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Multi-User CFOs Estimation for SC-FDMA System Over Frequency Selective Fading Channels

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ABSTRACT Frequency synchronization in single-carrier frequency division multiple access (SC-FDMA) uplink system is a challenging task due to the presence of different carrier frequency offsets (CFOs) for different users. In this paper, we propose a blind CFOs estimation algorithm for SC-FDMA uplink system by oversampling method in the presence of frequency selective fading channel. A cost function is derived which minimizes the power of the off-diagonal elements of a signal covariance matrix while estimating the correct CFOs. The off-diagonal elements are nothing but inter-carrier interference and multiple-access interference introduced in the presence of multiple CFOs. A complete mathematical model has been presented for multiple CFOs estimation under multiple access scenarios. The proposed CFOs estimation method does not require channel-state information which results in lower computational complexity. The higher iteration complexity of grid search algorithm has been further reduced by a *deterministic approach*. We also derive the Cramer–Rao bound for CFO estimation. The simulation results show that the proposed CFOs estimation method outperforms the existing subspace theory-based methods, specifically in the low signal-to-noise ratio region.

INDEX TERMS Covariance method, *deterministic approach*, inter-carrier interference (ICI), multiple access interference (MAI), multiple CFOs estimation, single-carrier frequency division multiple access (SC-FDMA).

I. INTRODUCTION

Single-carrier frequency division multiple access (SC-FDMA) has been adopted for the long-term evolution (LTE) uplink transmission due to its low peak to average power ratio property [1]–[3]. It helps in avoiding high signal distortion, simplifying transceiver design and reducing power consumption and thus increases battery life of mobile terminals [4], [5].

Similar to orthogonal frequency division multiplexing (OFDM) based systems, SC-FDMA is also sensitive to carrier frequency offset (CFO) [6]. It happens due to the oscillator mismatch and/or presence of Doppler shift [7]. In case of multi-user uplink SC-FDMA system, the received signal is a combination of multiple signals coming from different users where each user may experience a different CFO due to different hardware setup or different mobility of the transceivers. Because of the imperfect frequency synchronization between multiple mobile stations (or transmitters) and base station (BS), there is a loss of orthogonality among the subcarriers. It results in inter-carrier interference (ICI) for the same user and multiple access interference (MAI) with the other users deteriorating the overall performance of the system considerably [8]. Therefore, estimation and compensation of multiple CFOs are crucial tasks at the BS to improve the system performance. In the presence of multiple CFOs, the carrier frequency synchronization at BS cannot be achieved by simply adjusting the oscillator, as the CFO compensation for one user causes new CFOs for other users [9]. There are several methods that use iterative interference cancelation or successive interference cancelation [10]–[12] to mitigate the effect of ICI and MAI. In [10], successive-iterative ICI cancelation technique is used by exploiting pre-detected data pairs. Soft-in soft-out parallel interference/minimum mean square error (MMSE) multicarrier detector and convolutional channel decoder are introduced in [11] to reduce ICI. Similarly, iterative joint processing of equalization and decoding with the help of Turbo equalizer and maximum a posteriori (MAP) detector are used in [12] for ICI reduction. A low complexity joint equalization and CFO compensation scheme for the SC-FDMA system is presented in [13]. It combines the benefits of MMSE equalizers and removes the noise amplification problem of zero-forcing (ZF). Pragmatic frequency domain equalizer (FDE) receiver is proposed for offset modulation in [14] which minimizes the ISI and in-phase/quadrature-phase interference in the uplink. In case of a severe frequency selective fading channel [15], the phase rotation caused by multiple CFOs is directly estimated and compensated without explicitly estimating the CFOs.

Several data-aided CFO estimation and compensation schemes have been proposed for SC-FDMA, but most of them rely on periodically transmitted pilot symbols [16]–[19]. For instance, Shamaei and Sabbaghian [16] have used pilot signals for estimating CFOs. They have dedicated a part of each SC-FDMA symbol to pilot signals and developed an estimator based on MAP criterion. Block type pilots have been used in [17] to suppress the effect of CFO and phase noise in an SC-FDMA system with space frequency block coding. A similar method is discussed in [18], whereby CFOs are compensated using block type pilots. But the authors have assumed that different users start to communicate with the BS at different symbol periods, and CFO of each user is quasi-static during the pilot block. In [19], constant amplitude zero autocorrelation (*CAZAC*) property of Zadoff-Chu sequence is used for CFO estimation in case of SC-FDMA uplink.

In addition to data-aided estimation, there are blind synchronization methods which utilize the periodic structure of the signal to estimate CFOs [20]–[24]. These methods do not require pilot symbols or training sequences for CFO synchronization and thus improve the spectral efficiency of the transmission [25]–[27]. In [20], the inherent signal structure of the interleaved OFDMA uplink system is observed to obtain a correlation matrix and multiple signal classification (*MUSIC*) approach is used to blindly estimate multiple CFOs by searching peaks of the spectral function. Although *MUSIC* approach has good estimation performance; it fails to separate peaks at low signal-to-noise (SNR) when CFOs of different users are close to each other. In [21], CFO is estimated for OFDMA by utilizing rotational invariance techniques (*ESPRIT*). It has much lower computational complexity than *MUSIC*. A blind CFO estimation method is discussed in [22] which uses least-order cyclic moments based on the inner cyclic period of the signal. A blind maximum likelihood CFO estimation method for interleaved OFDMA is presented in [23], which uses root finding approach for the series expansion of the correlation matrix. However, it utilizes only half of the subchannels for data transmission. In [24], a specific periodic structure of interleaved mapping in SC-FDMA, also known as SC-IFDMA, is utilized to compensate CFOs of all users in the time domain and then the signals are equalized in frequency domain. However, localized mapping for SC-FDMA, also known as SC-LFDMA, does not possess periodic structure in the time domain, hence, a frequency guard band is used between subbands to separate the users. In [28], a filter bank has been used in case of SC-LFDMA to extract the signal of each user and then conventional CFO estimation method [29]–[31] is applied to estimate the CFO of multiple users.

In this paper, we study the effect of multiple CFOs in terms of signal-to-interference-plus-noise ratio (SINR) for SC-LFDMA uplink system. We propose a blind CFOs estimation method with the help of oversampling and *deterministic approach*. The proposed method is based on the fact that the information about CFOs is present in the off-diagonal elements of the received signal covariance matrix in frequency domain. A cost function is derived to minimize the total off-diagonal power produced by ICI and MAI in presence of multiple CFOs. The main contributions of the proposed method are summarized as follows:

- The proposed method works for both interleaved (SC-IFDMA) and localized (SC-LFDMA) schemes, whereas subspace theory based methods *MUSIC* [20], and *ESPRIT* [21] work only for SC-IFDMA.
- It is a blind method and can be applied to the fully loaded systems, which leads to higher bandwidth utilization compared to *CAZAC* [19] and subspace theory based methods.
- It presents the detailed mathematical derivation of each term of the covariance matrix and shows how the non-diagonal elements of the matrix get affected in presence of MAI and ICI.
- The CFO estimation for SC-FDMA is based on the *deterministic approach* which evaluates the cost function at only three points from the CFO range and thus, minimizes the search complexity of the algorithm.
- The Cramer-Rao bound (CRB) for the estimation of CFOs is derived and used to characterize the efficiency of the proposed estimation method.

The rest of the paper is organized as follows. Section II presents the system model and derives the expression for SINR in the presence of multiple CFOs. Section III describes the proposed method for multiple CFOs estimation. Simulation results are provided in Section IV and finally, Section V concludes this work.

Notations: $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$, and $\mathbb{E}[\cdot]$, denote transpose, conjugate, Hermitian, and expectation operation of a matrix respectively. $(\cdot)^k$ denotes kth user. Upper case boldface letters are used for matrices, and lower case boldface letters are used for vectors. $\mathbf{I}_N$ denotes $N \times N$ identity matrix, $\mathbf{0}_{N\times M}$ denotes zero matrix of dimension $N \times M$. diag($\mathbf{a}$) represents a diagonal matrix with its diagonal elements given by vector $\mathbf{a}$. $\mathbf{s}^u_{i,l}$ denotes the element of the ith row and lth column of a matrix $\mathbf{S}$ for uth user, and tr$(\cdot)$ denotes trace of a matrix.

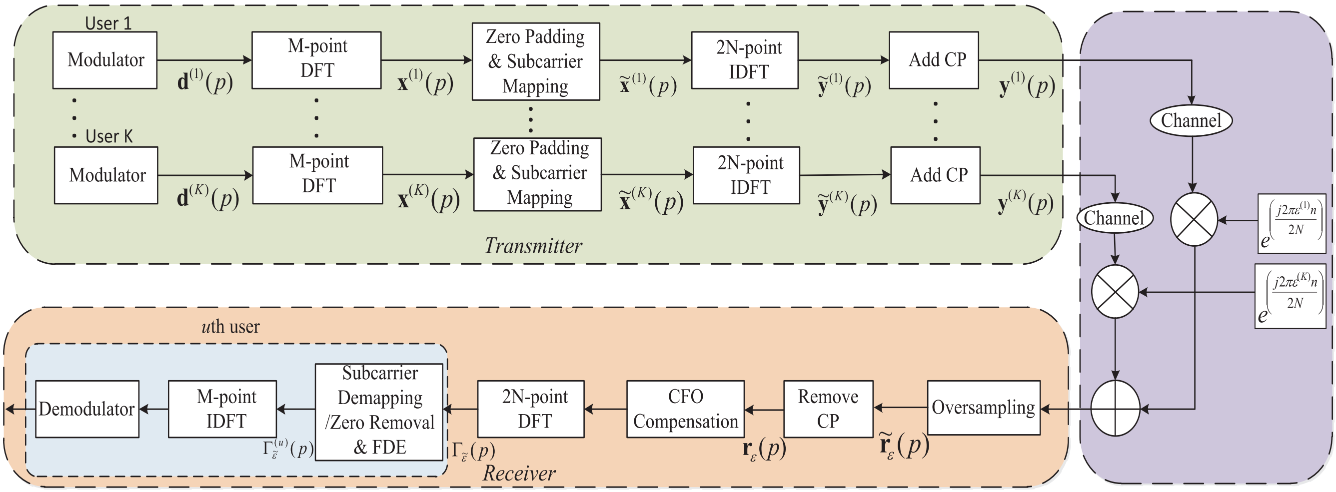

FIGURE 1. SC-FDMA system block diagram in the presence of multiple CFOs .

II. SYSTEM MODEL

We consider a baseband equivalent SC-FDMA uplink transmission system with localized subcarrier mapping as shown in Fig. 1. The system has N subcarriers which can support maximum K users at a time. The N subcarriers are divided into Q ($Q \leq K$) sub-channels, such that each sub-channel consists of $M = N/Q$ subcarriers assigned to each potential user. Let the pth symbol block of kth user be

$$\mathbf{d}^k(p) = \left[d_0^k, d_1^k, \cdots, d_{M-1}^k\right]^T \in \mathbb{C}^M, \tag{1}$$

where $k = 1, 2, \cdots, K$ and d_m^k is the mth information symbol for the kth user drawn from a quadrature phase shift keying (QPSK) constellation with $\mathbb{E}\left[|d_m^k|^2\right] = 1$. $\mathscr{S}^k$ is the set of subcarriers assigned to the kth user satisfying

$$\bigcup_{k=1}^{K} \mathscr{S}^k = \{0, 1, \cdots, N-1\} \quad \text{and } \mathscr{S}^i \bigcap \mathscr{S}^j = \emptyset \text{ for } i \neq j.$$

Here $\mathbf{d}^k(p)$ is processed by M point discrete Fourier transform (DFT) to generate the DFT-spread symbol of the kth user,

$$\mathbf{x}^k(p) = \mathbf{D}^M \mathbf{d}^k(p), \tag{2}$$

where $\mathbf{D}^M$ is M-point DFT matrix.

Oversampling is achieved by padding zeros in the frequency domain of the baseband signal. The zero padding pattern is related to the adopted mapping scheme. For instance, if the frequency mapping is localized, zeros are placed at the edge of contiguous subcarrier frequency block. If the frequency mapping is interleaved, then a zero is padded at the right of every subcarrier allocated for the user. The procedure helps in reducing MAI from neighboring users. Zeros are padded with the help of a matrix represented as $\mathbf{M}_{zp} = [\mathbf{I}_M; \mathbf{0}_M]$ for localized SC-FDMA. Similarly, zero padding matrix can be formulated for interleaved SC-FDMA based on the allocation of subcarriers for the user. Thus, the detailed derivation is omitted for interleaved SC-FDMA. Localized mapping is being used in LTE uplink, and it has the advantage of multi-user diversity [32]. So, the subcarrier mapping matrix of the kth user for localized SC-FDMA can be expressed as

$$\mathbf{M}^k = [\mathbf{0}_{2M(k-1)\times 2M}; \mathbf{I}_{2M}; \mathbf{0}_{2M(K-k)\times 2M}]. \tag{3}$$

The transmitted signal for the pth SC-FDMA block of kth user can be given as

$$\mathbf{y}^k(p) = \mathbf{Z}_{CP}\tilde{\mathbf{D}}^{2N}\mathbf{M}^k\mathbf{M}_{zp}\mathbf{x}^k(p), \tag{4}$$

where $\tilde{\mathbf{D}}^{2N}$ denotes 2N-point inverse-DFT (IDFT) matrix, $\mathbf{x}^k(p)_{N\times 1}$ is the frequency domain signal of the pth block and $\mathbf{Z}_{CP((2N+N_{cp})\times 2N)}$ is a matrix, which adds a cyclic prefix (CP) of length N_{cp}. The length of CP should be greater than or equal to the multipath delay spread of the channel to overcome the effect of inter-symbol interference (ISI), i.e., $N_{cp} \geqslant \max\{L^k\}$ where L^k is the number of multipaths for kth user, $\mathbf{Z}_{CP}$ can be given as

$$\mathbf{Z}_{CP} = [\mathbf{V}, \mathbf{I}_{2N}]^T, \tag{5}$$

where $\mathbf{V} = [\mathbf{0}_{N_{cp}\times(2N-N_{cp})}, \mathbf{I}_{N_{cp}}]^T$ and $\mathbf{I}_{2N}$ is $2N \times 2N$ identity matrix. It is assumed that timing synchronization has been taken care of by the receiver before performing CFOs estimation. Thus, the received signal at the BS is a superposition of signals from all active users and can be obtained as

$$\tilde{\mathbf{r}}(p) = \sum_{k=1}^{K} \mathbf{y}^k(p) \otimes \mathbf{h}^k + \mathbf{w}, \tag{6}$$

where $\otimes$ denotes circular convolution operation, $\mathbf{h}^k$ is frequency selective fading channel coefficients experienced by kth user. These channel taps are modeled as independent and identically distributed (iid) complex zero mean Gaussian random variables which are defined as

$\mathbf{h}^k \triangleq [h_0^k, h_1^k, \cdots, h_{L-1}^k]^T \in \mathbb{C}^L$ and $\mathbf{w}$ is circular complex additive white Gaussian noise (AWGN) with covariance matrix $\sigma^2\mathbf{I}_{2N}$.

When CFO is taken into consideration, the received signal after discarding the CP can be represented as

$$\mathbf{r}_\epsilon(p) = \tilde{Z}_{CP}\tilde{\mathbf{r}}_\epsilon(p) = \sum_{k=1}^{K} \phi(\epsilon^k)\mathbf{E}(\epsilon^k)\mathbf{H}_c^k\tilde{\mathbf{y}}^k(p) + \mathbf{w}, \quad (7)$$

where

- $\tilde{Z}_{CP} = [\mathbf{0}_{2N\times N_{cp}}, I_{2N}]$ is the matrix used to remove CP.
- ϵ^k is the normalized CFO of the kth user defined as the ratio between actual CFO and subcarrier spacing such that $\epsilon^k \in (-0.5, 0.5)$.
- The phase shift due to discarding of $P-1$ number of symbol blocks and removal of CP can be expressed as
$$\phi(\epsilon^k) = e^{j2\pi\epsilon^k\{(p-1)(2N+N_{cp})+N_{cp}\}/2N}. \quad (8)$$
- The CFO matrix for kth user can be defined as
$$\mathbf{E}(\epsilon^k) = \text{diag}\left(e^{j2\pi\epsilon^k n/2N}\right), \quad n = 0, 1, \cdots, 2N-1. \quad (9)$$
- $\mathbf{H}_c^k$ is channel circulant matrix of size $2N \times 2N$, with its first column as $\tilde{\mathbf{h}}^k = \left[h_0^k, h_1^k, \cdots, h_{L-1}^k, \mathbf{0}_{(2N-L)\times 1}\right]^T$.

Circulant channel matrix has a property that it gives a diagonal matrix when pre-multiplied by DFT and post-multiplied by IDFT matrix, i.e., $\mathbf{D}^{2N}\mathbf{H}_c^k\tilde{\mathbf{D}}^{2N} = \Pi^k$, where Π^k is a diagonal matrix whose elements are DFT coefficient of the channel impulse response.

The CP removed signal is converted into frequency domain by applying $2N$-point DFT, i.e.,

$$\begin{aligned}\Gamma(p) &= \sum_{k=1}^{K} \phi(\epsilon^k)\mathbf{D}^{2N}\mathbf{E}(\epsilon^k)\mathbf{H}_c^k\tilde{\mathbf{y}}^k(p) + \mathbf{w}_F \\ &= \sum_{k=1}^{K} \phi(\epsilon^k)\tilde{\mathbf{E}}(\epsilon^k)\Pi^k\tilde{\mathbf{x}}^k(p) + \mathbf{w}_F, \end{aligned} \quad (10)$$

where $\tilde{\mathbf{E}}(\epsilon^k) = \mathbf{D}^{2N}\mathbf{E}(\epsilon^k)\tilde{\mathbf{D}}^{2N}$ is a circulant matrix and $\mathbf{w}_F = \mathbf{D}^{2N}\mathbf{w}$. After performing equalization and subcarrier demapping, the detected signal for the desired uth ($u = 1, 2, \cdots, K$) user can be written as

$$\begin{aligned}\hat{\mathbf{d}}^u(p) = \tilde{\mathbf{D}}^M\tilde{\mathbf{M}}_{zp}\tilde{\mathbf{M}}^u\tilde{\mathbf{H}}^u\Big(&\phi(\epsilon^u)\tilde{\mathbf{E}}(\epsilon^u)\Pi^u\mathbf{M}^u\mathbf{M}_{zp}\mathbf{D}^M\mathbf{d}^u(p) \\ &+ \sum_{\substack{k=1\\k\neq u}}^{K} \phi(\epsilon^k)\tilde{\mathbf{E}}(\epsilon^k)\Pi^k\mathbf{M}^k\mathbf{M}_{zp}\mathbf{D}^M\mathbf{d}^k(p) + \mathbf{w}_F\Big), \end{aligned} \quad (11)$$

where $\tilde{\mathbf{M}}^u$ is demapping matrix for uth user defined as $\tilde{\mathbf{M}}^u = (\mathbf{M}^u)^T$, $\tilde{\mathbf{M}}_{zp}$ is zero removing matrix which has been obtained due to oversampling the signal, $\tilde{\mathbf{H}}^u$ is the diagonal MMSE equalizer matrix of the uth user obtained as

$$\tilde{\mathbf{H}}_{n,n}^u = \frac{(\mathbf{H}_{n,n}^u)^*}{|\mathbf{H}_{n,n}^u|^2 + \frac{1}{\text{SNR}}}, \quad (12)$$

where $\mathbf{H}^u$ is a diagonal matrix defined as

$$\mathbf{H}_{n,n}^u = \sum_{l=0}^{L-1} h_l^u e^{\frac{-j2\pi(i-1)l}{N}}. \quad (13)$$

Hence, the estimated time domain data symbols of the uth user can be given as

$$\hat{\mathbf{d}}^u = \mathbf{A}^u\mathbf{d}^u + \bar{\mathbf{A}}^u\mathbf{d}^u + \sum_{\substack{k=1\\k\neq u}}^{K} \mathbf{B}^k\mathbf{d}^k + \tilde{\mathbf{w}}, \quad (14)$$

where the symbols in (14) are given as follows

$$\begin{aligned}\mathbf{A}^u &= \text{diag}(\phi(\epsilon^u)\tilde{\mathbf{D}}^M\tilde{\mathbf{M}}_{zp}\tilde{\mathbf{M}}^u\tilde{\mathbf{H}}^u\tilde{\mathbf{E}}(\epsilon^u)\Pi^u\mathbf{M}^u\mathbf{D}^M), \\ \bar{\mathbf{A}}^u &= \phi(\epsilon^u)\tilde{\mathbf{D}}^M\tilde{\mathbf{M}}_{zp}\tilde{\mathbf{M}}^u\tilde{\mathbf{H}}^u\tilde{\mathbf{E}}(\epsilon^u)\Pi^u\mathbf{M}^u\mathbf{D}^M - \mathbf{A}^u, \\ \mathbf{B}^k &= \phi(\epsilon^k)\tilde{\mathbf{D}}^M\tilde{\mathbf{M}}_{zp}\tilde{\mathbf{M}}^u\tilde{\mathbf{H}}^u\tilde{\mathbf{E}}(\epsilon^k)\Pi^k\mathbf{M}^k\mathbf{D}^M, \\ \tilde{\mathbf{w}} &= \tilde{\mathbf{D}}^M\tilde{\mathbf{M}}_{zp}\tilde{\mathbf{M}}^u\tilde{\mathbf{H}}^u\mathbf{w}_F. \end{aligned} \quad (15)$$

Here detection and decoding are carried out in the time domain. It can be observed from (14) that the first term corresponds to the desired data. The second term corresponds to ICI caused by the other subcarriers of the same uth user. The third term is an MAI which occurs due to loss of orthogonality among the subcarriers of the desired user with the other users, and the fourth term is noise. So, SINR on mth symbol for the uth user's data can be formulated as

$$\text{SINR}_m^u(p) = \frac{|a_{m,m}^u|^2}{\sum_{\substack{\tilde{m}=1\\\tilde{m}\neq m}}^{M} |\bar{a}_{m,\tilde{m}}^u|^2 + \sum_{\substack{k=1\\k\neq u}}^{K}\sum_{\tilde{m}=1}^{M} |b_{m,\tilde{m}}^k|^2 + \sigma^2}. \quad (16)$$

It can be observed that the SINR can be increased if denominator of (16) is minimized which is nothing but the terms contributing to off-diagonal elements of the covariance matrix of (14). Thus, the objective can be fulfilled if phase shift because of multiple CFOs in the off-diagonal elements is compensated by introducing trial CFOs at the receiver.

III. PROPOSED METHOD

In the proposed estimator, phase shift because of multiple CFOs is being compensated to maximize the SINR or to minimize the off-diagonal elements of the covariance matrix. Let the trial CFO be denoted by $\tilde{\epsilon}$, then the compensated received signal vector after CP removal and DFT operation is given as

$$\Gamma_{\tilde{\epsilon}}(p) = \phi^*(\tilde{\epsilon})\mathbf{D}^{2N}\mathbf{E}^*(\tilde{\epsilon})\mathbf{r}_\epsilon(p). \quad (17)$$

After performing the localized demapping operation for the uth user, (17) can be expressed as

$$\begin{aligned}\Gamma_{\tilde{\epsilon}}^u(p) = \phi^*(\tilde{\epsilon})\tilde{\mathbf{M}}^u\mathbf{D}^{2N}\mathbf{E}^*(\tilde{\epsilon})\Big(&\phi(\epsilon^u)\mathbf{E}(\epsilon^u)\mathbf{H}_c^u\tilde{\mathbf{y}}^u(p) \\ &+ \sum_{\substack{k=1\\k\neq u}}^{K} \phi(\epsilon^k)\mathbf{E}(\epsilon^k)\mathbf{H}_c^k\tilde{\mathbf{y}}^k(p) + \mathbf{w}\Big). \end{aligned} \quad (18)$$

To find the covariance matrix of compensated SC-FDMA signal from (18), we have

$$Cov(\Gamma_{\tilde{\epsilon}}^{u}(p)) = \mathbb{E}[\Gamma_{\tilde{\epsilon}}^{u}(p)(\Gamma_{\tilde{\epsilon}}^{u}(p))^{H}] = \mathbf{S}^{u}(\tilde{\epsilon}). \quad (19)$$

When CFO is perfectly compensated for the desired user, i.e., $\tilde{\epsilon} = \epsilon^{u}$, ICI becomes zero for that user and MAI gets reduced on account of oversampling the signal with the help of zero padding pattern described in the system model. But when CFO is not compensated perfectly, there are ICI and MAI which cause off-diagonal elements to be non-zero. Hence, we have defined a cost function to minimize the total off-diagonal power,

$$\Lambda^{u}(\tilde{\epsilon}) = \text{Total off-diagonal power of } \mathbf{S}^{u}(\tilde{\epsilon}). \quad (20)$$

Then, CFO can be estimated by finding $\tilde{\epsilon}$ that minimizes the value of cost function $\Lambda^{u}(\tilde{\epsilon})$. Therefore, the estimated CFO for the uth user can be obtained as

$$\hat{\epsilon}^{u} = \arg \min_{\tilde{\epsilon}} \Lambda^{u}(\tilde{\epsilon}), \quad (21)$$

where $\tilde{\epsilon}$ is the trial CFO value of uth user. For the desired user, MAI from other users is considered as noise. We have considered K users in the system, so, there will be K different CFOs, the estimation of CFOs of all other users can be performed in the similar way.

A. SIGNAL COVARIANCE MATRIX

Alternatively, to find the covariance of $\Gamma_{\tilde{\epsilon}}^{u}(p)$, (18) can be expressed as the sum of three terms,

$$\text{Cov}(\Gamma_{\tilde{\epsilon}}^{u}(p)) = \mathbb{E}\left[(T_1 + T_2 + T_3)(T_1 + T_2 + T_3)^{H}\right]. \quad (22)$$

where

$$T_1 = \phi(\epsilon^{u} - \tilde{\epsilon})\tilde{\mathbf{M}}^{u}\mathbf{D}^{2N}\mathbf{E}(\epsilon^{u} - \tilde{\epsilon})\mathbf{H}_{c}^{u}\tilde{\mathbf{y}}^{u}(p),$$

$$T_2 = \tilde{\mathbf{M}}^{u}\mathbf{D}^{2N}\sum_{\substack{k=1\\k\neq u}}^{K}\phi(\epsilon^{k} - \tilde{\epsilon})\mathbf{E}(\epsilon^{k} - \tilde{\epsilon})\mathbf{H}_{c}^{k}\tilde{\mathbf{y}}^{k}(p),$$

and,

$$T_3 = \phi^{*}(\tilde{\epsilon})\tilde{\mathbf{M}}^{u}\mathbf{D}^{2N}\mathbf{E}^{*}(\tilde{\epsilon})\mathbf{w}.$$

Thus, the covariance matrix of the first term can be expressed as

$$\mathbb{E}[T_1T_1^{H}] = \tilde{\mathbf{M}}^{u}\mathbf{D}^{2N}\mathbf{E}(\epsilon^{u} - \tilde{\epsilon})\mathbf{H}_{c}^{u}\mathbf{H}_{c}^{uH}\mathbf{E}^{*}(\epsilon^{u} - \tilde{\epsilon}) \times \mathbf{D}^{2N^{H}}\tilde{\mathbf{M}}^{uH}. \quad (23)$$

Similarly, the covariance matrices corresponding to the second and the third term can be obtained as

$$\mathbb{E}[T_2T_2^{H}] = \tilde{\mathbf{M}}^{u}\mathbf{D}^{2N}\sum_{\substack{k=1\\k\neq u}}^{K}\left\{\mathbf{E}(\epsilon^{k} - \tilde{\epsilon})\mathbf{H}_{c}^{k}\mathbf{H}_{c}^{k^{H}}\mathbf{E}^{*}(\epsilon^{k} - \tilde{\epsilon})\right\} \times \mathbf{D}^{2N^{H}}\tilde{\mathbf{M}}^{uH}, \quad (24)$$

and

$$\mathbb{E}[T_3T_3^{H}] = \sigma^{2}\tilde{\mathbf{M}}^{u}\mathbf{D}^{2N}\mathbf{D}^{2N^{H}}\tilde{\mathbf{M}}^{uH} = \sigma^{2}\mathbf{I}_{2N}. \quad (25)$$

Since users are independent, the covariance of cross-product terms of different users is zero, i.e.,

$$\mathbb{E}[T_1T_2] = \mathbb{E}[T_1T_3] = \mathbb{E}[T_2T_3] = 0. \quad (26)$$

Thus, the covariance matrix of the compensated received signal for the uth user is obtained as

$$\mathbf{S}^{u}(\tilde{\epsilon}) = \tilde{\mathbf{M}}^{u}\mathbf{D}^{2N}\Bigg[\mathbf{E}(\epsilon^{u} - \tilde{\epsilon})\mathbf{H}_{c}^{u}\mathbf{H}_{c}^{uH}\mathbf{E}^{*}(\epsilon^{u} - \tilde{\epsilon}) + \sum_{\substack{k=1\\k\neq u}}^{K}\mathbf{E}(\epsilon^{k} - \tilde{\epsilon})\mathbf{H}_{c}^{k}\mathbf{H}_{c}^{k^{H}}\mathbf{E}^{*}(\epsilon^{k} - \tilde{\epsilon}) + \sigma^{2}\mathbf{I}_{2N}\Bigg] \times \mathbf{D}^{2N^{H}}\tilde{\mathbf{M}}^{uH}. \quad (27)$$

When the CFO is perfectly compensated for the desired user, i.e., $\tilde{\epsilon} = \epsilon^{u}$, $\mathbf{E}(\epsilon^{u} - \tilde{\epsilon})$ becomes an identity matrix and (27) converts to

$$\mathbf{S}^{u}(\tilde{\epsilon}) = \tilde{\mathbf{M}}^{u}\mathbf{D}^{2N}\left(\mathbf{H}_{c}^{u}\mathbf{H}_{c}^{uH}\right)\mathbf{D}^{2N^{H}}\tilde{\mathbf{M}}^{uH} + \tilde{\mathbf{M}}^{u}\mathbf{D}^{2N}\sum_{\substack{k=1\\k\neq u}}^{K}\mathbf{E}(\epsilon^{k} - \epsilon^{u})\left(\mathbf{H}_{c}^{k}\mathbf{H}_{c}^{k^{H}}\right)\mathbf{E}^{*}(\epsilon^{k} - \epsilon^{u}) \times \mathbf{D}^{2N^{H}}\tilde{\mathbf{M}}^{uH} + \sigma^{2}\mathbf{I}_{2M}. \quad (28)$$

It has been observed from (28) that the first term of covariance matrix $\mathbf{S}^{u}(\tilde{\epsilon})$ is a diagonal matrix which shows the absence of ICI. The second term indicates the presence of MAI and the third term corresponds to the noise power which is represented by a diagonal matrix.

B. EXPRESSION FOR THE SIGNAL COVARIANCE MATRIX TERMS

In order to obtain the expression for the elements of covariance matrix of uth user, we start with an example: Let the total number of subcarriers $N = 8$, subcarriers per user $M = 2$, thus the total number of users $K = N/M = 4$. The total number of sub-channels is equal to the number of users ($K = Q$) and each user occupies only one sub-channel in one SC-FDMA block. Let kth sub-channel be assigned to the kth user. We have assumed a four-tap multipath channel $\mathbf{h}^{k} = [h_1^{k}, h_2^{k}, h_3^{k}, h_4^{k}]$. We have not included Gaussian noise component in the covariance matrix as its off-diagonal elements are zero.

Case 1: Let's say the desired user be the first user, i.e., $u = 1$, so it can be assigned to the first sub-channel.

By taking respective values for the first user in (27) and simplifying it, covariance matrix elements, $s_{i,l}^{u}$, can be obtained as

$$s_{1,1}^{1} = \frac{1}{2N}\sum_{k=1}^{4}\sum_{n=0}^{2N-1} e^{\frac{j2\pi n}{2N}(\tilde{\epsilon}-\epsilon^{k})}\bar{\mathbf{h}}^{k}\Omega^{k}(1,1)^{\{-n\}}\bar{\mathbf{h}}^{k^{H}}$$

$$s_{1,2}^{1} = \frac{1}{2N}\sum_{k=1}^{4}\sum_{n=0}^{2N-1} e^{\frac{-j2\pi n}{2N}} e^{\frac{j2\pi n}{2N}(\tilde{\epsilon}-\epsilon^{k})}\bar{\mathbf{h}}^{k}\Omega^{k}(1,1)^{\{-n\}}\bar{\mathbf{h}}^{k^{H}}$$

$$s_{2,1}^{1} = \frac{1}{2N}\sum_{k=1}^{4}\sum_{n=0}^{2N-1} e^{\frac{j2\pi n}{2N}(\tilde{\epsilon}-\epsilon^{k})}\bar{\mathbf{h}}^{k}\Omega^{k}(2,1)^{\{-n\}}\bar{\mathbf{h}}^{k^{H}}$$
$$s_{2,2}^{1} = \frac{1}{2N}\sum_{k=1}^{4}\sum_{n=0}^{2N-1} e^{\frac{-j2\pi n}{2N}} e^{\frac{j2\pi n}{2N}(\tilde{\epsilon}-\epsilon^{k})}\bar{\mathbf{h}}^{k}\Omega^{k}(2,1)^{\{-n\}}\bar{\mathbf{h}}^{k^{H}} \tag{29}$$

where the above notations are defined as follows:

- $\bar{\mathbf{h}}^{k}$ is a $1 \times 2N$ row vector comprising the complex channel coefficients, if the length of channel matrix $L < 2N$ then $2N - L$ zeros need to be padded.
- $\Omega^{k}(i,u)$ is a circulant matrix with its first column as $\left[\omega_{0}^{k}(i,u), \omega_{1}^{k}(i,u), \ldots, \omega_{N-1}^{k}(i,u)\right]$ and it is defined as,

$$\Omega^{k}(i,u) = \begin{bmatrix} \omega_{0}^{k}(i,u) & \omega_{2N-1}^{k}(i,u) & \cdots & \omega_{1}^{k}(i,u) \\ \omega_{1}^{k}(i,u) & \omega_{0}^{k}(i,u) & \cdots & \omega_{2}^{k}(i,u) \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{2N-1}^{k}(i,u) & \omega_{2N-2}^{k}(i,u) & \cdots & \omega_{0}^{k}(i,u) \end{bmatrix} \tag{30}$$

where

$$\omega_{g}^{k}(i,u) = e^{-\frac{j2\pi g}{2NK}\{K((\tilde{\epsilon}-\epsilon^{k})-(i-1))-2N(u-1)\}},$$

$0 \leqslant g \leqslant 2N-1$, and $1 \leqslant i \leqslant 2M$.

- In the symbol $\Omega^{k}(i,u)^{\{-n\}}$, superscript $\{-n\}$ denotes the circular shift of n elements. For an example, if $n = 1$, $\Omega^{k}(i,u)^{\{-1\}}$ is a circulant matrix whose first column will be $\left[\omega_{1}^{k}(i,u), \omega_{2}^{k}(i,u), \cdots, \omega_{2N-1}^{k}(i,u), \omega_{0}^{k}(i,u)\right]$.

It can be observed from (29) that if only column index increases by a unit in the covariance matrix, $e^{-j2\pi n/2N}$ appears as a product in each term of the summation for that column element. Similarly, when only row index increases by a unit, $e^{j2\pi g/2N}$ gets included as a product in each term of the circulant matrix, $\Omega^{k}(i,u)$.

Therefore, the generalized expression of the covariance matrix elements for the first user can be given by

$$s_{i,l}^{1} = \frac{1}{2N}\sum_{k=1}^{4}\sum_{n=0}^{2N-1} e^{-\frac{j2\pi(l-1)n}{2N}} e^{\frac{j2\pi n}{2N}(\tilde{\epsilon}-\epsilon^{k})}\bar{\mathbf{h}}^{k}\Omega^{k}(i,1)^{\{-n\}}\bar{\mathbf{h}}^{k^{H}}, \tag{31}$$

where row $1 \leqslant i \leqslant 2M$ and column $1 \leqslant l \leqslant 2M$.

Case 2: For the second user, i.e., $u = 2$, the user can be assigned to the second sub-channel

$$s_{i,l}^{2} = \frac{1}{2N}\sum_{k=1}^{4}\sum_{n=0}^{2N-1} e^{-\frac{j2\pi n}{K}} e^{-\frac{j2\pi(l-1)n}{2N}} e^{\frac{j2\pi n}{2N}(\tilde{\epsilon}-\epsilon^{k})}\bar{\mathbf{h}}^{k} \times \Omega^{k}(i,2)^{\{-n\}}\bar{\mathbf{h}}^{k^{H}}. \tag{32}$$

For this case, we observe that when the user index increases by a unit, $e^{-j2\pi n/K}$ gets multiplied with each term of the summation for the element in covariance matrix and all elements of the circulant matrix gets multiplied by $e^{j2\pi g/K}$.

So, we can obtain the generalized expression for the covariance matrix terms for the uth user as

$$s_{i,l}^{u} = \frac{1}{2N}\sum_{k=1}^{K}\sum_{n=0}^{2N-1} e^{\frac{j2\pi n}{2NK}\{K((\tilde{\epsilon}-\epsilon^{k})-(l-1))-2N(u-1)\}}\bar{\mathbf{h}}^{k} \times \Omega^{k}(i,u)^{\{-n\}}\bar{\mathbf{h}}^{k^{H}} + s_{w}, \tag{33}$$

where s_{w} is noise covariance matrix terms, given as

$$s_{w} = \begin{cases} \sigma^{2}, & \text{if } i = l \\ 0, & \text{otherwise}. \end{cases}$$

Thus, the total off-diagonal power,

$$\Lambda^{u}(\tilde{\epsilon}) = \sqrt{\sum_{i=1}^{2M}\sum_{\substack{l=1\\ l\neq i}}^{2M}\left|s_{i,l}^{u}(\tilde{\epsilon})\right|^{2}}. \tag{34}$$

Since a covariance matrix is symmetric, we can use either upper triangle or lower triangle to find the total off-diagonal power. The cost function is obtained after receiving P number of SC-FDMA symbols, so the resulting sample cost function can be written as

$$\Lambda_{P}^{u}(\tilde{\epsilon}) = \sqrt{2\sum_{i=1}^{2M}\sum_{l=i+1}^{2M}\left|\mathbf{s}_{P(i,l)}^{u}(\tilde{\epsilon})\right|^{2}}. \tag{35}$$

Therefore, the estimated CFO is obtained by

$$\hat{\epsilon}^{u} = \arg\min_{\tilde{\epsilon}} \Lambda_{P}^{u}(\tilde{\epsilon}). \tag{36}$$

Note that solving (36) by a grid search method would suffer from high computational complexity. It is because in order to get a good estimate of CFO we need to evaluate the cost function iteratively with a minimum step size of searching parameter $\tilde{\epsilon}$. Thus, we have adopted the *deterministic approach* [30] for which the cost function follows the sinusoidal form. Fig. 2 presents cost function $\Lambda^{1}(\tilde{\epsilon})$ for the first user at different SNRs. It is clear that the cost function is almost similar to the sinusoidal function at 10 dB SNR. So, CFO can be deterministically estimated by utilizing the property of sinusoidal function where we have calculated cost function at three points, i.e., at 0, 0.25, and 0.5. From these points, a complex variable z is defined as follows,

$$z = \left\{\frac{\Lambda_{P}^{u}(0.5)-\Lambda_{P}^{u}(0)}{2}\right\} + j\left\{\frac{\Lambda_{P}^{u}(0)+\Lambda_{P}^{u}(0.5)}{2} - \Lambda_{P}^{u}(0.25)\right\}, \tag{37}$$

then the CFO can be deterministically estimated as,

$$\hat{\epsilon}^{u} = \frac{1}{2\pi}\measuredangle\{z\}, \tag{38}$$

where $\measuredangle\{z\}$ denotes the phase of complex number z.

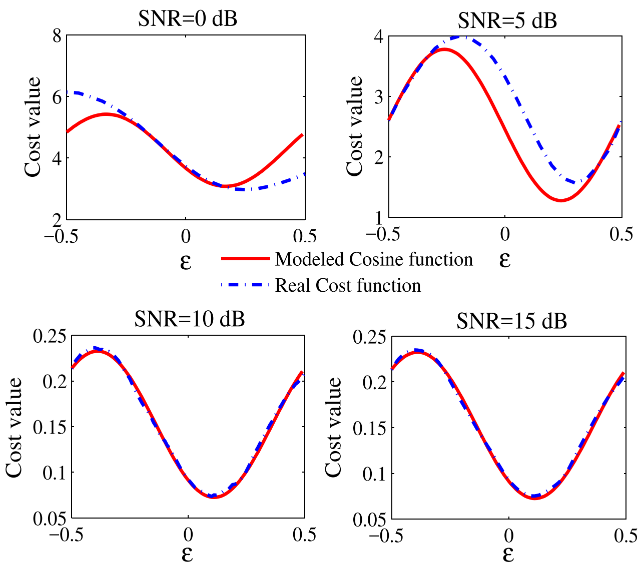

FIGURE 2. Comparison between modeled cosine function and real cost function $\Lambda_P^1(\bar{\epsilon})$ of user 1 with CFO of $\epsilon^1 = 0.1$ at SNR= 0 dB, 5 dB, 10, and 15 dB.

C. CRB ANALYSIS

We assume that a large number of subcarriers is taken so that the time domain signal can be considered as white Gaussian sequence. The probability density function of the received signal $\mathbf{r}_\epsilon$ is given as [33], [34]

$$f(\mathbf{r}_\epsilon, \epsilon) = ((2\pi)^{2N} \det(\mathbf{R}_{\mathbf{r}_\epsilon}))^{-1/2} \exp\left(-\frac{1}{2}\mathbf{r}_\epsilon^H \mathbf{R}_{\mathbf{r}_\epsilon}^{-1} \mathbf{r}_\epsilon\right). \tag{39}$$

The autocorrelation matrix, $\mathbf{R}_{\mathbf{r}_\epsilon}$ of the received signal is defined as

$$\begin{aligned}\mathbf{R}_{\mathbf{r}_\epsilon} &= \mathbb{E}\{\mathbf{r}_\epsilon \mathbf{r}_\epsilon^H\} \\ &= \mathbb{E}\left\{\left(\sum_{k=1}^{K} \bar{\Phi}(\epsilon^k)\mathbf{s}^k\right)\left(\sum_{k=1}^{K} \bar{\Phi}(\epsilon^k)\mathbf{s}^k\right)^H\right\} \\ &= \sum_{k=1}^{K} \bar{\Phi}(\epsilon^k)\mathbf{R_s}\bar{\Phi}(\epsilon^k)^H,\end{aligned} \tag{40}$$

where $\mathbf{R}_\mathbf{s}^k = \mathbb{E}\{\mathbf{s}^k {\mathbf{s}^k}^H\}$, $\mathbf{s}^k = \mathbf{H}_c^k \tilde{\mathbf{y}}^k$, and $\bar{\Phi}(\epsilon^k) = \phi(\epsilon^k)\mathbf{E}(\epsilon^k)$.

The log-likelihood function can be given as

$$\ln(f(\mathbf{r}_\epsilon, \epsilon)) = -N \ln 2\pi - \frac{1}{2}\ln\det(\mathbf{R}_{\mathbf{r}_\epsilon}) - \frac{1}{2}\mathbf{r}_\epsilon^H \mathbf{R}_{\mathbf{r}_\epsilon}^{-1}\mathbf{r}_\epsilon, \tag{41}$$

we can express determinant of $\mathbf{R}_{\mathbf{r}_\epsilon}$ as

$$\begin{aligned}\det(\mathbf{R}_{\mathbf{r}_\epsilon}) &= \sum_{k=1}^{K} \det(\bar{\Phi}(\epsilon^k))\det(\mathbf{R_s})\det(\bar{\Phi}(\epsilon^k)^H) \\ &= \sum_{k=1}^{K} \det(\mathbf{R_s}).\end{aligned} \tag{42}$$

After dropping the terms independent of ϵ^k from (41), the log-likelihood function is reduced to

$$\ln(f(\mathbf{r}_\epsilon, \epsilon)) = -\frac{1}{2}\mathbf{r}_\epsilon^H \mathbf{R}_{\mathbf{r}_\epsilon}^{-1}\mathbf{r}_\epsilon. \tag{43}$$

Fisher information matrix can be given as [34]

$$\mathbf{F}_{u,v} = -\mathbb{E}\left\{\frac{\partial^2 \ln(f(\mathbf{r}_\epsilon, \epsilon))}{\partial\epsilon^u \partial\epsilon^v}\right\} = \frac{1}{2}\,\mathrm{tr}\left(\mathbf{R}_{\mathbf{r}_\epsilon}^{-1}\frac{\partial \mathbf{R}_{\mathbf{r}_\epsilon}}{\partial\epsilon^u}\mathbf{R}_{\mathbf{r}_\epsilon}^{-1}\frac{\partial \mathbf{R}_{\mathbf{r}_\epsilon}}{\partial\epsilon^v}\right) \tag{44}$$

where $1 \le u, v \le K$ and

$$\frac{\partial \mathbf{R}_{\mathbf{r}_\epsilon}}{\partial\epsilon^u} = \frac{\partial\bar{\Phi}(\epsilon^u)}{\partial\epsilon^u}\mathbf{R_s}\bar{\Phi}^H(\epsilon^u) + \bar{\Phi}(\epsilon^u)\mathbf{R_s}\frac{\partial\bar{\Phi}^H(\epsilon^u)}{\partial\epsilon^u}, \tag{45}$$

$$\frac{\partial\bar{\Phi}(\epsilon^u)}{\partial\epsilon^u} = j2\pi\,\mathrm{diag}\left(\left(\frac{n+N_{cp}}{2N}\right)e^{j2\pi\epsilon^u(n+N_{cp})/2N}\right). \tag{46}$$

So, we obtain CRB for ϵ^i as follows

$$\mathrm{CRB}(\epsilon^i) = [F^{-1}]_{i,i} \tag{47}$$

where $i = 1, 2, \cdots, K$.

D. COMPUTATIONAL COMPLEXITY

We calculate the computational complexity of the proposed CFO estimator in terms of complex multiplications. There are three steps involved in the proposed method to estimate CFO. In the first step, we convert received signal into the frequency domain. The computational complexity for this step is $\mathcal{O}(Nlog_2N)$. In the second step, we obtain the covariance matrix of the compensated received signal using (19). The computational complexity for obtaining covariance matrix based on P SC-FDMA blocks is $\mathcal{O}(PM^2)$. Finally, we calculate the total off-diagonal power of the covariance matrix using (34) having a computational complexity of $\mathcal{O}(M^2 - M)$. Therefore, considering only the dominant terms, overall complexity of the proposed method using *deterministic approach* is approximately $\mathcal{O}(P(M^2 + Nlog_2N))$. However, the grid search method has more computational complexity than *deterministic approach* and can be given as $\mathcal{O}(\alpha P(M^2 + Nlog_2N))$, where α is the number of trial CFO values which has to be very large for accurate CFO estimation. We have compared the computational complexity of the proposed method with that of subspace theory based methods in Table 1.

TABLE 1. Computational complexity comparison.

Estimator	Complexity
Proposed method with *deterministic approach*	$\mathcal{O}(P(M^2 + Nlog_2N))$
Proposed method with grid search	$\mathcal{O}(\alpha P(M^2 + Nlog_2N))$
MUSIC [20]	$\mathcal{O}(Q^3 + 2\alpha Q(Q - K))$
ESPRIT [21]	$\mathcal{O}(Q^3 + K^3))$

IV. SIMULATION RESULTS

In this section, we provide simulation results to validate the proposed multi-user CFOs estimation method. We consider an SC-FDMA system with a total number of subcarriers

$N = 128$ and number of users $K = 4$ where each user is allocated $M = 32$ subcarriers. The data symbols are taken from QPSK constellations. The presence of normalized CFO in the received signal for the users 1, 2, 3, and 4 are $\epsilon^1 = 0.1$, $\epsilon^2 = 0.2$, $\epsilon^3 = 0.3$, and $\epsilon^4 = 0.04$, respectively. The channel for each user is a multipath Rayleigh fading channel of order four with different path delay and gain. In the simulation results, independent iterations of the algorithm are performed over P SC-FDMA blocks.

Fig. 3 shows the CFO estimation performance in terms of mean square error (MSE) versus SNR for $P = 1000$ SC-FDMA blocks. The simulation has been carried out for both different mapping schemes namely interleaved and localized. It has been observed that the performance of interleaved mapping is better than that of localized mapping. It happens because the average value of ICI for localized mapping is higher than the interleaved mapping. The estimation performance improves with an increase in SNR but after 12 dB it becomes almost independent with SNR. It happens because the performance of the proposed method is independent of noise level. It can be observed from (25) that the covariance matrix for complex white noise is a diagonal matrix if noise is uncorrelated. But, practically, the sample estimates of the covariance matrix experience disturbance from noise and it degrades the performance at low SNR. The performance of the proposed method is also constrained because of the presence of MAI, which again increases with the number of users for a given set of subcarriers. Thus, the performance of both mapping schemes decreases with an increase in the number of users.

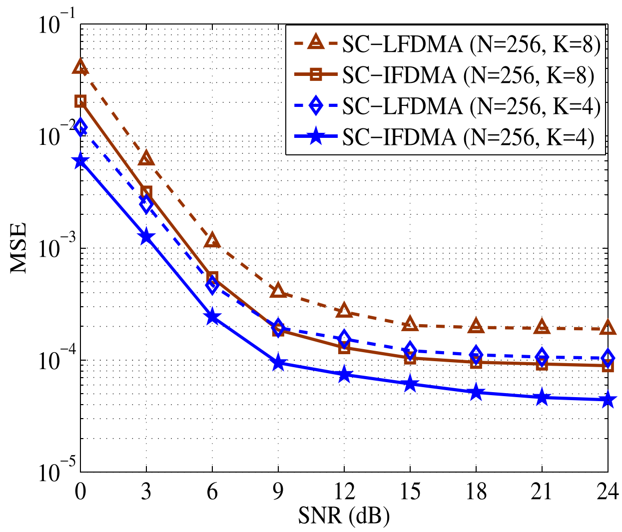

FIGURE 3. CFO estimation performance versus SNR for localized and interleaved mapping under frequency selective fading channel: $L = 4$, ($N = 256$, $K = 4$); and ($N = 256$, $K = 8$).

Fig. 4 illustrates CFO estimation accuracy comparison between the proposed scheme and subspace theory based methods [20], [21] in terms of MSE. The proposed method works for both SC-LFDMA and SC-IFDMA. The comparison for SC-IFDMA is made with that of *CAZAC* sequence

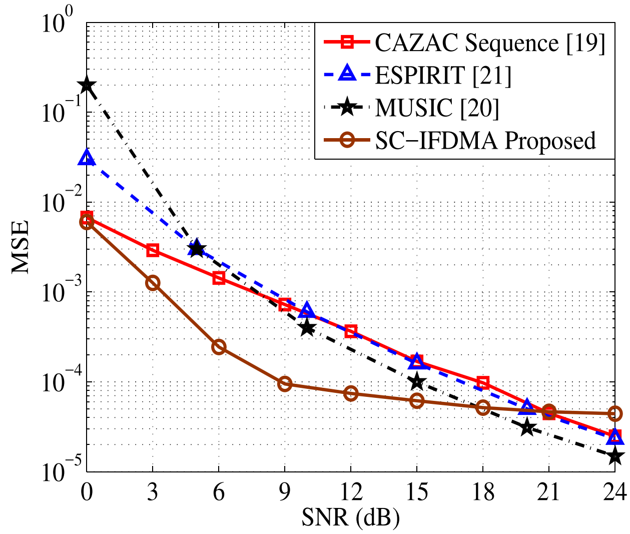

FIGURE 4. Accuracy of different CFO estimation methods versus SNR in Rayleigh fading channel with $L = 4$, $N = 256$, $K = 4$, $\epsilon^k = [0.1, 0.2, 0.3, 0.04]$.

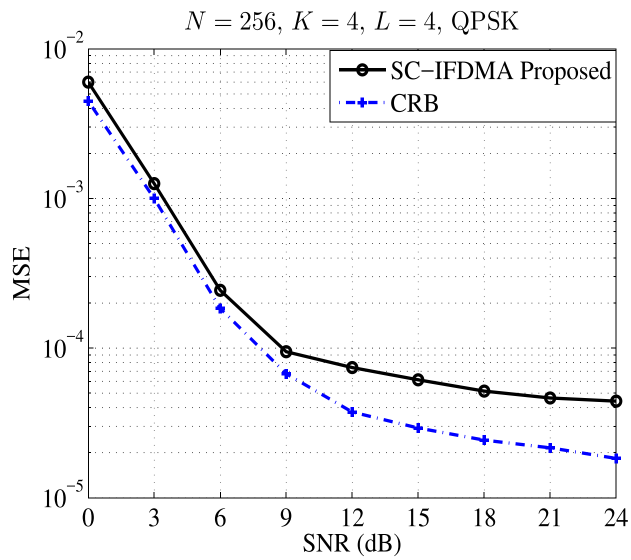

FIGURE 5. MSE and CRB versus SNR for QPSK.

method [19], *MUSIC* [20], and *ESPRIT* [21]. In [19], Zadoff-Chu sequence has been used for CFO estimation and its accuracy is better than that of *MUSIC* and *ESPRIT* methods when SNR is below 9 dB. The proposed method outperforms the mentioned existing methods when SNR is below 18 dB. As discussed in the above paragraph, the performance of the proposed method is independent with noise power. So, its performance is almost constant above 12 dB SNR. Fig. 5 presents the CRB and the MSE of the estimator.

Fig. 6 provides the outage probabilities of CFO which is the probability of estimated CFO value when it is out of the predefined interval for a certain percentage of residual error. The outage probability of CFO against SNR is provided for two different predefined cases: 2.5% and 5% error in CFO.

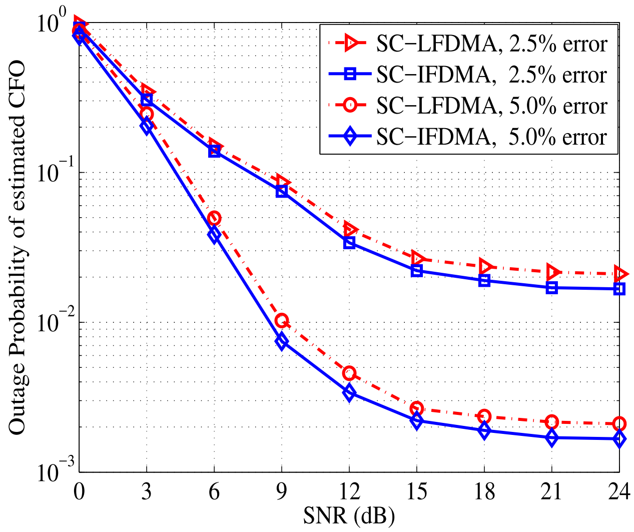

FIGURE 6. CFO Estimation outage probability for two situations: 5% and 2.5% error in CFO estimation.

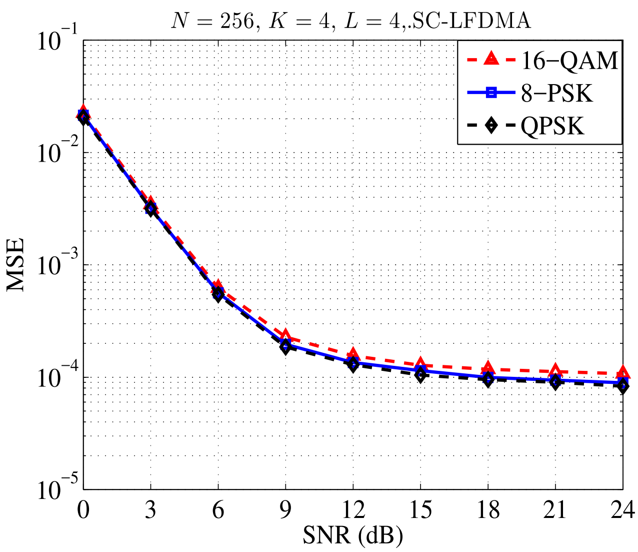

FIGURE 7. MSE of CFO for different modulation schemes.

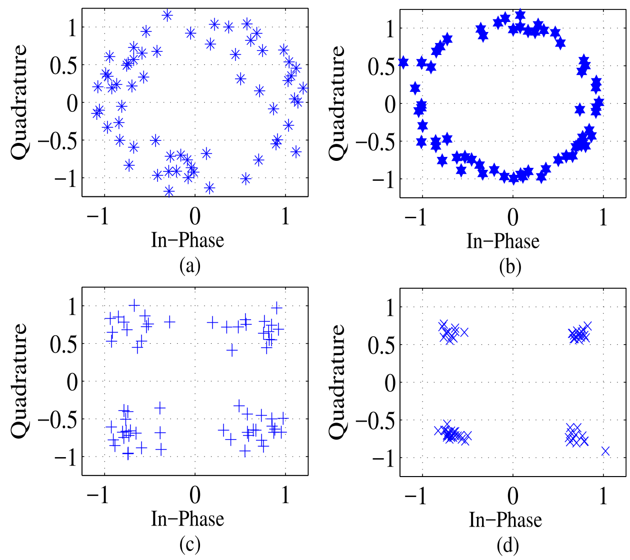

FIGURE 8. Constellation diagrams of QPSK modulated symbols: (a) Before CFO compensation and equalization; (b) After equalization but without CFO compensation; (c) CFO compensated but without equalization; (d) After CFO compensation and equalization.

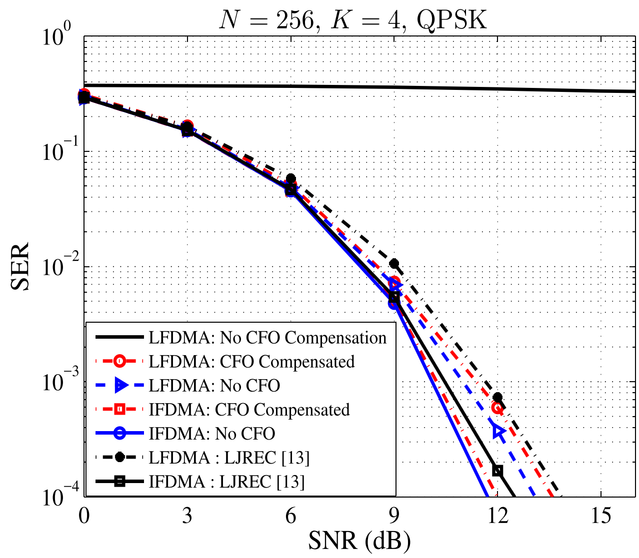

FIGURE 9. SER results for different mapping schemes in AWGN channel.

For 2.5% residual error, the outage probability for both mapping schemes is close to 0.02. For 5% residual error, the outage probability improves to 2×10^{-3}. The QPSK modulation can sustain up to 5% residual error in CFO estimation for a loss of 0.5 dB in SNR [35].

We have shown all the discussions considering QPSK modulation, however, the proposed method works for other symmetric modulation schemes as well. Fig. 7, illustrates the performance of the proposed method in terms of MSE for QPSK, 8-PSK, and 16-QAM modulation formats.

Fig. 8 presents the effect of CFO and equalization on the received QPSK constellations for SC-FDMA system where effect of multipath and AWGN have been considered. Fig. 8(a) shows phase shift and amplitude distortion on received constellation points due to the presence of CFO and multi-path effects. In Fig. 8(b), constellations have a phase shift due to the presence of CFO only. Fig. 8(c) shows amplitude distortion due to the presence of multipath where CFO has been compensated after the estimation. In above all cases, there is a high probability of error in demodulating the received signal because of phase shift and amplitude distortion. Fig. 8(d) shows the constellation point after CFO compensation and equalization which lead to minimize the probability of error while demodulating the signal.

Fig. 9 and Fig. 10 illustrate the usefulness of the proposed estimation method in terms of symbol error rate (SER) performance over frequency selective fading channel in the presence of AWGN. We use MMSE frequency domain equalizer to remove the channel effects. We compare the proposed

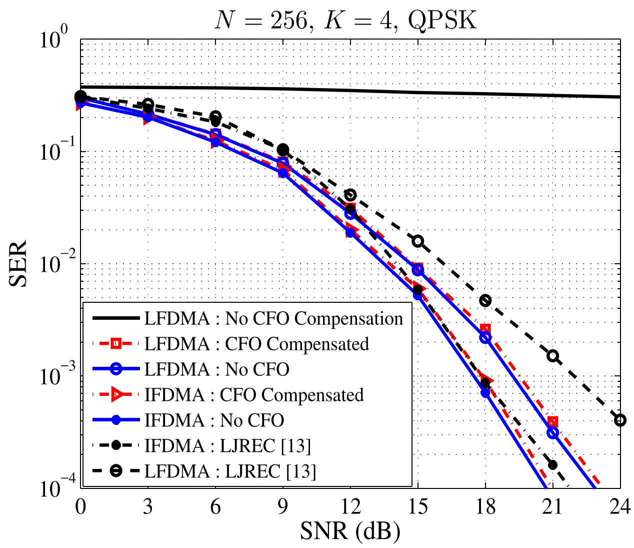

FIGURE 10. SER results for different mapping schemes in Rayleigh fading channel.

SER with low-complexity joint regularized equalization and carrier frequency offset compensation (LJREC) scheme [13]. In both the plots, if CFO is not compensated, SER is maximum and bounded by 0.3 for every SNR. Moreover, we also include the optimal SER results without the presence of CFOs as a benchmark. We observe that SER performance of the proposed method is almost the same as that of no CFO case and SNR gap between them is only around 0.5 dB.

V. CONCLUSION

In this paper, we have derived SINR expression in the presence of multi-user CFOs. It has been observed that SINR is degraded due to the presence of ICI and MAI caused by multiple CFOs. We have proposed a blind CFO estimator to compensate ICI and MAI in order to improve SINR for the SC-FDMA uplink system. The proposed method is mainly analyzed for localized mapping SC-LFDMA, but it also works for the SC-IFDMA. It is based on a property that the information about CFO is present in the covariance matrix of the desired user's received signal in the frequency domain. Thus, at the receiver, a trial CFO is used to minimize the off-diagonal elements of the covariance matrix for the desired user. As a result, ICI and MAI get reduced by compensating with the proper CFO. Similarly, CFOs of K users can be estimated. Simulation results show that at low SNR performance of the proposed method is better than the subspace theory based existing methods. The proposed estimator does not require information of the channels and there is no use of training sequences.

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