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A Compromise Solution for the Fully Fuzzy Multiobjective Linear Programming Problems

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ABSTRACT A new approach is undertaken to solve the fully fuzzy multiobjective linear programming (FFMLP) problem. The coefficients of the objective functions, constraints, right-hand-side parameters, and variables are of the triangular fuzzy number (T_rFN)s. A solution strategy, called compromise solution algorithm (CSA), is presented using a three-step procedure. First, a revised simplex method together with Gaussian elimination in the environment of the linear ranking function is used to convert the FFMLP problems partially into semi fully fuzzy multiobjective linear programming (SFFMLP) problems. Then, the obtained SFFMLP problems are gathered together as a single problem. Finally, the gathered problem is solved by one of four different methods to find a fuzzy compromise solution for the FFMLP problems. The CSA is then numerically applied to a FFMLP problem to illustrate the practicability of the proposed procedure.

INDEX TERMS FFMLP problems, SFFMLP problems, FFLP problem, SFFLP problem, CSA, simplex method and Gaussian elimination, fuzzy simplex method, linear ranking function, and fuzzy compromise solution.

I. INTRODUCTION

Zimmermann [1] first modelled fuzzy linear programming (FLP) problem and later applied it to fuzzy environment [2], which then evolved qualitatively and continuously [3]. Maleki *et al.* [4] proposed a method for solving FLP problem with uncertain vagueness constraints by using linear ranking functions. Ranking fuzzy numbers and their common methods were then reviewed by Wang and Kerre [5]. Gasimov and Yenilmez [6] discussed the solution of the FLP problems via linear ranking function. Nehi *et al.* [7] proposed the lexicographic ranking function of the fuzzy numbers to solve FLP problems using fuzzy numbers.

Buckley and Feuring [8] as well as Maleki [9] each considered a kind of FLP problems, and separately proposed an approach to solve them. The duality of the FLP problems in the fuzziness relations was considered by Inuiguchi *et al.* [10]. The fuzzy primal problems for linear programming(LP) problems and its fuzzy duality were modelled by Wu [11]. Special classes of the FLP problems through fuzzy relationship and based on the duality concept were discussed by Ramik [12].

Mahdavi-Amiri and Nasseri [13] considered the FLP problems and solved them by using a certain linear ranking function using comparison fuzzy numbers. Ganesan and Veeramani [14] considered types of LP problems and multiobjective linear programming (MLP) problems where the right hand side of the constraints and the variables are fuzzy assertions. Mahdavi-Amiri and Nasseri [15] applied a linear ranking function to order trapezoidal fuzzy numbers. They established the dual problem of the LP programming problem with trapezoidal fuzzy variables. Methods were proposed to convert the FLP problem to its corresponding deterministic LP problem, based on the attained values of fuzzy numbers [16]–[18].

The optimality conditions for LP problems had been derived for the FLP problems by Wu [19]. Multiobjective fuzzy linear programming (MFLP) problems were then converted into vector optimization programming problems via defuzzification functions [20]. Iskander [21] utilized possibilistic programming to convert MFLP problems into their modified equivalent crisp problems.

A fully fuzzy linear programming(FFLP) with T_rFNs was considered by Lotfi *et al.* [22]. Several researchers proposed methods to solve some types of FLP problems, through a comparison concept of fuzzy numbers via linear ranking functions [23]–[27]. Kumar *et al.* [28] used a ranking function to convert the FFLP problem to its corresponding crisp equivalent LP problem. A weighted maxmin method was used by Amid *et al.* [29] to solve MFLP problems. Gupta and Kumar [30] overcame the shortcomings for solving MFLP problems earlier proposed by Chiang [31]. Khan *et al.* [32] proposed a method to solve FFLP problem, while Bhardwaj and Kumar [33] refuted the claim that an FFLP can be solved without converting it into a crisp problem based on the incorrect mathematical assumptions used. Various kinds of MFLP problems had been reviewed [34], regarding solutions to MFLP problems. An approach has been proposed by Luhandjula and Rangoaga [35] to solve MFLP problem via nearest interval approximation operator.

Ebrahimnejad *et al.* [36] proposed using the fuzzy number comparisons with ranking functions to convert the FLP problems into equivalent crisp LP problems. They satisfied demands at certain nodes by using available supplies at other nodes to find the minimum fuzzy cost of a commodity. Later, Ebrahimnejad and Tavana [37] proposed a new method for solving FLP problems. They converted the FLP problem into an equivalent crisp LP problem, then solved the obtained problem through the standard primal simplex method. This was followed by a duality approach for solving special kind of FLP based on ranking functions [38]. Hamadameen [39] considered the MFLP problems in which the coefficients of the objective functions are T_rFNs. He utilized a linear ranking function through simplex method, and proposed a novel method to transform the MFLP problems into single FLP problem, then found a compromise solution for the original problem.

We will propose a method to solve the FFMLP problem. The proposed method converts the FFMLP problem to a SFFMLP problem, which was then solved via the revised simplex method using Gaussian eliminations through the proposed linear ranking function. The procedure is then illustrated by a numerical example.

The rest of this paper is structured as follows. Section II defines fuzzy concepts and algebra properties of T_rFNs, and types of ranking functions with their strengths and weaknesses, in addition to the comparison among fuzzy numbers (FNs). Section III considers the FFMLP problem and gives the mathematical formulation. Section IV deals with partially converting the FFLP problem into its equivalent SFFLP. In addition, it offers a procedure to solve the FFMLP problems within the frame of the CSA. In Section V, the three-step procedure of the CSA is applied to a numerical example to illustrate its practicality in solving FFMLP problems. Section VI analyzes the obtained results and interpreting them logically. Section VII lists the advantages of the proposed method over current existing methods. Conclusions are discussed in Section VIII.

II. CONCEPTS OF FUZZY NUMBERS, RANKING FUNCTIONS, AND FUZZY ALGEBRA PROPERTIES

In order to set our comments orderly, we start this section with the concept of fuzzy numbers, the most commonly used types of those numbers, and the relations between them. In addition, a brief discussion is made on fuzzy algebra on fuzzy numbers.

A. DEFINITIONS OF FUZZY NUMBERS

Fuzzy numbers are a kind of numbers with a continuity of grades of membership [40]. This kind of numbers is characterized by a function which assigns to each number a grade of membership within a range in the closed unit interval [41]. In other words, the membership function of a fuzzy number is set in a universal set, specified for each element in the fuzzy set a value in the closed unit interval. There are two common kinds of fuzzy numbers, Tapezoidal Fuzzy Numbers(T_pFNs), and Triangular Fuzzy Number (T_rFNs) [13]–[15].

1) THE TRAPEZOIDAL FUZZY NUMBER (T_pFN)

Let $\widetilde{A} = (a^L, a^U, \alpha, \beta)$ be the T_pFN, where $[a^L, a^U]$ is the modal set of $\widetilde{A}$, and $[a^L - \alpha, a^U + \beta]$ its support part [13]–[15] (Figure 1).

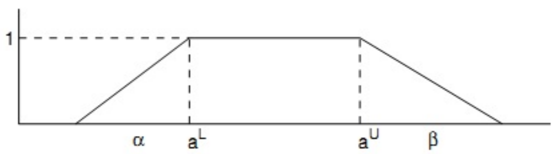

FIGURE 1. Trapezoidal fuzzy number.

2) THE TRIANGULAR FUZZY NUMBER (T_rFN)

If $a = a^L = a^U \in \widetilde{A}$ then the T_pFN is reduced to T_rFN, and denoted by $\widetilde{A} = (a, \alpha, \beta)$ (Figure 2).

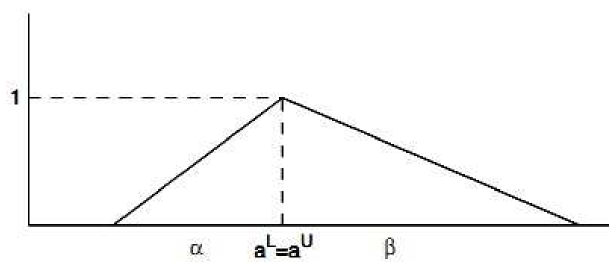

FIGURE 2. Triangular fuzzy number.

Thus $\widetilde{A} = (a, \alpha, \beta) \subset (a^L, a^U, \alpha, \beta)$. Hence, T_rFN is a special case of the general case (T_pFN). Since the study is focused on the FFMLP problems with T_rFNs, the next subsection lists fuzzy algebra properties specific to such FNs.

B. FUZZY ALGEBRA PROPERTIES OF T_rFNs

Since we are going to formulate the FFMLP problem with T_rFNs, we insert some arithmetic properties on these fuzzy numbers as follows.

Let $\widetilde{A}_1, \widetilde{A}_2 \in$ T$_r$FNs, such that $\widetilde{A}_1 = (a_1, \alpha_1, \beta_1)$ and $\widetilde{A}_2 = (a_2, \alpha_2, \beta_2)$. Based on [40] and [42], [43]; as well as [3], [13], [15], [44], the following rules apply.

1) Addition: $\widetilde{A}_1 \oplus \widetilde{A}_2 = (a_1, \alpha_1, \beta_1) \oplus (a_2, \alpha_2, \beta_2) = (a_1 + a_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)$.
2) Image $\widetilde{A}_1 = \text{Image}(a_1, \alpha_1, \beta_1) = -\widetilde{A}_1 = -(a_1, \alpha_1, \beta_1) = (-a_1, \beta_1, \alpha_1)$.
3) Subtraction: $\widetilde{A}_1 \ominus \widetilde{A}_2 = (a_1, \alpha_1, \beta_1) \ominus (a_2, \alpha_2, \beta_2) = (a_1, \alpha_1, \beta_1) \oplus (-a_2, \beta_2, \alpha_2) = (a_1 - a_2, \alpha_1 + \beta_2, \beta_1 + \alpha_2)$.
4) Multiplication:

$$\widetilde{A}_1 \otimes \widetilde{A}_2 = (a_1, \alpha_1, \beta_1) \otimes (a_2, \alpha_2, \beta_2) \cong \begin{cases} (a_1a_2, a_1\alpha_2 + a_2\alpha_1, a_1\beta_2 + a_2\beta_1); & A_1 > \widetilde{0}, \quad A_2 > \widetilde{0} \\ (a_1a_2, a_2\alpha_1 - a_1\beta_2, a_2\beta_1 - a_1\alpha_2); & A_1 < \widetilde{0}, \quad A_2 > \widetilde{0} \\ (a_1a_2, -a_2\beta_1 - a_1\beta_2, -a_2\alpha_1 - a_1\alpha_2); & A_1 < \widetilde{0}, \quad A_2 < \widetilde{0} \end{cases}$$

5) Scalar multiplication:

$$\delta \otimes \widetilde{A}_1 = \delta \otimes (a_1, \alpha_1, \beta_1) = \begin{cases} (\delta a_1, \delta\alpha_1, \delta\beta_1); & \delta > 0 \\ (\delta a_1, -\delta\beta_1, -\delta\alpha_1); & \delta < 0 \end{cases}$$

6) Inverse: $(\widetilde{A}_1)^{-1} = (a_1, \alpha_1, \beta_1)^{-1} = (a_1^{-1}, \beta_1\, a_1^{-2}, \alpha_1\, a_1^{-2})$
7) Division: $\widetilde{A}_1 \oslash \widetilde{A}_2 = \widetilde{A}_1 \otimes (\widetilde{A}_2)^{-1} = (a_1, \alpha_1, \beta_1) \oslash (a_2, \alpha_2, \beta_2) = (\frac{a_1}{a_2}, \frac{\beta_2 a_1 + \alpha_1 a_2}{a_2{}^2}, \frac{\alpha_2 a_1 + \beta_1 a_2}{a_2{}^2}); \forall \widetilde{A}_1, \widetilde{A}_2 > \widetilde{0}$.
Note that similar formulas hold when $\widetilde{A}_1$ and or $\widetilde{A}_2$ are negative.
8) $\widetilde{A}_1 = \widetilde{0} \Leftrightarrow \widetilde{A}_1 = (0, 0, 0)$

C. LINEAR RANKING FUNCTION AND ITS TYPES

One of the most convenient methods to defuzzify an FLP problem into its deterministic form and the comparison of fuzzy numbers is by using the linear ranking function [3], [4], [9], [13], [15].

Since this study deals with the FFLP problem in the fuzziness environment through the linear ranking function, we put forward in this section a definition of a linear ranking function and its properties. We describe the related ranking functions, their strengths and weaknesses.

Linear Ranking Function: is a map which transforms each fuzzy number into its corresponding real line, where a natural order exists, mathematically, $\Re : \widetilde{A} \rightarrow \mathbb{R}$; $\forall \widetilde{A}$ and $\mathbb{R}$ is the set of all real numbers [41], [45].

Furthermore, the $\Re$'s fuzzy algebra rules on the fuzzy numbers are as follows, for all $\widetilde{A}_1, \widetilde{A}_2, \widetilde{A}_3, \widetilde{A}_4 \in$ FNs, and $\delta \in \mathbb{R}$:

1) $\widetilde{A}_1 \underset{\Re}{\geq} \widetilde{A}_2 \Leftrightarrow \Re(\widetilde{A}_1) \geq \Re(\widetilde{A}_2)$,
2) $\widetilde{A}_1 \underset{\Re}{>} \widetilde{A}_2 \Leftrightarrow \Re(\widetilde{A}_1) > \Re(\widetilde{A}_2)$,
3) $\widetilde{A}_1 \underset{\Re}{=} \widetilde{A}_2 \Leftrightarrow \Re(\widetilde{A}_1) = \Re(\widetilde{A}_2)$,
4) $\widetilde{A}_1 \underset{\Re}{\leq} \widetilde{A}_2 \Leftrightarrow \Re(\widetilde{A}_1) \leq \Re(\widetilde{A}_2)$,
5) $\delta\widetilde{A}_1 \underset{\Re}{+} \widetilde{A}_2 = \delta\Re(\widetilde{A}_1) + \Re(\widetilde{A}_2)$,
6) $\widetilde{A}_1 \underset{\Re}{=} \widetilde{0} \Leftrightarrow \Re(\widetilde{A}_1) = \Re(\widetilde{0}) = 0$,
7) $\widetilde{A}_1 \underset{\Re}{\geq} \widetilde{A}_2 \Leftrightarrow \widetilde{A}_1 - \widetilde{A}_2 \underset{\Re}{\geq} 0 \Leftrightarrow -\widetilde{A}_2 \underset{\Re}{\geq} -\widetilde{A}_1$, and
8) $\widetilde{A}_1 \underset{\Re}{\geq} \widetilde{A}_2 \wedge \widetilde{A}_3 \underset{\Re}{\geq} \widetilde{A}_4 \Rightarrow \widetilde{A}_1 + \widetilde{A}_3 \underset{\Re}{\geq} \widetilde{A}_2 + \widetilde{A}_4$.

Finally, we describe the concepts of ranking functions, their strengths, and weaknesses as in Table 1. We identify that the type which is most suitable for our work is the one used by [15].

III. PROBLEM FORMULATION

A Fully Fuzzy Multiobjective Linear Programming (FFMLP) problem can be formulated as follows:

$$\begin{aligned} &\text{Max. } \widetilde{Z}_i(\widetilde{x}) \underset{\Re}{=} \sum_{j=1}^{n} \widetilde{c}_{ij}\widetilde{x}_j; \quad i = 1, \ldots, r \\ &\text{Min. } \widetilde{Z}_i(\widetilde{x}) \underset{\Re}{=} \sum_{j=1}^{n} \widetilde{c}_{ij}\widetilde{x}_j; \quad i = r+1, \ldots, s \\ &\text{s.t. } \sum_{j=1}^{n} \widetilde{a}_{ij}\widetilde{x}_j \begin{bmatrix} \leq \\ \geq \\ = \end{bmatrix} \widetilde{b}_i, \quad i = 1, 2, \ldots, m, \\ &\widetilde{x}_j \geq \widetilde{0}, \quad j = 1, \ldots, n. \end{aligned} \tag{1}$$

where $\widetilde{a}_{ij} = (a_{ij}^L, a_{ij}^U, \alpha_{ij}, \beta_{ij}), \widetilde{b}_i = (b_i^L, b_i^U, \alpha_i, \beta_i), \widetilde{c}_j = (c_j^L, c_j^U, \omega_j, \eta_j)$ and $\widetilde{x}_j = (x_j^L, x_j^U, \alpha_j, \beta_j)$ are in the set of all T$_\text{p}$FNs, $i = 1, \ldots, m,\ j = 1, \ldots, n$.

To solve (1), we have to find a set of basic feasible solution $\widetilde{x} = \{\widetilde{x}_1, \widetilde{x}_2, \ldots, \widetilde{x}_{n+m}\}$ of fuzzy variables which satisfies the set of all constraints, non-negative restrictions and optimizes (maximizes or minimizes) the objective functions in (1). Finding the optimal solution to the FFLP problem in (1) without converting it into its equivalent deterministic is still an open research problem as emphasized by some researchers [33]. Thus, it can be said that we cannot solve FFMLP problem in (1) without converting it into its equivalent deterministic problem.

In what follows we shall consider the FFMLP problem in (1), and try to find a compromise solution for it within a range of the proposed research methodologies.

IV. PARTIALLY CONVERTING THE FFLP PROBLEM INTO ITS EQUIVALENT SFFLP AND SOLUTION ALGORITHM OF THE FFMLP PROBLEMS

We now consider the questions in Section III logically and will attempt to find convincing answers to them. Since finding the optimal solution to the FFLP problem without converting it into its equivalent deterministic version is still an open research problem [33] we will try to establish an alternate methodology to solve FFMLP problem in (1) by using revised simplex method together with Gaussian elimination in the

TABLE 1. The solution of the objective functions.

Author and Date	Description	Strength	Weakness
Khan *et al.*[32].	$\Re(\widetilde{A}) = \alpha + \beta - \sigma$; $\sigma = \frac{\beta-\alpha}{6}$ (Variance between α and β)	This ranking function is good for directions, or shifting the fuzzy number toward right or left by a known number of units. In addition, it has been used specially to T_rFNs, to know the variance between the extreme values around the central value of the fuzzy number.	When applied to the set of real number, this ranking function yields values which may lie outside the interval of the fuzzy number. This linear ranking function is specified to T_rFNs, and it is to defuzzify and order fuzzy numbers.
Mahdavi-Amiri and Nasseri [15].	$\Re(\widetilde{A}) = \frac{a^L+a^U}{2} + \frac{\beta-\alpha}{4}$	This ranking function transforms the fuzzy set into a value within the interval. This type is more suitable for defuzzifing, because its real value remains in the fuzzy number's interval. This type is more suitable for our work, because its real value remains in the fuzzy number's interval. Furthermore, this is a special version of a linear ranking function proposed essentially by Yager [47].	—
Mahdavi-Amiri and Nasseri [13].	$\Re(\widetilde{A}) = a^L + a^U + \frac{\beta-\alpha}{2}$	This ranking function is good for directions, or shifting the fuzzy number toward right or left direction by a known number of units, and similar to the ranking function adopted by Maleki [9].	When applied to the set of real number, this ranking function yields values which may lie outside the interval of the fuzzy number.
Ebrahimnejad [25].	$\Re(\widetilde{A}) = a^L + a^U + \frac{\beta-\alpha}{2}$	This ranking function is good for directions, or shifting the fuzzy number toward right or left direction by a known number of units, and similar to the ranking function adopted by Maleki [9].	When applied to the set of real number, this ranking function yields values which may lie outside the interval of the fuzzy number.

environment of the linear ranking function as described in Section II, in which the methodology converts the system (1) partially into Semi-Fully Fuzzy Multiobjective Linear Programming (SFFMLP) problem. Next each objective function in the system is solved individually, then the SFFMLP problem is converted into Semi-Fully Fuzzy Linear Programming (SFFLP) problem. Finally the obtained problem is solved by an interactive method to arrive at a compromise solution to the original problem in (1).

The following procedure is used to solve FFMLP problem through the linear ranking function as follows:

Consider the FFMLP problem in (1) as

$$\begin{aligned} &\text{Max. } \widetilde{Z}_i(\widetilde{x}) \underset{\Re}{=} \sum_{j=1}^{n} \widetilde{c}_{ij}\widetilde{x}_j; \quad i = 1, \ldots, r \\ &\text{Min. } \widetilde{Z}_i(\widetilde{x}) \underset{\Re}{=} \sum_{j=1}^{n} \widetilde{c}_{ij}\widetilde{x}_j; \quad i = r+1, \ldots, s \\ &\text{s.t. } \sum_{j=1}^{n} \widetilde{a}_{ij}\widetilde{x}_j \leq \widetilde{b}_i, \quad i = 1, 2, \ldots, m, \\ &\quad \widetilde{x}_j \geq \widetilde{0}, \quad j = 1, \ldots, n. \end{aligned} \tag{2}$$

TABLE 2. The SFFLP problem.

	Objective function value : $\tilde{z}$	R.H.S.	Basic variable : $\tilde{x}_B$	Nonbasic variable : $\tilde{s}_N$
$\tilde{x}_B$	$\tilde{0}$	$\tilde{I}$	$B^{-1}\tilde{b}$	$B^{-1}N$
$\tilde{z}$	$\tilde{1}$	$\tilde{c}_B B^{-1}\tilde{b}$	$\tilde{0}$	$\tilde{c}_B B^{-1}N - \tilde{c}_N$

where $\tilde{a}_{ij} = (a_{ij}, \alpha_{ij}, \beta_{ij}), \tilde{b}_i = (b_i, \alpha_i, \beta_i)$, $\tilde{c}_j = (c_j, \omega_j, \eta_j)$ and $\tilde{x}_j = (x_j, \alpha_j, \beta_j)$ are in the set of all T$_r$FNs, $i = 1, \ldots, m$, $j = 1, \ldots, n$.

Now, we convert each $\tilde{a}_{ij}, \forall i, j$ into their deterministic a_{ij} by using [15] linear ranking function;

$$\Re(\tilde{A}) = \frac{a^L + a^U}{2} + \frac{\beta - \alpha}{4} = \frac{2a^L(a^U)}{2} + \frac{\beta - \alpha}{4} = a + \frac{\beta - \alpha}{4} \tag{3}$$

The obtained SFFMLP problem will be as:

$$\begin{aligned} &\text{Max. } \tilde{Z}_i(\tilde{x}) \underset{\Re}{=} \sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_j; \quad i = 1, \ldots, r \\ &\text{Min. } \tilde{Z}_i(\tilde{x}) \underset{\Re}{=} \sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_j; \quad i = r+1, \ldots, s \\ &\text{s.t.} \sum_{j=1}^{n} a_{ij}\tilde{x}_j \leq \tilde{b}_i, \quad i = 1, 2, \ldots, m, \\ &\quad \tilde{x}_j \geq \tilde{0}, \quad j = 1, \ldots, n. \end{aligned} \tag{4}$$

Now, we solve each SFFLP problem Max./Min. $\tilde{Z}_i(\tilde{x})$; $\forall i \in [1, s]$ in (4) individually. For convenient, let us, rename Max. $\tilde{Z}_i(\tilde{x}) =$ Max. $\tilde{z}(\tilde{x})$. Thus, the FFLP problem in (4) can be written as:

$$\begin{aligned} &\text{Max. } \tilde{z}(\tilde{x}) \underset{\Re}{=} \sum_{j=1}^{n} \tilde{c}_j\tilde{x}_j, \\ &\text{s.t.} \sum_{j=1}^{n} \tilde{a}_{ij}\tilde{x}_j \underset{\Re}{\leq} \tilde{b}_i, \quad i = 1, 2, \ldots, m, \\ &\quad \tilde{x}_j \underset{\Re}{\geq} \tilde{0}, \quad j = 1, \ldots, n. \end{aligned} \tag{5}$$

Convert (5) into the standard form as:

$$\begin{aligned} &\text{Max. } \tilde{z}(\tilde{x}) \underset{\Re}{=} \sum_{j=1}^{n} \tilde{c}_j\tilde{x}_j + \tilde{0}(\sum_{i=1}^{m} \tilde{s}_i), \\ &\text{s.t.} \sum_{j=1}^{n} a_{ij}\tilde{x}_j + \tilde{s}_i \underset{\Re}{=} \tilde{b}_i, \quad i = 1, 2, \ldots, m, \\ &\quad \tilde{x}_j \,\&\, \tilde{s}_i \underset{\Re}{\geq} \tilde{0}, \quad j = 1, \ldots, n \,\&\, i = 1, \ldots, m. \end{aligned} \tag{6}$$

where $\tilde{s}_i$ are fuzzy identity slack variables.

Express (6) by its basic and non-basic variables as follows:

$$\begin{aligned} &\text{Max. } \tilde{z}(\tilde{x}) \underset{\Re}{=} \tilde{c}_B\tilde{x} + \tilde{c}_N\tilde{s} \\ &\text{s.t. } B\tilde{x}_B + N\tilde{s}_N \underset{\Re}{=} \tilde{b} \\ &\quad \tilde{x}_B \,\&\, \tilde{s}_N \underset{\Re}{\geq} \tilde{0} \end{aligned} \tag{7}$$

where $A = [B, N]$, the nonsingular matrix $B = (m, m)$, and rank $(B) = m$, where $\tilde{x}_B = B^{-1}\tilde{b}, \tilde{x}_N = \tilde{0}$; the basic solution point $\tilde{x} = (\tilde{x}_B^T, \tilde{x}_N^T)^T$ is called the Basic Feasible Solution (BFS) for the system in (6), where B and N are basic matrix, and non-basic matrix respectively [3], [46].

Now, (7) can be expressed by its initial simplex tableau in Table 2.

Based on [46] and other studies [3], [4], [15], in this table we have:

1) The fuzzy objective row $\tilde{\gamma}_j \underset{\Re}{=} (\tilde{c}_B B^{-1} a_j - \tilde{c}_i)_{j \neq B_j}$ contains the $\tilde{\gamma}_j \underset{\Re}{=} \tilde{z}_j - \tilde{c}_j$ for the nonbasic variables.
2) For the feasible optimal solution it should be $\tilde{\gamma}_j \underset{\Re}{=}\geq 0, \forall j \neq B_i$.
3) If, $\tilde{\gamma}_k \underset{\Re}{<} 0, \forall k \neq B_i$, then exchange $\tilde{x}_{B_r}$ by $\tilde{x}_k$. Then $\tilde{\gamma}_k \underset{\Re}{=} B^{-1} a_k$.
4) If, $\tilde{\gamma}_k \underset{\Re}{\leq} 0$, then $\tilde{x}_k$ is an unbounded solution for the problem.
5) If an m exist such that $\tilde{z}_m - \tilde{c}_m \underset{\Re}{<} \tilde{0}$ and there exist a basic index i in which $y_{im>0}$, then a pivoting row p can be found in which the pivoting y_{pm} yields a feasible tableau corresponding fuzzy objective value.
6) For any feasible solution to FLP problem, if there are some columns not in the basic solution in which $\tilde{z}_m - \tilde{c}_m \underset{\Re}{<} \tilde{0}$ and $y_{im} \leq 0$, $i = 1, \ldots, s$, then the problem is unbounded.

Now, we have to verify the fuzzy feasibility as well as the fuzzy optimality solution for the FFLP problem in (5) through the linear ranking function $\Re$. After passing the steps (1-6) logically and successfully, the optimal solution for the Max./Min. $\tilde{Z}_i$; $\forall i$ the optimal solution is;

$$\begin{aligned} &\{\text{Max. } \tilde{Z}_i, \tilde{X}(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n); 1 \leq i \leq r\} \\ &\qquad = \{\tilde{\phi}_i; \tilde{X}(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n); 1 \leq i \leq r\} \end{aligned}$$

and

$$\begin{aligned} &\{\text{Min. } \tilde{Z}_i, \tilde{X}(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n); r+1 \leq i \leq s\} \\ &\qquad = \{\tilde{\phi}_i; \tilde{X}(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n); r+1 \leq i \leq s\}. \end{aligned}$$

Now, after each objective function in the system (4) has been solved individually, and the value of each one has been obtained, we can transfer (4) to its unique FFLP problem by an interactive method.

1) For solving MFLP Problems, [39] used a new technique to transform the multiple optimization problems into a single FLP problem, and found a compromise solution for the resulted problem by using linear ranking

function through simplex method. Here, in this study, we extend his proposed technique [39] into FFMLP problem to convert (4) into SFFLP problem as follows:

$$\textbf{Max. } \tilde{Z}_{\tilde{\phi}}(\tilde{x}) \underset{\Re}{=} \sum_{j=1}^{n} \frac{\text{Max.}\tilde{c}_{ij}\tilde{x}_j}{\tilde{\phi}_i} \quad (i=1,\ldots,r) - \sum_{j=1}^{n} \frac{\text{Min.}\tilde{c}_{ij}\tilde{x}_j}{\tilde{\phi}_i}(i=r+1,\ldots,s); \quad \forall \tilde{\phi}_i \neq \tilde{0}$$
$$\text{s.t. } \sum_{j=1}^{n} a_{ij}\tilde{x}_j \leq \tilde{b}_i, \quad i=1,2,\ldots,m,$$
$$\tilde{x}_j \geq \tilde{0}, \quad j=1,\ldots,n. \tag{8}$$

Note that the SFFLP problem in (8) is equivalent to the SFFLP problem in (6). Thus, using (7), the initial simplex tableau expressed in Table 2 and equations (1) through (7), the optimal solution can be found for the system (8) and the obtained solution will be the compromise solution for the FFMLP problem in (2) and can be written as: $\left\{\textbf{Max.}\tilde{Z}_{\tilde{\phi}}(\tilde{x}), \tilde{X}(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)\right\}$.

2) Using Fuzzy Adaptive Average Arithmetic Method (FAAAM) as a transformation technique to transform (4) into SFFLP problem as: $\tilde{\phi}_{\text{max.}} = \text{max.}\left\{\tilde{\phi}_i; 1 \leq i \leq r\right\}; \tilde{\phi}_{\text{min.}} = \text{min.}\left\{\tilde{\phi}_i; r+1 \leq i \leq s\right\}$, $\tilde{\phi}_{\text{FAAA}} = \frac{\tilde{\phi}_{\text{max.}}+\tilde{\phi}_{\text{min.}}}{2}$.

$$\textbf{Max. } \tilde{Z}_{\tilde{\phi}_{\text{FAAA}}}(\tilde{x}) \underset{\Re}{=} \frac{\sum_{j=1}^{n}[(\text{Max.}\tilde{c}_{ij}\tilde{x}_j)_{i=1,\ldots,r} - (\text{Min.}\tilde{c}_{ij}\tilde{x}_j)_{i=r+1,\ldots,s})]}{\tilde{\phi}_{\text{FAAA}}}; \quad \tilde{\phi}_{\text{FAAA}} \neq \tilde{0}$$
$$\text{s.t. } \sum_{j=1}^{n} a_{ij}\tilde{x}_j \leq \tilde{b}_i, \quad i=1,2,\ldots,m,$$
$$\tilde{x}_j \geq \tilde{0}, \quad j=1,\ldots,n. \tag{9}$$

This obtained SFFLP problem in (9) is equivalent to the SFFLP problem in (6). Now, following the same steps of the methodology in the first case the optimal solution can be obtained. The optimal solution is a compromise solution for FFMLP problem in (2), and can be expressed as follows: $\left\{\textbf{Max.}\tilde{Z}_{\tilde{\phi}_{\text{FAAA}}}(\tilde{x}), \tilde{X}(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)\right\}$.

3) Using the point of the fuzzy balance or fuzzy average of a data set. i. e. fuzzy mean or fuzzy arithmetic mean. Mathematically, $\tilde{\text{mean}} = \frac{\sum_{i=1}^{n}\tilde{\phi}_i}{n}$. Let us assume that $\tilde{m}_1 = \frac{\sum_{i=1}^{r}\tilde{\phi}_i}{r}$, and $\tilde{m}_2 = \frac{\sum_{i=r+1}^{s}\tilde{\phi}_i}{s-r}$. Thus, the SFFMLP problem in (4) can be converted to its equivalent SFFLP problem as following:

$$\textbf{Max. } \tilde{Z}_{\tilde{\phi}_{\text{mean}}}(\tilde{x}) \underset{\Re}{=} \frac{\sum_{j=1}^{n}(\text{Max.}\tilde{c}_{ij}\tilde{x}_j)_{i=1,\ldots,r}}{\tilde{m}_1} - \frac{\sum_{j=1}^{n}(\text{Min.}\tilde{c}_{ij}\tilde{x}_j)_{i=r+1,\ldots,s}}{\tilde{m}_2}$$
$$\text{s.t. } \sum_{j=1}^{n} a_{ij}\tilde{x}_j \leq \tilde{b}_i, \quad i=1,2,\ldots,m,$$
$$\tilde{x}_j \geq \tilde{0}, \quad j=1,\ldots,n. \tag{10}$$

Note that, the last SFFLP problem in (10) is the same as the SFFLP problem in (6), and the fuzzy optimal solution can be found using the same technique. Thus, the compromise solution for the FFMLP problem in (2) is $\left\{\textbf{Max.}\tilde{Z}_{\tilde{\phi}_{\text{mean}}}(\tilde{x}), \tilde{X}(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)\right\}$.

4) Using fuzzy median point, or the fuzzy mid-point of the fuzzy data set when the fuzzy data set of fuzzy observations are placed in ascending order. For an odd number of fuzzy observations, the fuzzy median is the data point which falls in the middle, at location $\tilde{\phi}(i+1)/2; \forall i \in \mathbb{N}$ when values are placed in ascending order. For an even number of observations the median is defined by the fuzzy mean of the two fuzzy middle observations at location $\tilde{\phi}(i/2), \tilde{\phi}(i/2)+1; \forall i \in \mathbb{N}$. Thus, the fuzzy median is the fuzzy value represented by the fuzzy average of the fuzzy points at locations $\tilde{\phi}(i/2), \tilde{\phi}(i/2)+1; \forall i \in \mathbb{N}$. Suppose that $\tilde{M}_1 = \text{fuzzy median}\left\{\tilde{\phi}_i; 1 \leq i \leq r\right\}$, and $\tilde{M}_2 = \text{fuzzy median}\left\{\tilde{\phi}_i; r+1 \leq i \leq s\right\}$. Thus, the SFFMLP problem in (4) can be converted to its equivalent SFFLP problem as follows:

$$\textbf{Max. } \tilde{Z}_{\tilde{\phi}_{\text{median}}}(\tilde{x}) \underset{\Re}{=} \frac{\sum_{j=1}^{n}(\text{Max.}\tilde{c}_{ij}\tilde{x}_j)_{i=1,\ldots,r}}{\tilde{M}_1} - \frac{\sum_{j=1}^{n}(\text{Min.}\tilde{c}_{ij}\tilde{x}_j)_{i=r+1,\ldots,s}}{\tilde{M}_2}$$
$$\text{s.t. } \sum_{j=1}^{n} a_{ij}\tilde{x}_j \leq \tilde{b}_i, \quad i=1,2,\ldots,m,$$
$$\tilde{x}_j \geq \tilde{0}, \quad j=1,\ldots,n. \tag{11}$$

Since the SFFLP problem in (11) is also the same FFLP problem in (6) hence, again, recall the same techniques which have been used to solve the SFFLP problems in the cases (1 through 3), the fuzzy optimal solution of (10) can be found, and the solution is the compromise for the original FFOLP problem in (2), and can be expressed as: $\left\{\textbf{Max.}\tilde{Z}_{\tilde{\phi}_{\text{median}}}(\tilde{x}), \tilde{X}(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)\right\}$.

Now, we can express the above four cases in the following SFFLP problem as equation (12), as shown at the top of the next page

Now, let us describe the solution algorithm of the FFMLP problem step by step in the following Compromise Solution Algorithm (CSA).

CSA for the FFMLP problem:-

- STEP1: Consider the FFMLP problem in (2).
- STEP2: Use the linear ranking function formulation in (3) to transform $\tilde{a}_{ij}; i=1,2,\ldots,m, j=1,2,\ldots,n$ into its equivalent deterministic form a_{ij} to get the SFFMLP problem in (4).

$$\textbf{Max.}\begin{cases}\widetilde{Z}_{\widetilde{\phi}}(\widetilde{x}) \underset{\Re}{=} \sum_{j=1}^{n}\frac{\text{Max.}\widetilde{c}_{ij}\widetilde{x}_j}{\widetilde{\phi}_i}(i=1,\ldots,r) - \sum_{j=1}^{n}\frac{\text{Min.}\widetilde{c}_{ij}\widetilde{x}_j}{\widetilde{\phi}_i}(i=r+1,\ldots,s); \quad \forall\widetilde{\phi}_i \neq \widetilde{0} \\ \widetilde{Z}_{\widetilde{\phi}_{\text{FAAA}}}(\widetilde{x}) \underset{\Re}{=} \frac{\sum_{j=1}^{n}[(\text{Max.}\widetilde{c}_{ij}\widetilde{x}_j)_{i=1,\ldots,r} - (\text{Min.}\widetilde{c}_{ij}\widetilde{x}_j)_{i=r+1,\ldots,s})]}{\widetilde{\phi}_{\text{FAAA}}}; \quad \widetilde{\phi}_{\text{FAAA}} \neq \widetilde{0} \\ \widetilde{Z}_{\widetilde{\phi}_{\text{mean}}}(\widetilde{x}) \underset{\Re}{=} \frac{\sum_{j=1}^{n}(\text{Max.}\widetilde{c}_{ij}\widetilde{x}_j)_{i=1,\ldots,r}}{\widetilde{m}_1} - \frac{\sum_{j=1}^{n}(\text{Min.}\widetilde{c}_{ij}\widetilde{x}_j)_{i=r+1,\ldots,s}}{\widetilde{m}_2} \\ \widetilde{Z}_{\widetilde{\phi}_{\text{median}}}(\widetilde{x}) \underset{\Re}{=} \frac{\sum_{j=1}^{n}(\text{Max.}\widetilde{c}_{ij}\widetilde{x}_j)_{i=1,\ldots,r}}{\widetilde{M}_1} - \frac{\sum_{j=1}^{n}(\text{Min.}\widetilde{c}_{ij}\widetilde{x}_j)_{i=r+1,\ldots,s}}{\widetilde{M}_2}\end{cases}$$

$$\text{s.t.} \sum_{j=1}^{n} a_{ij}\widetilde{x}_j \leq \widetilde{b}_i, \quad i=1,2,\ldots,m,$$
$$\widetilde{x}_j \geq \widetilde{0}, \quad j=1,\ldots,n. \tag{12}$$

- STEP3: Solve each $\text{Max.}\widetilde{Z}_i(\widetilde{x}); i = 1,2,\ldots,r$ and $\text{Min.}\widetilde{Z}_i(\widetilde{x}); i = r+1, r+2,\ldots,s$ individually, as indicated in (5), by using the fuzzy simplex method in (5-7) as in the Table 2, using the Gaussian elimination process and the equations (1-6). Get the fuzzy values $\widetilde{\phi}_i; i=1,2,\ldots,r$ for $\text{Max.}\widetilde{Z}_i(\widetilde{x}); i=1,2,\ldots,r$ and $\widetilde{\phi}_i; i = r+1, r+2,\ldots,s$ for $\text{Min.}\widetilde{Z}_i(\widetilde{x}); i = r+1, r+2,\ldots,s$.
- STEP4: Convert the SFFMLP problem in (2) to the SFFLP problem in (8-11) and collect them in the SFFLP problem as modelled in (12).
- STEP5: Find the fuzzy optimal solution for each branch in (12). Or, solve the SFFLP problems in (8-11).
- STEP6: Compare among the fuzzy optimal solutions for the branches of (12), or among fuzzy optimal solution of the SFFLP problems in (8-11). Select a compromise solution among obtained fuzzy optimal solutions of (12) for the original FFMLP problem in (2).

V. NUMERICAL EXAMPLE

We present an example of the implementation of the research methodology in the problem statement within the framework of the MFFLP problem.

Solve the following MFFLP problem:

$$\begin{aligned}
&\text{Max. } \widetilde{Z}_1(\widetilde{x}) \underset{\Re}{=} (2, \tfrac{1}{2}, \tfrac{1}{3})\widetilde{x}_1 + (4, \tfrac{1}{3}, \tfrac{1}{2})\widetilde{x}_2 + (3, \tfrac{3}{4}, \tfrac{1}{4})\widetilde{x}_3,\\
&\text{Max. } \widetilde{Z}_2(\widetilde{x}) \underset{\Re}{=} (3, \tfrac{1}{3}, \tfrac{2}{3})\widetilde{x}_1 + (1, \tfrac{1}{2}, \tfrac{1}{3})\widetilde{x}_2 + (2, \tfrac{1}{4}, \tfrac{3}{4})\widetilde{x}_3,\\
&\text{Max. } \widetilde{Z}_3(\widetilde{x}) \underset{\Re}{=} (1, \tfrac{1}{5}, \tfrac{3}{5})\widetilde{x}_1 + (1, \tfrac{2}{3}, \tfrac{1}{4})\widetilde{x}_2 + (2, \tfrac{1}{2}, \tfrac{1}{2})\widetilde{x}_3,\\
&\text{Min. } \widetilde{Z}_4(\widetilde{x}) \underset{\Re}{=} (\tfrac{-5}{2}, \tfrac{1}{2}, \tfrac{2}{3})\widetilde{x}_1 + (-2, \tfrac{1}{3}, \tfrac{2}{3})\widetilde{x}_2 + (-2, \tfrac{1}{2}, \tfrac{1}{2})\widetilde{x}_3,\\
&\text{Min. } \widetilde{Z}_5(\widetilde{x}) \underset{\Re}{=} (-1, \tfrac{2}{3}, \tfrac{1}{3})\widetilde{x}_1 + (-3, \tfrac{1}{5}, \tfrac{3}{5})\widetilde{x}_2 + (-2, \tfrac{3}{4}, \tfrac{1}{4})\widetilde{x}_3,\\
&\text{s.t. } (3, \tfrac{1}{2}, \tfrac{1}{2})\widetilde{x}_1 + (4, \tfrac{1}{3}, \tfrac{2}{3})\widetilde{x}_2 + (2, \tfrac{1}{3}, \tfrac{1}{3})\widetilde{x}_3 \underset{\Re}{\leq} (40, \tfrac{1}{7}, \tfrac{2}{7}),\\
&\quad (2, \tfrac{2}{5}, \tfrac{3}{5})\widetilde{x}_1 + (1, \tfrac{1}{10}, \tfrac{1}{5})\widetilde{x}_2 + (2, \tfrac{1}{7}, \tfrac{2}{7})\widetilde{x}_3 \underset{\Re}{\leq} (20, \tfrac{1}{4}, \tfrac{1}{8}),\\
&\quad (1, \tfrac{1}{9}, \tfrac{2}{9})\widetilde{x}_1 + (3, \tfrac{1}{6}, \tfrac{1}{6})\widetilde{x}_2 + (2, \tfrac{3}{10}, \tfrac{1}{10})\widetilde{x}_3 \underset{\Re}{\leq} (30, \tfrac{1}{9}, \tfrac{1}{3}),\\
&\quad \widetilde{x}_j \underset{\Re}{\geq} \widetilde{0}, \quad j=1,2,3.
\end{aligned} \tag{13}$$

Solution: Now, we defuzzify and convert $\widetilde{a}_{ij}, i = 1,2,3; j = 1,2,3$ partially from (13) into their corresponding deterministic a_{ij} by utilizing the linear ranking function in (3), to convert the problem to its SFFMLP problem in the following optimization problem.

$$\begin{aligned}
&\text{Max. } \widetilde{Z}_1(\widetilde{x}) \underset{\Re}{=} (2, \tfrac{1}{2}, \tfrac{1}{3})\widetilde{x}_1 + (4, \tfrac{1}{3}, \tfrac{1}{2})\widetilde{x}_2 + (3, \tfrac{3}{4}, \tfrac{1}{4})\widetilde{x}_3,\\
&\text{Max. } \widetilde{Z}_2(\widetilde{x}) \underset{\Re}{=} (3, \tfrac{1}{3}, \tfrac{2}{3})\widetilde{x}_1 + (1, \tfrac{1}{2}, \tfrac{1}{3})\widetilde{x}_2 + (2, \tfrac{1}{4}, \tfrac{3}{4})\widetilde{x}_3,\\
&\text{Max. } \widetilde{Z}_3(\widetilde{x}) \underset{\Re}{=} (1, \tfrac{1}{5}, \tfrac{3}{5})\widetilde{x}_1 + (1, \tfrac{2}{3}, \tfrac{1}{4})\widetilde{x}_2 + (2, \tfrac{1}{2}, \tfrac{1}{2})\widetilde{x}_3,\\
&\text{Min. } \widetilde{Z}_4(\widetilde{x}) \underset{\Re}{=} (\tfrac{-5}{2}, \tfrac{1}{2}, \tfrac{2}{3})\widetilde{x}_1 + (-2, \tfrac{1}{3}, \tfrac{2}{3})\widetilde{x}_2 + (-2, \tfrac{1}{2}, \tfrac{1}{2})\widetilde{x}_3,\\
&\text{Min. } \widetilde{Z}_5(\widetilde{x}) \underset{\Re}{=} (-1, \tfrac{2}{3}, \tfrac{1}{3})\widetilde{x}_1 + (-3, \tfrac{1}{5}, \tfrac{3}{5})\widetilde{x}_2 + (-2, \tfrac{3}{4}, \tfrac{1}{4})\widetilde{x}_3,\\
&\text{s.t. } 3\widetilde{x}_1 + \tfrac{49}{12}\widetilde{x}_2 + 2\widetilde{x}_3 \underset{\Re}{\leq} (40, \tfrac{1}{7}, \tfrac{2}{7}),\\
&\quad \tfrac{41}{20}\widetilde{x}_1 + \tfrac{41}{40}\widetilde{x}_2 + \tfrac{57}{28}\widetilde{x}_3 \underset{\Re}{\leq} (20, \tfrac{1}{4}, \tfrac{1}{8}),\\
&\quad \tfrac{37}{36}\widetilde{x}_1 + 3\widetilde{x}_2 + \tfrac{39}{20}\widetilde{x}_3 \underset{\Re}{\leq} (30, \tfrac{1}{9}, \tfrac{1}{3}),\\
&\quad \widetilde{x}_j \underset{\Re}{\geq} \widetilde{0}, \quad j=1,2,3.
\end{aligned} \tag{14}$$

There are five objective functions in the SFFMLP problems in (14) competing in the same semi fuzzy constraints environments. We solve each objective function subject to the constraints, and rewriting the obtained FFLP problem as a standard form as follows:

$$\begin{aligned}\text{Max. } \widetilde{Z}_1(\widetilde{x}) &\underset{\Re}{=} (2, \tfrac{1}{2}, \tfrac{1}{3})\widetilde{x}_1 + (4, \tfrac{1}{3}, \tfrac{1}{2})\widetilde{x}_2\\ &\quad + (3, \tfrac{3}{4}, \tfrac{1}{4})\widetilde{x}_3 + (0,0,0)\sum_{j=1}^{3}\widetilde{s}_j,\end{aligned}$$

TABLE 3. The status of the solution-i.

			$(2,\frac{1}{2},\frac{1}{3})$	$(4,\frac{1}{3},\frac{1}{2})$	$(3,\frac{3}{4},\frac{1}{4})$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$	
$\widetilde{B}$	$\widetilde{c}_i$	R.H.S.	$\widetilde{x}_1$	$\widetilde{x}_2$	$\widetilde{x}_3$	$\widetilde{s}_1$	$\widetilde{s}_2$	$\widetilde{s}_3$	Min ratio
$\widetilde{s}_1$	$(0,0,0)$	$(40,\frac{1}{7},\frac{2}{7})$	3	$\frac{49}{12}$*	2	1	0	0	$(\frac{480}{49},\frac{12}{343},\frac{24}{343})\leftarrow$
$\widetilde{s}_2$	$(0,0,0)$	$(20,\frac{1}{4},\frac{1}{8})$	$\frac{41}{20}$	$\frac{41}{40}$	$\frac{57}{28}$	0	1	0	$(\frac{800}{41},\frac{10}{41},\frac{5}{41})$
$\widetilde{s}_3$	$(0,0,0)$	$(30,\frac{1}{9},\frac{1}{3})$	$\frac{37}{36}$	3	$\frac{39}{20}$	0	0	1	$(10,\frac{1}{27},\frac{1}{9})$
	$\widetilde{z}$	$(0,0,0)$	$(-2,\frac{1}{3},\frac{1}{2})$	$(-4,\frac{1}{2},\frac{1}{3})$	$(-3,\frac{1}{4},\frac{3}{4})$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$	$\Re$
				$\uparrow$		$\downarrow$			

TABLE 4. The status of the solution-ii.

			$(2,\frac{1}{2},\frac{1}{3})$	$(4,\frac{1}{3},\frac{1}{2})$	$(3,\frac{3}{4},\frac{1}{4})$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$	
$\widetilde{B}$	$\widetilde{c}_i$	R.H.S.	$\widetilde{x}_1$	$\widetilde{x}_2$	$\widetilde{x}_3$	$\widetilde{s}_1$	$\widetilde{s}_2$	$\widetilde{s}_3$	Min ratio
$\widetilde{x}_2$	$(4,\frac{1}{3},\frac{1}{2})$	$(\frac{480}{49},\frac{12}{343},\frac{24}{343})$	$\frac{36}{49}$	1	$\frac{24}{49}$	$\frac{12}{49}$	0	0	$(20,\frac{1}{14},\frac{1}{7})$
$\widetilde{s}_2$	$(0,0,0)$	$(\frac{488}{49},\frac{2207}{6860},\frac{2207}{13720})$	$\frac{1271}{980}$	0	$\frac{1503}{980}$	$\frac{-123}{490}$	1	0	$(\frac{9760}{1503},\frac{2207}{10521},\frac{2207}{21042})$
$\widetilde{s}_3$	$(0,0,0)$	$(\frac{30}{49},\frac{991}{3087},\frac{451}{1029})$	$\frac{-2075}{1764}$	0	$\frac{471}{980}$*	$\frac{-36}{49}$	0	1	$(\frac{200}{157},\frac{1217}{1822},\frac{611}{670})\leftarrow$
	$\widetilde{z}$	$(\frac{1920}{49},\frac{1168}{343},\frac{1776}{343})$	$(\frac{46}{49},\frac{85}{147},\frac{85}{98})$	$(0,0,0)$	$(\frac{-51}{49},\frac{81}{196},\frac{195}{196})$	$(\frac{48}{49},\frac{4}{49},\frac{6}{49})$	$(0,0,0)$	$(0,0,0)$	$\Re$
					$\uparrow$			$\downarrow$	

TABLE 5. The status of the solution-iii.

			$(2,\frac{1}{2},\frac{1}{3})$	$(4,\frac{1}{3},\frac{1}{2})$	$(3,\frac{3}{4},\frac{1}{4})$	$(0,0,0)$	$0,0,0)$	$(0,0,0)$	
$\widetilde{B}$	$\widetilde{c}_i$	R.H.S.	$\widetilde{x}_1$	$\widetilde{x}_2$	$\widetilde{x}_3$	$\widetilde{s}_1$	$\widetilde{s}_2$	$\widetilde{s}_3$	Min ratio
$\widetilde{x}_2$	$(4,\frac{1}{3},\frac{1}{2})$	$(\frac{1440}{157},\frac{1693}{3515},\frac{4260}{10727})$	$\frac{901}{466}$	1	0	$\frac{156}{157}$	0	$\frac{-160}{157}$	$(\frac{1333}{281},\frac{1471}{5905},\frac{845}{4114})$
$\widetilde{s}_2$	$(0,0,0)$	$(\frac{27851}{3479},\frac{203}{117},\frac{499}{421})$	$\frac{5687}{1126}$*	0	0	$\frac{2173}{1038}$	1	$\frac{-501}{157}$	$(\frac{1081}{682},\frac{685}{1994},\frac{157}{669})\leftarrow$
$\widetilde{x}_3$	$(3,\frac{3}{4},\frac{1}{4})$	$(\frac{200}{157},\frac{1217}{1822},\frac{611}{670})$	$\frac{-5409}{2210}$	0	1	$\frac{-240}{157}$	0	$\frac{980}{471}$	$-$
	$\widetilde{z}$	$(\frac{6360}{157},\frac{2437}{454},\frac{8563}{1285})$	$(\frac{-1118}{695},\frac{1019}{641},\frac{1671}{506})$	$(0,0,0)$	$(0,0,0)$	$(\frac{-1014}{1493},\frac{112}{157},\frac{258}{157})$	$(0,0,0)$	$(\frac{946}{419},\frac{325}{157},\frac{135}{157})$	$\Re$
			$\uparrow$				$\downarrow$		

$$\begin{aligned}\text{s.t. } & 3\widetilde{x}_1+\frac{49}{12}\widetilde{x}_2+2\widetilde{x}_3+\widetilde{s}_1 \underset{\Re}{=} (40,\frac{1}{7},\frac{2}{7}),\\ & \frac{41}{20}\widetilde{x}_1+\frac{41}{40}\widetilde{x}_2+\frac{57}{28}\widetilde{x}_3+\widetilde{s}_2 \underset{\Re}{=} (20,\frac{1}{4},\frac{1}{8}),\\ & \frac{37}{36}\widetilde{x}_1+3\widetilde{x}_2+\frac{39}{20}\widetilde{x}_3+\widetilde{s}_3 \underset{\Re}{=} (30,\frac{1}{9},\frac{1}{3}),\\ & \widetilde{x}_j,\widetilde{s}_j \underset{\Re}{\geq} \widetilde{0}, \quad \forall j,\ i=1,2,3. \end{aligned} \tag{15}$$

The initial tableau of (15) is given in Table 3 above:

From the initial tableau of Table 3, we have;

$$\{(\widetilde{z}_j-\widetilde{c}_j)\} \underset{\Re}{=} \left\{(-2,\frac{1}{3},\frac{1}{2}),(-4,\frac{1}{2},\frac{1}{3}),(-3,\frac{1}{4},\frac{3}{4}),\widetilde{0},\widetilde{0},\widetilde{0}\right\};\quad j=1,\ldots,6.$$

Since, min $\{\widetilde{\gamma}_j\}=\min\{\Re(\widetilde{\gamma}_j)\}=\min\left\{-1\frac{23}{24},-4\frac{1}{24},-2\frac{7}{8},0,0,0\right\}=-4\frac{1}{24}, j=1,\ldots,6$, thus, $\widetilde{x}_2$ should enter and becomes the basic variable.

From Table 3 also, in the min ratio's column we have; min $\Re\left\{(\frac{480}{49},\frac{12}{343},\frac{24}{343}),(\frac{800}{41},\frac{10}{41},\frac{5}{41}),(10,\frac{1}{27},\frac{1}{9})\right\}$. Since, min $\left\{9\frac{276}{343},19\frac{159}{164},10\frac{1}{54}\right\}=9\frac{276}{343}$, the leaving variable is $\widetilde{s}_1$.

The pivotal element is $\frac{49}{12}$. Thus, by using the Gaussian elimination process and the row operations on the Table 3 as follows; $\frac{12}{49}R_1\to R_1'$, $-\frac{41}{40}R_1'+R_2\to R_2'$, $-3R_1'+R_3\to R_3'$ and $(4,\frac{1}{3},\frac{1}{2})R_1'+R_0\to R_0'$. The result is as in Table 4.

Currently, from the simplex tableau of Table 4, we have; $\{(\widetilde{z}_j-\widetilde{c}_j)\} \underset{\Re}{=} \left\{(\frac{46}{49},\frac{85}{147},\frac{85}{98}),(0,0,0)(\frac{-51}{49},\frac{81}{196},\frac{195}{196}),(\frac{48}{49},\frac{4}{49},\frac{6}{49}),(0,0,0),(0,0,0)\right\}; j=1,\ldots,6$. Since, min $\{\widetilde{\gamma}_j\}=\min\{\Re(\widetilde{\gamma}_j)\}=\min\left\{1\frac{13}{1176},0,\frac{-351}{392},\frac{97}{98},0,0\right\}=\frac{-35}{302}, j=1,\ldots,6$, thus, $\widetilde{x}_3$ should enter and becomes the basic variable.

Again, from Table 4, and in the min ratio's column we have; min $\Re\left\{(20,\frac{1}{14},\frac{1}{7}),(\frac{9760}{1503},\frac{2207}{10521},\frac{2207}{21042}),(\frac{200}{157},\frac{1217}{1822},\frac{611}{670})\right\}$. Since, min $\left\{20\frac{1}{56},6\frac{79}{169},1\frac{72}{215}\right\}=1\frac{72}{215}$, hence, the leaving variable is $\widetilde{s}_3$.

The pivotal element is $\frac{471}{980}$. Thus, by using the Gaussian elimination process and the row operations on Table 4 as follows; $\frac{980}{471}R_3\to R_3'$, $\frac{-24}{49}R_3'+R_1\to R_1'$, $\frac{-1503}{980}R_3'+R_2\to R_2'$ and $(\frac{51}{49},\frac{195}{196},\frac{81}{196})R_3'+R_0\to R_0'$. The result is as in Table 5.

Since, $\{(\widetilde{z}_j-\widetilde{c}_j)\} \underset{\Re}{=} \left\{(\frac{-1118}{695},\frac{1019}{641},\frac{1671}{506}),(0,0,0),(0,0,0),(\frac{-1014}{1493},\frac{112}{157},\frac{258}{157}),(0,0,0),(\frac{946}{419},\frac{325}{157},\frac{135}{157})\right\}; j=1,\ldots,6$, and $\{\widetilde{\gamma}_j\}=\{\Re(\widetilde{\gamma}_j)\}=\left\{\frac{-556}{471},0,0,\frac{-2547}{5702},0,\frac{1091}{546}\right\}$.;

TABLE 6. The status of the solution-iv.

			$(2,\frac{1}{2},\frac{1}{3})$	$(4,\frac{1}{3},\frac{1}{2})$	$(3,\frac{3}{4},\frac{1}{4})$	$(0,0,0)$	$0,0,0)$	$(0,0,0)$	
$\widetilde{B}$	$\widetilde{c}_i$	$\widetilde{\text{R.H.S.}}$	$\widetilde{x}_1$	$\widetilde{x}_2$	$\widetilde{x}_3$	$\widetilde{s}_1$	$\widetilde{s}_2$	$\widetilde{s}_3$	Min ratio
$\widetilde{x}_2$	$(4,\frac{1}{3},\frac{1}{2})$	$(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451})$	0	0	0	$\frac{410}{2133}$	$\frac{-410}{1071}$	$\frac{1393}{6879}$	
$\widetilde{x}_1$	$(2,\frac{1}{2},\frac{1}{3})$	$(\frac{1081}{682},\frac{685}{1994},\frac{157}{669})$	1	0	0	$\frac{2082}{5023}$	$\frac{1126}{5687}$	$\frac{-1795}{2841}$	
$\widetilde{x}_3$	$(3,\frac{3}{4},\frac{1}{4})$	$(\frac{2891}{561},\frac{863}{572},\frac{1195}{804})$	0	0	1	$\frac{-909}{859}$	$\frac{-431}{460}$	$\frac{1435}{557}$	
$\widetilde{z}$		$(\frac{5081}{118},\frac{2521}{226},\frac{659}{69})$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$	$(\frac{-101}{8144},\frac{5751}{2762},\frac{1752}{761})$	$(\frac{859}{2697},\frac{3028}{4631},\frac{96}{305})$	$(\frac{4037}{3252},\frac{289}{94},\frac{1099}{373})$	$\Re$

TABLE 7. The solution of the objective functions.

Object function$(\widetilde{Z}_i)$	$\widetilde{\phi}_i$	$\widetilde{X}_i\left\{\widetilde{x}_1,\widetilde{x}_2,\widetilde{x}_3\right\}$
Max.$\widetilde{Z}_1$	$(\frac{5081}{118},\frac{2521}{226},\frac{659}{69})$	$\widetilde{X}_1\left\{(\frac{1081}{682},\frac{685}{1994},\frac{157}{669}),(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451}),(\frac{2891}{561},\frac{863}{572},\frac{1195}{804})\right\}$
Max.$\widetilde{Z}_2$	$(\frac{1200}{41},\frac{445}{123},\frac{1645}{246})$	$\widetilde{X}_2\left\{(\frac{400}{41},\frac{5}{41},\frac{5}{82}),(0,0,0),(0,0,0)\right\}$
Max.$\widetilde{Z}_3$	$(\frac{1120}{57},\frac{98}{19},\frac{287}{57})$	$\widetilde{X}_3\left\{(0,0,0),(0,0,0)(\frac{560}{57},\frac{7}{57},\frac{7}{114})\right\}$
Min.$\widetilde{Z}_4$	$(\frac{-2338}{85},\frac{3277}{515},\frac{15671}{1463})$	$\widetilde{X}_4\left\{(\frac{9760}{1271},\frac{2207}{8897},\frac{2207}{17794}),(\frac{5280}{1271},\frac{1122}{8897},\frac{2244}{8897}),(0,0,0)\right\}$
Min.$\widetilde{Z}_5$	$(\frac{-6647}{220},\frac{6661}{679},\frac{4435}{497})$	$\widetilde{X}_5\left\{(\frac{848}{535},\frac{187}{549},\frac{157}{669}),(\frac{4837}{792},\frac{1028}{1099},\frac{1156}{1095}),(\frac{1479}{287},\frac{464}{309},\frac{1195}{804})\right\}$

$j=1,\ldots,6=\frac{-556}{471}$. Thus, $\widetilde{x}_1$ should enter and becomes the basic variable.

Again, from Table 5, and in the min ratio's column we have; min $\Re\left\{(\frac{1333}{281},\frac{1471}{5905},\frac{845}{4114}),(\frac{1081}{682},\frac{685}{1994},\frac{157}{669})\right\}$. Since, min $\left\{\frac{1931}{408},\frac{1293}{830}\right\}=\frac{1293}{830}$, hence, the leaving variable is $\widetilde{s}_2$.

The pivotal element is $\frac{5687}{1126}$. Thus, by using the Gaussian elimination process and the row operations on Table 5 as follows; $\frac{1126}{5687}R_2\rightarrow R_2'$, $\frac{-901}{466}R_2'+R_1\rightarrow R_1'$, $\frac{5409}{2210}R_2'+R_3\rightarrow R_3'$ and $(\frac{1118}{695},\frac{1671}{506},\frac{1019}{641})R_2'+R_0\rightarrow R_0'$. The result is as in Table 6:

Now, in the current solution in Table 6, $\left\{(\widetilde{z}_j-\widetilde{c}_j)\right\}\underset{\Re}{=}\left\{(0,0,0)(0,0,0)(0,0,0)(\frac{-101}{8144},\frac{5751}{2762},\frac{1752}{761})(\frac{859}{2697},\frac{3028}{4631},\frac{96}{305})(\frac{4037}{3252},\frac{289}{94},\frac{1099}{373})\right\};j=1,\ldots,6$, and $\left\{\widetilde{\gamma}_j\right\}=\left\{\Re(\widetilde{\gamma}_j)\right\}=\left\{0,0,0,\frac{367}{8613},\frac{772}{3303},\frac{2143}{1772}\right\}\geq 0;j=1,\ldots,6$.

Thus, according to the optimality feasible condition of STEP5 of the CSA, no more variable may enter the basis. In addition, $\left\{\Re(\widetilde{x}_{B_i})\right\}=\Re\left\{(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451}),(\frac{1081}{682},\frac{685}{1994},\frac{157}{669}),(\frac{2891}{561},\frac{863}{572},\frac{1195}{804})\right\}\geq 0;\ i=1,2,3.$

The fuzzy set, $\left\{\widetilde{Z};\widetilde{X}(\widetilde{x}_1,\widetilde{x}_2,\widetilde{x}_3)\right\}=\left\{(\frac{5081}{118},\frac{2521}{226},\frac{659}{69});\widetilde{X}((\frac{1081}{682},\frac{685}{1994},\frac{157}{669}),(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451}),(\frac{2891}{561},\frac{863}{572},\frac{1195}{804}))\right\}$ is the fuzzy optimal solution for the problem (15), because it is the fuzzy feasible solution through the environment of the linear ranking function in (3) and its properties for the comparison of fuzzy numbers in Section II, as verified below.

$$(3,\frac{1}{2},\frac{1}{2})\widetilde{x}_1+(4,\frac{1}{3},\frac{2}{3})\widetilde{x}_2+(2,\frac{1}{3},\frac{1}{3})\widetilde{x}_3\underset{\Re}{\leq}(40,\frac{1}{7},\frac{2}{7}),$$
$$(2,\frac{2}{5},\frac{3}{5})\widetilde{x}_1+(1,\frac{1}{10},\frac{1}{5})\widetilde{x}_2+(2,\frac{1}{7},\frac{2}{7})\widetilde{x}_3\underset{\Re}{\leq}(20,\frac{1}{4},\frac{1}{8}),$$
$$(1,\frac{1}{9},\frac{2}{9})\widetilde{x}_1+(3,\frac{1}{6},\frac{1}{6})\widetilde{x}_2+(2,\frac{3}{10},\frac{1}{10})\widetilde{x}_3\underset{\Re}{\leq}(30,\frac{1}{9},\frac{1}{3}),$$
$$\widetilde{x}_j\underset{\Re}{\geq}\widetilde{0},\quad j=1,2,3.\Rightarrow$$

$$(3,\frac{1}{2},\frac{1}{2})(\frac{1081}{682},\frac{685}{1994},\frac{157}{669})+(4,\frac{1}{3},\frac{2}{3})(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451})+(2,\frac{1}{3},\frac{1}{3})(\frac{2891}{561},\frac{863}{572},\frac{1195}{804})\underset{\Re}{\leq}(40,\frac{1}{7},\frac{2}{7}),$$
$$(2,\frac{2}{5},\frac{3}{5})(\frac{1081}{682},\frac{685}{1994},\frac{157}{669})+(1,\frac{1}{10},\frac{1}{5})(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451})+(2,\frac{1}{7},\frac{2}{7})(\frac{2891}{561},\frac{863}{572},\frac{1195}{804})\underset{\Re}{\leq}(20,\frac{1}{4},\frac{1}{8}),$$
$$(1,\frac{1}{9},\frac{2}{9})(\frac{1081}{682},\frac{685}{1994},\frac{157}{669})+(3,\frac{1}{6},\frac{1}{6})(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451})+(2,\frac{3}{10},\frac{1}{10})(\frac{2891}{561},\frac{863}{572},\frac{1195}{804})\underset{\Re}{\leq}(30,\frac{1}{9},\frac{1}{3}),$$
$$\widetilde{x}_j\underset{\Re}{\geq}\widetilde{0},\quad j=1,2,3.\Rightarrow$$

$$\Re((3,\frac{1}{2},\frac{1}{2})(\frac{1081}{682},\frac{685}{1994},\frac{157}{669})+(4,\frac{1}{3},\frac{2}{3})(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451})+(2,\frac{1}{3},\frac{1}{3})(\frac{2891}{561},\frac{863}{572},\frac{1195}{804}))\leq\Re(40,\frac{1}{7},\frac{2}{7}),$$
$$\Re((2,\frac{2}{5},\frac{3}{5})(\frac{1081}{682},\frac{685}{1994},\frac{157}{669})+(1,\frac{1}{10},\frac{1}{5})(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451})+(2,\frac{1}{7},\frac{2}{7})(\frac{2891}{561},\frac{863}{572},\frac{1195}{804}))\leq\Re(20,\frac{1}{4},\frac{1}{8}),$$
$$\Re((1,\frac{1}{9},\frac{2}{9})(\frac{1081}{682},\frac{685}{1994},\frac{157}{669})+(3,\frac{1}{6},\frac{1}{6})(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451})+(2,\frac{3}{10},\frac{1}{10})(\frac{2891}{561},\frac{863}{572},\frac{1195}{804}))\leq\Re(30,\frac{1}{9},\frac{1}{3}),$$
$$\widetilde{x}_j\underset{\Re}{\geq}\widetilde{0},\quad j=1,2,3.\Rightarrow$$

$$\Re((3,\frac{1}{2},\frac{1}{2})(\frac{1081}{682},\frac{685}{1994},\frac{157}{669}))+\Re((4,\frac{1}{3},\frac{2}{3})(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451}))+\Re((2,\frac{1}{3},\frac{1}{3})(\frac{2891}{561},\frac{863}{572},\frac{1195}{804}))\leq\Re(40,\frac{1}{7},\frac{2}{7}),$$
$$\Re((2,\frac{2}{5},\frac{3}{5})(\frac{1081}{682},\frac{685}{1994},\frac{157}{669}))+\Re((1,\frac{1}{10},\frac{1}{5})(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451}))+\Re((2,\frac{1}{7},\frac{2}{7})(\frac{2891}{561},\frac{863}{572},\frac{1195}{804}))\leq\Re(20,\frac{1}{4},\frac{1}{8}),$$
$$\Re((1,\frac{1}{9},\frac{2}{9})(\frac{1081}{682},\frac{685}{1994},\frac{157}{669}))+\Re((3,\frac{1}{6},\frac{1}{6})(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451}))+\Re((2,\frac{3}{10},\frac{1}{10})(\frac{2891}{561},\frac{863}{572},\frac{1195}{804}))\leq\Re(30,\frac{1}{9},\frac{1}{3}),$$
$$\widetilde{x}_j\underset{\Re}{\geq}\widetilde{0},\quad j=1,2,3.\Rightarrow$$

$$\Re(3,\frac{1}{2},\frac{1}{2})\Re(\frac{1081}{682},\frac{685}{1994},\frac{157}{669})+\Re(4,\frac{1}{3},\frac{2}{3})\Re(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451})+\Re(2,\frac{1}{3},\frac{1}{3})\Re(\frac{2891}{561},\frac{863}{572},\frac{1195}{804})\leq\Re(40,\frac{1}{7},\frac{2}{7}),$$
$$\Re(2,\frac{2}{5},\frac{3}{5})\Re(\frac{1081}{682},\frac{685}{1994},\frac{157}{669})+\Re(1,\frac{1}{10},\frac{1}{5})\Re(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451})+\Re(2,\frac{1}{7},\frac{2}{7})\Re(\frac{2891}{561},\frac{863}{572},\frac{1195}{804})\leq\Re(20,\frac{1}{4},\frac{1}{8}),$$
$$\Re(1,\frac{1}{9},\frac{2}{9})\Re(\frac{1081}{682},\frac{685}{1994},\frac{157}{669})+\Re(3,\frac{1}{6},\frac{1}{6})\Re(\frac{2333}{382},\frac{1028}{1099},\frac{1540}{1451})+\Re(2,\frac{3}{10},\frac{1}{10})\Re(\frac{2891}{561},\frac{863}{572},\frac{1195}{804})\leq\Re(30,\frac{1}{9},\frac{1}{3}),$$
$$\widetilde{x}_j\underset{\Re}{\geq}\widetilde{0},\quad j=1,2,3.\Rightarrow$$

$$(3)(\frac{1293}{830})+(\frac{49}{12})(\frac{2388}{389})+(2)(\frac{1673}{325})\leq\frac{1121}{28},$$
$$(\frac{41}{20})(\frac{1293}{830})+(\frac{41}{40})(\frac{2388}{389})+(\frac{57}{28})(\frac{1673}{325})\leq\frac{639}{32},$$
$$(\frac{37}{36})(\frac{1293}{830})+(3)(\frac{2388}{389})+(\frac{39}{20})(\frac{1673}{325})\leq\frac{541}{18},$$
$$\widetilde{x}_j\underset{\Re}{\geq}\widetilde{0},\quad j=1,2,3.\Rightarrow$$
$$\frac{1121}{28}\leq\frac{1121}{28},$$
$$19\frac{221}{229}\leq19\frac{31}{32},$$
$$\frac{541}{18}\leq\frac{541}{18},\quad \widetilde{x}_j\underset{\Re}{\geq}\widetilde{0},\quad j=1,2,3.$$

As the same way to solve (15), we can obtain the solution for Max. $\widetilde{Z}_i(\widetilde{x})$, $i=2,3$ and Min. $\widetilde{Z}_i(\widetilde{x})$, $i=4,5$. The solution is shown in Table 7.

Now, by employing the model (8) in the CSA, we can convert the system problem in (13) to the SFFLP problem into a unique objective function in the form of the first part of the compromise problem in (12), as follows:

$$\textbf{Max. } \widetilde{Z}_{\widetilde{\phi}}(\widetilde{x})$$
$$\underset{\Re}{=}[((2,\frac{1}{2},\frac{1}{3})\widetilde{x}_1+(4,\frac{1}{3},\frac{1}{2})\widetilde{x}_2$$
$$+(3,\frac{3}{4},\frac{1}{4})\widetilde{x}_3)/(\frac{5081}{118},\frac{2521}{226},\frac{659}{69})$$
$$+((3,\frac{1}{3},\frac{2}{3})\widetilde{x}_1+(1,\frac{1}{2},\frac{1}{3})\widetilde{x}_2$$
$$+(2,\frac{1}{4},\frac{3}{4})\widetilde{x}_3)/(\frac{1200}{41},\frac{445}{123},\frac{1645}{246})+((1,\frac{1}{5},\frac{3}{5})\widetilde{x}_1$$
$$+(1,\frac{2}{3},\frac{1}{4})\widetilde{x}_2+(2,\frac{1}{2},\frac{1}{2})\widetilde{x}_3)/(\frac{1120}{57},\frac{98}{19},\frac{287}{57})]$$
$$-[((\frac{-5}{2},\frac{1}{2},\frac{2}{3})\widetilde{x}_1+(-2,\frac{1}{3},\frac{2}{3})\widetilde{x}_2$$
$$+(-2,\frac{1}{2},\frac{1}{2})\widetilde{x}_3)/(\frac{-2338}{85},\frac{3277}{515},\frac{15671}{1463})$$
$$+((-1,\frac{2}{3},\frac{1}{3})\widetilde{x}_1+(-3,\frac{1}{5},\frac{3}{5})\widetilde{x}_2$$
$$+(-2,\frac{3}{4},\frac{1}{4})\widetilde{x}_3)/(\frac{-6647}{220},\frac{6661}{679},\frac{4435}{497})]$$
$$\text{s.t. } 3\widetilde{x}_1+\frac{49}{12}\widetilde{x}_2+2\widetilde{x}_3\underset{\Re}{\leq}(40,\frac{1}{7},\frac{2}{7}),$$
$$\frac{41}{20}\widetilde{x}_1+\frac{41}{40}\widetilde{x}_2+\frac{57}{28}\widetilde{x}_3\underset{\Re}{\leq}(20,\frac{1}{4},\frac{1}{8}),$$
$$\frac{37}{36}\widetilde{x}_1+3\widetilde{x}_2+\frac{39}{20}\widetilde{x}_3\underset{\Re}{\leq}(30,\frac{1}{9},\frac{1}{3}),$$
$$\widetilde{x}_j\underset{\Re}{\geq}\widetilde{0},\quad j=1,2,3. \tag{16}$$

By applying the algebra arithmetic processes on the SFFLP problem in (16) within the framework of the fuzzy algebra properties on T_rFNs, we get the following model in (17) as;

$$\textbf{Max. } \widetilde{Z}_{\widetilde{\phi}}(\widetilde{x})$$
$$\underset{\Re}{=}(\frac{1762}{3567},\frac{402}{2735},\frac{699}{3788})\widetilde{x}_1$$
$$+(\frac{99}{16639},\frac{233}{1205},\frac{377}{2452})\widetilde{x}_2+(\frac{137}{1358},\frac{699}{4033},\frac{377}{1876})\widetilde{x}_3$$
$$\text{s.t. } 3\widetilde{x}_1+\frac{49}{12}\widetilde{x}_2+2\widetilde{x}_3\underset{\Re}{\leq}(40,\frac{1}{7},\frac{2}{7}),$$
$$\frac{41}{20}\widetilde{x}_1+\frac{41}{40}\widetilde{x}_2+\frac{57}{28}\widetilde{x}_3\underset{\Re}{\leq}(20,\frac{1}{4},\frac{1}{8}),$$
$$\frac{37}{36}\widetilde{x}_1+3\widetilde{x}_2+\frac{39}{20}\widetilde{x}_3\underset{\Re}{\leq}(30,\frac{1}{9},\frac{1}{3}),$$
$$\widetilde{x}_j\underset{\Re}{\geq}\widetilde{0},\quad j=1,2,3. \tag{17}$$

Utilizing fuzzy simplex method through the proposed solution algorithm of the FFLP problem in the CSA, the solution of (17) can be obtained as follows;

$$\left\{\textbf{Max. } \widetilde{Z}_{\widetilde{\phi}}(\widetilde{x});\ \widetilde{X}_1(\widetilde{x}_1,\widetilde{x}_2,\widetilde{x}_3)\right\} = \left\{(\frac{1653}{343},\frac{647}{433},\frac{2299}{1256});\ \widetilde{X}_1((\frac{400}{41},\frac{5}{41},\frac{5}{82}),(0,0,0),(0,0,0))\right\}.$$

This obtained solution is a fuzzy compromise solution for the FFMLP problem in (13).

Using FAAAM in (9) in the CSA, we can convert the system problem in (13) to the SFFLP problem into a unique objective function in the form of the second part of the fuzzy compromise problem in (12), as follows:

$$\begin{aligned}
&\widetilde{\textbf{Max.}}_{\widetilde{\phi}_{\text{FAAA}}}(\widetilde{x})\\
&\underset{\Re}{=} [((2,\frac{1}{2},\frac{1}{3})\widetilde{x}_1+(4,\frac{1}{3},\frac{1}{2})\widetilde{x}_2+(3,\frac{3}{4},\frac{1}{4})\widetilde{x}_3\\
&\quad+(3,\frac{1}{3},\frac{2}{3})\widetilde{x}_1+(1,\frac{1}{2},\frac{1}{3})\widetilde{x}_2+(2,\frac{1}{4},\frac{3}{4})\widetilde{x}_3\\
&\quad+(1,\frac{1}{5},\frac{3}{5})\widetilde{x}_1+(1,\frac{2}{3},\frac{1}{4})\widetilde{x}_2+(2,\frac{1}{2},\frac{1}{2})\widetilde{x}_3)\\
&\quad-((\frac{-5}{2},\frac{1}{2},\frac{2}{3})\widetilde{x}_1+(-2,\frac{1}{3},\frac{2}{3})\widetilde{x}_2+(-2,\frac{1}{2},\frac{1}{2})\widetilde{x}_3\\
&\quad+(-1,\frac{2}{3},\frac{1}{3})\widetilde{x}_1+(-3,\frac{1}{5},\frac{3}{5})\widetilde{x}_2\\
&\quad+(-2,\frac{3}{4},\frac{1}{4})\widetilde{x}_3)]\oslash(\frac{2081}{324},\frac{5671}{541},\frac{5025}{544})\\
&\quad\text{s.t. } 3\widetilde{x}_1+\frac{49}{12}\widetilde{x}_2+2\widetilde{x}_3 \underset{\Re}{\leq} (40,\frac{1}{7},\frac{2}{7}),\\
&\qquad\frac{41}{20}\widetilde{x}_1+\frac{41}{40}\widetilde{x}_2+\frac{57}{28}\widetilde{x}_3 \underset{\Re}{\leq} (20,\frac{1}{4},\frac{1}{8}),\\
&\qquad\frac{37}{36}\widetilde{x}_1+3\widetilde{x}_2+\frac{39}{20}\widetilde{x}_3 \underset{\Re}{\leq} (30,\frac{1}{9},\frac{1}{3}),\\
&\qquad\widetilde{x}_j \underset{\Re}{\geq} \widetilde{0},\quad j=1,2,3.
\end{aligned}\tag{18}$$

$$\begin{aligned}
&\widetilde{\textbf{Max.}}_{\widetilde{\phi}_{\text{FAAA}}}(\widetilde{x}) \underset{\Re}{=} (\frac{810}{2081},\frac{1949}{2160},\frac{675}{649})\widetilde{x}_1\\
&\quad+(\frac{324}{2081},\frac{307}{568},\frac{757}{1221})\widetilde{x}_2+(\frac{972}{2081},\frac{1145}{1041},\frac{820}{737})\widetilde{x}_3\\
&\text{s.t. } 3\widetilde{x}_1+\frac{49}{12}\widetilde{x}_2+2\widetilde{x}_3 \underset{\Re}{\leq} (40,\frac{1}{7},\frac{2}{7}),\\
&\text{s.t. } \frac{41}{20}\widetilde{x}_1+\frac{41}{40}\widetilde{x}_2+\frac{57}{28}\widetilde{x}_3 \underset{\Re}{\leq} (20,\frac{1}{4},\frac{1}{8}),\\
&\text{s.t. } \frac{37}{36}\widetilde{x}_1+3\widetilde{x}_2+\frac{39}{20}\widetilde{x}_3 \underset{\Re}{\leq} (30,\frac{1}{9},\frac{1}{3}),\\
&\text{s.t. } \widetilde{x}_j \underset{\Re}{\geq} \widetilde{0},\quad j=1,2,3.
\end{aligned}\tag{19}$$

Again, utilizing fuzzy simplex method through the proposed solution algorithm of the FFLP problem in the CSA, the solution of (19) can be obtained as follows;

$$\left\{\textbf{Max.}\widetilde{Z}_{\widetilde{\phi}_{\text{FAAA}}}(\widetilde{x}) \underset{\Re}{=} (\widetilde{x});\ \widetilde{X}_2(\widetilde{x}_1,\widetilde{x}_2,\widetilde{x}_3)\right\} = \left\{(\frac{413}{90},\frac{2466}{227},\frac{1359}{124});\ \widetilde{X}_2((0,0,0),(0,0,0),(\frac{560}{57},\frac{7}{57},\frac{7}{114}))\right\}.$$

This obtained solution is a fuzzy compromise solution for the FFMLP problem in (13).

Using the fuzzy arithmetic mean in (10) in the CSA, we can convert the system problem in (13) to the SFFLP problem into a unique objective function again in the form of the third part of the fuzzy compromise problem in (12), as follows:

$$\begin{aligned}
&\widetilde{\textbf{Max.}}_{\widetilde{\phi}_{\text{mean}}}(\widetilde{x})\\
&\underset{\Re}{=} [(6,\frac{31}{30},\frac{8}{5})\widetilde{x}_1+(6,\frac{3}{2},\frac{13}{12})\widetilde{x}_2\\
&\quad+(7,\frac{3}{2},\frac{3}{2})\widetilde{x}_3]\oslash(\frac{3955}{129},\frac{4119}{620},\frac{25109}{3541})\ominus[(\frac{-7}{2},\frac{7}{6},1)\widetilde{x}_1\\
&\quad+(-5,\frac{8}{15},\frac{19}{15})\widetilde{x}_2+(-4,\frac{5}{4},\frac{3}{4})\widetilde{x}_3)]\\
&\quad\oslash(\frac{-4733}{164},\frac{3550}{439},\frac{2027}{203})\\
&\text{s.t. } 3\widetilde{x}_1+\frac{49}{12}\widetilde{x}_2+2\widetilde{x}_3 \underset{\Re}{\leq} (40,\frac{1}{7},\frac{2}{7}),\\
&\quad\frac{41}{20}\widetilde{x}_1+\frac{41}{40}\widetilde{x}_2+\frac{57}{28}\widetilde{x}_3 \underset{\Re}{\leq} (20,\frac{1}{4},\frac{1}{8}),\\
&\quad\frac{37}{36}\widetilde{x}_1+3\widetilde{x}_2+\frac{39}{20}\widetilde{x}_3 \underset{\Re}{\leq} (30,\frac{1}{9},\frac{1}{3}),\\
&\quad\widetilde{x}_j \underset{\Re}{\geq} \widetilde{0},\quad j=1,2,3.
\end{aligned}\tag{20}$$

$$\begin{aligned}
&\widetilde{\textbf{Max.}}_{\widetilde{\phi}_{\text{mean}}}(\widetilde{x})\\
&\underset{\Re}{=} (\frac{502}{6745},\frac{249}{1687},\frac{1124}{6351})\widetilde{x}_1\\
&\quad+(\frac{357}{15902},\frac{466}{2497},\frac{290}{1857})\widetilde{x}_2+(\frac{219}{2441},\frac{251}{1507},\frac{257}{1355})\widetilde{x}_3\\
&\text{s.t. } 3\widetilde{x}_1+\frac{49}{12}\widetilde{x}_2+2\widetilde{x}_3 \underset{\Re}{\leq} (40,\frac{1}{7},\frac{2}{7}),\\
&\quad\frac{41}{20}\widetilde{x}_1+\frac{41}{40}\widetilde{x}_2+\frac{57}{28}\widetilde{x}_3 \underset{\Re}{\leq} (20,\frac{1}{4},\frac{1}{8}),\\
&\quad\frac{37}{36}\widetilde{x}_1+3\widetilde{x}_2+\frac{39}{20}\widetilde{x}_3 \underset{\Re}{\leq} (30,\frac{1}{9},\frac{1}{3}),\\
&\quad\widetilde{x}_j \underset{\Re}{\geq} \widetilde{0},\quad j=1,2,3.
\end{aligned}\tag{21}$$

Again, utilizing fuzzy simplex method through the proposed solution algorithm of the FFLP problem in the CSA, the solution of (21) can be obtained as follows;

$$\left\{\textbf{Max. } \widetilde{Z}_{\widetilde{\phi}_{\text{mean}}}(\widetilde{x}) \underset{\Re}{=} (\widetilde{x});\ \widetilde{X}_3(\widetilde{x}_1,\widetilde{x}_2,\widetilde{x}_3)\right\} = \left\{(\frac{1353}{1535},\frac{967}{587},\frac{499}{267});\ \widetilde{X}_3((0,0,0),(0,0,0),(\frac{560}{57},\frac{7}{57},\frac{7}{114}))\right\}.$$

TABLE 8. The different fuzzy compromise solutions.

$\textbf{Max.}\widetilde{Z}_{\widetilde{\phi}_{(.)}}(\widetilde{x})$	$\left\{\widetilde{\phi}_i; \widetilde{X}_i(\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3)\right\}, i = 1, 2, 3, 4$
$\textbf{Max.}\widetilde{Z}_{\widetilde{\phi}}(\widetilde{x})$	$\left\{(\frac{1653}{343}, \frac{647}{433}, \frac{2299}{1256}); \widetilde{X}_1((\frac{400}{41}, \frac{5}{41}, \frac{5}{82}), (0,0,0), (0,0,0))\right\}$
$\textbf{Max.}\widetilde{Z}_{\widetilde{\phi}_{\text{FAAA}}}(\widetilde{x})$	$\left\{(\frac{413}{90}, \frac{2466}{227}, \frac{1359}{124}); \widetilde{X}_2((0,0,0), (0,0,0), (\frac{560}{57}, \frac{7}{57}, \frac{7}{114}))\right\}$
$\textbf{Max.}\widetilde{Z}_{\widetilde{\phi}_{\text{mean}}}(\widetilde{x})$	$\left\{(\frac{1353}{1535}, \frac{967}{587}, \frac{499}{267}); \widetilde{X}_3((0,0,0), (0,0,0), (\frac{560}{57}, \frac{7}{57}, \frac{7}{114}))\right\}$
$\textbf{Max.}\widetilde{Z}_{\widetilde{\phi}_{\text{median}}}(\widetilde{x})$	$\left\{(\frac{412}{417}, \frac{664}{393}, \frac{901}{531}); \widetilde{X}_4((0,0,0), (0,0,0), (\frac{560}{57}, \frac{7}{57}, \frac{7}{114}))\right\}$

This obtained solution is a fuzzy compromise solution for the FFMLP problem in (13). Using the fuzzy arithmetic median in (11) in the CSA, we can convert the system problem in (13) to the SFFLP problem into a unique objective function again in the form of the fourth part of the fuzzy compromise problem in (12), as follows:

$$\begin{aligned}
&\widetilde{\textbf{Max.}}_{\widetilde{\phi}_{\text{median}}}(\widetilde{x}) \\
&\underset{\Re}{=} [(6, \frac{31}{30}, \frac{8}{5})\widetilde{x}_1 + (6, \frac{3}{2}, \frac{13}{12})\widetilde{x}_2 \\
&\quad + (7, \frac{3}{2}, \frac{3}{2})\widetilde{x}_3] \oslash (\frac{1200}{41}, \frac{445}{123}, \frac{1645}{246}) \ominus [(\frac{-7}{2}, \frac{7}{6}, 1)\widetilde{x}_1 \\
&\quad + (-5, \frac{8}{15}, \frac{19}{15})\widetilde{x}_2 + (-4, \frac{5}{4}, \frac{3}{4})\widetilde{x}_3)] \\
&\quad \oslash(\frac{-4733}{164}, \frac{3550}{439}, \frac{2027}{203}) \\
&\text{s.t. } 3\widetilde{x}_1 + \frac{49}{12}\widetilde{x}_2 + 2\widetilde{x}_3 \underset{\Re}{\leq} (40, \frac{1}{7}, \frac{2}{7}), \\
&\quad \frac{41}{20}\widetilde{x}_1 + \frac{41}{40}\widetilde{x}_2 + \frac{57}{28}\widetilde{x}_3 \underset{\Re}{\leq} (20, \frac{1}{4}, \frac{1}{8}), \\
&\quad \frac{37}{36}\widetilde{x}_1 + 3\widetilde{x}_2 + \frac{39}{20}\widetilde{x}_3 \underset{\Re}{\leq} (30, \frac{1}{9}, \frac{1}{3}), \\
&\quad \widetilde{x}_j \underset{\Re}{\geq} \widetilde{0}, \quad j = 1, 2, 3.
\end{aligned} \tag{22}$$

$$\begin{aligned}
&\widetilde{\textbf{Max.}}_{\widetilde{\phi}_{\text{median}}}(\widetilde{x}) \\
&\underset{\Re}{=} (\frac{268}{3201}, \frac{438}{2905}, \frac{3920}{24139})\widetilde{x}_1 \\
&\quad + (\frac{217}{6835}, \frac{390}{2047}, \frac{561}{3985})\widetilde{x}_2 + (\frac{338}{3361}, \frac{281}{1646}, \frac{482}{2801})\widetilde{x}_3 \\
&\text{s.t. } 3\widetilde{x}_1 + \frac{49}{12}\widetilde{x}_2 + 2\widetilde{x}_3 \underset{\Re}{\leq} (40, \frac{1}{7}, \frac{2}{7}), \\
&\quad \frac{41}{20}\widetilde{x}_1 + \frac{41}{40}\widetilde{x}_2 + \frac{57}{28}\widetilde{x}_3 \underset{\Re}{\leq} (20, \frac{1}{4}, \frac{1}{8}), \\
&\quad \frac{37}{36}\widetilde{x}_1 + 3\widetilde{x}_2 + \frac{39}{20}\widetilde{x}_3 \underset{\Re}{\leq} (30, \frac{1}{9}, \frac{1}{3}), \\
&\quad \widetilde{x}_j \underset{\Re}{\geq} \widetilde{0}, \quad j = 1, 2, 3.
\end{aligned} \tag{23}$$

Finally, when the fuzzy simplex method employed in the environment of the proposed solution algorithm of the FFLP problem in the CSA, the solution of (23) can be obtained as follows;

$$\begin{aligned}
&\left\{\textbf{Max.}\widetilde{Z}_{\widetilde{\phi}_{\text{median}}}(\widetilde{x}) \underset{\Re}{=} (\widetilde{x}); \widetilde{X}_4(\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3)\right\} \\
&= \left\{(\frac{412}{417}, \frac{664}{393}, \frac{901}{531}); \widetilde{X}_4((0, 0, 0), \right. \\
&\quad \left. (0, 0, 0), (\frac{560}{57}, \frac{7}{57}, \frac{7}{114}))\right\}.
\end{aligned}$$

This obtained solution is a fuzzy compromise solution for the FFMLP problem in (13).

Now, we collect the obtained compromise solutions in the four different cases in the following table to discuss and analyze the results in the next section.

VI. ANALYSIS OF THE RESULTS AND LOGICAL INTERPRETATION

This section deals with analytic results and tries to find logical justifications for different obtained results relatively. It explores how changes in an FLP problem's coefficients in objective functions, right hand sides and technological coefficients affect the fuzzy compromise optimal solution. In other words, we are checking the sensitivity analysis of our illustrated numerical example.

Now, let us analyze and discuss how the two following types of changes in an MFLP problem's parameters impact in the fuzzy compromise solution. These types are:

1) changing the parameters by;
   - changing the objective function coefficient of a basic variables,
   - changing the objective function coefficient of a nonbasic variables,
   - changing the right-hand side of a constraints, and
2) adding a new variables or constraints by;
   - adding a new variables,
   - adding a new constraints.

With respect to the addition of a new variables or anew constraints, nothing like those happened. With respect to the changing of the parameters, the coefficients of the basic variables of the objective functions had been changed. Thus in:

Case 1: $\textbf{Max.}\widetilde{Z}_{\widetilde{\phi}}(\widetilde{x})$ in (17), the initial tableau is given in Table 9, while the corresponding optimal tableau is in Table 10.

TABLE 9. The status of the solution.

			$\widetilde{n}_1$	$\widetilde{n}_2$	$\widetilde{n}_3$	$\widetilde{n}_7$	$\widetilde{n}_7$	$\widetilde{n}_7$	
$\widetilde{B}$	$\widetilde{c}_i$	$\widetilde{\text{R.H.S.}}$	$\widetilde{x}_1$	$\widetilde{x}_2$	$\widetilde{x}_3$	$\widetilde{s}_1$	$\widetilde{s}_2$	$\widetilde{s}_3$	Min ratio
$\widetilde{s}_1$	$\widetilde{n}_7$	$(40, \frac{1}{7}, \frac{2}{7})$	3	$\frac{49}{12}$	2	1	0	0	$(\frac{40}{3}, \frac{1}{21}, \frac{2}{21})$
$\widetilde{s}_2$	$\widetilde{n}_7$	$(20, \frac{1}{4}, \frac{1}{8})$	$\frac{41}{20}$ *	$\frac{41}{40}$	$\frac{57}{28}$	0	1	0	$(\frac{400}{41}, \frac{5}{41}, \frac{5}{82}) \leftarrow$
$\widetilde{s}_3$	$\widetilde{n}_7$	$(30, \frac{1}{9}, \frac{1}{3})$	$\frac{37}{36}$	3	$\frac{39}{20}$	0	0	1	$(\frac{1081}{37}, \frac{4}{37}, \frac{12}{37})$
	$\widetilde{z}$	$(0, 0, 0)$	$\widetilde{n}_4$ $\uparrow$	$\widetilde{n}_5$	$\widetilde{n}_6$	$\widetilde{n}_7$	$\widetilde{n}_7$ $\downarrow$	$\widetilde{n}_7$	$\Re$

$\widetilde{n}_1 = (\frac{1762}{3567}, \frac{402}{2735}, \frac{699}{3788})$, $\widetilde{n}_2 = (\frac{99}{16639}, \frac{233}{1205}, \frac{377}{2452})$, $\widetilde{n}_3 = (\frac{137}{1358}, \frac{699}{4033}, \frac{377}{1876})$, $\widetilde{n}_4 = (\frac{-1762}{3567}, \frac{699}{3788}, \frac{402}{2735})$, $\widetilde{n}_5 = (\frac{-99}{16639}, \frac{377}{2452}, \frac{233}{1205})$, $\widetilde{n}_6 = (\frac{-137}{1358}, \frac{377}{1876}, \frac{699}{4033})$, $\widetilde{n}_7 = (0, 0, 0)$.

TABLE 10. The status of the solution.

			$\widetilde{n}_1$	$\widetilde{n}_2$	$\widetilde{n}_3$	$\widetilde{n}_7$	$\widetilde{n}_7$	$\widetilde{n}_7$	
$\widetilde{B}$	$\widetilde{c}_i$	$\widetilde{\text{R.H.S.}}$	$\widetilde{x}_1$	$\widetilde{x}_2$	$\widetilde{x}_3$	$\widetilde{s}_1$	$\widetilde{s}_2$	$\widetilde{s}_3$	Min ratio
$\widetilde{s}_1$	$\widetilde{n}_7$	$(\frac{440}{41}, \frac{187}{574}, \frac{187}{287})$	0	$\frac{31}{12}$	$\frac{-281}{287}$	1	$\frac{-60}{41}$	0	
$\widetilde{x}_1$	$\widetilde{n}_1$	$(\frac{400}{41}, \frac{5}{41}, \frac{5}{82})$	1	$\frac{1}{2}$	$\frac{285}{287}$	0	$\frac{20}{41}$	0	
$\widetilde{s}_3$	$\widetilde{n}_7$	$(\frac{7370}{369}, \frac{57}{328}, \frac{677}{1476})$	0	$\frac{179}{72}$	$\frac{4001}{4305}$	0	$\frac{-185}{369}$	1	
	$\widetilde{z}$	$(\frac{1653}{343}, \frac{647}{433}, \frac{2299}{1256})$	$\widetilde{n}_7$	$\widetilde{n}_4$	$\widetilde{n}_5$	$\widetilde{n}_7$	$\widetilde{n}_6$	$\widetilde{n}_7$	$\Re$

$\widetilde{n}_1 = (\frac{1762}{3567}, \frac{402}{2735}, \frac{699}{3788})$, $\widetilde{n}_2 = (\frac{99}{16639}, \frac{233}{1205}, \frac{377}{2452})$, $\widetilde{n}_3 = (\frac{137}{1358}, \frac{699}{4033}, \frac{377}{1876})$, $\widetilde{n}_4 = (\frac{921}{3821}, \frac{357}{1571}, \frac{463}{1621})$, $\widetilde{n}_5 = (\frac{1257}{3226}, \frac{349}{1006}, \frac{220}{617})$, $\widetilde{n}_6 = (\frac{1753}{7275}, \frac{227}{3166}, \frac{183}{2033})$, $\widetilde{n}_7 = (0, 0, 0)$.

The fuzzy compromise solution for (17) was;

$$\left\{\textbf{Max.}\widetilde{Z}_{\widetilde{\phi}}(\widetilde{x}); \widetilde{X}_1(\widetilde{s}_1, \widetilde{x}_1, \widetilde{s}_3)\right\} = \left\{(\frac{1653}{343}, \frac{647}{433}, \frac{2299}{1256}); \widetilde{X}_1((\frac{400}{41}, \frac{5}{41}, \frac{5}{82}), (0,0,0), (0,0,0))\right\}.$$

Thus, the $\widetilde{BV} = \{\widetilde{s}_1, \widetilde{x}_1, \widetilde{s}_3\}$, and $\widetilde{NBV} = \{\widetilde{x}_2, \widetilde{x}_3, \widetilde{s}_2\}$.

$\Re$(the coefficient of $\widetilde{s}_1$) $= \Re(0,0,0) = 0$, $\Re(\widetilde{c}_1) = \Re(\frac{400}{41}, \frac{5}{41}, \frac{5}{82}) = 0.5034$ and $\Re(\widetilde{c})_2 = -0.00395$. Now, we will compute $c_{BV}B^{-1}$ if $\Re(\widetilde{c}_1) + \triangle = 0.5034 + \triangle$:

$$\begin{bmatrix} 0 & 0.5034+\triangle & 0 \end{bmatrix} \begin{bmatrix} 1 & \frac{-60}{41} & 0 \\ 0 & \frac{20}{41} & 0 \\ 0 & \frac{-185}{369} & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0.2455+\frac{20}{41}\triangle & 0 \end{bmatrix},$$

$\Re(\widetilde{c})_2 = -0.00395$, $\Re(\widetilde{c})_3 = 0.1078$ and $\Re$(the coefficient of $\widetilde{s}_2$) $= \Re(0,0,0) = 0$.

Since

$$a_1 = \begin{bmatrix} 3 \\ \frac{41}{20} \\ \frac{37}{36} \end{bmatrix}, \quad a_2 = \begin{bmatrix} \frac{49}{12} \\ \frac{41}{40} \\ 3 \end{bmatrix} \text{ and } a_3 = \begin{bmatrix} 2 \\ \frac{57}{28} \\ \frac{39}{20} \end{bmatrix}$$

hence,

$$\bar{c}_2 = c_{BV}B^{-1}a_2 - c_2 = \begin{bmatrix} 0 & 0.2455+\frac{20}{41}\triangle & 0 \end{bmatrix} \begin{bmatrix} \frac{49}{12} \\ \frac{41}{40} \\ 3 \end{bmatrix} - (-0.00395)0.25569 + 0.5\triangle$$

$$\bar{c}_3 = c_{BV}B^{-1}a_3 - c_3 = \begin{bmatrix} 0 & 0.2455+\frac{20}{41}\triangle & 0 \end{bmatrix} \begin{bmatrix} 2 \\ \frac{57}{28} \\ \frac{39}{20} \end{bmatrix} - 0.1078 = 0.3922 + \frac{285}{287}\triangle$$

coefficient of $s_2$in row 0= second element of $c_{BV}B^{-1} = 0.2455 + \frac{20}{41}\triangle$.

Now, row $\widetilde{0}$ of the optimal tableau (Table 10) is:

$$\widetilde{z} + (0.25569 + 0.5\triangle)\widetilde{x}_2 + (0.3922 + \frac{285}{287}\triangle)\widetilde{x}_3 + (0.2455 + \frac{20}{41}\triangle)\widetilde{s}_3 = (\text{Some amount})? \quad (24)$$

From the row 0 of (24), BV will remain optimal if and only if the following hold:

$$0.25569 + 0.5\triangle \Leftrightarrow \triangle \geq -0.51138$$
$$0.3922 + \frac{285}{287}\triangle \Leftrightarrow \triangle \geq -0.3950$$
$$0.2455 + \frac{20}{41}\triangle \Leftrightarrow \triangle \geq -0.5033 \quad (25)$$

TABLE 11. The different fuzzy compromise solutions and their sensitivities.

$\mathbf{Max.}\widetilde{Z}_{\widetilde{\phi}_{(.)}}(\widetilde{x})$	$\left\{\widetilde{\phi}_i; \widetilde{X}_i(\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3)\right\}, i = 1,2,3,4$	Sensitivity analysis of the optimality ($\triangle_i; i = 1,2,3,4$)
$\mathbf{Max.}\widetilde{Z}_{\widetilde{\phi}}(\widetilde{x})$	$\left\{(\frac{1653}{343}, \frac{647}{433}, \frac{2299}{1256}); \widetilde{X}_1((\frac{400}{41}, \frac{5}{41}, \frac{5}{82}), (0,0,0), (0,0,0))\right\}$	Optimal $\Leftrightarrow \triangle_1 \geq -0.3950$
$\mathbf{Max.}\widetilde{Z}_{\widetilde{\phi}_{\text{FAAA}}}(\widetilde{x})$	$\left\{(\frac{413}{90}, \frac{2466}{227}, \frac{1359}{124}); \widetilde{X}_2((0,0,0), (0,0,0), (\frac{560}{57}, \frac{7}{57}, \frac{7}{114}))\right\}$	Optimal $\Leftrightarrow \triangle_2 \geq -0.049$
$\mathbf{Max.}\widetilde{Z}_{\widetilde{\phi}_{\text{mean}}}(\widetilde{x})$	$\left\{(\frac{1353}{1535}, \frac{967}{587}, \frac{499}{267}); \widetilde{X}_3((0,0,0), (0,0,0), (\frac{560}{57}, \frac{7}{57}, \frac{7}{114}))\right\}$	Optimal $\Leftrightarrow \triangle_3 \geq -0.0143$
$\mathbf{Max.}\widetilde{Z}_{\widetilde{\phi}_{\text{median}}}(\widetilde{x})$	$\left\{(\frac{412}{417}, \frac{664}{393}, \frac{901}{531}); \widetilde{X}_4((0,0,0), (0,0,0), (\frac{560}{57}, \frac{7}{57}, \frac{7}{114}))\right\}$	Optimal $\Leftrightarrow \triangle_4 \geq -0.01489$

Thus, from (25), the current basis will remain optimal if and only if $\triangle \geq -0.3950$. The determination of range of values on $\Re(\widetilde{c}_1) \geq -0.3950$ for which the current basic remains optimal. By the same method and logic, the same conclusion applies to all the other cases.

Case 2: For $\mathbf{Max.}\widetilde{Z}_{\widetilde{\phi}_{\text{FAAA}}}(\widetilde{x})$ in (19), the current basis will remain optimal if and only if $\triangle \geq -0.049$. In other words, the values on $\Re(\widetilde{c}_3) \geq -0.049$ for which its current basic remains optimal.

Case 3: For $\mathbf{Max.}\widetilde{Z}_{\widetilde{\phi}_{\text{mean}}}(\widetilde{x})$ in (21), the current basis will remain optimal if and only if $\triangle \geq -0.0143$. In other words, the values on $\Re(\widetilde{c}_3) \geq -0.0143$ for which its current basic remains optimal.

Case 4: For $\mathbf{Max.}\widetilde{Z}_{\widetilde{\phi}_{\text{median}}}(\widetilde{x})$ in (23), the current basis will remain optimal if and only if $\triangle \geq -0.01489$, or, the values on $\Re(\widetilde{c}_3) \geq -0.01489$ for which its current basic remains optimal.

Table 11 collects all four different cases of the fuzzy compromise solutions and their sensitivities. Note that the optimality of all cases will remain optimum if and only if $\triangle \geq -0.0143$.

VII. ADVANTAGES OF THE PROPOSED METHOD OVER THE EXISTING METHODS

In this section, we will list recent existing methods on solving fully fuzzy linear programing (FFLP) problems in order to compare with our proposed method. In 2017, Das *et al.* [48] proposed a lexicographic ordering method which depends on ordering trapezoidal fuzzy numbers. The method suggested the auxiliary MLP model to be used to solve the corresponding LP. Hence it is not applicable to solve complicated problem areas in risk investment, engineering management, supply chain management and transportation problem. This method is an improvement of Das [49] published in the same year 2017 which modified Lotfi *et al.* [22] method established in 2009. It converted the FLP problems into MLP problems, in which the solution cannot be obtained without converting the fuzzy system into its corresponding multiple deterministic of objective functions. This method is limited to single fuzzy objective function and did not cover MFLP problems. A year earlier in 2016, Hosseinzadeh and Edalatpanah [50] suggested the use of L-R fuzzy numbers method using both the lexicography and linear programming models. Even though the method gave promising results in terms of computing performance, it is only limited to nonnegative fuzzy numbers. Hence our CSA method is proposed to rectify these deficiencies. As mentioned in the preceding section, the CSA method is able to define FFMLP problems. The variables can be either triangular or trapezoidal fuzzy numbers, with practical computational applications. The methodology used is novel yet simple by converting FFMLP to SFFMLP and solved interactively to find a fuzzy compromise solution. Hence our proposed method is able to solve FFMLP problems with variables being triangular or trapezoidal fuzzy numbers, deficient in other previous models. Previous models are of MLP problems and are limited to variables being trapezoidal [48], [49] or nonnegative fuzzy numbers [50].

VIII. CONCLUSION

In this paper, the fully fuzzy multiobjective linear programming (FFMLP) problems have been defined, where the coefficients of the objective functions, constraints, right hand side parameters, and variables are of the type of the Triangular Fuzzy Number (T_rFN)s. A solution strategy of such FFMLP was presented in the circumstance of certain linear ranking function, namely, Compromise Solution Algorithm (CSA). The revised simplex method together with Gaussian elimination in the environment of the linear ranking function which was described in the proposed was first converted from FFMLP problem to partially SFFMLP problem. The obtained SFFMLP problems were gathered together, and then was solved by four different methods to find a fuzzy compromise solution. The results show that the proposed CSA is applicable and practicable within computational applications. We intend to expand this research further to practical contributions. The work will be able to help solve real life and industrial problems which are usually complicated, uncertain and continuously subject to changes, by considering the fuzziness in the formulation of the model. In addition, potential research will be the application of our proposed solution procedure to real life problems which usually involve many variables and require quick optimum solutions. We believe that this study will spur other research so as to have positive impacts on organization productivity and competitiveness in many industries.

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