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Robust Adaptive Sliding Mode Observer Design for T-S Fuzzy Descriptor Systems With Time-Varying Delay

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ABSTRACT This paper addresses the problems of robust adaptive sliding mode observer (SMO) design and SMO-based sliding mode control (SMC) for T-S fuzzy descriptor systems with time-varying delay. Until now, there are a few results about the adaptive SMO design problem and adaptive SMO-based SMC for T-S fuzzy descriptor systems with time-varying delay. Therefore, we are motivated to study this issue. First, two integral-type sliding surfaces, which involve the SMO gain matrix, are constructed for the error system and the SMO system, respectively. Then, some delay-dependent sufficient conditions are established, such that the sliding motions are robustly admissible with $H\infty$ performance. New adaptive sliding mode controllers, which need not to use the assumption that the fast subsystem of descriptor system is observable, are synthesized for the error system and the SMO system, such that the reachability conditions can be guaranteed. In addition, the adaptive control strategy is applied to estimate the unknown parameters. Finally, simulation examples are discussed to show the effectiveness of our approach.

INDEX TERMS Adaptive SMO, integral-type sliding surface, T-S fuzzy descriptor systems, time-varying delay.

I. INTRODUCTION

Descriptor system [1] is also referred to as singular system, generalized state-space system, differential-algebraic system or semistate system, which can be expressed by a set of differential and algebraic equations in mathematics [2]. Descriptor model can be used in a large class of fields, such as, electrical circuits, mechanical systems and moving robots. It can describe the behavior and maintain the physical characteristics of a lot of physical systems better than the standard state-space model. Over the past few years, a lot of research results about descriptor system have been reported, such as, dissipativity analysis [3], [4], stability and stabilization problems [5]–[7], SMC [8], [9] and observer design [10]–[12].

It is generally known that T-S fuzzy model [13], [14] has been proved to be an effective strategy for the control of nonlinear systems. We can describe a nonlinear system by a family of local linear models. Then through the use of fuzzy blending, the overall T-S fuzzy model can be obtained. Moreover, in order to extend the T-S fuzzy model to a more general case, Taniguchi put forward the T-S fuzzy descriptor model, and it has provided an effective way of controlling nonlinear descriptor systems. During the past few years, a lot of results related to T-S fuzzy models have been reported. For example, stability and stabilization [15], [16], sliding mode control [17], fault tolerant control [18] and so on. On the other hand, time-delay occurs in many physical, industrial and engineering systems [19], [20]. It has become a hot topic since it is often the major source of instability and usually unavoidable. Recently, there are also many systems can be described by T-S fuzzy descriptor models with time-delay [5], [8].

This paper addresses the problem of robust adaptive SMO design for T-S fuzzy descriptor systems. In fact, SMO is based on SMC approach [21]–[24]. It has a nonlinear input which is designed to ensure that the state trajectories of the error system can be driven onto the sliding surface in finite time. As we all know, SMC [25]–[28] has been proven to be an effective robust control scheme for systems with uncertainties, nonlinearities and disturbances. It has many attractive features, such as fast response, good transient and strong robustness. Based on the characteristics of SMC Strategy, the authors in [29]–[33] developed adaptive SMC methods to estimate the unknown parameters. In [34] and [35], the SMC strategies were introduced for extracting maximum wind power. In addition, the essence of the SMO is to design a SMC strategy for the error system. Therefore, according to the advantages of SMC, the SMO has better robustness to uncertainties, nonlinearities and disturbances. Over the past few decades, a lot of research results related to SMO have been extensively reported, such as, SMO design for stochastic systems [9], SMO design for T-S fuzzy systems [36], SMO design for normal systems [37] and so on.

In this paper, we investigate the problems of robust adaptive SMO design and SMO based SMC for T-S fuzzy descriptor systems with time-varying delay. There are a lot of research results, which related to SMC or SMO design, have been reported. Therefore, it is necessary to point out the differences between our work and the existing works. In [17] and [38]–[40] , the authors developed SMC strategies for a class of normal or descriptor Markovian jump systems. In [26], [41], and [42], the authors developed SMC strategies for nonlinear descriptor systems and multi-input-multi-output discrete-time system respectively. The authors in [43]–[46] studied the SMOs for normal systems to estimate the faults, they did not developed them in descriptor systems. Furthermore, the authors in [31]–[33] considered the adaptive SMC for T-S fuzzy systems. In [47], the authors developed a robust $H\infty$ SMO for a class of uncertain nonlinear T-S fuzzy descriptor systems with time-varying delay. The assumption that the fast subsystem of descriptor system is observable is required in sliding mode controller design. In our paper, this assumption is removed, which reduces the requirements of the descriptor systems. In addition, we developed adaptive SMC strategy to estimate the unknown parameters which is also different from the result in [47]. By the way, in [19] and [48], the authors developed dynamic SMC for T-S fuzzy systems. However, dynamic SMC strategy can not be used to design fuzzy SMO. As mentioned above, there are few results about the adaptive SMO design problem for T-S fuzzy descriptor systems with time-varying delay. Therefore, we are motivated to study this issue. The main contributions of this paper are summarized below.

(1) Two integral-type sliding surfaces, which involve the SMO gain matrices L_i, are constructed for the error system and the SMO system.
(2) New adaptive sliding mode controllers, which need not to use the assumption that the fast subsystem of descriptor system is observable, are synthesized for the error system and the SMO system such that the reachability conditions can be guaranteed.
(3) The adaptive control strategy is applied to estimate the unknown parameters especially the bounds of $e(t)$ and $e(t-\tau(t))$. This is more appropriate and practical since the estimation error variables are unknown.
(4) In this paper, we also develop a SMO based SMC strategy.

This paper is organized as follows. Section II describes the T-S fuzzy descriptor model. In section III, we focus on the SMO design and admissibility analysis. In section IV, a SMO based SMC strategy is developed. Finally, simulation examples and conclusions are given in section V and section VI respectively.

Notations: In this paper, $\bar{A}^T$ and $\bar{A}^{-1}$ denote the transpose and inverse of matrix $\bar{A}$. $\|.\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix. I_n denotes the $n \times n$ identity matrix. $\bar{A} > 0$ or ($\bar{A} < 0$) means that $\bar{A}$ is symmetric and positive (negative) definite. $sym(\bar{A})$ stands for $\bar{A} + \bar{A}^T$. $*$ denotes the transposed element in the symmetric positions of a matrix.

II. PROBLEM FORMULATION

In this paper, we focus on the following T-S fuzzy descriptor system.

Fuzzy rule i: IF $\bar{\theta}_1(t)$ is $\bar{M}_{i1}$ and ... and $\bar{\theta}_g(t)$ is $\bar{M}_{ig}$, THEN

$$\begin{aligned} E\dot{x}(t) &= A_i x(t) + A_{di}x(t-\tau(t)) \\ &\quad + B(u(t) + f_i(t, x(t), x(t-\tau(t)))) \\ &\quad + (H_i + \Delta H_i(t))w(t) \\ y(t) &= Cx(t) \\ x(t) &= \tilde{\varphi}(t), \quad t \in [-\tau_M, 0] \end{aligned} \tag{1}$$

where $i = 1, 2, ..., l$, l is the number of IF-THEN rules; $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the control input vector, $f_i(t, x(t), x(t-\tau(t)))$ represent the system nonlinearities; $w(t) \in \Re^q$ is the external disturbance which is assumed to belong to $L_2[0, \infty)$ and $y(t) \in \Re^p$ is the measured output; $\bar{\theta}_j(t)$ $(j = 1, 2, ..., g)$ and $\bar{M}_{ij}$ are the premise variables and the fuzzy sets respectively; We assume that $rank(E) = r \le n$; A_i, A_{di}, B, H_i and C are known real matrices; $\tau(t)$ is the time-varying delay which satisfies $0 < \tau_m \le \tau(t) \le \tau_M$ and $\dot{\tau}(t) \le \tau_a < 1$, where τ_m, τ_M τ_a are positive constants; $\tilde{\varphi}(t)$ is the initial function on $[-\tau_M, 0]$; $\Delta H_i(t)$ represent parameter uncertainties. In this paper, we assume that $\bar{\theta}_j(t)$ are available. Then based on (1), the overall T-S fuzzy descriptor model can be expressed as follows:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{l} \bar{h}_i(\bar{\theta}(t))\{A_i x(t) + A_{di}x(t-\tau(t)) \\ &\quad + B(u(t) + f_i(t, x(t), x(t-\tau(t)))) \\ &\quad + (H_i + \Delta H_i(t))w(t)\} \\ y(t) &= Cx(t) \\ x(t) &= \tilde{\varphi}(t), t \in [-\tau_M, 0] \end{aligned} \tag{2}$$

where

$$\bar{h}_i(\bar{\theta}(t)) = \frac{\omega_i(\bar{\theta}(t))}{\sum_{i=1}^{l}\omega_i(\bar{\theta}(t))}, \quad \omega_i(\bar{\theta}(t)) = \prod_{j=1}^{g} \bar{M}_{ij}(\bar{\theta}_j(t)) \tag{3}$$

$\bar{M}_{ij}(\bar{\theta}_j(t))$ represents the membership degree of $\bar{\theta}_j(t)$ in the fuzzy set $\bar{M}_{ij}$. $\bar{h}_i(\bar{\theta}(t))$ satisfies

$$\bar{h}_i(\bar{\theta}(t)) \geq 0, \quad \sum_{i=1}^{l} \bar{h}_i(\bar{\theta}(t)) = 1 \tag{4}$$

Here, we make the following assumptions.

Assumption 1: The nonlinearities $f_i(t, x(t), x(t-\tau(t)))$ are assumed to satisfy the following condition:

$$\|f_i(t, x(t), x(t-\tau(t)))\| \leq \sigma_{1i} + \sigma_{2i}\|y(t)\| + \sigma_{3i}\|y(t-\tau(t))\| \tag{5}$$

where σ_{1i}, σ_{2i} and σ_{3i} are unknown positive constants.

Assumption 2: $\Delta H_i(t)$ are time-varying matrix representing norm-bounded parameter uncertainty and are assumed to be of the following form:

$$\Delta H_i(t) = M_i F(t) N_{wi} \tag{6}$$

where M_i and N_{wi} are known real constant matrices. $F(t) \in \Re^{f_1 \times f_2}$ is unknown matrix function satisfying $F^T(t)F(t) \leq I$.

Assumption 3: The external disturbance $w(t)$ is bounded and satisfies

$$\|w(t)\| \leq \vartheta \tag{7}$$

where ϑ is an unknown positive real constant.

In order to facilitate the development of the subsequent results, we introduce some definitions and lemmas. Firstly, consider the following descriptor system

$$E\dot{x}(t) = Ax(t) + A_d x(t-\tau(t))$$
$$x(t) = \varphi(t), t \in [-\tau_M, 0] \tag{8}$$

Definition 1 [1]:

(1) The system (8) is said to be regular if $\det(sE - A)$ is not identically zero.
(2) The system (8) is said to be impulse free if $\deg(\det(sE - A)) = rank(E)$.
(3) The system (8) is said to be admissible if it is regular, impulse-free and stable.

Definition 2 [49]: The system (8) is said to be regular and impulse free if the pair (E, A) is regular and impulse free. The system (8) is said to be asymptotically stable if for any $\bar{\varepsilon} > 0$ there exists a scalar $\delta(\bar{\varepsilon}) > 0$ such that, for any compatible initial condition $\chi(t)$ with $\sup_{-\tau_M < t \leq 0} \|\chi(t)\| < \delta(\bar{\varepsilon})$, the solution $x(t)$ of the system (8) satisfies $\|x(t)\| < \bar{\varepsilon}$ for $t \geq 0$. Furthermore $x(t) \to 0$, $t \to \infty$.

The system (8) may have an impulsive solution, however, the regularity and non-impulse of (E, A) guarantee the existence and uniqueness of impulse free solution to (8) on $[0, \infty)$.

Lemma 1 [20]: For any constant matrix $H > 0$, any scalar τ_M and τ_m with $0 < \tau_m < \tau_M$, and vector function $x(t) : [-\tau_M, -\tau_m] \to \Re^n$ such that the integrals concerned are well defined, then the following holds

$$-(\tau_M - \tau_m)\int_{t-\tau_M}^{t-\tau_m} x^T(z)Hx(z)dz \leq -\int_{t-\tau_M}^{t-\tau_m} x^T(z)dzH\int_{t-\tau_M}^{t-\tau_m} x(z)dz \tag{9}$$

Lemma 2 [50]: Given matrices $\tilde{M}$, $\tilde{N}$ and $\bar{Q}$ of appropriate dimensions and with $\bar{Q}$ symmetrical

$$\bar{Q} + \tilde{M}\Delta\tilde{N} + \tilde{N}^T\Delta^T\tilde{M}^T < 0$$

for any Δ satisfying $\Delta^T\Delta \leq I$, if and only if there exists a scalar $\xi > 0$ such that

$$\bar{Q} + \xi\tilde{M}\tilde{M}^T + \xi^{-1}\tilde{N}^T\tilde{N} < 0 \tag{10}$$

Lemma 3 [51]: For a symmetric matrix $P \in \Re^{n\times n}$ and a singular matrix $E = E_L E_R^T$. $E_L \in \Re^{n\times r}$, $E_R \in \Re^{n\times r}$ are full column rank. There exist nonsingular matrix $X \in \Re^{(n-r)\times(n-r)}$ such that $PE + \bar{U}^T X \bar{V}^T$ is nonsingular where $\bar{U}^T \in \Re^{n\times(n-r)}$, $\bar{V} \in \Re^{n\times(n-r)}$ have full column rank and satisfying $E^T\bar{U}^T = 0$, $E\bar{V} = 0$ respectively. Then we can get the following equation

$$(PE + \bar{U}^T X \bar{V}^T)^{-1} = \bar{P}E^T + \bar{V}\bar{X}\bar{U}$$

where $\bar{P}$ is a symmetric matrix and $\bar{X}$ is a nonsingular matrix, and

$$\bar{X} = (\bar{V}^T\bar{V})^{-1}X^{-1}(UU^T)^{-1}, \quad E_R^T\bar{P}E_R = (E_L^T P E_L)^{-1}$$

III. SLIDING MODE OBSERVER DESIGN

In this section, a SMO design method is proposed for the T-S fuzzy descriptor system, which involves four steps. The first one is to construct a SMO for system(2). In the second, a novel sliding surface is designed for the error system. Thirdly, sufficient conditions are proposed to ensure the admissibility of the sliding mode dynamic. Finally, a SMC law is synthesized such that the reachability condition can be guaranteed.

A. CONSTRUCTION OF SLIDING MODE OBSERVER

In this paper, we will consider the following sliding mode observer:

Fuzzy rule i: IF $\bar{\theta}_1(t)$ is $\bar{M}_{i1}$ and ... and $\bar{\theta}_g(t)$ is $\bar{M}_{ig}$, THEN

$$E\dot{\hat{x}}(t) = A_i\hat{x}(t) + A_{di}\hat{x}(t-\tau(t)) + Bu(t) + L_i(y(t) - \hat{y}(t)) + Bv(t)$$
$$\hat{y}(t) = C\hat{x}(t)$$
$$\hat{x}(t) = \tilde{\phi}(t), \quad t \in [-\tau_M, 0] \tag{11}$$

where $\hat{x}(t)$ is the state estimation of $x(t)$, $\hat{y}(t)$ is the output of the observer, $L_i \in \Re^{n\times p}$ are the SMO gain to be designed, $\tilde{\phi}(t)$ is the initial function and $v(t)$ is the nonlinear input. By fuzzy blending, the overall T-S fuzzy descriptor SMO system can be expressed as follows:

$$\begin{aligned} E\dot{\hat{x}}(t) &= \sum_{i=1}^{l} \bar{h}_i(\bar{\theta}(t))\{A_i\hat{x}(t) + A_{di}\hat{x}(t-\tau(t)) + Bu(t) \\ &\quad + L_i(y(t) - \hat{y}(t)) + Bv(t)\} \\ \hat{y}(t) &= C\hat{x}(t) \\ \hat{x}(t) &= \tilde{\phi}(t), \quad t \in [-\tau_M, 0] \end{aligned} \tag{12}$$

We define $e(t) = x(t)-\hat{x}(t)$ and $e_y(t) = y(t)-\hat{y}(t) = Ce(t)$, which represent the estimation error and the output error respectively. Then the error system can be obtained.

$$\begin{aligned} E\dot{e}(t) &= \sum_{i=1}^{l} \bar{h}_i(\bar{\theta}(t))\{(A_i - L_iC)e(t) + A_{di}e(t-\tau(t)) \\ &\quad + Bf_i(t, x(t), x(t-\tau(t))) - Bv(t) \\ &\quad + (H_i + \Delta H_i(t))w(t)\} \end{aligned} \tag{13}$$

B. SLIDING SURFACE DESIGN FOR THE ERROR SYSTEM

Here, the following integral-type sliding surface is constructed for the error system.

$$s(t) = \bar{\mathcal{G}}Ee(t) + \bar{\mathcal{G}}\int_0^t \sum_{i=1}^{l} \bar{h}_i(\bar{\theta}(z))(L_iCe(z))dz \tag{14}$$

where $\bar{\mathcal{G}} \in \Re^{m\times n}$ is known matrix satisfying $\det(\bar{\mathcal{G}}B) \neq 0$ and $rank\begin{bmatrix}\bar{\mathcal{G}}E \\ C\end{bmatrix} = rank(C)$.

Remark 1: By setting the initial value of the integrator, the initial state of the system is on the sliding surface from the beginning such that the robustness and fast response of the system can be guaranteed. In addition, for the convenience of calculating, we add $L_iCe(t)$ to $s(t)$. Following these reasons, we design the integral-type sliding surface as (14).

Remark 2: $rank\begin{bmatrix}\bar{\mathcal{G}}E \\ C\end{bmatrix} = rank(C)$ is provided to guarantee that there exists a matrix $\mathcal{S}$ such that $\bar{\mathcal{G}}E = \mathcal{S}C$ [9]. In practical applications, the state variables are unknown. In other words, we can not use the estimation error directly. Since $e_y(t) = y(t) - \hat{y}(t) = Ce(t)$. Therefore we can use $e_y(t)$ instead of $e(t)$, such that

$$s(t) = \mathcal{S}e_y(t) + \bar{\mathcal{G}}\int_0^t \sum_{i=1}^{l} \bar{h}_i(\bar{\theta}(z))(L_ie_y(z))dz \tag{15}$$

can be used in the SMO design.

Remark 3: From (15), we can get the following one.

$$\begin{aligned} s(t) &= \mathcal{S}y(t) - \mathcal{S}C\hat{x}(t) \\ &\quad + \bar{\mathcal{G}}\int_0^t \sum_{i=1}^{l} \bar{h}_i(\bar{\theta}(z))(L_iy(z) - L_iC\hat{x}(z))dz \end{aligned}$$

Since $y(t)$ is measurable, $s(t)$ depends on $\hat{x}(t)$. By designing L_i, the solution of $\hat{x}(t)$ is unique. Therefore, the result of the solution of $s(t)$ is the only one.

According to SMC theory, when the sliding motion takes place, we have $s(t) = 0$ and $\dot{s}(t) = 0$.

$$\begin{aligned} \dot{s}(t) &= \bar{\mathcal{G}}\sum_{i=1}^{l} \bar{h}_i(\bar{\theta}(t))\{A_ie(t) + A_{di}e(t-\tau(t)) - Bv(t) \\ &\quad + Bf_i(t, x(t), x(t-\tau(t))) + (H_i + \Delta H_i(t))w(t)\} \end{aligned}$$

Then, from $\dot{s}(t) = 0$, the equivalent control law can be given by:

$$\begin{aligned} v_{eq}(t) &= (\bar{\mathcal{G}}B)^{-1}\bar{\mathcal{G}}\sum_{i=1}^{l} \bar{h}_i(\bar{\theta}(t))\{A_ie(t) + A_{di}e(t-\tau(t)) \\ &\quad + (H_i + \Delta H_i(t))w(t)\} \\ &\quad + \sum_{i=1}^{l} \bar{h}_i(\bar{\theta}(t))f_i(t, x(t), x(t-\tau(t))) \end{aligned} \tag{16}$$

By substituting the equivalent control law (16) into (13), the sliding mode dynamic can be obtained.

$$\begin{aligned} E\dot{e}(t) &= \sum_{i=1}^{l} \bar{h}_i(\bar{\theta}(t))\{(\mathcal{R}A_i - L_iC)e(t) + \mathcal{R}A_{di}e(t-\tau(t)) \\ &\quad + \mathcal{R}(H_i + \Delta H_i(t))w(t)\} \end{aligned} \tag{17}$$

where $\mathcal{R} = I - B(\bar{\mathcal{G}}B)^{-1}\bar{\mathcal{G}}$.

C. ADMISSIBILITY ANALYSIS

In this subsection, we will discuss the admissibility of system (17) and present some delay-dependent sufficient LMI conditions.

1) NOMINAL CASE

First of all, we analyze the nominal case of (17) is admissible with $H\infty$ performance.

Theorem 1: For given positive scalars τ_m, τ_a, τ_M and $\bar{\gamma}$, the nominal case of (17) is admissible with $H\infty$ performance index γ, if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Z_1 > 0$, $Z_2 > 0$, X, such that the following LMI hold

$$\tilde{\Gamma}_i = \begin{bmatrix} \Gamma_i & \tau_m\tilde{\Gamma}_{12i}Z_1 & \tau_d\tilde{\Gamma}_{12i}Z_2 \\ * & -Z_1 & 0 \\ * & * & -Z_2 \end{bmatrix} < 0 \tag{18}$$

where

$$\Gamma_i = \begin{bmatrix} \Gamma_{11i} & 0 & \Gamma_{13i} & E^TZ_1E & \Gamma_{15i} \\ * & \Gamma_{22} & E^TZ_2E & 0 & 0 \\ * & * & \Gamma_{33} & E^TZ_2E & 0 \\ * & * & * & \Gamma_{44} & 0 \\ * & * & * & * & -\bar{\gamma}I \end{bmatrix}$$

and $\Gamma_{11i} = sym(\mathcal{P}^T(\mathcal{R}A_i - L_iC)) - E^TZ_1E + C^TC + Q_1 + Q_2 + Q_3$, $\Gamma_{13i} = \mathcal{P}^T\mathcal{R}A_{di}$, $\Gamma_{15i} = \mathcal{P}^T\mathcal{R}H_i$, $\Gamma_{22} = -Q_1 - E^TZ_2E$, $\Gamma_{33} = -(1-\tau_a)Q_2 - 2E^TZ_2E$, $\Gamma_{44} = -Q_3 - E^TZ_1E - E^TZ_2E$, $\mathcal{P} = PE + \bar{U}^TX\bar{V}^T$, $\bar{\gamma} = \gamma^2$, $\tilde{\Gamma}_{12i} = \begin{bmatrix}\mathcal{R}A_i - L_iC & 0 & \mathcal{R}A_{di} & 0 & \mathcal{R}H_i\end{bmatrix}^T$.

Proof: Firstly, we prove the regularity and the impulse-free of the the nominal case (17). From (18), it is easy to see that

$$\Gamma_{11i} = sym(\mathcal{P}^T(\mathcal{R}A_i - L_iC)) - E^TZ_1E + C^TC + (Q_1 + Q_2 + Q_3) < 0 \quad (19)$$

Since $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$ and $C^TC \geq 0$, we can get

$$sym(\mathcal{P}^T(\mathcal{R}A_i - L_iC)) - E^TZ_1E < 0 \quad (20)$$

In addition, we know that $rank(E) = r$, there must exist two invertible matrices $\mathcal{U} \in \Re^{n\times n}$ and $\mathcal{V} \in \Re^{n\times n}$ such that

$$\bar{E} = \mathcal{U}E\mathcal{V} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

Then, we define $\mathscr{A}_i = \mathcal{R}A_i - L_iC$ and

$$\mathcal{U}\mathscr{A}_i\mathcal{V} = \begin{bmatrix} \bar{\mathscr{A}}_{i,11} & \bar{\mathscr{A}}_{i,12} \\ \bar{\mathscr{A}}_{i,21} & \bar{\mathscr{A}}_{i,22} \end{bmatrix},$$

$$\mathcal{U}^{-T}P\mathcal{U}^{-1} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix}$$

$$\mathcal{U}^{-T}Z_1\mathcal{U}^{-1} = \begin{bmatrix} \bar{Z}_{1,11} & \bar{Z}_{1,12} \\ \bar{Z}_{1,21} & \bar{Z}_{1,22} \end{bmatrix}, \quad \mathcal{V}^T\bar{V} = \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix}$$

$$\bar{U}\mathcal{U}^{-1} = \bar{U}_1 \begin{bmatrix} 0 & I_{n-r} \end{bmatrix}$$

Pre- and post-multiplying (20) by $\mathcal{V}^T$ and $\mathcal{V}$, respectively, we can get the following inequality.

$$\begin{bmatrix} \bullet & \bullet \\ \bullet & \bar{V}_2X^T\bar{U}_1\bar{\mathscr{A}}_{i,22} + \bar{\mathscr{A}}_{i,22}^T\bar{U}_1^TX\bar{V}_2^T \end{bmatrix} < 0 \quad (21)$$

where $\bullet$ represents the terms that are not relevant to our discussion. It is obvious that

$$\bar{V}_2X^T\bar{U}_1\bar{\mathscr{A}}_{i,22} + \bar{\mathscr{A}}_{i,22}^T\bar{U}_1^TX\bar{V}_2^T < 0$$

Since (4) holds, we have $\bar{V}_2X^T\bar{U}_1\left(\sum_{i=1}^{l}\bar{h}_i(\bar{\theta}(t))\bar{\mathscr{A}}_{i,22}\right) + \left(\sum_{i=1}^{l}\bar{h}_i(\bar{\theta}(t))\bar{\mathscr{A}}_{i,22}^T\right)\bar{U}_1^TX\bar{V}_2^T < 0$. Therefore, we can deduce that $\sum_{i=1}^{l}\bar{h}_i(\bar{\theta}(t))\bar{\mathscr{A}}_{i,22}$ is nonsingular. According to Definition 1, the nominal case of system (17) is regular and impulse free.

Next, we will show that the nominal case of system (17) with $w(t) = 0$ is asymptotically stable. Construct a Lyapunov-Krasovskii functional as:

$$\begin{aligned} V(t) &= e^T(t)E^TPEe(t) + \int_{t-\tau_M}^{t} e^T(z)Q_1e(z)dz \\ &+ \int_{t-\tau(t)}^{t} e^T(z)Q_2e(z)dz + \int_{t-\tau_m}^{t} e^T(z)Q_3e(z)dz \\ &+ \tau_m\int_{-\tau_m}^{0}\int_{t+\theta}^{t} \dot{e}^T(z)E^TZ_1E\dot{e}(z)dzd\theta \\ &+ \tau_d\int_{-\tau_M}^{-\tau_m}\int_{t+\theta}^{t} \dot{e}^T(z)E^TZ_2E\dot{e}(z)dzd\theta \end{aligned} \quad (22)$$

where $\tau_d = \tau_M - \tau_m$. Taking the time derivative of $V(t)$, we have

$$\begin{aligned} \dot{V}(t) \leq\ & 2e^T(t)E^TPE\dot{e}(t) + e^T(t)(Q_1 + Q_2 + Q_3)e(t) \\ & - e^T(t-\tau_M)Q_1e(t-\tau_M) - e^T(t-\tau_m)Q_3e(t-\tau_m) \\ & - (1-\tau_a)e^T(t-\tau(t))Q_2e(t-\tau(t)) \\ & + \tau_m^2\dot{e}^T(t)E^TZ_1E\dot{e}(t) + \tau_d^2\dot{e}^T(t)E^TZ_2E\dot{e}(t) \\ & - \tau_m\int_{t-\tau_m}^{t} \dot{e}^T(z)E^TZ_1E\dot{e}(z)dz \\ & - \tau_d\int_{t-\tau_M}^{t-\tau_m} \dot{e}^T(z)E^TZ_2E\dot{e}(z)dz \end{aligned} \quad (23)$$

According to (23), the following inequality can be obtained.

$$\begin{aligned} -\tau_d\int_{t-\tau_M}^{t-\tau_m}\bar{\mathcal{H}}(z)dz &= -\tau_d\int_{t-\tau_M}^{t-\tau(t)}\bar{\mathcal{H}}(z)dz - \tau_d\int_{t-\tau(t)}^{t-\tau_m}\bar{\mathcal{H}}(z)dz \\ &\leq -(\tau_M - \tau(t))\int_{t-\tau_M}^{t-\tau(t)}\bar{\mathcal{H}}(z)dz \\ &\quad - (\tau(t) - \tau_m)\int_{t-\tau(t)}^{t-\tau_m}\bar{\mathcal{H}}(z)dz \end{aligned} \quad (24)$$

where $\bar{\mathcal{H}}(z) = \dot{e}^T(z)E^TZ_2E\dot{e}(z)$. Then according to Lemma 1 and Newton-Leibniz formula, we have

$$\begin{aligned} -\tau_m\int_{t-\tau_m}^{t} \dot{e}^T(z)E^TZ_1E\dot{e}(z)dz &\leq -e^T(t)E^TZ_1Ee(t) \\ &\quad + 2e^T(t)E^TZ_1Ee(t-\tau_m) \\ &\quad - e^T(t-\tau_m)E^TZ_1Ee(t-\tau_m) \end{aligned} \quad (25)$$

and

$$\begin{aligned} -\tau_d\int_{t-\tau_M}^{t-\tau_m}\bar{\mathcal{H}}(z)dz &\leq -2e^T(t-\tau(t))E^TZ_2Ee(t-\tau(t)) \\ &\quad - e^T(t-\tau_M)E^TZ_2Ee(t-\tau_M) \\ &\quad - e^T(t-\tau_m)E^TZ_2Ee(t-\tau_m) \\ &\quad + 2e^T(t-\tau(t))E^TZ_2Ee(t-\tau_M) \\ &\quad + 2e^T(t-\tau(t))E^TZ_2Ee(t-\tau_m) \end{aligned} \quad (26)$$

In addition, according to Lemma 3 and (23), (25), (26), we can obtain

$$\dot{V}(t) \leq \sum_{i=1}^{l}\bar{h}_i(\bar{\theta}(t))\eta_1^T(t)(\Phi_i + \tau_m\tilde{\Phi}_{12i}Z_1\tau_m\tilde{\Phi}_{12i}^T + \tau_d\tilde{\Phi}_{12i}Z_2\tau_d\tilde{\Phi}_{12i}^T)\eta_1(t)$$

where

$$\Phi_i = \begin{bmatrix} \Phi_{11i} & 0 & \Phi_{13i} & E^TZ_1E \\ * & \Phi_{22} & E^TZ_2E & 0 \\ * & * & \Phi_{33} & E^TZ_2E \\ * & * & * & \Phi_{44} \end{bmatrix}$$

and $\Phi_{11i} = sym(\mathcal{P}^T(\mathcal{R}A_i - L_iC)) + (Q_1 + Q_2 + Q_3) - E^TZ_1E$, $\Phi_{13i} = \mathcal{P}^T\mathcal{R}A_{di}$, $\Phi_{22} = -Q_1 - E^TZ_2E$, $\Phi_{33} = -(1-\tau_a)Q_2 - 2E^TZ_2E$, $\Phi_{44} = -Q_3 - E^TZ_1E - E^TZ_2E$, $\tilde{\Phi}_{12i} = \begin{bmatrix}\mathcal{R}A_i - L_iC & 0 & \mathcal{R}A_{di} & 0\end{bmatrix}^T$, $\eta_1(t) = \begin{bmatrix}e^T(t) & e^T(t-\tau_M) & e^T(t-\tau(t)) & e^T(t-\tau_m)\end{bmatrix}^T$.

Using Schur complement, we can obtain that if $\tilde{\Phi}_i < 0$ hold, then $\dot{V}(t) < 0$.

$$\tilde{\Phi}_i = \begin{bmatrix} \Phi_i & \tau_m\tilde{\Phi}_{12i}Z_1 & \tau_d\tilde{\Phi}_{12i}Z_2 \\ * & -Z_1 & 0 \\ * & * & -Z_2 \end{bmatrix}$$

From (18), it is easy to see that $\tilde{\Phi}_i < 0$. Hence, $\dot{V}(t) < 0$, which implies that the nominal case of system (17) with $w(t) = 0$ is asymptotically stable.

Let us analyze the $H\infty$ performance. Our purpose is to ensure the nominal case of system (17) is admissible with the following specified $H\infty$ norm upper bound

$$\int_{t_0}^{\infty} e_y^T(t)e_y(t)dt < \int_{t_0}^{\infty} \gamma^2 w^T(t)w(t)dt \tag{27}$$

for all nonzero $w(t)$ belong to $L_2[0,\infty)$ under zero initial condition $e(\tilde{t}) = 0$, for all $\tilde{t} \in [t_r, t_0]$ where $t_0 = t_r + \tau_M$ and t_r is the time when the sliding surface is reached.

For this purpose, consider the performance index:

$$J(t) = \dot{V}(t) + e_y^T(t)e_y(t) - \gamma^2 w^T(t)w(t) \tag{28}$$

Following the same procedure as used above, we can obtain that if $\tilde{\Gamma}_i < 0$ hold, then $J(t) < 0$. Under the zero initial condition, integrating both sides of the inequality $\dot{V}(t) < -e_y^T(t)e_y(t) + \gamma^2 w^T(t)w(t)$ in t form t_0 to ∞, we obtain that

$$\int_{t_0}^{\infty} e_y^T(t)e_y(t)dt < \int_{t_0}^{\infty} \gamma^2 w^T(t)w(t) - V(\infty)$$

Since $V(\infty) \geq 0$, so we have

$$\int_{t_0}^{\infty} e_y^T(t)e_y(t)dt < \int_{t_0}^{\infty} \gamma^2 w^T(t)w(t)$$

Thus, the nominal case of system (17) is asymptotically stable with $H\infty$ performance. Thus completes the proof. □

2) UNCERTAIN CASE

Based on Theorem 1, we develop a delay-dependent sufficient condition such that system (17) is admissible with $H\infty$ performance.

Theorem 2: For given positive scalars τ_m, τ_a, τ_M and $\bar{\gamma}$, system (17) is admissible with $H\infty$ performance index γ, if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Z_1 > 0$, $Z_2 > 0$, X and positive scalar ε, such that the following LMI hold

$$\begin{bmatrix} \tilde{\Gamma}_i & \bar{\Xi}_{12i} & \varepsilon\bar{\Xi}_{13i}^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0 \tag{29}$$

where

$$\bar{\Xi}_{12i} = \begin{bmatrix}\mathcal{P} & 0 & 0 & 0 & 0 & \tau_m Z_1 & \tau_d Z_2\end{bmatrix}^T \mathcal{R}M_i$$
$$\bar{\Xi}_{13i} = \begin{bmatrix}0 & 0 & 0 & 0 & N_{wi} & 0 & 0\end{bmatrix}$$

Proof: Replacing H_i by $H_i + \Delta H_i(t)$, we can obtain that system (17) is admissible with $H\infty$ performance if

$$\tilde{\Gamma}_i + sym(\bar{\Xi}_{12i}F(t)\bar{\Xi}_{13i}) < 0 \tag{30}$$

Then, by using Lemma 2 and the Schur complement, (29) can be obtained. Thus completes the proof. □

3) ANALYSIS OF GAIN MATRIX

In this subsection, we focus on the determination of gain matrix L_i.

Theorem 3: For given positive scalars τ_m, τ_a, τ_M, $\bar{\gamma}$, system (17) is admissible with $H\infty$ performance index γ, if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Z_1 > 0$, $Z_2 > 0$, X, $\mathcal{K}_i$ and positive scalars ε, such that the following LMI hold

$$\begin{bmatrix} \Omega_i & \tau_m\tilde{\Omega}_{12i} & \tau_d\tilde{\Omega}_{12i} & \tilde{\Theta}_{1i} & \tilde{\Theta}_{2i} \\ * & \tilde{\Omega}_{22} & 0 & \tau_m\mathcal{P}^T\mathcal{R}M_i & 0 \\ * & * & \tilde{\Omega}_{33} & \tau_d\mathcal{P}^T\mathcal{R}M_i & 0 \\ * & * & * & -\varepsilon I & 0 \\ * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{31}$$

where

$$\Omega_i = \begin{bmatrix} \Omega_{11i} & 0 & \Omega_{13i} & E^TZ_1E & \Omega_{15i} \\ * & \Omega_{22} & E^TZ_2E & 0 & 0 \\ * & * & \Omega_{33} & E^TZ_2E & 0 \\ * & * & * & \Omega_{44} & 0 \\ * & * & * & * & -\bar{\gamma}I \end{bmatrix}$$

and $\Omega_{11i} = sym(\mathcal{P}^T\mathcal{R}A_i - \mathcal{K}_i^TC) - E^TZ_1E + C^TC + Q_1 + Q_2 + Q_3$, $\Omega_{13i} = \mathcal{P}^T\mathcal{R}A_{di}$, $\Omega_{15i} = \mathcal{P}^T\mathcal{R}H_i$, $\Omega_{22} = -Q_1 - E^TZ_2E$, $\Omega_{33} = -(1-\tau_a)Q_2 - 2E^TZ_2E$, $\Omega_{44} = -Q_3 - E^TZ_1E - E^TZ_2E$, $\tilde{\Omega}_{22} = Z_1 - sym(\mathcal{P})$, $\tilde{\Omega}_{33} = Z_2 - sym(\mathcal{P})$, $\tilde{\Theta}_{1i} = \begin{bmatrix}M_i^T\mathcal{R}^T\mathcal{P} & 0 & 0 & 0 & 0\end{bmatrix}^T$, $\tilde{\Theta}_{2i} = \begin{bmatrix}0 & 0 & 0 & 0 & \varepsilon N_{wi}\end{bmatrix}^T$, $\mathcal{K}_i = L_i^T\mathcal{P}$, $\tilde{\Omega}_{12i} = \begin{bmatrix}\mathcal{P}^T\mathcal{R}A_i - \mathcal{K}_i{}^TC & 0 & \mathcal{P}^T\mathcal{R}A_{di} & 0 & \mathcal{P}^T\mathcal{R}H_i\end{bmatrix}^T$.

Proof: Since $Z_{\bar{i}} > 0$, $(\bar{i} = 1, 2)$, it is easy to see that $(\mathcal{P} - Z_{\bar{i}})^T Z_{\bar{i}}^{-1}(\mathcal{P} - Z_{\bar{i}}) \geq 0$, which implies that

$$-\mathcal{P}^T Z_{\bar{i}}^{-1}\mathcal{P} \leq Z_{\bar{i}} - sym(\mathcal{P}) \tag{32}$$

By utilizing (32) and performing congruence transformations to (31) by

$$diag\{I_{(4n+q)}, \mathcal{P}^{-T}, \mathcal{P}^{-T}, I_{f_1}, I_{f_2}\}$$

and

$$diag\{I_{(4n+q)}, Z_1, Z_2, I_{f_1}, I_{f_2}\}$$

We can obtain the inequality (29). Then the SMO gain matrix L_i can be obtained by $L_i = \mathcal{P}^{-T}\mathcal{K}_i^T$. Thus completes the proof. □

D. ADAPTIVE SLIDING MODE CONTROL LAW SYNTHESIS

In this subsection, we will design a novel adaptive SMC law, such that the trajectories of the system (13) can be driven onto the sliding surface $s(t) = 0$. If (31) is solvable, then it implies that $e(t)$ and $e(t-\tau(t))$ are bounded, and for some small $\lambda_1 > 0$ and $\lambda_2 > 0$, we have

$$\sup_{0\le t<\infty} \|e(t)\| \le \lambda_1, \quad \sup_{0\le t<\infty} \|e(t-\tau(t))\| \le \lambda_2 \tag{33}$$

Here the adaptive control method is applied to estimate the unknown parameters. Firstly, we define the adaptive parameters $\hat{\lambda}_1(t)$, $\hat{\lambda}_2(t)$, $\hat{\sigma}_{1i}(t)$, $\hat{\sigma}_{2i}(t)$, $\hat{\sigma}_{3i}(t)$, $\hat{\vartheta}(t)$ to estimate λ_1, λ_2, σ_{1i}, σ_{2i}, σ_{3i}, ϑ. Then the estimation errors are denoted as $\tilde{\lambda}_1(t) = \hat{\lambda}_1(t) - \lambda_1$, $\tilde{\lambda}_2(t) = \hat{\lambda}_2(t) - \lambda_2$, $\tilde{\sigma}_{1i}(t) = \hat{\sigma}_{1i}(t) - \sigma_{1i}$, $\tilde{\sigma}_{2i}(t) = \hat{\sigma}_{2i}(t) - \sigma_{2i}$, $\tilde{\sigma}_{3i}(t) = \hat{\sigma}_{3i}(t) - \sigma_{3i}$ and $\tilde{\vartheta}(t) = \hat{\vartheta}(t) - \vartheta$.

Theorem 4: For given appropriate matrix $\mathcal{T}$, the trajectories of the T-S fuzzy descriptor system (13) can be driven onto the sliding surface $s(t) = 0$ by the following adaptive SMC law:

$$v(t) = (\bar{\mathcal{G}}B)^{-1}\sum_{i=1}^{l}\bar{h}_i(\bar{\theta}(t))\{-\bar{\mathcal{G}}A_i\mathcal{T}Ce(t) - \bar{\mathcal{G}}A_{di}\mathcal{T}Ce(t-\tau(t)) + \rho_i(t)\frac{s(t)}{\|s(t)\|}\} \tag{34}$$

where

$$\begin{aligned}\rho_i(t) =& \left\|\bar{\mathcal{G}}A_i + \bar{\mathcal{G}}A_i\mathcal{T}C\right\|\hat{\lambda}_1(t) + \left\|\bar{\mathcal{G}}A_{di} + \bar{\mathcal{G}}A_{di}\mathcal{T}C\right\|\hat{\lambda}_2(t)\\ &+ \left\|\bar{\mathcal{G}}B\right\|\hat{\sigma}_{1i}(t) + \left\|\bar{\mathcal{G}}B\right\|\|y(t)\|\hat{\sigma}_{2i}(t)\\ &+ \left\|\bar{\mathcal{G}}B\right\|\|y(t-\tau(t))\|\hat{\sigma}_{3i}(t)\\ &+ (\left\|\bar{\mathcal{G}}H_i\right\| + \left\|\bar{\mathcal{G}}M_i\right\|\|N_{wi}\|)\hat{\vartheta}(t) + \epsilon_0\end{aligned} \tag{35}$$

with the adaptive laws

$$\begin{aligned}\dot{\hat{\lambda}}_1(t) &= q_1\sum_{i=1}^{l}\bar{h}_i(\bar{\theta}(t))\{\left\|\bar{\mathcal{G}}A_i + \bar{\mathcal{G}}A_i\mathcal{T}C\right\|\}\|s(t)\|\\ \dot{\hat{\lambda}}_2(t) &= q_2\sum_{i=1}^{l}\bar{h}_i(\bar{\theta}(t))\{\left\|\bar{\mathcal{G}}A_{di} + \bar{\mathcal{G}}A_{di}\mathcal{T}C\right\|\}\|s(t)\|\\ \dot{\hat{\sigma}}_{1i}(t) &= \bar{h}_i(\bar{\theta}(t))q_{3i}\left\|\bar{\mathcal{G}}B\right\|\|s(t)\|\\ \dot{\hat{\sigma}}_{2i}(t) &= \bar{h}_i(\bar{\theta}(t))q_{4i}\left\|\bar{\mathcal{G}}B\right\|\|y(t)\|\|s(t)\|\\ \dot{\hat{\sigma}}_{3i}(t) &= \bar{h}_i(\bar{\theta}(t))q_{5i}\left\|\bar{\mathcal{G}}B\right\|\|y(t-\tau(t))\|\|s(t)\|\\ \dot{\hat{\vartheta}}(t) &= q_6\sum_{i=1}^{l}\bar{h}_i(\bar{\theta}(t))\{(\left\|\bar{\mathcal{G}}H_i\right\| + \left\|\bar{\mathcal{G}}M_i\right\|\|N_{wi}\|)\}\|s(t)\|\end{aligned} \tag{36}$$

and ϵ_0 is a small positive constant, q_1, q_2, q_{3i}, q_{4i}, q_{5i}, q_6 are designed constants.

Proof: We choose the following Lyapunov functional candidate

$$\begin{aligned}V_s(t) =& \frac{1}{2}s^T(t)s(t) + \frac{1}{2q_1}\tilde{\lambda}_1^2(t) + \frac{1}{2q_2}\tilde{\lambda}_2^2(t)\\ &+ \sum_{i=1}^{l}\{\frac{1}{2q_{3i}}\tilde{\sigma}_{1i}^2(t)\} + \sum_{i=1}^{l}\{\frac{1}{2q_{4i}}\tilde{\sigma}_{2i}^2(t)\}\\ &+ \sum_{i=1}^{l}\{\frac{1}{2q_{5i}}\tilde{\sigma}_{3i}^2(t)\} + \frac{1}{2q_6}\tilde{\vartheta}^2(t)\end{aligned} \tag{37}$$

Then, based on (17) and (37), we can obtain that

$$\begin{aligned}\dot{V}_s(t) =& s^T(t)\sum_{i=1}^{l}\bar{h}_i(\bar{\theta}(t))\{\bar{\mathcal{G}}A_ie(t) + \bar{\mathcal{G}}A_i\mathcal{T}e_y(t)\\ &+ \bar{\mathcal{G}}A_{di}e(t-\tau(t)) + \bar{\mathcal{G}}A_{di}\mathcal{T}e_y(t-\tau(t))\\ &+ \bar{\mathcal{G}}Bf_i(t, x(t), x(t-\tau(t))) + \bar{\mathcal{G}}(H_i + \Delta H_i)w(t)\\ &- \bar{\mathcal{G}}A_i\mathcal{T}e_y(t) - \bar{\mathcal{G}}A_{di}\mathcal{T}e_y(t-\tau(t))\} - s^T(t)\bar{\mathcal{G}}Bv(t)\\ &+ \frac{1}{q_1}\dot{\hat{\lambda}}_1(t)\tilde{\lambda}_1(t) + \frac{1}{q_2}\dot{\hat{\lambda}}_2(t)\tilde{\lambda}_2(t) + \frac{1}{q_6}\dot{\hat{\vartheta}}(t)\tilde{\vartheta}(t)\\ &+ \sum_{i=1}^{l}\{\frac{1}{q_{3i}}\dot{\hat{\sigma}}_{1i}(t)\tilde{\sigma}_{1i}(t)\} + \sum_{i=1}^{l}\{\frac{1}{q_{4i}}\dot{\hat{\sigma}}_{2i}(t)\tilde{\sigma}_{2i}(t)\}\\ &+ \sum_{i=1}^{l}\{\frac{1}{q_{5i}}\dot{\hat{\sigma}}_{3i}(t)\tilde{\sigma}_{3i}(t)\}\end{aligned} \tag{38}$$

It follows from (34), (35), (36) and (38) that

$$\dot{V}_s(t) \le -\epsilon_0\|s(t)\| < 0, \quad \forall\|s(t)\| \ne 0 \tag{39}$$

which implies that the trajectories of the T-S fuzzy descriptor system (15) can be driven onto the sliding surface $s(t) = 0$ in finite time, thus ends the proof. □

Remark 4: In this paper, our main purpose is to design a state feedback control law for the error system such that the closed-loop system is admissible. However, in engineering applications, the estimation error variables are unknown. So we have to use $e_y(t)$ in stead of $e(t)$. Here, we do not use the assumption that the fast subsystem of descriptor system is observable [47]. We introduce an appropriate matrix $\mathcal{T}$ such that $\bar{\mathcal{G}}A_i\mathcal{T}e_y(t)$ and $\bar{\mathcal{G}}A_{di}\mathcal{T}e_y(t-\tau(t))$ can be obtained. This reduces the system's constraints.

Remark 5: Our theorems proposed in this paper are also feasible for normal system. In fact, the normal system is a special case of the descriptor system, and when we set $E = I$, the descriptor system becomes a normal system.

IV. SMO BASED SLIDING MODE CONTROL

In this section, a SMO based SMC strategy is developed for the T-S fuzzy descriptor system (2).

A. SLIDING SURFACE DESIGN FOR THE SMO SYSTEM

Here, we consider the following novel integral-type sliding function for the SMO system (12).

$$s_1(t) = G_1E\hat{x}(t) - G_1\int_0^t\sum_{i=1}^{l}\bar{h}_i(\bar{\theta}(t))\{(A_i + B\mathcal{Q}_i)\hat{x}(z) + L_iCe(z)\}dz \tag{40}$$

where $G_1 \in \Re^{m\times n}$ is known matrix satisfying $\det(G_1B) \ne 0$ and L_i are defined in (12). $\mathcal{Q}_i \in \Re^{m\times n}$ is chosen so that $A_i + B\mathcal{Q}_i$ is Hurwitz. This requires that (A_i, B) are controllable.

Remark 6: In order to avoid the complex nonlinear terms in the following derivations, we have added $L_iCe(t)$ in (40). Since $e_y(t) = y(t) - \hat{y}(t) = Ce(t)$. Therefore the sliding surface (40) can be written as

$$s_1(t) = G_1E\hat{x}(t) - G_1\int_0^t \sum_{i=1}^{l} \bar{h}_i(\bar{\theta}(t))\{(A_i + B\mathcal{Q}_i)\hat{x}(z) + L_i(y(z) - C\hat{x}(z))\}dz$$

According to SMC theory, when the sliding motion takes place, we have $s_1(t) = 0$ and $\dot{s}_1(t) = 0$. Thus, from $\dot{s}_1(t) = 0$, the equivalent control law can be given by:

$$u_{eq}(t) = -(G_1B)^{-1}G_1\sum_{i=1}^{l} \bar{h}_i(\bar{\theta}(t))\{-B\mathcal{Q}_i\hat{x}(t) + A_{di}\hat{x}(t-\tau(t))\} - v(t) \quad (41)$$

Furthermore, by substituting the equivalent control law (41) into (12), we can obtain the following equation.

$$E\dot{\hat{x}}(t) = \sum_{i=1}^{l} \bar{h}_i(\bar{\theta}(t))\{(A_i + B\mathcal{Q}_i)\hat{x}(t) + R_1A_{di}\hat{x}(t-\tau(t)) + L_iCe(t)\} \quad (42)$$

where $R_1 = I - B(G_1B)^{-1}G_1$.

B. ADMISSIBILITY ANALYSIS OF THE SLIDING MOTION

In this subsection, we will analyze the admissibility of (17) and (42) simultaneously.

Theorem 5: For given positive scalars τ_m, τ_a, τ_M and $\bar{\gamma}_1$, system (17) and (42) are admissible with $H\infty$ performance index γ_1, if there exist matrices $P_1 > 0$, $\bar{Q}_1 > 0$, $\bar{Q}_2 > 0$, $\bar{Q}_3 > 0$, $\bar{Q}_4 > 0$, $\bar{Z}_1 > 0$, $\bar{Z}_2 > 0$, $\tilde{X}$ and positive scalar $\bar{\varepsilon}$, such that the following LMI hold

$$\begin{bmatrix} \Upsilon_i & \tilde{\Xi}_{12i} & \bar{\varepsilon}\tilde{\Xi}_{13i}^T \\ * & -\bar{\varepsilon}I & 0 \\ * & * & -\bar{\varepsilon}I \end{bmatrix} < 0 \quad (43)$$

where

$$\Upsilon_i = \begin{bmatrix} \tilde{\Upsilon}_{1i} & \tau_m\tilde{\Upsilon}_{2i}^T\bar{Z}_1 & \tau_m\tilde{\Upsilon}_{3i}^T\bar{Z}_2 \\ * & -\bar{Z}_1 & 0 \\ * & * & -\bar{Z}_2 \end{bmatrix}$$

$$\tilde{\Xi}_{12i} = \begin{bmatrix} \bar{\mathcal{P}} & 0 & 0 & 0 & 0 & 0 & 0 & \tau_m\bar{Z}_1 & 0 \end{bmatrix}^T \mathcal{R}M_i$$

$$\tilde{\Xi}_{13i} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & N_{wi} & 0 & 0 \end{bmatrix}.$$

and $\tilde{\Upsilon}_{1i}$, as shown at the top of the next page, with $\Upsilon_{11i} = sym(\bar{\mathcal{P}}^T(\mathcal{R}A_i - L_iC)) + \bar{Q}_1 + \bar{Q}_2 - E^T\bar{Z}_1E + C^TC$, $\Upsilon_{22} = -(1-\tau_a)\bar{Q}_1$, $\Upsilon_{33} = -\bar{Q}_2 - E^T\bar{Z}_1E$, $\Upsilon_{44i} = sym(\bar{\mathcal{P}}^T(A_i + B\mathcal{Q}_i)) + \bar{Q}_3 + \bar{Q}_4 - E^T\bar{Z}_2E$, $\Upsilon_{55} = -(1-\tau_a)\bar{Q}_3$, $\Upsilon_{66} = -\bar{Q}_4 - E^T\bar{Z}_2E$, $\bar{\mathcal{P}} = P_1E + \bar{U}^T\tilde{X}\bar{V}^T$, $\bar{\gamma}_1 = \gamma_1^2$, $\tilde{\Upsilon}_{2i} = \begin{bmatrix} \mathcal{R}A_i - L_iC & \mathcal{R}A_{di} & 0 & 0 & 0 & 0 & \mathcal{R}H_i \end{bmatrix}$, $\tilde{\Upsilon}_{3i} = \begin{bmatrix} L_iC & 0 & 0 & A_i + B\mathcal{Q}_i & R_1A_{di} & 0 & 0 \end{bmatrix}$.

Proof: Firstly, we prove the regularity and the no-impulse of (17) and (42). According to (43), we have $\Upsilon_{11i} < 0$ and $\Upsilon_{44i} < 0$. Since $\bar{Q}_1 > 0$, $\bar{Q}_2 > 0$, $\bar{Q}_3 > 0$, $\bar{Q}_4 > 0$, $\bar{Z}_1 > 0$, $\bar{Z}_2 > 0$ and $C^TC \geq 0$, we can get the following inequalities.

$$sym(\bar{\mathcal{P}}^T(\mathcal{R}A_i - L_iC)) - E^T\bar{Z}_1E < 0 \quad (44)$$

and

$$sym(\bar{\mathcal{P}}^T(A_i + B\mathcal{Q}_i)) - E^T\bar{Z}_2E < 0 \quad (45)$$

In addition, since $rank(E) = r$, there must exist two invertible matrices $\bar{\mathcal{U}} \in \Re^{n\times n}$ and $\bar{\mathcal{V}} \in \Re^{n\times n}$ such that

$$\bar{\mathcal{U}}E\bar{\mathcal{V}} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

Then we define $\mathscr{A}_i = \mathcal{R}A_i - L_iC$, $\mathcal{A}_i = A_i + B\mathcal{Q}_i$ and

$$\bar{\mathcal{U}}\mathscr{A}_i\bar{\mathcal{V}} = \begin{bmatrix} \tilde{\mathscr{A}}_{i,11} & \tilde{\mathscr{A}}_{i,12} \\ \tilde{\mathscr{A}}_{i,21} & \tilde{\mathscr{A}}_{i,22} \end{bmatrix},$$

$$\bar{\mathcal{U}}\mathcal{A}_i\bar{\mathcal{V}} = \begin{bmatrix} \tilde{\mathcal{A}}_{i,11} & \tilde{\mathcal{A}}_{i,12} \\ \tilde{\mathcal{A}}_{i,21} & \tilde{\mathcal{A}}_{i,22} \end{bmatrix}$$

$$\bar{\mathcal{U}}^{-T}P_1\bar{\mathcal{U}}^{-1} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix}, \quad \bar{\mathcal{V}}^T\tilde{V} = \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{bmatrix}$$

$$\bar{\mathcal{U}}^{-T}\bar{Z}_2\bar{\mathcal{U}}^{-1} = \begin{bmatrix} \tilde{Z}_{2,11} & \tilde{Z}_{2,12} \\ \tilde{Z}_{2,21} & \tilde{Z}_{2,22} \end{bmatrix}, \quad \tilde{U}\bar{\mathcal{U}}^{-1} = \tilde{U}_1\begin{bmatrix} 0 & I_{n-r} \end{bmatrix}$$

$$\bar{\mathcal{U}}^{-T}\bar{Z}_1\bar{\mathcal{U}}^{-1} = \begin{bmatrix} \tilde{Z}_{1,11} & \tilde{Z}_{1,12} \\ \tilde{Z}_{1,21} & \tilde{Z}_{1,22} \end{bmatrix}$$

Pre- and post-multiplying (44) and (45) by $\mathcal{V}^T$ and $\mathcal{V}$ respectively, we can get the following inequality.

$$\begin{bmatrix} \bullet & \bullet \\ \bullet & \tilde{V}_2\tilde{X}^T\tilde{U}_1\tilde{\mathscr{A}}_{i,22} + \tilde{\mathscr{A}}_{i,22}^T\tilde{U}_1^T\tilde{X}\tilde{V}_2^T \end{bmatrix} < 0 \quad (46)$$

$$\begin{bmatrix} \bullet & \bullet \\ \bullet & \tilde{V}_2\tilde{X}^T\tilde{U}_1\tilde{\mathcal{A}}_{i,22} + \tilde{\mathcal{A}}_{i,22}^T\tilde{U}_1^T\tilde{X}\tilde{V}_2^T \end{bmatrix} < 0 \quad (47)$$

where $\bullet$ represents the terms that are not relevant to our discussion. (46) implies that $\tilde{\mathscr{A}}_{i,22}$ and $\tilde{\mathcal{A}}_{i,22}$ are nonsingular. Therefore according to Definition 1, system (17) and (42) are regular and impulse free.

Next, we will prove that (17) and (42) are asymptotically stable. Construct the following Lyapunov-Krasovskii functional.

$$\mathcal{V}(t) = \mathcal{V}_1(t) + \mathcal{V}_2(t) \quad (48)$$

where

$$\mathcal{V}_1(t) = e^T(t)E^TP_1Ee(t) + \int_{t-\tau(t)}^{t} e^T(z)\bar{Q}_1e(z)dz + \int_{t-\tau_m}^{t} e^T(z)\bar{Q}_2e(z)dz + \tau_m\int_{-\tau_m}^{0}\int_{t+\theta}^{t} \dot{e}^T(z)E^T\bar{Z}_1E\dot{e}(z)dzd\theta$$

$$\mathcal{V}_2(t) = \hat{x}^T(t)E^TP_1E\hat{x}(t) + \int_{t-\tau(t)}^{t} \hat{x}^T(z)\bar{Q}_3\hat{x}(z)dz + \int_{t-\tau_m}^{t} \hat{x}^T(z)\bar{Q}_4\hat{x}(z)dz + \tau_m\int_{-\tau_m}^{0}\int_{t+\theta}^{t} \dot{\hat{x}}^T(z)E^T\bar{Z}_2E\dot{\hat{x}}(z)dzd\theta$$

$$\tilde{\Upsilon}_{1i} = \begin{bmatrix} \Upsilon_{11i} & \bar{\mathcal{P}}^T\mathcal{R}A_{di} & 2E^T\bar{Z}_1E & C^TL_i^T\bar{\mathcal{P}} & 0 & 0 & \bar{\mathcal{P}}^T\mathcal{R}H_i \\ * & \Upsilon_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & \Upsilon_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Upsilon_{44i} & \bar{\mathcal{P}}^TR_1A_{di} & 2E^T\bar{Z}_2E & 0 \\ * & * & * & * & \Upsilon_{55} & 0 & 0 \\ * & * & * & * & * & \Upsilon_{66} & 0 \\ * & * & * & * & * & * & -\bar{\gamma}_1 \end{bmatrix}$$

Then, the time-derivative of $\mathcal{V}(t)$ can be obtained.

$$\begin{aligned}\dot{\mathcal{V}}(t) \le\ & 2e^T(t)E^TP_1E\dot{e}(t) + e^T(t)\bar{Q}_1e(t) + e^T(t)\bar{Q}_2e(t) \\ & - (1-\tau_a)e^T(t-\tau(t))\bar{Q}_1e(t-\tau(t)) \\ & - e^T(t-\tau_m)\bar{Q}_2e(t-\tau_m) + \tau_m^2\dot{e}^T(t)E^T\bar{Z}_1E\dot{e}(t) \\ & - \tau_m\int_{t-\tau_m}^{t}\dot{e}^T(z)E^T\bar{Z}_1E\dot{e}(z)dz \\ & + 2\hat{x}^T(t)E^TP_1E\dot{\hat{x}}(t) + \hat{x}^T(t)\bar{Q}_3\hat{x}(t) + \hat{x}^T(t)\bar{Q}_4\hat{x}(t) \\ & - (1-\tau_a)\hat{x}^T(t-\tau(t))\bar{Q}_2\hat{x}(t-\tau(t)) \\ & - \hat{x}^T(t-\tau_m)\bar{Q}_4\hat{x}(t-\tau_m) + \tau_m^2\dot{\hat{x}}^T(t)E^T\bar{Z}_2E\dot{\hat{x}}(t) \\ & - \tau_m\int_{t-\tau_m}^{t}\dot{\hat{x}}^T(z)E^T\bar{Z}_2E\dot{\hat{x}}(z)dz\end{aligned}$$

Considering the following specified $H\infty$ norm upper bound

$$\int_{t_3}^{\infty} e_y^T(t)e_y(t)dt < \int_{t_3}^{\infty}\gamma_1^2w^T(t)w(t)dt$$

for all nonzero $w(t)$ belong to $L_2[0,\infty)$ under zero initial condition $e(\tilde{t}_1) = 0$ and $\hat{x}(\tilde{t}_1) = 0$, for all $\tilde{t}_1 \in [t_2, t_3]$ where $t_3 = t_1 + \tau_M$, t_1 is the time when the sliding surface $s_1(t)$ is reached for $\hat{x}(t)$ and t_2 is the time when the sliding surface $s(t)$ is reached for $e(t)$. We also assume that $t_1 \ge t_2$. Of course, the case of $t_1 < t_2$ is also the same processing method. For this purpose, consider the performance index:

$$\mathcal{J}(t) = \dot{\mathcal{V}}(t) + e_y^T(t)e_y(t) - \gamma_1^2w^T(t)w(t)$$

Following the same procedure as used above, we can obtain that if (43) hold, then $\mathcal{J}(t) < 0$. Under the zero initial condition, integrating both sides of the inequality $\dot{\mathcal{V}}(t) < -e_y^T(t)e_y(t) + \gamma_1^2w^T(t)w(t)$ in t form t_3 to ∞, we obtain that

$$\int_{t_3}^{\infty} e_y^T(t)e_y(t)dt < \int_{t_3}^{\infty}\gamma_1^2w^T(t)w(t) - V(\infty)$$

Since $V(\infty) \ge 0$, so we have

$$\int_{t_3}^{\infty} e_y^T(t)e_y(t)dt < \int_{t_3}^{\infty}\gamma_1^2w^T(t)w(t)$$

Hence (17) and (42) are asymptotically stable with $H\infty$ performance. According to Definition 1 and Definition 2, (17) and (42) are admissible. Thus completes the proof. □

Theorem 6: For given positive scalars τ_m, τ_a, τ_M, $\bar{\gamma}_1$, system (17) and (42) are admissible with $H\infty$ performance index γ_1, if there exist matrices $P_1 > 0$, $\bar{Q}_1 > 0$, $\bar{Q}_2 > 0$, $\bar{Q}_3 > 0$, $\bar{Q}_4 > 0$, $\bar{Z}_1 > 0$, $\bar{Z}_2 > 0$, $\tilde{X}$ and positive scalars $\bar{\varepsilon}$, such that the following LMI hold

$$\begin{bmatrix} \tilde{\Upsilon}_{1i} & \tau_m\tilde{\Upsilon}_{2i}^T\bar{P} & \tau_m\tilde{\Upsilon}_{3i}^T\bar{P} & \tilde{\Upsilon}_{4i} & \bar{\varepsilon}\tilde{\Upsilon}_{5i} \\ * & \tilde{\Upsilon}_{22} & 0 & \tau_m\bar{\mathcal{P}}^T\mathcal{R}M_i & 0 \\ * & * & \tilde{\Upsilon}_{33} & 0 & 0 \\ * & * & * & -\bar{\varepsilon}I & 0 \\ * & * & * & * & -\bar{\varepsilon}I \end{bmatrix} < 0 \tag{49}$$

where $\tilde{\Upsilon}_{4i} = \left[M_i^T\mathcal{R}^T\bar{\mathcal{P}}\ 0\ 0\ 0\ 0\ 0\ 0\right]^T$, $\tilde{\Upsilon}_{22} = \bar{Z}_1 - sym(\bar{\mathcal{P}})$, $\tilde{\Upsilon}_{5i} = \left[0\ 0\ 0\ 0\ 0\ 0\ N_{wi}\right]^T$, $\tilde{\Upsilon}_{33} = \bar{Z}_2 - sym(\bar{\mathcal{P}})$.

Proof: The proof is similar to Theorem 3. Here, we do not make a detailed proof. By the way, (49) has $\bar{\mathcal{P}}^TL_i$ and $L_i^T\bar{\mathcal{P}}$, we can use the variable $\mathcal{K}_{1i} = L_i^T\bar{\mathcal{P}}$. Thus the SMO gain matrix can be obtained by $L_i = \bar{\mathcal{P}}^{-T}\mathcal{K}_{1i}^T$. □

C. SLIDING MODE CONTROL LAW SYNTHESIS

In this subsection, we will design a SMC law, such that the trajectories of (12) can be driven onto the sliding surface $s_1(t) = 0$.

Theorem 7: For given appropriate matrix $\mathcal{T}$, the trajectories of the T-S fuzzy descriptor system (12) can be driven onto the sliding surface $s_1(t) = 0$ by the following adaptive SMC law:

$$\begin{aligned}u(t) = \sum_{i=1}^{l}\bar{h}_i(\bar{\theta}(t))\{&\mathcal{Q}_i\hat{x}(t) - (G_1B)^{-1}G_1A_{di}\hat{x}(t-\tau(t)) \\ &+ (\bar{\mathcal{G}}B)^{-1}\bar{\mathcal{G}}A_i\mathcal{T}e_y(t) + (\bar{\mathcal{G}}B)^{-1}\bar{\mathcal{G}}A_{di}\mathcal{T}e_y(t-\tau(t)) \\ &- (G_1B)^{-1}\beta_i\frac{s_1(t)}{\|s_1(t)\|}\}\end{aligned} \tag{50}$$

where $\beta_i = \left\|(G_1B)(\bar{\mathcal{G}}B)^{-1}\right\|\rho_i + \beta_0$

Proof: We choose the following Lyapunov functional candidate $\mathcal{V}_s(t) = \frac{1}{2}s_1^T(t)s_1(t)$ and take the time-derivative of $\mathcal{V}_s(t)$.

$$\begin{aligned}\dot{\mathcal{V}}_s(t) = & -s_1^T\sum_{i=1}^{l}\bar{h}_i(\bar{\theta}(t))\beta_i\frac{s_1(t)}{\|s_1(t)\|} \\ & + s_1^T\sum_{i=1}^{l}\bar{h}_i(\bar{\theta}(t))(G_1B)(\bar{\mathcal{G}}B)^{-1}\rho_i\frac{s(t)}{\|s(t)\|}\end{aligned}$$

It follows from (34), (35), (36) and (50) that

$$\dot{\mathcal{V}}_s(t) \le -\beta_0\|s_1(t)\| < 0, \quad \forall\|s_1(t)\| \ne 0 \tag{51}$$

which implies that the trajectories of the T-S fuzzy descriptor system (12) can be driven onto the sliding surface $s_1(t) = 0$ in finite time, thus ends the proof. □

V. ILLUSTRATIVE EXAMPLES

In this section, we will introduce three examples to show the feasibility of our results.

Example 1: Here, we consider a truck-trailer system [18].

$$\begin{aligned}
\dot{x}_1(t) &= -a\frac{v_1\bar{t}}{L_l t_0}x_1(t) - (1-a)\frac{v_1\bar{t}}{L_l t_0}x_1(t-h) + \frac{v_1\bar{t}}{l t_0}u(t)\\
\dot{x}_2(t) &= a\frac{v_1\bar{t}}{L_l t_0}x_1(t) + (1-a)\frac{v_1\bar{t}}{L_l t_0}x_1(t-h)\\
\dot{x}_3(t) &= \frac{v_1\bar{t}}{t_0}\sin\left(x_2(t) + a\frac{v_1\bar{t}}{2L_l}x_1(t) + (1-a)\frac{v_1\bar{t}}{2L_l}x_1(t-h)\right)\\
0 &= x_2(t) - a\frac{v_1\bar{t}}{L_l t_0}x_1(t) - (1-a)\frac{v_1\bar{t}}{L_l t_0}x_1(t-h) - x_4(t)
\end{aligned}$$

where $x_1(t)$ is the angle difference between the truck and the trailer, $x_2(t)$ is the angle of the trailer, $x_3(t)$ is the vertical position of the rear end of the trailer, $x_4(t)$ is a new variable for the descriptor system, $u(t)$ is the steering angle, $a = 0.7$ is the retarded coefficient, $v_1 = -1$ is the constant speed of backing up, $l = 2.8$ is the length of the truck, and $L_l = 5.5$ is the length of the trailer. Furthermore $\bar{t} = 2.0$, $t_0 = 0.5$.

In this model, we consider time-varying delay, external disturbance, nonlinearities and uncertainty simultaneously. Therefore the following T-S fuzzy descriptor system can be established.

$$\begin{aligned}
E\dot{x}(t) &= \sum_{i=1}^{2}\bar{h}_i(\bar{\theta}(t))\{A_i x(t) + A_{di}x(t-\tau(t))\\
&\quad + B_i(u(t)+f_i(t)) + (H_i + \Delta H_i(t))w(t)\}\\
y(t) &= Cx(t)
\end{aligned} \tag{52}$$

where

$$E = \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -a\frac{v_1\bar{t}}{L_l t_0} & 0 & 0 & 0\\ a\frac{v_1\bar{t}}{L_l t_0} & 0 & 0 & 0\\ -a\frac{v_1^2\bar{t}^2}{2L_l t_0} & \frac{v_1\bar{t}}{t_0} & 0 & 0\\ -a\frac{v_1\bar{t}}{L_l t_0} & 1 & 0 & -1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -a\frac{v_1\bar{t}}{L_l t_0} & 0 & 0 & 0\\ a\frac{v_1\bar{t}}{L_l t_0} & 0 & 0 & 0\\ -a\frac{\bar{\varphi}v_1^2\bar{t}^2}{2L_l t_0} & \frac{\bar{\varphi}v_1\bar{t}}{t_0} & 0 & 0\\ -a\frac{v_1\bar{t}}{L_l t_0} & 1 & 0 & -1 \end{bmatrix}, \quad B_i = \begin{bmatrix} \frac{v_1\bar{t}}{l t_0}\\ 0\\ 0\\ 0 \end{bmatrix}$$

$$A_{d1} = \begin{bmatrix} -(1-a)\frac{v_1\bar{t}}{L_l t_0} & 0 & 0 & 0\\ (1-a)\frac{v_1\bar{t}}{L_l t_0} & 0 & 0 & 0\\ (1-a)\frac{v_1^2\bar{t}^2}{2L_l t_0} & 0 & 0 & 0\\ -(1-a)\frac{v_1\bar{t}}{L_l t_0} & 0 & 0 & 0 \end{bmatrix}, \quad H_i = \begin{bmatrix} 0\\ 0\\ -1\\ 0 \end{bmatrix}$$

$$A_{d2} = \begin{bmatrix} -(1-a)\frac{v_1\bar{t}}{L_l t_0} & 0 & 0 & 0\\ (1-a)\frac{v_1\bar{t}}{L_l t_0} & 0 & 0 & 0\\ (1-a)\frac{\bar{\varphi}v_1^2\bar{t}^2}{2L_l t_0} & 0 & 0 & 0\\ -(1-a)\frac{v_1\bar{t}}{L_l t_0} & 0 & 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0\\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$$\bar{\varphi} = \frac{10t_0}{\pi}, \quad M_i = \begin{bmatrix} 0\\ 0\\ 0\\ 1 \end{bmatrix}, \; N_{wi} = 0.1$$

For simulation purpose, we take the time-varying delay $\tau(t) = 0.3 + 0.1sin(t)$, the external disturbance $w(t) = 4\,sin(t)e^{-0.3t}$, the system nonlinearities $f_1(t) = sin(x_1(t))$ and $f_2(t) = sin(x_1(t-\tau(t)))$. Furthermore we define

$$\begin{aligned}
\bar{h}_1(\bar{\theta}(t)) &= \left(1 - \frac{1}{1+e^{(-3(\bar{\theta}(t)-0.5\pi))}}\right)\left(\frac{1}{1+e^{(-3(\bar{\theta}(t)+0.5\pi))}}\right)\\
\bar{h}_2(\bar{\theta}(t)) &= 1 - \bar{h}_1(\bar{\theta}(t))
\end{aligned}$$

where $\bar{\theta}(t) = x_2(t) + a\frac{v_1\bar{t}}{2L_l}x_1(t) + (1-a)\frac{v_1\bar{t}}{2L_l}x_1(t-\tau(t))$.

By solving the inequality (36), we can obtain that a feasible solution is

$$L_1 = \begin{bmatrix} 1.1570 & 1.1938\\ 1.2789 & -1.0430\\ -2.3716 & 3.6694\\ -0.5632 & 0.3185 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 0.8723 & 1.4911\\ 1.6510 & -1.4270\\ -2.5100 & 3.4389\\ -0.7978 & 0.5554 \end{bmatrix}$$

Here, we set $\mathcal{G} = \begin{bmatrix} -\frac{1}{1.429} & 0 & 0 & 0 \end{bmatrix}$. Thus $\mathcal{G}B = 1$ is nonsingular. As we can see, we need to design an adaptive sliding mode control law $v(t)$ as given in (34) (35) and (36) such that the closed loop system (17) is admissible with $H\infty$ performance. Therefore, we take the matrices

$$\mathcal{S} = \begin{bmatrix} -\frac{1}{1.429} & 0 \end{bmatrix}, \quad \mathcal{T} = \begin{bmatrix} 0.5 & 0\\ 0 & 0\\ 0 & 0\\ -0.5 & 1 \end{bmatrix}$$

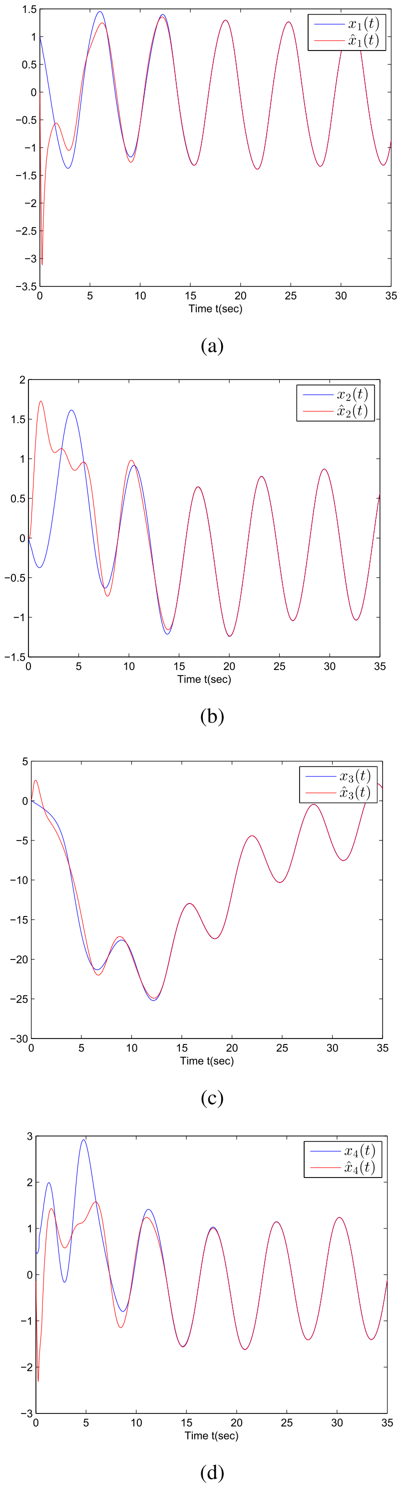


FIGURE 1. **State trajectories. (a) Response and estimation of state $x_1(t)$. (b) Response and estimation of state $x_2(t)$. (c) Response and estimation of state $x_3(t)$.**

and set the initial conditions $\tilde{\varphi}(t) = \begin{bmatrix} 1 \; 0 \; 0 \; 0.509 \end{bmatrix}^T$, $\tilde{\phi}(t) = \begin{bmatrix} 0 \; 0 \; 0.1 \; -0.1365 \end{bmatrix}^T$. The states trajectories of the system (52) and the observer system are displayed in Fig. 1.

It is observed from Fig. 1 that the SMO system can accurately track the original system.

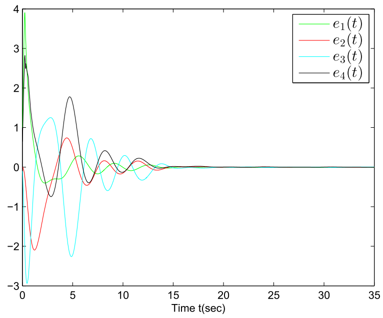


FIGURE 2. **State trajectories of the error system.**

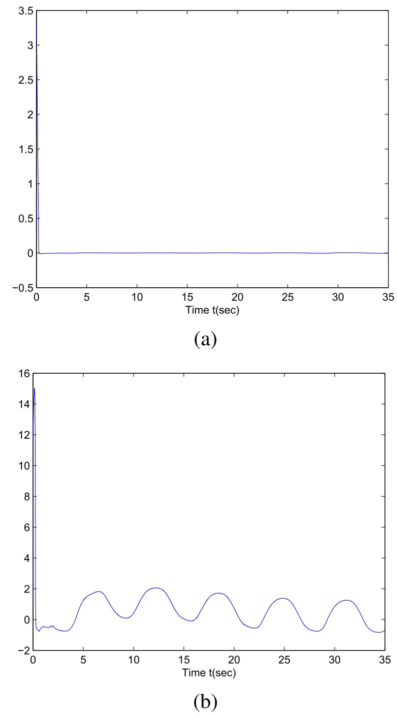


FIGURE 3. **Switching surface and controller. (a) Sliding mode controller $v(t)$. (b) Switching surface function $s(t)$.**

Fig. 2 depicts the trajectories of the error system. It is observed from Figure 2 that the estimation error variables converge to zero quickly. In other words, the error system can be stabilized by the proposed method. Furthermore, Fig. 3 plots the switching surface function $s(t)$ and the sliding mode controller $v(t)$ respectively. Fig. 4 plots the adaptive parameters. Regarding these results, we can conclude that the proposed SMO design method in this paper is feasible.

Example 2: We have mentioned that our method is suitable for normal systems. Therefore, we are going to make a comparison with the different SMO design method in [22]. Consider the following normal system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B(u(t) + f(t, x(t))) + (H + \Delta H)w(t) \\ y(t) &= Cx(t) \end{aligned} \tag{53}$$

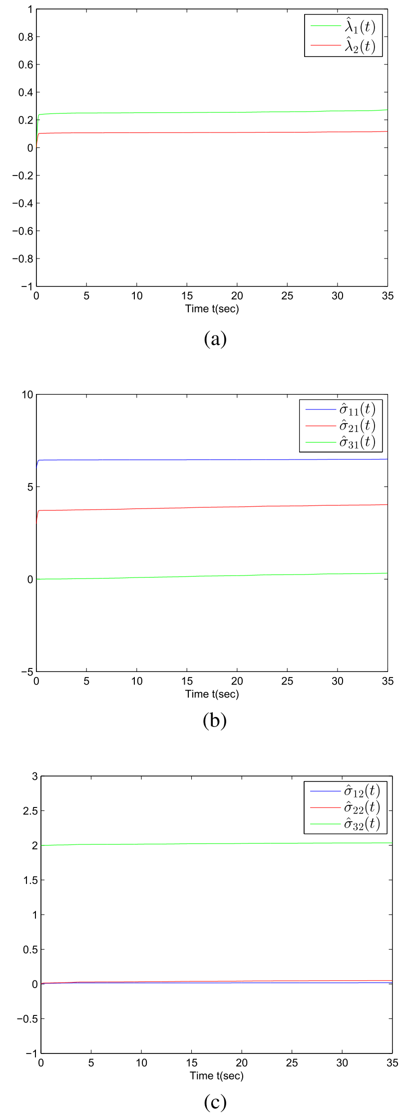


FIGURE 4. Adaptive parameters. (a) $\hat{\lambda}_1(t)$ and $\hat{\lambda}_2(t)$. (b) $\hat{\sigma}_{11}(t)$, $\hat{\sigma}_{21}(t)$ and $\hat{\sigma}_{31}(t)$. (c) $\hat{\sigma}_{12}(t)$, $\hat{\sigma}_{22}(t)$ and $\hat{\sigma}_{32}(t)$.

where

$$A = \begin{bmatrix} -3 & 0 & 0 & -14.7 \\ 0 & 0 & 0 & 1.2 \\ 1 & 15 & 0 & 2 \\ 1 & 0 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

In addition, we take for simulation purpose $f(t, x(t)) = \sin(x_1(t))$. The initial conditions $\tilde{\varphi}(t) = \begin{bmatrix} 12 \; 3.3 \; 2 \; 2 \end{bmatrix}^T$, $\tilde{\phi}(t) = \begin{bmatrix} 0 \; -1 \; 4 \; 1 \end{bmatrix}^T$. In order to compare with the SMO design method in [22], we set $w(t) = 0$. Then the state trajectories of the error systems can be shown in Fig. 5.

From Fig. 5, we can find that the estimation error variables in both methods converge to zero quickly. However, in our method, the estimation error variables converge to zero faster.

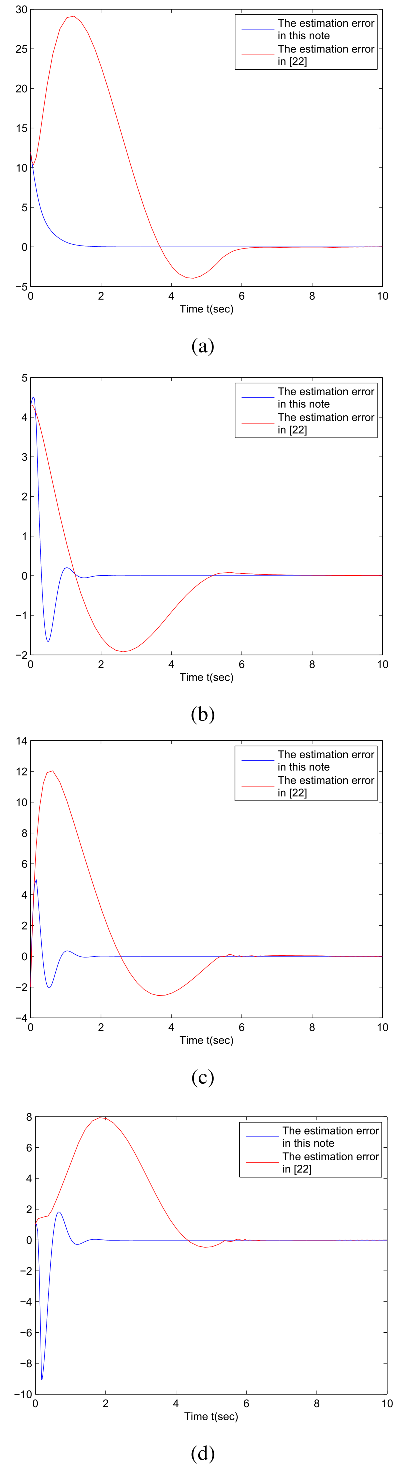


FIGURE 5. State trajectories of the error systems. (a) Estimation error variables $e_1(t)$. (b) Estimation error variables $e_2(t)$. (c) Estimation error variables $e_3(t)$. (d) Estimation error variables $e_4(t)$.

From Fig. 7, it is easy to see that the resulting closed-loop system of (54) is asymptotically stable.

Example 3: In order to illustrate the feasibility of the SMO based SMC method. Consider the following T-S fuzzy

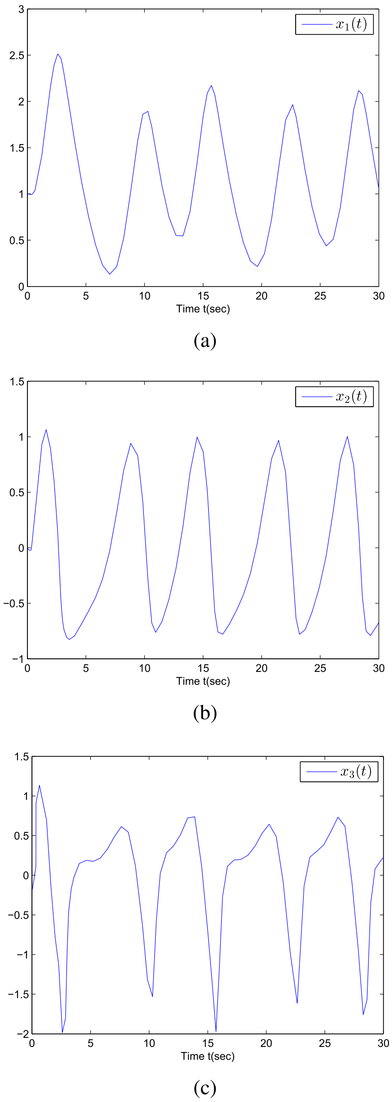


FIGURE 6. **State trajectories of the open-loop system. (a) Response of state $x_1(t)$. (b) Response of state $x_2(t)$. (c) Response of state $x_3(t)$.**

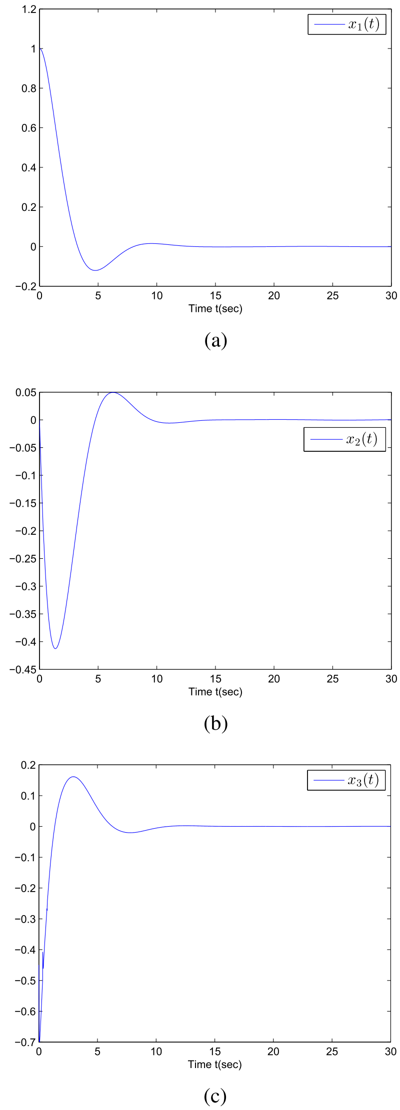


FIGURE 7. **State trajectories of the closed-loop system. (a) Response of state $x_1(t)$. (b) Response of state $x_2(t)$. (c) Response of state $x_3(t)$.**

descriptor system:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{3} \bar{h}_i(\bar{\theta}(t))\{A_i x(t) + A_{di}x(t-\tau(t)) \\ &\quad + B(u(t)+f_i(t)) + (H_i + \Delta H_i)w(t)\} \\ y(t) &= Cx(t) \end{aligned} \tag{54}$$

where

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_1 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -6 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & -6 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

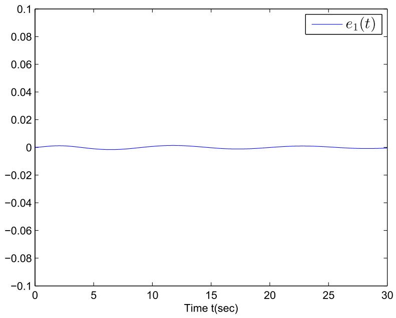
$$A_{di} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad H_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -0.5 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \quad M_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; N_{wi} = 1$$

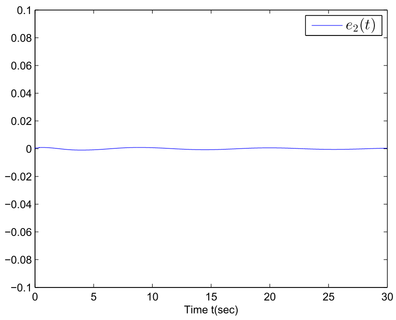
$$\bar{h}_1(\bar{\theta}(t)) = \frac{x_2^2(t)}{6}, \quad \bar{h}_2(\bar{\theta}(t)) = \frac{1+\cos(x_1(t))}{6}$$

$$\bar{h}_3(\bar{\theta}(t)) = 1 - \bar{h}_1(\bar{\theta}(t)) - \bar{h}_2(\bar{\theta}(t))$$

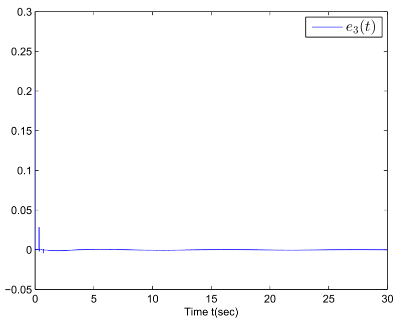
In addition, we assume that the nonlinear functions $f_1(t) = sin(x_1(t))$, $f_2(t) = sin(x_1(t-\tau(t)))$, $f_3(t) = sin(2x_1(t))$, the external disturbance $w(t) = sin(t)e^{-0.3t}$. The time-varying delay $\tau(t) = 0.3 + 0.1\sin(t)$, thus, $\tau_m = 0.2$, $\tau_M = 0.4$ and $\tau_a = 0.1$. Then, we give the initial condition $\tilde{\varphi}(t) = \begin{bmatrix} 1 & 0 & -0.18 \end{bmatrix}^T$, the state trajectories of the open-loop system are displayed in Fig. 6.



(a)



(b)



(c)

FIGURE 8. **State trajectories of the error system. (a) Estimation error variable $e_1(t)$. (b) Estimation error variable $e_2(t)$. (c) Estimation error variable $e_3(t)$.**

It is observed from Fig. 6 that state trajectories of the open-loop system are not converge to zero. In other words, the open-loop system (54) is not stable.

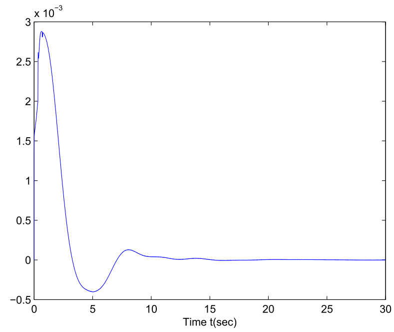
By solving the LMI (49), we can obtain the following feasible solutions of L_i.

$$L_1 = \begin{bmatrix} 0.2235 & -0.2235 \\ -0.3270 & 0.2437 \\ -0.9633 & 0.9205 \end{bmatrix},$$

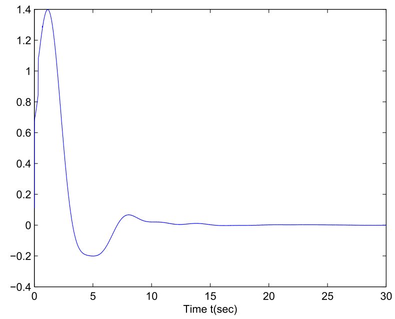
$$L_2 = \begin{bmatrix} -0.0004 & 0.0275 \\ 0.1150 & -0.4695 \\ -1.7823 & 3.6617 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} -0.0526 & 0.0811 \\ -0.3849 & 0.2763 \\ -1.0851 & 1.2368 \end{bmatrix}$$

For convenience, we set $\bar{\mathcal{G}} = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$, $G_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$, $\mathcal{Q}_1 = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix}$, $\mathcal{Q}_2 = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}$, $\mathcal{Q}_3 = \begin{bmatrix} -1 & -3 & -1 \end{bmatrix}$.

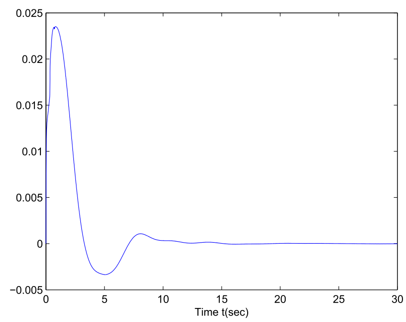


(a)

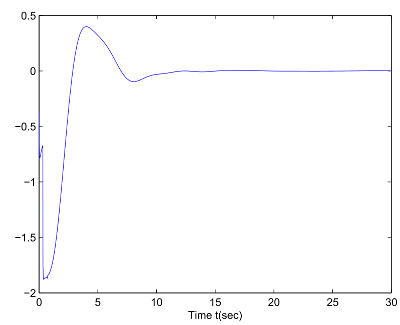


(b)

FIGURE 9. **Switching surface and controller. (a) Switching surface function $s(t)$. (b) Sliding mode controller $v(t)$.**



(a)



(b)

FIGURE 10. **Switching surface and controller. (a) Switching surface function $s_1(t)$. (b) Sliding mode controller $u(t)$.**

Taking the matrices

$$\mathcal{S} = \begin{bmatrix} 0 & 1 \end{bmatrix}, \mathcal{T} = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \\ -1 & 0.5 \end{bmatrix}$$

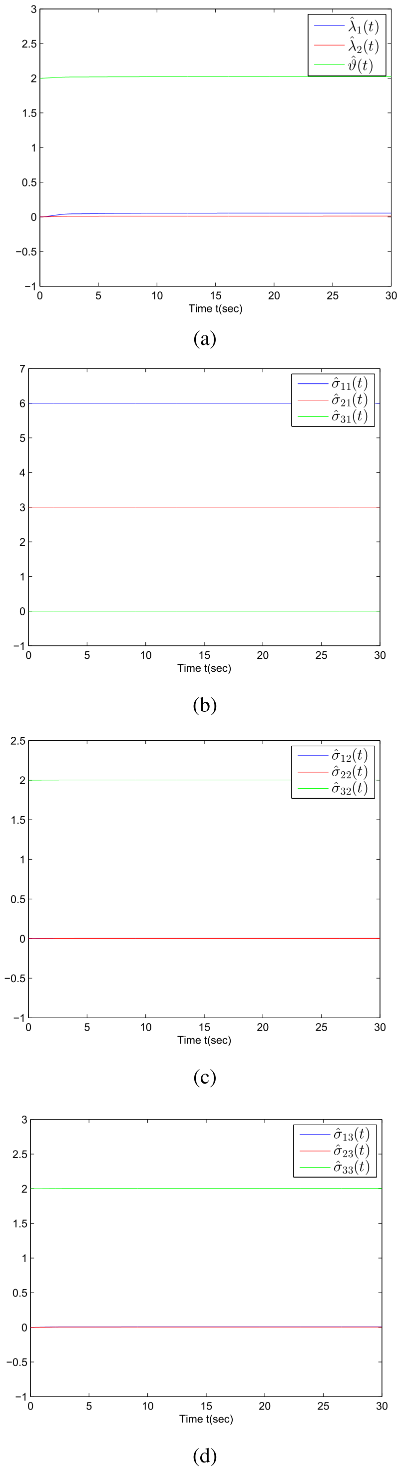


FIGURE 11. **Adaptive parameters. (a) $\hat{\lambda}_1(t)$, $\hat{\lambda}_2(t)$ and $\hat{\vartheta}(t)$. (b) $\hat{\sigma}_{11}(t)$, $\hat{\sigma}_{21}(t)$ and $\hat{\sigma}_{31}(t)$. (c) $\hat{\sigma}_{12}(t)$, $\hat{\sigma}_{22}(t)$ and $\hat{\sigma}_{32}(t)$. (d) $\hat{\sigma}_{13}(t)$, $\hat{\sigma}_{23}(t)$ and $\hat{\sigma}_{33}(t)$.**

Then given the initial condition $\tilde{\phi}(t) = \begin{bmatrix} 1 & 0 & -0.55 \end{bmatrix}^T$, we can obtain the state trajectories of the closed-loop system and the error system which are depicted in Fig. 7 and Fig. 8.

It can be seen from Fig. 8 that the error system is asymptotically stable. Therefore the designed SMO in this paper can successfully track the original system.

Fig. 9 and Fig. 10 plot the switching surface function $s(t)$, the sliding mode controller $v(t)$ and the switching surface function $s_1(t)$, the sliding mode controller $u(t)$ respectively. Fig. 11 plots the adaptive parameters. Regarding these results, we can conclude that the proposed results in this paper is feasible.

VI. CONCLUSION

In this paper, we have presented a robust adaptive SMO design method and a SMO based adaptive SMC strategy for T-S fuzzy descriptor systems with time-varying delay. By taking the SMO gain matrix into account, two integral-type sliding surfaces were constructed. Then some delay-dependent sufficient conditions have been obtained, which guaranteed the sliding motions to be admissible with $H\infty$ performance. New adaptive sliding mode controllers, which need not to use the assumption that the fast subsystem of descriptor system is observable, are synthesized for the error system and the SMO system such that the reachability conditions can be guaranteed. Furthermore, the adaptive control strategy is applied to estimate the unknown parameters especially the bounds of $e(t)$ and $e(t-\tau(t))$. Finally, simulation examples are provided to support our results.

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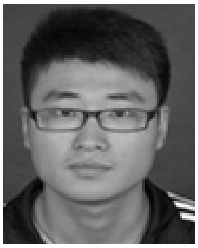

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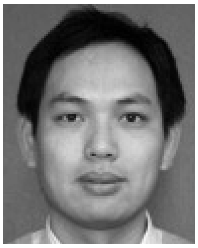

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