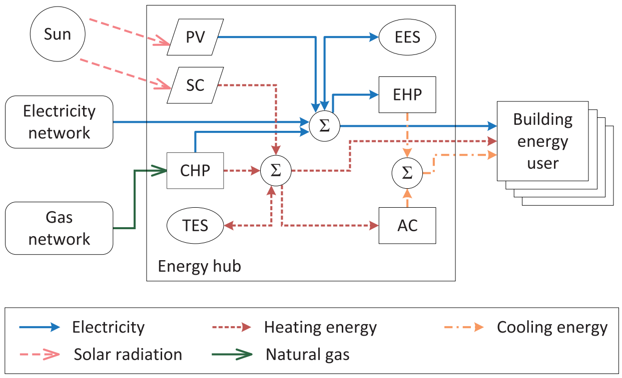


FIGURE 1. Energy hub serving building energy users.

of various energy conversion and storage devices, including a combined heat and power (CHP) unit, an electric heat pump (EHP), an absorption chiller (AC), a photovoltaic (PV) system, a solar combisystem (SC), an electric energy storage (EES), and a thermal energy storage (TES). A hub manager is responsible for optimizing multi-energy scheduling of the energy hub and allocating energy to users.

B. ENERGY HUB MODEL

An operating day is divided into a series of time slots, and each of them is indexed by $t \in \mathcal{T} = \{1, \ldots, T\}$. The set of energy types on the user side is denoted by $\mathcal{E} = \{\mathrm{e}, \mathrm{h}, \mathrm{c}\}$, including electricity, heating energy, and cooling energy. The following device models in the energy hub are based on [3] and [24]. Table 1 summarizes the notions of the models.

1) CHP

A CHP unit generates electricity and heating energy by consuming gas, which is described by

$$y_{\mathrm{e},t}^{\mathrm{chp}} = \eta_{\mathrm{ge}}^{\mathrm{chp}} \cdot y_{\mathrm{g},t}^{\mathrm{chp}}, \tag{1}$$
$$y_{\mathrm{h},t}^{\mathrm{chp}} = \eta_{\mathrm{gh}}^{\mathrm{chp}} \cdot y_{\mathrm{g},t}^{\mathrm{chp}}, \tag{2}$$

where $y_{\mathrm{e},t}^{\mathrm{chp}}$, $y_{\mathrm{h},t}^{\mathrm{chp}}$, and $y_{\mathrm{g},t}^{\mathrm{chp}}$ (kWh) denote the electricity production, heat production, and gas consumption of the CHP unit at time slot t, respectively; $\eta_{\mathrm{ge}}^{\mathrm{chp}}$ and $\eta_{\mathrm{gh}}^{\mathrm{chp}}$ represent the gas-to-electricity and gas-to-heat efficiencies, respectively. Energy production bounds of the CHP are provided by

$$0 \le y_{\mathrm{e},t}^{\mathrm{chp}} \le y_{\mathrm{e}}^{\mathrm{chp,max}}, \tag{3}$$
$$0 \le y_{\mathrm{h},t}^{\mathrm{chp}} \le y_{\mathrm{h}}^{\mathrm{chp,max}}. \tag{4}$$

2) EHP

An EHP is used for space cooling. Let $y_{\mathrm{c},t}^{\mathrm{ehp}}$ and $y_{\mathrm{e},t}^{\mathrm{ehp}}$ denote the cooling energy production and electricity consumption of the EHP, respectively. We have

$$y_{\mathrm{c},t}^{\mathrm{ehp}} = \eta^{\mathrm{ehp}} \cdot y_{\mathrm{e},t}^{\mathrm{ehp}}, \tag{5}$$
$$0 \le y_{\mathrm{c},t}^{\mathrm{ehp}} \le y_{\mathrm{c}}^{\mathrm{ehp,max}}, \tag{6}$$

where η^{ehp} stands for the cooling coefficient of the EHP.

TABLE 1. Descriptions of notations.

Notation	Description
e	Electricity
h	Heating energy
c	Cooling energy
chp	Combined heat and power
ehp	Electric heat pump
ac	Absorption chiller
sc	Solar combisystem
pv	Photovoltaics
ch	Charging of a storage device
dis	Discharging of a storage device
t	Index for time slots
T	Number of time slots
$\mathcal{T}$	Set of time slots
i	Index for users
N	Number of users
$\mathcal{N}$	Set of users
ϵ	Index for energy type
$\mathcal{E}$	Set of energy types of users
y	Energy consumption or production of a device
η	Efficiency or coefficient of a device
s	Energy stored in a storage device
r	Solar radiation
x	Energy consumption of a user

3) AC

An AC uses heating energy to drive the cooling process. Let $y_{\mathrm{c},t}^{\mathrm{ac}}$ and $y_{\mathrm{h},t}^{\mathrm{ac}}$ denote the cooling energy production and heating energy consumption of the AC, respectively; let η^{ac} be the cooling coefficient of the AC. We have

$$y_{\mathrm{c},t}^{\mathrm{ac}} = \eta^{\mathrm{ac}} \cdot y_{\mathrm{h},t}^{\mathrm{ac}}, \tag{7}$$
$$0 \le y_{\mathrm{c},t}^{\mathrm{ac}} \le y_{\mathrm{c}}^{\mathrm{ac,max}}. \tag{8}$$

4) EES

Let $s_{\mathrm{e},t}$ denote the amount of electric energy stored in the EES; let $y_{\mathrm{e},t}^{\mathrm{ch}}$ and $y_{\mathrm{e},t}^{\mathrm{dis}}$ be the charging and discharging energy of the EES, respectively; let $\eta_{\mathrm{e}}^{\mathrm{ch}}$ and $\eta_{\mathrm{e}}^{\mathrm{dis}}$ denote the charging and discharging efficiencies of the EES, respectively. The EES model is described by

$$s_{\mathrm{e},t} = s_{\mathrm{e},t-1} + \eta_{\mathrm{e}}^{\mathrm{ch}} \cdot y_{\mathrm{e},t}^{\mathrm{ch}} - (1/\eta_{\mathrm{e}}^{\mathrm{dis}}) y_{\mathrm{e},t}^{\mathrm{dis}}, \tag{9}$$
$$s_{\mathrm{e},1} = s_{\mathrm{e},T}, \tag{10}$$
$$0 \le y_{\mathrm{e},t}^{\mathrm{ch}} \le y_{\mathrm{e}}^{\mathrm{ch,max}}, \tag{11}$$
$$0 \le y_{\mathrm{e},t}^{\mathrm{dis}} \le y_{\mathrm{e}}^{\mathrm{dis,max}}, \tag{12}$$
$$s_{\mathrm{e}}^{\mathrm{min}} \le s_{\mathrm{e},t} \le s_{\mathrm{e}}^{\mathrm{max}}. \tag{13}$$

5) TES

The model of the TES is similar to that of the EES. Let $s_{\mathrm{h},t}$ denote the amount of heating energy stored in the TES; let $y_{\mathrm{h},t}^{\mathrm{ch}}$ and $y_{\mathrm{h},t}^{\mathrm{dis}}$ be the charging and discharging energy of the TES, respectively; let $\eta_{\mathrm{h}}^{\mathrm{ch}}$ and $\eta_{\mathrm{h}}^{\mathrm{dis}}$ denote the charging and discharging efficiencies of the TES, respectively. We have

$$s_{\mathrm{h},t} = s_{\mathrm{h},t-1} + \eta_{\mathrm{h}}^{\mathrm{ch}} \cdot y_{\mathrm{h},t}^{\mathrm{ch}} - (1/\eta_{\mathrm{h}}^{\mathrm{dis}}) y_{\mathrm{h},t}^{\mathrm{dis}}, \tag{14}$$
$$s_{\mathrm{h},1} = s_{\mathrm{h},T}, \tag{15}$$
$$0 \le y_{\mathrm{h},t}^{\mathrm{ch}} \le y_{\mathrm{h}}^{\mathrm{ch,max}}, \tag{16}$$
$$0 \le y_{\mathrm{h},t}^{\mathrm{dis}} \le y_{\mathrm{h}}^{\mathrm{dis,max}}, \tag{17}$$
$$s_{\mathrm{h}}^{\mathrm{min}} \le s_{\mathrm{h},t} \le s_{\mathrm{h}}^{\mathrm{max}}. \tag{18}$$

6) SC

An SC is used to generate heating energy from solar thermal collectors. Let r_t (kWh/m^2) denote the solar radiation on a horizontal surface at time slot t; let α^{sc} (m^2) be the horizontal area of solar thermal collectors; let η^{sc} be the efficiency of the SC. The heating energy production of the SC is described by

$$y^{\text{sc}}_{\text{h},t} = \eta^{\text{sc}} \cdot \alpha^{\text{sc}} \cdot r_t, \tag{19}$$
$$0 \le y^{\text{sc}}_{\text{h},t} \le y^{\text{sc,max}}_{\text{h}}. \tag{20}$$

7) PV

An PV system uses solar panels to generate electricity. Let α^{pv} denote the horizontal area of the solar panels, and η^{pv} be the efficiency of the PV system. The electricity production of the PV is described by

$$y^{\text{pv}}_{\text{e},t} = \eta^{\text{pv}} \cdot \alpha^{\text{pv}} \cdot r_t, \tag{21}$$
$$0 \le y^{\text{pv}}_{\text{e},t} \le y^{\text{pv,max}}_{\text{e}}. \tag{22}$$

8) ELECTRICITY AND GAS NETWORKS

Let $y^{\text{buy}}_{\text{e},t}$ denote the amount of electricity bought from the distribution network, and let $y^{\text{buy}}_{\text{g},t}$ denote the amount of gas bought from the gas network at slot t. They are bounded by

$$0 \le y^{\text{buy}}_{\text{e},t} \le y^{\text{buy,max}}_{\text{e}}, \tag{23}$$
$$0 \le y^{\text{buy}}_{\text{g},t} \le y^{\text{buy,max}}_{\text{g}}. \tag{24}$$

9) USER LOADS

Index each building energy user by $i \in \mathcal{N} = \{1, \ldots, N\}$. Let $x_{i,\epsilon,t}$ denote the ϵ-type energy allocated to user i at time slot t. We have

$$x^{\min}_{i,\epsilon,t} \le x_{i,\epsilon,t} \le x^{\max}_{i,\epsilon,t}, \quad \forall \epsilon \in \mathcal{E}, \tag{25}$$
$$\sum_{t\in\mathcal{T}} x_{i,\epsilon,t} = x^{\text{day}}_{i,\epsilon}, \quad \forall \epsilon \in \mathcal{E}, \tag{26}$$

where $x^{\min}_{i,\epsilon,t}$ stands for the minimal amount of load that must be satisfied at each time slot, and $x^{\text{day}}_{i,\epsilon}$ denotes the total amount of load that must be satisfied in one day.

10) ENERGY CONSERVATION

In the energy hub, the energy conservations of gas, electricity, heating energy, and cooling energy are described by

$$y^{\text{chp}}_{\text{g},t} - y^{\text{buy}}_{\text{g},t} = 0, \tag{27}$$
$$\sum_{i\in\mathcal{N}} x_{i,\text{e},t} + y^{\text{ch}}_{\text{e},t} + y^{\text{ehp}}_{\text{e},t} - y^{\text{chp}}_{\text{e},t} - y^{\text{pv}}_{\text{e},t} - y^{\text{dis}}_{\text{e},t} - y^{\text{buy}}_{\text{e},t} = 0, \tag{28}$$
$$\sum_{i\in\mathcal{N}} x_{i,\text{h},t} + y^{\text{ac}}_{\text{h},t} + y^{\text{ch}}_{\text{h},t} - y^{\text{chp}}_{\text{h},t} - y^{\text{sc}}_{\text{h},t} - y^{\text{dis}}_{\text{h},t} = 0, \tag{29}$$
$$\sum_{i\in\mathcal{N}} x_{i,\text{c},t} - y^{\text{ehp}}_{\text{c},t} - y^{\text{ac}}_{\text{c},t} = 0. \tag{30}$$

In (27)–(30), stack all the variables represented as y in vector $\boldsymbol{y}_t$, which denotes the decision vector for the energy hub operation at time slot t. Let $\boldsymbol{x}_{i,t} = [x_{i,\text{e},t}, x_{i,\text{h},t}, x_{i,\text{c},t}]$ which denotes the energy allocation to user i at time slot t.

III. AUCTION MECHANISM FOR ENERGY HUB SCHEDULING

In this section, we design a centralized auction mechanism for multi-energy scheduling of the energy hub. At first, we provide the definition of a centralized mechanism in which energy users are auction participants, and the hub manager acts as a center collecting the users' demand information, called *types*.

Definition 1 (Standard Mechanism [25]): A centralized (direct-revelation) mechanism $M = (f, \Theta)$ defines an outcome function f and a type space $\Theta = \Theta_1 \times \ldots \times \Theta_N$.

In the proposed auction mechanism, users are directly asked to reveal their demand information $\hat{\boldsymbol{\theta}} \in \Theta$, and the hub manager centrally determines an outcome $f(\hat{\boldsymbol{\theta}}) = (\boldsymbol{x}, \boldsymbol{p})$, where $\boldsymbol{x}$ is the energy allocation to users and $\boldsymbol{p}$ is the payment by users to the hub manager. In the following, we will present the content of users' demand information and the rules for determining the energy allocation and payment.

A. UTILITIES

We first define the utilities of the users and hub manager in the auction. Consider that each user has an increasing concave function to describe its satisfaction degree of energy consuming [26]. The satisfaction function of user i at time slot t is defined as

$$v_{i,t}(\boldsymbol{x}_{i,t}) = \sum_{\epsilon\in\mathcal{E}} \left(\beta_{i,\epsilon,t} \cdot x_{i,\epsilon,t} - \frac{\beta_{i,\epsilon,t}}{2 \cdot x^{\max}_{i,\epsilon,t}} (x_{i,\epsilon,t})^2 \right), \tag{31}$$

where $\beta_{i,\epsilon,t}$ is a positive constant.

At the beginning of the auction, each user needs to report its demand information (i.e., type) to the hub manager. A user's type includes parameters of its energy consumption constraints (25), (26) and satisfaction function (31). We define the type of user i as

$$\boldsymbol{\theta}_i = \{\boldsymbol{\beta}_i, \boldsymbol{x}^{\text{mim}}_i, \boldsymbol{x}^{\max}_i, \boldsymbol{x}^{\text{day}}_i\}, \tag{32}$$

where $\boldsymbol{\beta}_i = \{\beta_{i,\epsilon,t} | \epsilon \in \mathcal{E}, t \in \mathcal{T}\}$, $\boldsymbol{x}^{\text{mim}}_i = \{x^{\text{mim}}_{i,\epsilon,t} | \epsilon \in \mathcal{E}, t \in \mathcal{T}\}$, $\boldsymbol{x}^{\max}_i = \{x^{\max}_{i,\epsilon,t} | \epsilon \in \mathcal{E}, t \in \mathcal{T}\}$, and $\boldsymbol{x}^{\text{day}}_i = \{x^{\text{day}}_{i,\epsilon} | \epsilon \in \mathcal{E}\}$. Since $\boldsymbol{\theta}_i$ is only known by user i itself, users can misreport their types. Let $\hat{\boldsymbol{\theta}}_i = \{\hat{\boldsymbol{\beta}}_i, \hat{\boldsymbol{x}}^{\text{mim}}_i, \hat{\boldsymbol{x}}^{\max}_i, \hat{\boldsymbol{x}}^{\text{day}}_i\}$ denote the reported type of user i. Receiving $\hat{\boldsymbol{\theta}} = \{\hat{\boldsymbol{\theta}}_i | i \in \mathcal{N}\}$, the hub manager determines an outcome including energy allocation $\boldsymbol{x} = \{\boldsymbol{x}_{i,t} | i \in \mathcal{N}, t \in \mathcal{T}\}$ and payment $\boldsymbol{p} = \{p_{i,t} | i \in \mathcal{N}, t \in \mathcal{T}\}$ according to the outcome rule f. On the operating day, user i will consume energy according to $\boldsymbol{x}_{i,t}$ and make payments according to $p_{i,t}$.

The *utility* of user i is defined as its satisfaction minus the payment that it makes, which is denoted by

$$u_i(f(\hat{\boldsymbol{\theta}}), \boldsymbol{\theta}_i) = \sum_{t\in\mathcal{T}} v_{i,t}(\boldsymbol{x}_{i,t}) - \sum_{t\in\mathcal{T}} p_{i,t}(\hat{\boldsymbol{\theta}}), \tag{33}$$

where $v_{i,t}(\boldsymbol{x}_{i,t}) = v_{i,t}(\boldsymbol{\theta}_i, \boldsymbol{x}_{i,t})$.

The cost of the hub manager at time slot t is given by

$$c_t(\boldsymbol{y}_t) = c^{\text{d}}_t(\boldsymbol{y}_t) + c^{\text{b}}_t(\boldsymbol{y}_t), \tag{34}$$

which shows that the hub's cost includes two parts: the energy hub operating cost and the cost of buying energy from electricity and gas networks. An increasing convex function is used to estimate the operating cost [16], which is given by

$$c_t^{\mathrm{d}}(\boldsymbol{y}_t) = \sum_{k=1}^{10} \gamma_{1,k}(y_{t,k}^{\mathrm{d}})^2 + \gamma_{2,k} \cdot y_{t,k}^{\mathrm{d}}, \tag{35}$$

where $\gamma_{1,k}$ and $\gamma_{2,k}$ are positive constants; $y_{t,k}^{\mathrm{d}}$ denotes the kth component of $\boldsymbol{y}_t^{\mathrm{d}} = [y_{\mathrm{e},t}^{\mathrm{chp}}, y_{\mathrm{h},t}^{\mathrm{chp}}, y_{\mathrm{c},t}^{\mathrm{ehp}}, y_{\mathrm{c},t}^{\mathrm{ac}}, y_{\mathrm{e},t}^{\mathrm{ch}}, y_{\mathrm{e},t}^{\mathrm{dis}}, y_{\mathrm{h},t}^{\mathrm{ch}}, y_{\mathrm{h},t}^{\mathrm{dis}}, y_{\mathrm{h},t}^{\mathrm{sc}}, y_{\mathrm{e},t}^{\mathrm{pv}}]$. The energy purchase cost is given by

$$c_t^{\mathrm{b}}(\boldsymbol{y}_t) = \rho_{\mathrm{e},t} \cdot y_{\mathrm{e},t}^{\mathrm{buy}} + \rho_{\mathrm{g},t} \cdot y_{\mathrm{g},t}^{\mathrm{buy}}, \tag{36}$$

where $\rho_{\mathrm{e},t}$ and $\rho_{\mathrm{g},t}$ denotes the prices of electricity and gas at time slot t, respectively. The utility of the hub manager is defined as the total payment received minus the total cost, which is denoted by

$$\sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} p_{i,t}(\hat{\boldsymbol{\theta}}) - \sum_{t\in\mathcal{T}} c_t(\boldsymbol{y}_t). \tag{37}$$

B. ENERGY ALLOCATION RULE

Temporarily suppose that all users truthful report their types. The social welfare is defined as the hub manager's utility (37) plus the sum of (33) over all users. Let $\boldsymbol{x}_i = [\boldsymbol{x}_{i,1}, \ldots, \boldsymbol{x}_{i,T}]$, $\boldsymbol{y} = [\boldsymbol{y}_1, \ldots, \boldsymbol{y}_T]$, $v_i(\boldsymbol{x}_i) = \sum_{t\in\mathcal{T}} v_{i,t}(\boldsymbol{x}_{i,t})$, and $c(\boldsymbol{y}) = \sum_{t\in\mathcal{T}} c_t(\boldsymbol{y}_t)$. The energy allocation rule of the auction mechanism is described by the following social welfare maximization problem, denoted as problem $\mathscr{M}$.

$$\mathscr{M}: \max_{\boldsymbol{x},\boldsymbol{y}} \sum_{i\in\mathcal{N}} v_i(\boldsymbol{x}_i) - c(\boldsymbol{y}), \tag{38a}$$

$$\text{s.t. } \boldsymbol{x}_i \in \mathcal{X}_i, \quad \forall i \in \mathcal{N}, \tag{38b}$$

$$\boldsymbol{y} \in \mathcal{Y}, \tag{38c}$$

$$\sum_{i\in\mathcal{N}} A_i\boldsymbol{x}_i + B\boldsymbol{y} = 0, \tag{38d}$$

where $\mathcal{X}_i$ is the constraint set of user i for meeting (25) and (26) $\forall t \in \mathcal{T}$; $\mathcal{Y}$ is the constraint set of the energy hub for satisfying (1)–(24), (27) $\forall t \in \mathcal{T}$; A_i and B are the coefficient matrices, making constraint (38d) equivalent to (28)–(30) $\forall t \in \mathcal{T}$. The optimal solution to problem $\mathscr{M}$ is denoted by $\{\boldsymbol{x}^*, \boldsymbol{y}^*\}$, in which $\boldsymbol{x}^*$ is the outcome of energy allocation to users.

C. PAYMENT RULE

Based on VCG payment rule [21], [27], we define the payment by user i at time slot t as

$$p_{i,t}(\boldsymbol{\theta}) = \sum_{j\neq i} v_{j,t}(\boldsymbol{x}_{j,t}^{-i}) - \sum_{j\neq i} v_{j,t}(\boldsymbol{x}_{j,t}^{*}) + c_t(\boldsymbol{y}_t^*), \tag{39}$$

where we have $\boldsymbol{x}^{-i} = \{\boldsymbol{x}_{j,t}^{-i} | j \in \mathcal{N}\backslash\{i\}, t \in \mathcal{T}\}$, which is the optimal solution to the following maximization problem, denoted as problem $\mathscr{M}_{-i}$.

$$\mathscr{M}_{-i}: \max_{\boldsymbol{x}} \sum_{j\neq i} v_j(\boldsymbol{x}_j), \tag{40a}$$

$$\text{s.t. } \boldsymbol{x}_j \in \mathcal{X}_j, \forall j \in \mathcal{N}/\{i\}. \tag{40b}$$

Notice that user i is excluded in problem $\mathscr{M}_{-i}$.

D. ANALYSIS OF THE AUCTION MECHANISM

Consider that users know the energy allocation rule (38) and payment rule (39), and each user will strategically choose the reported type $\hat{\boldsymbol{\theta}}_i$ to maximize its utility (33). The proposed auction mechanism is able to achieve that each user has a maximum utility only when it truthfully reports $\boldsymbol{\theta}_i$. Formally, a mechanism is characterized by *incentive compatibility* if truth-revelation of users is achieved in an equilibrium [21], [25]. In the following, we introduce the definition of a dominant-strategy equilibrium and then prove that the proposed auction mechanism is incentive compatible in this equilibrium. Let $s_i(\boldsymbol{\theta}_i)$ denote the strategy of user i given $\boldsymbol{\theta}_i$. Since a user's strategy in the auction is to choose a reported type, we have $s_i(\boldsymbol{\theta}_i) \in \Theta_i$. Let $\boldsymbol{\theta}_{-i} = \{\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_{i-1}, \boldsymbol{\theta}_{i+1}, \ldots, \boldsymbol{\theta}_N\}$.

Definition 2 (Dominant-Strategy Equilibrium [28]): A strategy profile $s^* = \{s_1^*, \ldots, s_N^*\} \in \Theta$ is in a dominant-strategy equilibrium, if

$$u_i(f(s_i^*(\boldsymbol{\theta}_i), s_{-i}(\boldsymbol{\theta}_{-i})), \boldsymbol{\theta}_i) \geq u_i(f(s_i(\boldsymbol{\theta}_i), s_{-i}(\boldsymbol{\theta}_{-i})), \boldsymbol{\theta}_i) \tag{41}$$

holds $\forall i \in \mathcal{N}$, $\forall s_i \neq s_i^*$, $\forall \boldsymbol{\theta}_i$, and $\forall \boldsymbol{\theta}_{-i}$.

Thus, a mechanism is incentive compatible in a dominant-strategy equilibrium if it can achieve

$$u_i(f(\boldsymbol{\theta}_i, \hat{\boldsymbol{\theta}}_{-i}), \boldsymbol{\theta}_i) \geq u_i(f(\hat{\boldsymbol{\theta}}_i, \hat{\boldsymbol{\theta}}_{-i}), \boldsymbol{\theta}_i), \tag{42}$$

which means that truthfully reporting type $\boldsymbol{\theta}_i$ is the best strategy of user i whatever the other users report. It is also said that a mechanism is *strategy-proof* if (42) holds.

Theorem 1: The proposed centralized auction mechanism with energy allocation rule (38) and payment rule (39) is strategy-proof.

Proof: See Appendix A. □

In addition, the auction can achieve *budget balance* of the hub manager, which means that the total received payment is no less than the total cost.

Theorem 2: The proposed auction mechanism with energy allocation rule (38) and payment rule (39) is budget-balanced.

Proof: See Appendix B. □

IV. DISTRIBUTED IMPLEMENTATION

In this section, we develop a distributed implementation of the centralized auction mechanism, offloading computation onto users. At first, we analyze the problems that the hub manager needs to solve in the centralized mechanism.

To determine the energy allocation and payment, the hub manager has to attain the optimal solution to each problem in $\{\mathscr{M}, \mathscr{M}_{-1}, \mathscr{M}_{-2}, \ldots, \mathscr{M}_{-N}\}$. But, we do not have to solve these problems individually. The objective function and constraints of problem $\mathscr{M}_{-i}$ are separable, which greatly reduces the amount of computation. Specifically, in the distributed implementation, let user $i \in \mathcal{N}$ solve the following maximization problem, called problem $\mathscr{M}_i$.

$$\mathscr{M}_i: \max_{\boldsymbol{x}_i \in \mathcal{X}_i} v_i(\boldsymbol{x}_i). \tag{43}$$

Let $\boldsymbol{x}_i^\star$ be the optimal solution to problem $\mathscr{M}_i$. Then, the optimal solution to problem $\mathscr{M}_{-i}$ is given by $\boldsymbol{x}^{-i} = \{\boldsymbol{x}_j^\star | j \in \mathcal{N} \backslash \{i\}\}$. Solving problem $\mathscr{M}_i$ only requires local information of user i, so the user is able to solve the problem independently. Therefore, by collecting $\boldsymbol{x}_i^\star$ from all users, the hub manager can attain the optimal solutions to $\mathscr{M}_{-1}, \mathscr{M}_{-2}, \ldots, \mathscr{M}_{-N}$, respectively.

Then, we focus on problem $\mathscr{M}$, in which constraint (38d) couples the hub manager and users altogether. To solve problem $\mathscr{M}$ in a distributed manner, we employ dual consensus ADMM [22], [23].

A. DUAL CONSENSUS ADMM

In the following, problem $\mathscr{M}$ is reformulated and solved in the framework of dual consensus ADMM. Let $\boldsymbol{\lambda}$ be the dual variable of constraint (38d). The Lagrange dual problem of $\mathscr{M}$ is provided by

$$\min_{\boldsymbol{\lambda}} \sum_{i \in \mathcal{N}} \phi_i(\boldsymbol{\lambda}) + \psi(\boldsymbol{\lambda}), \tag{44}$$

where

$$\phi_i(\boldsymbol{\lambda}) = \max_{\boldsymbol{x}_i \in \mathcal{X}_i} \left\{ v_i(\boldsymbol{x}_i) - \boldsymbol{\lambda}^T A_i \boldsymbol{x}_i \right\}, \quad \forall i \in \mathcal{N}, \tag{45}$$

$$\psi(\boldsymbol{\lambda}) = \max_{\boldsymbol{y} \in \mathcal{Y}} \left\{ - c(\boldsymbol{y}) - \boldsymbol{\lambda}^T B \boldsymbol{y} \right\}. \tag{46}$$

Problem $\mathscr{M}$ is a concave maximization problem, so strong duality holds. This indicates that solving problem (44) is equivalent to solving problem $\mathscr{M}$. Consider that each user only communicates with the hub manager. Let the hub manager manages the dual variable $\boldsymbol{\lambda}$, and let user i manages a copy of $\boldsymbol{\lambda}$, denoted by $\boldsymbol{\lambda}_i$. We construct the following problem:

$$\min_{\boldsymbol{\lambda}, \{\boldsymbol{\lambda}_i\}, \{\boldsymbol{\lambda}_i'\}} \sum_{i \in \mathcal{N}} \phi_i(\boldsymbol{\lambda}_i) + \psi(\boldsymbol{\lambda}) \tag{47a}$$

$$\text{s.t. } \boldsymbol{\lambda}_i = \boldsymbol{\lambda}_i', \quad \forall i \in \mathcal{N}, \tag{47b}$$

$$\boldsymbol{\lambda} = \boldsymbol{\lambda}_i', \tag{47c}$$

where $\boldsymbol{\lambda}_i'$ is a slack variable. Constraints (47b) and (47c) ensure a global consensus of the dual variable, which makes problem (47) equivalent to problem (44). Next, consensus ADMM technique [29] is employed to solve problem (47).

According to [22] and [23], the variable update steps of the hub manager at iteration τ are presented as

$$\boldsymbol{\mu}^{[\tau]} = \boldsymbol{\mu}^{[\tau-1]} + q \sum_{i \in \mathcal{N}} (\boldsymbol{\lambda}^{[\tau-1]} - \boldsymbol{\lambda}_i^{[\tau-1]}), \tag{48}$$

$$\boldsymbol{y}^{[\tau]} = \arg\min_{\boldsymbol{y} \in \mathcal{Y}} \Big\{ c(\boldsymbol{y}) + \frac{q}{4N} \Big\| \frac{1}{q} B\boldsymbol{y} - \frac{1}{q} \boldsymbol{\mu}^{[\tau]} + \sum_{i \in \mathcal{N}} (\boldsymbol{\lambda}^{[\tau-1]} + \boldsymbol{\lambda}_i^{[\tau-1]}) \Big\|_2^2 \Big\}, \tag{49}$$

$$\boldsymbol{\lambda}^{[\tau]} = \frac{1}{2N} \Big(\frac{1}{q} B\boldsymbol{y}^{[\tau]} - \frac{1}{q} \boldsymbol{\mu}^{[\tau]} + \sum_{i \in \mathcal{N}} (\boldsymbol{\lambda}^{[\tau-1]} + \boldsymbol{\lambda}_i^{[\tau-1]}) \Big), \tag{50}$$

where $\boldsymbol{\mu}$ is associated with the dual variables of constraints (47b) and (47c), and q is a positive constant. The variable update steps of user i at iteration τ are given by

$$\boldsymbol{\mu}_i^{[\tau]} = \boldsymbol{\mu}_i^{[\tau-1]} + q(\boldsymbol{\lambda}_i^{[\tau-1]} - \boldsymbol{\lambda}^{[\tau-1]}), \tag{51}$$

$$\boldsymbol{x}_i^{[\tau]} = \arg\min_{\boldsymbol{x}_i \in \mathcal{X}_i} \Big\{ - v_i(\boldsymbol{x}_i) + \frac{q}{4} \Big\| \frac{1}{q} A_i \boldsymbol{x}_i - \frac{1}{q} \boldsymbol{\mu}_i^{[\tau]} + \boldsymbol{\lambda}^{[\tau-1]} + \boldsymbol{\lambda}_i^{[\tau-1]} \Big\|_2^2 \Big\}, \tag{52}$$

$$\boldsymbol{\lambda}_i^{[\tau]} = \frac{1}{2q} A_i \boldsymbol{x}_i^{[\tau]} - \frac{1}{2q} \boldsymbol{\mu}_i^{[\tau]} + \frac{1}{2} (\boldsymbol{\lambda}^{[\tau-1]} + \boldsymbol{\lambda}_i^{[\tau-1]}). \tag{53}$$

Notice that the update steps of the hub (or user i) only requires local information and iterative exchange of $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}_i$. Algorithm 1 shows the dual consensus ADMM framework for solving problem $\mathscr{M}$. To allow the hub manager to be aware of the success of convergence, the stopping criteria are defined as

$$\| \boldsymbol{\lambda}^{[\tau]} - \bar{\boldsymbol{\lambda}}^{[\tau]} \|_2^2 + \sum_{i \in \mathcal{N}} \| \boldsymbol{\lambda}_i^{[\tau]} - \bar{\boldsymbol{\lambda}}^{[\tau]} \|_2^2 \leq \varepsilon_1, \tag{54}$$

$$\| \bar{\boldsymbol{\lambda}}^{[\tau]} - \bar{\boldsymbol{\lambda}}^{[\tau-1]} \|_2^2 \leq \varepsilon_2, \tag{55}$$

where $\bar{\boldsymbol{\lambda}}^{[\tau]} = (\boldsymbol{\lambda}^{[\tau]} + \sum_{i \in \mathcal{N}} \boldsymbol{\lambda}_i^{[\tau]})/(N + 1)$, and ε_1 and ε_2 are very small positive constants. The stopping criteria (54) and (55) are based on the primal and dual residuals of consensus ADMM [29].

Algorithm 1 Dual Consensus ADMM for Solving $\mathscr{M}$

Set $\tau = 0$. For the hub manager, set $\boldsymbol{\mu}^{[0]} = \boldsymbol{0}$, $\boldsymbol{y}^{[0]} \in \mathbb{R}^{15T}$, $\boldsymbol{\lambda}^{[0]} \in \mathbb{R}^{3T}$. For user $i \in \mathcal{N}$, set $\boldsymbol{\mu}_i^{[0]} = \boldsymbol{0}$, $\boldsymbol{x}_i^{[0]} \in \mathbb{R}^{3T}$, $\boldsymbol{\lambda}_i^{[0]} \in \mathbb{R}^{3T}$.

repeat

- $\tau \leftarrow \tau + 1$.
- The hub manager sends $\boldsymbol{\lambda}^{[\tau-1]}$ to each user and updates $\boldsymbol{\mu}^{[\tau]}$, $\boldsymbol{y}^{[\tau]}$, and $\boldsymbol{\lambda}^{[\tau]}$ according to (48)–(50).
- **for** $i \in \mathcal{N}$ *(in parallel)* **do**
  - User i sends $\boldsymbol{\lambda}_i^{[\tau-1]}$ to the hub manager and updates $\boldsymbol{\mu}_i^{[\tau]}$, $\boldsymbol{x}_i^{[\tau]}$, and $\boldsymbol{\lambda}_i^{[\tau]}$ according to (51)–(53).

until *stopping criteria (54) and (55) are satisfied*;

B. DISTRIBUTED MECHANISM

Based on the aforementioned distributed approaches for solving problems $\{\mathscr{M}, \mathscr{M}_{-1}, \mathscr{M}_{-2}, \ldots, \mathscr{M}_{-N}\}$, we develop a distributed mechanism that can produce the same outcome as the centralized mechanism. The distributed mechanism can be summarized into the following steps.

Step 1: User $i \in \mathcal{N}$ submits type $\hat{\boldsymbol{\theta}}_i$ to the hub manager.

Step 2: User $i \in \mathcal{N}$ solves problem $\mathscr{M}_i$ and sends the optimal solution $\boldsymbol{x}_i^\star$ to the hub manager. Then, the manager determines $\boldsymbol{x}^{-i} = \{\boldsymbol{x}_j^\star | j \in \mathcal{N}\backslash\{i\}\}$, the optimal solution to problem $\mathscr{M}_{-i}$, for all $i \in \mathcal{N}$.

Step 3: Run Algorithm 1 to get the optimal solution to problem $\mathscr{M}$. At the end of the algorithm, the hub manager has $\boldsymbol{y}^*$ and $\boldsymbol{\lambda}^*$, and user i has $\boldsymbol{x}_i^*$ and $\boldsymbol{\lambda}_i^*$. Then, user $i \in \mathcal{N}$ sends $\boldsymbol{x}_i^*$ to the hub.

Step 4: Given $\boldsymbol{x}^*$ and $\boldsymbol{x}^{-i}, \forall i \in \mathcal{N}$, the hub manager calculates payment according to (39), and executes energy allocation $\boldsymbol{x}^*$ on the operating day.

Note that users are obliged to submit their types (in Step 1) even if the hub manager does not need to users' information to complete its computation in the distributed mechanism. Collecting users' types reserves the ability of the hub manager to check computation results from users. In Step 3, for example, the hub manager can obtain $\boldsymbol{x}_i^*$ by solving (45) given $\boldsymbol{\lambda}^*$, and check the $\boldsymbol{x}_i^*$ sent by user i. But, checking every user's results leads to additional computation burden of the hub manager. We prefer that the distributed mechanism can incentive users to faithfully complete the assigned computation.

C. ANALYSIS OF THE DISTRIBUTED MECHANISM

In the centralized mechanism, the only action that a user takes is to report its type. In contrast, a distributed mechanism allows users to participate in auction computation, which gives users opportunities to strategically change computational results and in turn manipulate the auction outcome. In the proposed distributed mechanism, the actions of user i include 1) reporting $\hat{\boldsymbol{\theta}}_i$, 2) solving $\mathscr{M}_i$, 3) sending $\boldsymbol{x}_i^\star$, 4) updating $\boldsymbol{\mu}^{[\tau]}$, $\boldsymbol{y}^{[\tau]}$, and $\boldsymbol{\lambda}^{[\tau]}$, 5) sending $\boldsymbol{\lambda}^{[\tau]}$, and 6) sending $\boldsymbol{x}_i^*$. In the following, we prove that a user will faithfully complete all of these actions in the distributed mechanism. At first, the definition of a distributed mechanism is given.

Definition 3 (Distributed Mechanism [25]): A distributed mechanism $M^d = (g, \Sigma, s^M)$ defines an outcome function g, a feasible strategy space $\Sigma = \Sigma_1 \times \ldots \times \Sigma_N$, and an intended strategy profile $s^M = \{s_1^M, \ldots, s_N^M\}$.

The outcome function g determines an outcome $g(s(\boldsymbol{\theta}))$ when users have types $\boldsymbol{\theta}$ and take strategy $s \in \Sigma$. A strategy $s_i \in \Sigma_i$ is a mapping from user i's type to the aforementioned 6 types of actions.

Definition 4 (Intended Strategy [28]): Given a strategy-proof direct-revelation mechanism M that implements outcome $f(\boldsymbol{\theta})$, s^M is the intended strategy of a distributed mechanism M^d when

$$g(s^{\mathrm{M}}(\boldsymbol{\theta})) = f(\boldsymbol{\theta}), \quad \forall \boldsymbol{\theta} \in \Theta. \tag{56}$$

The intended strategy s^{M} can be taken as a strategy that the mechanism designer expects every user to follow. In the proposed distributed mechanism, the intended strategy is that all users faithfully complete the 6 types of actions, producing the same outcome as the centralized mechanism. Next, we introduce the concept of *faithful implementation*.

Definition 5 (Faithful Implementation [25]): A distributed mechanism $M^d = (g, \Sigma, s^M)$ is a faithful implementation of outcome $g(s^M(\boldsymbol{\theta}))$ when the intended strategy s^M is in an ex-post Nash equilibrium.

Definition 6 (Ex-Post Nash Equilibrium [25], [28]): A strategy profile $s^* = \{s_1^*, \ldots, s_N^*\} \in \Sigma$ is in an ex-post Nash equilibrium, if

$$u_i(g(s_i^*(\boldsymbol{\theta}_i), s_{-i}^*(\boldsymbol{\theta}_{-i})), \boldsymbol{\theta}_i) \geq u_i(g(s_i(\boldsymbol{\theta}_i), s_{-i}^*(\boldsymbol{\theta}_{-i})), \boldsymbol{\theta}_i) \tag{57}$$

holds $\forall i \in \mathcal{N}$, $\forall s_i \neq s_i^*$, $\forall \boldsymbol{\theta}_i$, and $\forall \boldsymbol{\theta}_{-i}$.

In other words, user i will not deviate from s_i^* when other users are taking s_{-i}^*. It is shown that an ex-post Nash equilibrium depends on the common knowledge that (other) users are rational. In a faithful distributed mechanism, a user will follow the intended strategy if no unilateral deviation can increase its utility. Finally, we derive the following theorem.

Theorem 3: The proposed distributed mechanism (Section IV-B) is a faithful distributed implementation of energy allocation (38) and payment (39).

Proof: See Appendix C. □

V. PERFORMANCE EVALUATION

A. SIMULATION SETTING

Consider an energy hub serving 50 house buildings. Each house acts as an energy user in the auction, buying electricity, heating energy, and cooling energy from the hub manager to meet its energy demand on the next day. The length of one time slot is set to one hour, i.e., $\mathcal{T} = \{1, \ldots, 24\}$. Residential building hourly load data in the U.S. [30] is used to characterize the users' demands. Fig. 2 shows a sample of load profiles of a house in a day. Let $x_{i,\epsilon,t}^{\text{data}}$ be the load point given by the dataset. Then, we set $x_{i,\epsilon}^{\text{day}} = \sum_{t\in\mathcal{T}} x_{i,\epsilon,t}^{\text{data}}$, $x_{i,\epsilon,t}^{\min} = (1-\gamma)x_{i,\epsilon,t}^{\text{data}}$ and $x_{i,\epsilon,t}^{\max} = (1+\gamma)x_{i,\epsilon,t}^{\text{data}}$, where γ describes the flexibility of load. We set $\gamma = 0.1$ initially and will vary it in simulation. A user's satisfaction parameter $\beta_{i,\epsilon,t}$ is uniformly distributed over [80, 100]. The electricity and gas prices of outer energy networks are set according to [2]. The gas price is 16 Mu/kWh and unchanged in a day. Mu is a monetary unit [2]. The electricity prices are determined by time-of-use pricing, shown in Fig. 3. Consider three different scenarios in which the configurations of the energy hub are

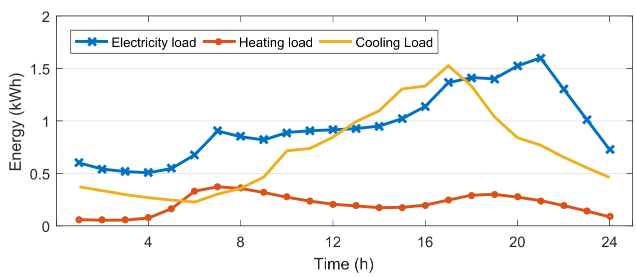


FIGURE 2. Load profiles of a house [30].

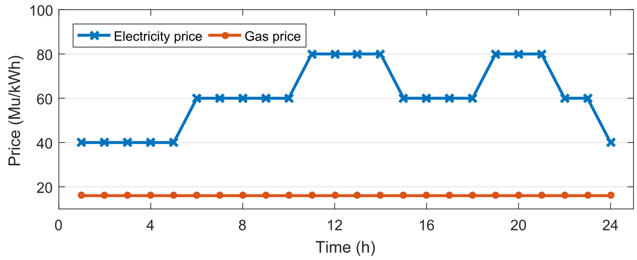


FIGURE 3. Electricity and gas buy prices [2].

TABLE 2. Energy hub configurations.

	CHP	EHP	AC	EES	TES	SC	PV
Scenario 1	✓	✓	✓	✓	✓	✓	✓
Scenario 2	✓	✓	✓	✓	✓	✗	✗
Scenario 3	✓	✓	✓	✗	✗	✗	✗

given in Table 2. The parameters of devices in the energy hub are set based on [3] and [16]. The solar radiation data is provided by U.S. solar radiation database [31].

B. SIMULATION RESULTS

1) MULTI-ENERGY SCHEDULING

Focus on scenario 1 at first. Fig. 4 shows the energy scheduling result of the proposed mechanism. As the gas price is much lower than the electricity prices, the CHP unit is used a lot to produce electricity and heat. In the daylight, the PV and SC provide solar power and solar heat, respectively. Around 12 h and 20 h, electricity prices are high, so the EES is scheduled to discharge to support the electricity load. Note that the electricity load in Fig. 4(a) includes the houses, EHP, and EES (when charging), and the heating load in Fig. 4(b) includes the houses, AC, and TES (when charging). At 17–18 h, the TES is discharged to release heating energy that is used by the AC to produce cooling energy during the cooling load peak. The cooling load in Fig. 4(c) is only contributed by the houses. As observed, the proposed auction mechanism is able to balance the energy supply and demand, and flexibly schedule the energy devices to optimize the social welfare.

2) LOAD FLEXIBILITY

Fig. 5 shows social welfares changing with load flexibility γ. Increasing γ means that the controllable range of user's loads at each time is increased. Hence, the growth of γ enlarges the feasible region of the social welfare maximization problem and in turn improves the social welfare. Observe that scenario 1 always achieves the highest welfares because it has a full configuration of the energy hub. Scenario 3 has no storage or renewable device, which largely reduces the elasticity of energy scheduling and thus leads to lower social welfares.

3) INCENTIVE COMPATIBILITY

Here, the incentive compatibility of the auction mechanism is verified. As discussed in the proof of Theorem 1, it is

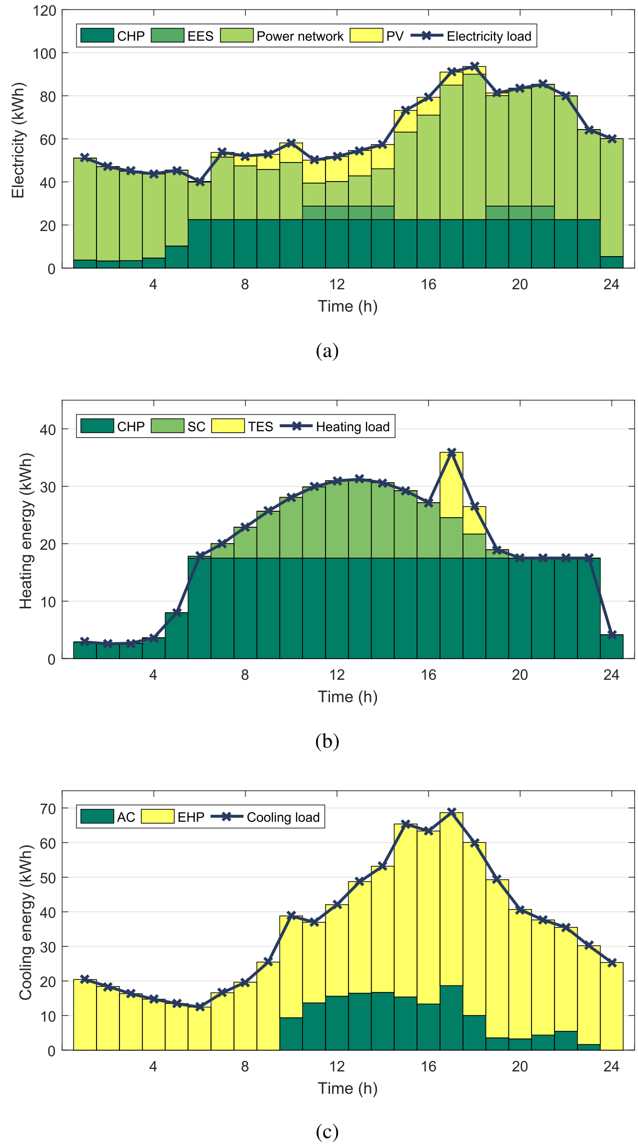


FIGURE 4. Energy production and consumption of the energy hub in scenario 1.

straightforward to prove that a user will truthfully report $\boldsymbol{x}_i^{\min}$, $\boldsymbol{x}_i^{\max}$, and $\boldsymbol{x}_i^{\text{day}}$. Also, any act of deviation in the distributed computation process may lead to deviation from the optimal social welfare and even the failure in convergence. Thus, it is clear that a rational user will faithfully complete the assigned computation. Here, we mainly analyze how the reported $\hat{\boldsymbol{\beta}}_i$ influences the utility of user i. In each of the three scenarios, select a user, fix its $\boldsymbol{\beta}_i$, and then make its reported $\hat{\boldsymbol{\beta}}_i$ deviate from $\boldsymbol{\beta}_i$. Each deviation is simulated in one trial, in which other users' $\boldsymbol{\beta}_{-i}$ is uniformly distributed over [80, 100], and $\hat{\boldsymbol{\beta}}_{-i} = \boldsymbol{\beta}_{-i}$. We repeat each trial 200 times and attain the probabilities of the selected users getting maximum utilities, which is shown in Fig. 6. The deviation of x axis is defined as $\sum_{\epsilon \in \mathcal{E}} \sum_{t \in \mathcal{T}} (\hat{\beta}_{i,\epsilon,t} - \beta_{i,\epsilon,t})/\beta_{i,\epsilon,t}$. As illustrated, the users have a 100% chance of getting maximum utilities only when they truthfully report $\boldsymbol{\beta}_i$, which verifies the incentive compatibility of the mechanism.

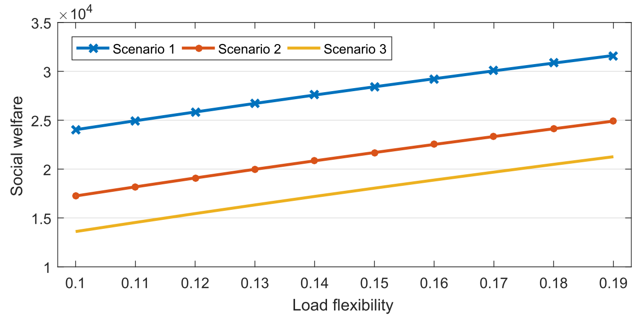


FIGURE 5. Social welfare versus load flexibility γ.

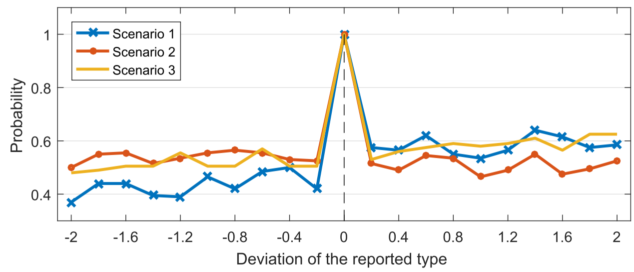


FIGURE 6. Probability of a user getting a maximum utility.

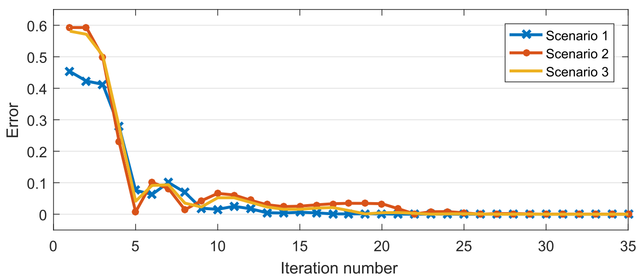


FIGURE 7. Convergence of Algorithm 1.

4) CONVERGENCE

Fig. 7 illustrates the convergence performance of Algorithm 1, in which we set $q = 0.01$. The error of y axis is defined as $(SW^{[\tau]} - SW^*)/SW^*$, where SW^* is the optimal social welfare obtained by solving problem $\mathscr{M}$ directly in a centralized manner, and $SW^{[\tau]}$ is the social welfare at the τth iteration of Algorithm 1. As shown in Fig. 7, the algorithm is able to converge to the optimum in a small number of iterations.

VI. CONCLUSION

This paper proposes a distributed auction mechanism for multi-energy scheduling of an energy hub. At first, we design a centralized auction, in which building energy users buy electricity, heating energy, and cooling energy from the energy hub manager. The buyers submit their demand information (i.e., types) to the hub manager. Given the reported types, the hub allocates energy to the users according to the optimal solution to the social welfare maximization problem. Users pay for the energy according to the VCG-based payment rule. With these energy allocation and payment rules, the proposed centralized mechanism can achieve incentive compatibility in a dominant-strategy equilibrium. Then, we develop a distributed implementation of the auction, in which the auction's computation is partitioned and assigned to users. Dual consensus ADMM is employed to enable the hub and users to jointly solve the social welfare maximization problem in a distributed manner. The proposed distributed mechanism is carefully designed so that users will faithfully complete the assigned computation, producing the same outcome as the centralized mechanism in an ex-post Nash equilibrium. In the simulation, we employ building load data and solar data to evaluate energy scheduling performance and verify the incentive compatibility of the mechanism.

APPENDIX A
PROOF OF THEOREM 1

A reported type contains 4 components: satisfaction parameter $\boldsymbol{\beta}_i$, lower load bound $\boldsymbol{x}_i^{\text{mim}}$, upper load bound $\boldsymbol{x}_i^{\text{max}}$, and daily load $\boldsymbol{x}_i^{\text{day}}$. To show that the proposed auction mechanism is strategy-proof, we need to prove that each user will truthfully report each of the type components.

A. SATISFACTION PARAMETER β_i

Let $\{\boldsymbol{x}^*, \boldsymbol{y}^*\}$ denote the optimal solution to problem (38) when user i truthfully reports $\boldsymbol{\theta}_i$; let $\{\boldsymbol{x}', \boldsymbol{y}'\}$ denote the optimal solution to problem (38) when user i reports $\hat{\boldsymbol{\theta}}_i$ in which only $\hat{\boldsymbol{\beta}}_i$ is misreported. Accordingly, we have

$$v_i(\boldsymbol{x}_i^*)+\sum_{j\neq i}\hat{v}_j(\boldsymbol{x}_j^*)-c(\boldsymbol{y}^*)\geq v_i(\boldsymbol{x}_i')+\sum_{j\neq i}\hat{v}_j(\boldsymbol{x}_j')-c(\boldsymbol{y}'), \quad (58)$$

where $\hat{v}_j(\cdot) = v_j(\hat{\boldsymbol{\beta}}_j, \cdot)$. Adding $-\sum_{j\neq i}\hat{v}_j(\boldsymbol{x}_j^{-i})$ to the both sides of (58), according to (39), we have

$$v_i(\boldsymbol{x}_i^*) - p_i(\boldsymbol{\theta}_i, \hat{\boldsymbol{\theta}}_{-i}) \geq v_i(\boldsymbol{x}_i') - p_i(\hat{\boldsymbol{\theta}}_i, \hat{\boldsymbol{\theta}}_{-i}), \quad (59)$$

which leads to (42), indicating that truthfully reporting $\boldsymbol{\beta}_i$ can attain a higher utility than misreporting. Therefore, user i will truthfully reveal $\boldsymbol{\beta}_i$.

B. LOWER LOAD BOUND x_i^{mim}

To ensure that the energy consumption minimum is satisfied, user i will not understate $\boldsymbol{x}_i^{\text{mim}}$. To prove that a user will not overstate $\boldsymbol{x}_i^{\text{mim}}$ neither, according to (39), we write the utility of user i as

$$u_i(f(\hat{\boldsymbol{\theta}}), \boldsymbol{\theta}_i)=v_i(\boldsymbol{x}_i^*)+\sum_{j\neq i}v_j(\boldsymbol{x}_j^*)-c(\boldsymbol{y}^*)-\sum_{j\neq i}v_j(\boldsymbol{x}_j^{-i}). \quad (60)$$

Notice that we use v_j instead of $\hat{v}_j$ in (60) since it has been proven that every user chooses to truthfully reveal $\boldsymbol{\beta}_j$. User i cannot influence the last term in (60) by changing $\hat{\boldsymbol{\theta}}_i$. Thus, a user aiming at utility maximization will try to maximize the first three terms, i.e, the social welfare. Overstating $\boldsymbol{x}_i^{\text{mim}}$ narrows the feasible range of $\boldsymbol{x}_i$, in turn limits the growth of the social welfare. Therefore, user i will not overstate $\boldsymbol{x}_i^{\text{mim}}$.

C. UPPER LOAD BOUND x_i^{max}

Similarly, understating $\boldsymbol{x}_i^{\max}$ limits the growth of the social welfare, so user i will not understate $\boldsymbol{x}_i^{\max}$. Overstating $\boldsymbol{x}_i^{\max}$ may lead to the allocated energy larger than the actual user demand. The hub manager can observe this situation on the operating day and then penalizes the user. To avoid the penalty of energy surplus, user i will not overstate $\boldsymbol{x}_i^{\max}$ neither.

D. DAILY LOAD x_i^{day}

To avoid the shortage of daily energy consumption, $\boldsymbol{x}_i^{\text{day}}$ will not be understated. To avoid the penalty of energy surplus, $\boldsymbol{x}_i^{\text{day}}$ will not be overstated neither.

APPENDIX B
PROOF OF THEOREM 2

The total payment that the hub manager receives can be denoted by

$$\sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} p_{i,t}(\boldsymbol{\theta})=\sum_{i\in\mathcal{N}}\sum_{j\neq i} v_j(\boldsymbol{x}_j^{-i})-\sum_{i\in\mathcal{N}}\sum_{j\neq i} v_j(\boldsymbol{x}_j^{*})+N\cdot c(\boldsymbol{y}^{*}). \tag{61}$$

Since $\boldsymbol{x}^{-i}$ is the optimal solution to problem (40), we have $\sum_{j\neq i} v_j(\boldsymbol{x}_j^{-i}) \geq \sum_{j\neq i} v_j(\boldsymbol{x}_j^{*})$. Further, according to the definition of the hub cost function, $c(\boldsymbol{y}^*) \geq 0$ holds. Therefore, we have

$$\sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} p_{i,t}(\boldsymbol{\theta}) \geq N\cdot c(\boldsymbol{y}^{*}) \geq c(\boldsymbol{y}^{*}), \tag{62}$$

which completes the proof.

APPENDIX C
PROOF OF THEOREM 3

The design of the distributed mechanism follows *static-partitioning* principle [25], which is a distributed algorithm design principle for VCG mechanism. The proposed distributed mechanism has the following three properties:

- Users only communicate with the hub manager.
- The computation of user i does not contribute to solving problem $\mathscr{M}_{-i}$.
- The computation of a user does not rely on the computation results from any other user.

With these properties, a rational user will follow the intended strategy, i.e., it will faithfully complete the following actions: 1) reporting $\hat{\boldsymbol{\theta}}_i$, 2) solving $\mathscr{M}_i$, 3) sending $\boldsymbol{x}_i^{\star}$, 4) updating $\boldsymbol{\mu}^{[\tau]}$, $\boldsymbol{y}^{[\tau]}$, and $\boldsymbol{\lambda}^{[\tau]}$, 5) sending $\boldsymbol{\lambda}^{[\tau]}$, and 6) sending $\boldsymbol{x}_i^{*}$.

Specifically, according to payment rule (39), the payment made by user i is related to $\sum_{j\neq i} v_j(\boldsymbol{x}_j^{-i})$, so a user would always want to tamper other users' $\boldsymbol{\theta}_{-i}$ and $\boldsymbol{x}^{-i}$ to lower its payment if this is possible. The first property ensures that users do not communicate with each other, which avoids the tampering. The second property makes sure that user i cannot influence the term $\sum_{j\neq i} v_j(\boldsymbol{x}_j^{-i})$ in the payment rule, so a rational user will faithfully complete actions 2) and 3). The third property prevents rational users' computation from being affected by potentially irrational users.

Moreover, as analyzed in the proof of Theorem 1, the centralized mechanism implements truth-revelation in a dominant-strategy equilibrium, which implies that truth-revelation can also be achieved in the ex-post Nash implementation. Thus, users will truthfully take action 1) as well. The proof of Theorem 1 also shows that a user can maximize its utility only by maximizing the social welfare. Therefore, user i will faithfully perform actions 4)–6) as well, since this is the only way to maximize the social welfare when other users are following the intended strategy. Once all users follow the intended strategy, the distributed mechanism yields the same outcome of the centralized mechanism.

REFERENCES

[1] (2018). *International Energy Outlook 2017*. [Online]. Available: https://www.eia.gov/outlooks/ieo/ieo_tables.php

[2] I. G. Moghaddam, M. Saniei, and E. Mashhour, "A comprehensive model for self-scheduling an energy hub to supply cooling, heating and electrical demands of a building," *Energy*, vol. 94, pp. 157–170, Jan. 2016.

[3] E. Fabrizio, V. Corrado, and M. Filippi, "A model to design and optimize multi-energy systems in buildings at the design concept stage," *Renew. Energy*, vol. 35, no. 3, pp. 644–655, 2010.

[4] E. M. Cesena and P. Mancarella, "Energy systems integration in smart districts: Robust optimisation of multi-energy flows in integrated electricity, heat and gas networks," *IEEE Trans. Smart Grid*, to be published, doi: 10.1109/TSG.2018.2828146.

[5] E. Dall'Anese, P. Mancarella, and A. Monti, "Unlocking flexibility: Integrated optimization and control of multienergy systems," *IEEE Power Energy Mag.*, vol. 15, no. 1, pp. 43–52, Jan./Feb. 2017.

[6] W. Zhong, K. Xie, Y. Liu, C. Yang, and S. Xie, "Auction mechanisms for energy trading in multi-energy systems," *IEEE Trans. Ind. Informat.*, vol. 14, no. 4, pp. 1511–1521, Apr. 2018.

[7] T. Capuder and P. Mancarella, "Techno-economic and environmental modelling and optimization of flexible distributed multi-generation options," *Energy*, vol. 71, pp. 516–533, Jul. 2014.

[8] M. Alipour, K. Zare, and M. Abapour, "Minlp probabilistic scheduling model for demand response programs integrated energy hubs," *IEEE Trans. Ind. Informat.*, vol. 14, no. 1, pp. 79–88, Jan. 2018.

[9] P. Mancarella and G. Chicco, "Real-time demand response from energy shifting in distributed multi-generation," *IEEE Trans. Smart Grid*, vol. 4, no. 4, pp. 1928–1938, Dec. 2013.

[10] A. Parisio, C. Del Vecchio, and A. Vaccaro, "A robust optimization approach to energy hub management," *Int. J. Elect. Power Energy Syst.*, vol. 42, no. 1, pp. 98–104, 2012.

[11] A. Najafi, H. Falaghi, J. Contreras, and M. Ramezani, "A stochastic bilevel model for the energy hub manager problem," *IEEE Trans. Smart Grid*, vol. 8, no. 5, pp. 2394–2404, Sep. 2017.

[12] M. Yazdani-Damavandi, M. P. Moghaddam, M.-R. Haghifam, M. Shafie-khah, and J. P. S. Catalão, "Modeling operational behavior of plug-in electric vehicles' parking lot in multienergy systems," *IEEE Trans. Smart Grid*, vol. 7, no. 1, pp. 124–135, Jan. 2016.

[13] A. Shabanpour-Haghighi and A. R. Seifi, "Energy flow optimization in multicarrier systems," *IEEE Trans. Ind. Informat.*, vol. 11, no. 5, pp. 1067–1077, Oct. 2015.

[14] X. Zhang, M. Shahidehpour, A. Alabdulwahab, and A. Abusorrah, "Optimal expansion planning of energy hub with multiple energy infrastructures," *IEEE Trans. Smart Grid*, vol. 6, no. 5, pp. 2302–2311, Sep. 2015.

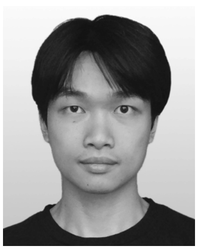
[15] S. Bahrami and A. Sheikhi, "From demand response in smart grid toward integrated demand response in smart energy hub," *IEEE Trans. Smart Grid*, vol. 7, no. 2, pp. 650–658, Mar. 2016.

[16] H. Zhang, Y. Li, D. W. Gao, and J. Zhou, "Distributed optimal energy management for energy Internet," *IEEE Trans Ind. Informat.*, vol. 13, no. 6, pp. 3081–3097, Dec. 2017.

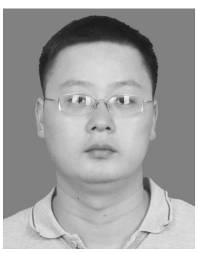
[17] E. A. M. Ceseña and P. Mancarella, "Distribution network support from multi-energy demand side response in smart districts," in *Proc. Innov. Smart Grid Technol.–Asia (ISGT-Asia)*, Nov./Dec. 2016, pp. 753–758.

[18] R. Yu, W. Zhong, S. Xie, C. Yuen, S. Gjessing, and Y. Zhang, "Balancing power demand through EV mobility in vehicle-to-grid mobile energy networks," *IEEE Trans. Ind. Informat*, vol. 12, no. 1, pp. 79–90, Feb. 2016.

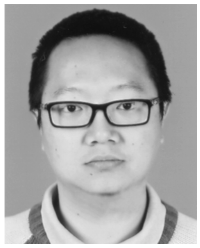
[19] S. Xie, W. Zhong, K. Xie, R. Yu, and Y. Zhang, "Fair energy scheduling for vehicle-to-grid networks using adaptive dynamic programming," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 27, no. 8, pp. 1697–1707, Aug. 2016.
[20] J. Kang, R. Yu, X. Huang, S. Maharjan, Y. Zhang, and E. Hossain, "Enabling localized peer-to-peer electricity trading among plug-in hybrid electric vehicles using consortium blockchains," *IEEE Trans Ind. Informat.*, vol. 13, no. 6, pp. 3154–3164, Dec. 2017.
[21] N. Nisan and A. Ronen, "Computationally feasible VCG mechanisms," *J. Artif. Intell. Res.*, vol. 29, pp. 19–47, Mar. 2007.
[22] T. H. Chang, M. Hong, and X. Wang, "Multi-agent distributed optimization via inexact consensus ADMM," *IEEE Trans. Signal Process.*, vol. 63, no. 2, pp. 482–497, Jan. 2015.
[23] T.-H. Chang, "A randomized dual consensus ADMM method for multi-agent distributed optimization," in *Proc. Int. Conf. Acoust., Speech Signal Process. (ICASSP)*, Apr. 2015, pp. 3541–3545.
[24] M. Yazdani-Damavandi, N. Neyestani, M. Shafie-khah, J. Contreras, and J. P. Catalão, "Strategic behavior of multi-energy players in electricity markets as aggregators of demand side resources using a bi-level approach," *IEEE Trans. Power Syst.*, vol. 33, no. 1, pp. 397–411, Jan. 2018.
[25] D. C. Parkes and J. Shneidman, "Distributed implementations of Vickrey-Clarke-groves mechanisms," in *Proc. 3rd Int. Joint Conf. Auton. Agents Multiagent Syst.*, vol. 1, 2004, pp. 261–268.
[26] P. Samadi, H. Mohsenian-Rad, R. Schober, and V. W. S. Wong, "Advanced demand side management for the future smart grid using mechanism design," *IEEE Trans. Smart Grid*, vol. 3, no. 3, pp. 1170–1180, Sep. 2012.
[27] W. Zhong, K. Xie, Y. Liu, C. Yang, and S. Xie, "Topology-aware vehicle-to-grid energy trading for active distribution systems," *IEEE Trans. Smart Grid*, to be published, doi: 10.1109/TSG.2018.2789940.
[28] J. Shneidman and D. C. Parkes, "Overcoming rational manipulation in mechanism implementations," Citeseer, Tech. Rep., 2004. [Online]. Available: http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.203.1414
[29] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Found. Trends Mach. Learn.*, vol. 3, no. 1, pp. 1–122, Jan. 2011.
[30] (2018). *Commercial and Residential Hourly Load Profiles for all TMY3 Locations in the United States*. [Online]. Available: https://openei.org/doe-opendata/dataset
[31] (2018). *National Solar Radiation Database (NSRDB)—NREL*. [Online]. Available: https://nsrdb.nrel.gov/current-version



WEIFENG ZHONG received the B.Eng. and M.Eng. degrees from the Guangdong University of Technology, Guangzhou, China, in 2013 and 2016, respectively, where he is currently pursuing the Ph.D. degree in control science and engineering. In 2016, he was a Visiting Student with The Hong Kong University of Science and Technology for six months. His research interest lies in the energy management of vehicle-to-grid, power system, and multi-energy system.



CHAO YANG received the Ph.D. degree in signal and information processing from the South China University of Technology, Guangzhou, China, in 2013. From 2014 to 2016, he was a Research Associate with the Department of Computing, The Hong Kong Polytechnic University. He is currently with the School of Automation, Guangdong University of Technology. His research interests focus on VANETs, smart grid, and edge computing.



KAN XIE received the Ph.D. degree in control science and engineering from the Guangdong University of Technology, Guangzhou, China, in 2017. He is currently a Post-Doctoral Fellow with the Institute of Intelligent Information Processing, Guangdong University of Technology. His current research interests include machine learning, non-negative signal processing, blind signal processing, and smart grid.

SHENGLI XIE (M'01–SM'02) received the M.S. degree in mathematics from the Central China Normal University, Wuhan, China, in 1992, and the Ph.D. degree in control theory and applications from the South China University of Technology, Guangzhou, China, in 1997. He is currently a Full Professor and the Head of the Institute of Intelligent Information Processing, Guangdong University of Technology, Guangzhou. He has authored or co-authored two books and over 150 scientific papers in journals and conference proceedings. His research interests include wireless networks, automatic control, and blind signal processing. He was a recipient of the Second Prize of the China's State Natural Science Award in 2009 for his research on blind source separation and identification.

YAN ZHANG (M'05–SM'10) received the Ph.D. degree from the School of Electrical and Electronics Engineering, Nanyang Technological University, Singapore. He is currently a Full Professor with the Department of Informatics, University of Oslo, Oslo, Norway. His current research interests include next-generation wireless networks leading to 5G, green, and secure cyber-physical systems, such as smart grid, healthcare, and transport. He is an IEEE Vehicular Technology Society (VTS) Distinguished Lecturer. He is also a Senior Member of the IEEE ComSoc, the IEEE CS, the IEEE PES, and the IEEE VTS. He is a fellow of the IET. He is an Associate Technical Editor of the *IEEE Communications Magazine*, an Editor of the *IEEE Network Magazine*, an Editor of the IEEE Transactions on Green Communications and Networking, an Editor of the IEEE Communications Surveys and Tutorials, an Editor of the IEEE Internet of Things Journal, and an Associate Editor of the IEEE Access. He serves as the chair for a number of conferences, including the IEEE GLOBECOM 2017, the IEEE PIMRC 2016, the IEEE CloudCom 2016, the IEEE ICCC 2016, the IEEE CCNC 2016, the WCSP 2016, the IEEE SmartGridComm 2015, and the IEEE CloudCom 2015. He serves as a TPC member for numerous international conferences, including the IEEE INFOCOM, the IEEE ICC, the IEEE GLOBECOM, and the IEEE WCNC.