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Energy Trading System in Microgrids With Future Forecasting and Forecasting Errors

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ABSTRACT In this paper, we propose a periodic energy trading system model in microgrids with future forecasting and forecasting errors. In the proposed model, retailers in a microgrid can purchase (sell) energy periodically from (to) other retailers in the same microgrid. In this regard, our proposed model uses stochastic processes that capture the series of forecasted energy generation/consumption over time and the forecasting errors as transitions among states. We show with the proposed model that it is enough to consider only one time period to maximize revenues of retailers over the whole time period. We further design a hierarchical algorithm that provides an equilibrium price of energy among retailers to balance between energy demand and supply in the microgrid. Some numerical examples show that our proposed model and algorithm outperform traditional ones by efficiently managing forecasting errors.

INDEX TERMS Microgrid, energy trading system, future forecasting, forecasting error.

I. INTRODUCTION

Due to growing concerns on environment damage from existing power grid and depletion of traditional fossil sources, a next generation power grid, which is known as a smart-grid, has led to a desire to use a large amount of renewable energy sources such as solar power and wind power. This innovation is enabled by improvements in modern communication technologies and continuous developments in distributed generation (DG) with energy storage as well as modern power electronics [1]–[5]. In parallel, in such an innovated system, previous technological limitations on metering no longer force peak power prices to be averaged out and no longer charge to all energy buyers equally. Hence, for efficiently and reliably managing and operating such a critical complex infrastructure in a smart-grid, the microgrid, which is an important building block of a smart-grid, is also widely believed to be a promising platform for optimal control of DGs and dynamic price management in a decentralized manner [6]–[8].

A microgrid is a small-scale regional power system that can distribute energy in a small geographic area. It utilizes DGs for meeting local demands without reliance on a macrogrid [9], [10], and reduces the risk to blackout by alleviating overload on a macrogrid [11]. Furthermore, it is more energy efficient than the conventional system due to reduced transmission and distribution loss by only covering a small geographical area. Nevertheless, because of highly variable, unpredictable, and intermittent generation from renewable energy sources used in microgrids, balancing between demand and supply has been a challenging issue in microgrids. As a promising solution to handle this issue, energy trading among retailers in microgrids has been considered; if some retailer with superfluous energy can sell the energy to other retailers at higher prices than to a macrogrid, while some retailer, who requires more energy, can purchase energy from other retailers at cheaper prices than from a macrogrid, it would be beneficial for the retailers to trade energy with one another instead of purchasing and selling energy from/to a macrogrid.

To design such an energy trading model in microgrids, a number of different models have been studied in the literature and their revenues have been analyzed [12]–[23]. The works in [12]–[14] suggested an energy trading mechanism in a basic situation where all retailers compete under identical conditions. However, recently, it becomes more important to encourage retailers to trade or to reflect some realistic conditions in energy trading. Tushar *et al.* [15] considered an energy trading system among retailers as well as with macro-grids, and modeled it as a singleleader-multifollower Stackelberg game because the buyer, the retailer who needs energy, has a strategic advantage over the sellers, the retailers who sell energy to the buyer. They proposed a Stackelberg equilibrium at which the retailer obtains its maximum revenue. In [16], an energy trading model with a hierarchical decision making scheme was suggested by using a multileader-multifollower Stackelberg game. A unique Stackelberg equilibrium was also provided where the energy distribution is proportional to the price of buyers' bids. In [17] and [18], the same bidding-based trading model as in [16] was considered and also formulated as a Stackelberg game. However, this work considered incomplete information conditions and proposed a reinforcement learning (RL) based algorithm for seeking the Nash equilibrium (NE). With slightly different viewpoints, Park *et al.* [19] proposed a non-pricing based energy trading model for microgrids using a contribution concept, which is available as a virtual currency, in order to encourage the retailers to sell energy. In [20], an energy trading system with an energy storage system (ESS) for a demand-side load management was modeled as a dynamic non-cooperative repeated game and a Pareto-efficient pure strategy was provided to suggest an optimal trading strategy. Zhang *et al.* [21] proposed a contract based direct energy trading model among one buyer and multiple sellers to suggest an optimal mechanism for short-term and long-term energy tradings. In [22], an optimal energy trading strategy was proposed by considering the risk from uncertain energy supply and demand and using a two-stage stochastic game model with the Cournot Nash pricing mechanism. Most recently, in [23], an event-driven energy trading mechanism was proposed in order to increase energy independence in a small geographical area by encouraging buyer-centric trading.

Even though many works related to energy trading in microgrids have been carried out, most of existing works suggest energy trading algorithms considering only energy shortage/surplus amount in one trading period, the period between the current energy trading time and the next energy trading time. In addition, most studies do not use the expected time series of energy consumption/generation but only use the sum of the energy consumption/generation amount over the entire period. So several issues arise with this traditional methodology. For instance, even if the total amount of energy produced is greater than the total amount of energy consumed by a retailer over one period, energy shortage moments for the retailer can happen due to the variability on renewable energy generation or instantaneous energy consumption increase. However, these situations cannot be observed if we only consider the total amounts of energy consumption/generation over the whole period.

To overcome these limitations, this paper proposes an optimal energy trading algorithm using time series data of energy consumption/generation. To do this, it requires a study of forecasting on the time series of energy consumption/generation, and there have been a lot of researches for this purpose. A number of different forecasting models were provided in a survey study [24], [25], a forecasting model for photovoltaic power through weather forecast was proposed in [26]–[28], and some forecasting models using neural networks were introduced in [28]–[30]. With the help of the results of these previous works, we can design and propose an energy trading system that is based on the forecasting in energy generation and consumption. That is, in our energy trading system, we assume that we use one of forecasting methods to obtain the forecasted energy generation and consumption rates, called the main stream.

When we use the forecasting in energy generation and consumption for our energy trading system, it is important to note that there exist some errors in forecasting due to the randomness of the feature of renewable energy source in energy generation side and the randomness of human behaviors in energy consumption side. Therefore, the forecasting errors should be considered in the design of an energy trading system. Otherwise, the performance of the energy trading system is not guaranteed due to the forecasting error. To capture the forecasting error in our energy trading system we construct a stochastic process. A stochastic process is a useful mathematical tool that helps understanding an environment of which state is changed randomly in time.

The contributions of this paper are summarized as follows:

- In this paper, we propose an energy trading system for retailers in a microgrid based on forecasted energy generation and consumption, and analyze the system. While previous long-term based energy trading studies consider only the total forecasted amount of energy generation and consumption over a time period of interest, we consider the energy generation and consumption as a forecasted time series, so that some temporal energy shortage and excess during a time period of interest can be well captured in the model.
- In practice, the forecasted time series for energy generation and consumption might contain some forecasting errors. Moreover, the forecasting errors are correlated in time. So we consider such correlated forecasting errors in our model. We investigate the impact of the forecasting errors on the performance of the system and show that the forecasting errors should be considered in the design of an energy trading system.
- We find an optimal policy for retailers in a microgrid to maximize their revenues. In our approach, we show that the optimal policy can be obtained by considering only a single period separately, not considering the whole time periods together, which greatly simplifies the algorithm to find an optimal policy.
- Based on the optimal policy for retailers we show that there exists an equilibrium trading price for retailers in a microgrid to balance the energy demand and supply. Moreover, we propose an algorithm to find the equilibrium trading price.

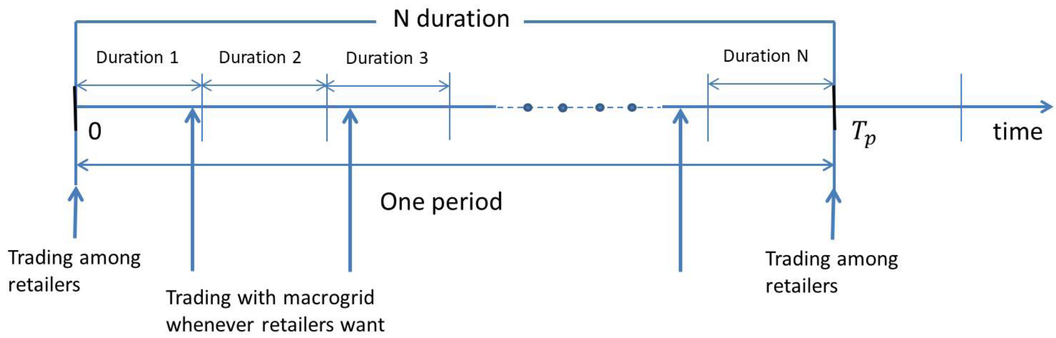

FIGURE 1. The trading among retailers occurs at the beginning of each period and the retailers can trade energy with the macrogrid whenever they want.

This paper is organized as follows. In section II, we first explain the system model considered in this paper. In section III we develop a stochastic model to consider forecasted main stream and forecasting errors and find an optimal policy for retailers, and in section IV, we propose an algorithm to find an equilibrium trading price. In section V, we investigate the system behaviors and discuss several useful results that can be applied in practice. Finally, we conclude the paper in section VI.

II. SYSTEM MODEL

In this section, we propose a periodic energy trading system among heterogeneous retailers in microgrids.

A. SYSTEM CONFIGURATION

There are M ***retailers*** with an index set $\mathbf{M} = \{1, \cdots, M\}$, a ***distributor***, and a ***macrogrid***. ***Retailers*** are defined by individual energy users that can generate energy themselves by using generators such as solar panels or wind turbines and consume energy at the same time. Here, retailer i has an energy storage system (ESS) that can store energy up to $E_{i,max}$. We assume that retailer i generates energy with rate $f_i(t)$ and consumes energy with rate of $g_i(t)$ at time t. So, the amount of energy generated by retailer i during the time interval $[a, b]$ is $\int_a^b f_i(t)dt$ and the amount of energy consumed by retailer i is $\int_a^b g_i(t)dt$. By combining these two rates, we define the energy change rate by $h_i(t) := f_i(t) - g_i(t)$. We assume that the energy generation rate $f_i(t)$ and energy consumption rate $g_i(t)$ can be obtained from the forecasting based on the environment factors (the amount of sunshine, wind speed, etc.) and the energy usage statistics from smart meters. Now, the retailers in the microgrid are allowed to use the following two types of energy trading.

- Retailers can ***periodically*** exchange energy with other retailers in the microgrid;
- Retailers can purchase energy or sell energy ***at anytime*** with fixed prices C_B and C_S, respectively, from/to the macrogrid whenever they want.

Through the trading, all retailers should be able to meet their energy consumption requirements at any time. In other words, retailer i's energy $e_{i,t}$ always has to satisfy $0 \leq e_{i,t} \leq E_{i,max}$ at any time t.

The ***distributor*** manages the energy trading among retailers in the microgrid based on the following steps.

1) The time axis is divided into intervals of equal length, called ***periods***.
2) Before the beginning of each period, the distributor suggests to all retailers an energy trading price among retailers. Refer to Fig. 2.
3) Each retailer determines the amount of energy to purchase/sell through the ***One Period Consideration Algorithm (OPCA)*** that will be suggested in section III. Then, all retailers inform the distributor their optimal actions.
4) The distributor computes the total energy demand/supply in the microgrid. If an equilibrium between energy demand and supply is not achieved, the distributor suggests another energy trading price based on the ***Equilibrium Trading Price Determination Algorithm*** suggested in section IV and repeats the above procedure until it achieves an equilibrium.
5) After achieving an equilibrium trading price, the distributor informs all the retailers their determined optimal actions and the energy trading among retailers begins. We call this epoch a ***trading epoch*** or a ***decision epoch***, and the period begins. Refer to Fig. 3.
6) After the energy trading among retailers, each retailer can purchase/sell their energy from/to the macrogrid whenever it occurs energy shortage or energy excess during the period. Refer to Fig. 4.

B. QUANTIZATION

Let the length of each period be T_p and the first trading epoch be $t = 0$. Then, the energy trading among retailers occurs only when $t \in \{0, T_p, 2T_p, \cdots\}$. Here, for simplicity, we do not consider the time length to determine an equilibrium trading price (step 2 to step 5 in the previous subsection).

Even though the amount of energy is continuous in $[0, E_{max}]$, for simplicity, it is much better to use discrete states and actions. So, we introduce how we can quantize a continuous function $\phi(t)$ through the following procedure.

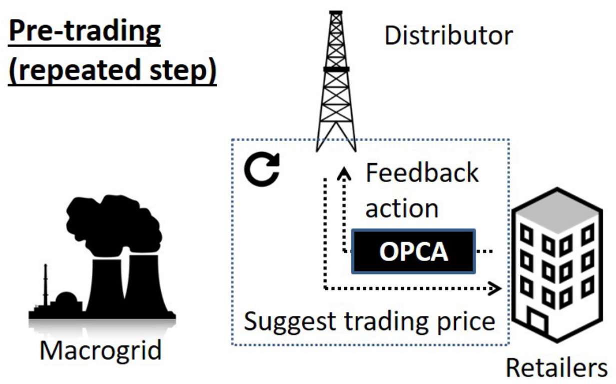

FIGURE 2. Before trading, the system should determine the trading price among retailers by *Equilibrium Trading Price Determination Algorithm*.

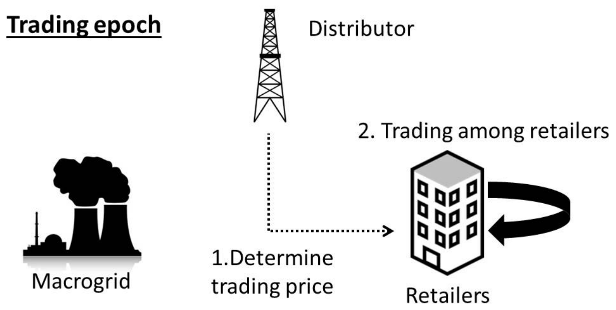

FIGURE 3. At the trading epoch, retailers trade energy with the determined equilibrium trading price.

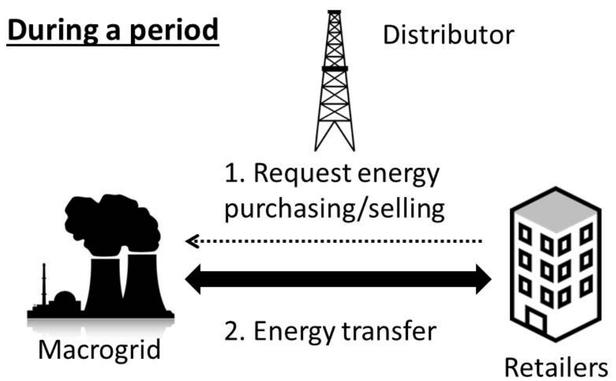

FIGURE 4. During the period, retailers can trade energy with the macrogrid whenever they want.

1) We define

$$\tilde{\phi}[n] := \frac{1}{2}\left\{ \max_{t\in\left[\frac{nT_p}{N}, \frac{(n+1)T_p}{N}\right]} \phi(t) + \min_{t\in\left[\frac{nT_p}{N}, \frac{(n+1)T_p}{N}\right]} \phi(t) \right\},$$

for $n = 0, 1, 2, \cdots$. In the sequence $\tilde{\phi}$, N elements represent quantized values in time based on the subintervals in a period of length T_p. We call this subinterval a ***duration***. So, one period consists of N durations.

2) To quantize the range of the function $\tilde{\phi}$, we define

$$\tilde{\phi}_\epsilon[n] := \left\lfloor \frac{\tilde{\phi}[n]}{\epsilon} \right\rfloor \epsilon,$$

for $n = 0, 1, 2, \cdots$, where $\lfloor x \rfloor$ is the largest integer not exceeding x. By this step, $\tilde{\phi}_\epsilon[n]$ is the largest value of the multiples of ϵ not exceeding $\tilde{\phi}[n]$ for all n, and the range of $\tilde{\phi}_\epsilon$ becomes the set of the multiples of ϵ.

3) For convenience, we define

$$\hat{\phi}(t) := \tilde{\phi}_\epsilon\left[\lfloor Nt/T_p \rfloor\right]$$

for any (nonnegative) time t. By this step, the function $\hat{\phi}$ has a domain of the set of nonnegative real values, is right continuous, and $\hat{\phi}\left(\frac{nT_p}{N}\right) = \tilde{\phi}_\epsilon[n]$.

By the above quantization procedure, we obtain an approximated function $\hat{\phi}$ of ϕ. In addition, $\hat{\phi}$ converges to the original function ϕ as $\epsilon \to 0$ and $N \to \infty$. So, by choosing suitable values of N and ϵ, we can control the quantization level of the function ϕ. For our model, we quantize the continuous energy change rate $h_i(t)$ to obtain an approximated function $\hat{h}_i(t)$ for all retailers by the above procedure. These energy change rates are constant in each duration, but they are not constant in the whole period. So this can still reflect the dynamics of the energy change rate and the shorter duration reflects better.

III. OPTIMAL TRADING POLICY DETERMINATION

In this section, we propose an algorithm to determine each retailer's optimal policy in a given environment using stochastic processes. The mathematical modeling in this section is similar to the Markov decision process based on [31]. In this section, subscript i will be omitted to concentrate on an individual retailer.

A. PROCESS MODELING

We use a stochastic process to model the amount of energy the retailer has in the ESS. This process consists of $\hat{h}_{forecast}(t)$, the forecasted energy change rate, obtained from the future forecast, and the error arising in forecasting.

1) STATE SPACE AND ACTION SET

Since we assume that we know the forecasted energy change rate $\hat{h}_{forecast}(t)$, it is important to capture the forecasting error in the stochastic process to determine each retailer's optimal policy. To this end we consider the forecasting error as the state of the process, that is, we let $X_{t,k}$ denotes the error between $\hat{h}$ and $\hat{h}_{forecast}$ at time $t + \frac{kT_p}{N}$ where $\hat{h}$ is the real energy change rate. The corresponding state space is then given by

$$\mathbf{S} := \{\cdots, -2\epsilon, -\epsilon, 0, \epsilon, 2\epsilon, \cdots\}$$

which is the set of error values that capture how far the real energy change rate $\hat{h}$ is away from the forecasted energy change rate $\hat{h}_{forecast}$ at the time of interest. By quantization, the spacing of $\mathbf{S}$ is ϵ.

The ***action set*** $\mathbf{A}$ is defined by the set of the amount of energy the retailer has in the ESS at the beginning of each period, i.e., the trading epoch. Then, the ***action set*** is given by

$$\mathbf{A} := \left\{0, \frac{T_p\epsilon}{N}, \frac{2T_p\epsilon}{N}, \cdots, E_{max}\right\}.$$

If a retailer has energy as much as e, the retailer can sell energy at most as much as e and can buy energy at most as much as $E_{max} - e$. So, the amount of energy at the beginning of each period can be between 0 and E_{max}. In addition, the spacing of $\mathbf{A}$ is also $T_p\epsilon/N$ because of the quantization.

2) STATE TRANSITIONS

Forecasting usually become more inaccurate and variable as time goes by. From this perspective, we introduce the spread random variable $Z_{t,k}$ defined by the error in forecasting occurred in the kth duration of time interval $[t, t + T_p]$. We also introduce a constant α, called a ***dependence factor***, that captures the dependency in inaccuracy between two adjacent durations. Since the errors in forecasting are accumulated and correlated, with these $Z_{t,k}$ and α, we can derive the following state transition:

$$X_{t,k+1} = round_\epsilon\left(\alpha X_{t,k} + Z_{t,k+1}\right). \tag{1}$$

In the recursion (1), $round_\epsilon$ is the function of the nearest multiple of ϵ because states are multiples of ϵ.

Since we can estimate the value of α and the distribution of $Z_{t,k}$ according to the characteristics of a practical system, from (1) and Algorithm 1, we can obtain the transition probability

$$p_t(s_{t+T_p}|s_t) := P\{X_{t,N} = s_{t+T_p}|X_{t,0} = s_t\}$$

for the energy change in one period.

Algorithm 1 State Transition in $[t, t + T_p]$

Initialization :
Set $X_{t,0} = s_t$ and $e_{t,0} = a_t$.
Repeat the iteration for $k = 0, 1, \cdots, N - 1$

$$\begin{aligned} X_{t,k+1} &= round_\epsilon\left(\alpha X_{t,k} + Z_{t,k+1}\right) \\ e'_{t,k+1} &= e_{t,k} + \frac{T_p}{N}\left\{\hat{h}_{forecast}\left(t + \frac{kT_p}{N}\right) + X_{t,k+1}\right\} \\ e_{t,k+1} &= \left[e'_{t,k+1}\right]_0^{E_{max}} \\ buy_{k+1} &= \left[e'_{t,k+1}\right]^+ - e'_{t,k+1} \\ sell_{k+1} &= \left[e'_{t,k+1} - E_{max}\right]^+ \end{aligned}$$

End iteration
Finalization : Set $s_{t+T_p} = X_{t,N}$.

In Algorithm 1,

$$[x]_0^{E_{max}} := \begin{cases} E_{max} & \text{if } x \geq E_{max} \\ x & \text{if } 0 \leq x \leq E_{max} \\ 0 & \text{if } x \leq 0 \end{cases}$$

$$[x]^+ := \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

The iteration in Algorithm 1 shows the state transitions and the changes of energy during a period. In the initialization step, s_t and a_t are the state and the action at decision epoch t, respectively. $e_{t,k}$ denotes the energy level at time $t + \frac{kT_p}{N}$. $e'_{t,k+1}$ denotes the virtual energy level at time $t + \frac{(k+1)T_p}{N}$ under an assumption that there is no energy trading with the macrogrid in the $(k + 1)$th duration, and $e_{t,k+1}$ is the actual energy level at time $t + \frac{(k+1)T_p}{N}$. Note that the energy to purchase or sell in a duration is given by the difference between the virtual energy level and the actual energy level at the end of the duration. So, buy_{k+1} denotes the energy amount purchased from the macrogrid in the $(k + 1)$th duration and $sell_{k+1}$ denotes the energy amount sold to the macrogrid in the $(k + 1)$th duration. After the iteration, $X_{t,N}$ becomes the state after time period $[t, t + T_p]$.

From (1) and Algorithm 1, we can obtain the transition probability between decision epoch t and $t + T_p$ as follows:

$$p_t\left(s_{t+T_p}|s_t, a_t\right) = p_t\left(s_{t+T_p}|s_t\right),$$

that is, ***for any choice of action, the transition probabilities are identical***. The reason is explained as follows. The state denotes the error between the real energy change rate and the forecasted energy change rate, while the action denotes the amount of the retailer's energy at the trading epoch. Even though choosing an action affects to the amount of energy purchased/sold from/to the macrogrid during the period after the action, it does not affect the energy consumption/generation rate during the period. So the transition of the states is independent of the action.

For analysis, we first show the following lemma. From now, let $buy_j(a_t)$ and $sell_j(a_t)$ be buy_j and $sell_j$ from Algorithm 1 with action a_t.

Lemma 1 (Monotonicity of Energy Level): When $a_t > a'_t$, $buy_j(a_t) \leq buy_j(a'_t)$ and $sell_j(a_t) \geq sell_j(a'_t)$ for all $j = 1, 2, \cdots, N$.

This lemma is obvious. If the action a_t, the initial energy level of Algorithm 1 at time t, is greater, the amount of energy sold to the macrogrid during the period is more. Also, the amount of energy purchased from the macrogrid during the period is less. The opposite situation also holds.

3) REWARD FUNCTION

Rational retailers always want to get their revenues as much as they can. We define the reward function at time t, R_t, as ***the expectation of the change in the retailer's asset*** between time t and time $t + T_p$. To this end, we consider the two kinds of assets that the energy amount and the money each retailer owns.

In Table 1, we tabulate the energy and money changes of the retailer between time t and time $t + T_p$. From this table, we can obtain the following two equations regarding two assets.

$$e_{t+T_p} = a_t + \sum_{j=1}^{N} buy_j(a_t) - \sum_{j=1}^{N} sell_j(a_t), \tag{2}$$

TABLE 1. Change of asset in period $[t, t + T_p]$.

	time t (before trading)	trading	during period	time $t + T_p$
energy	e_t	$+(a_t - e_t)$	$\sum_{j=1}^{N} buy_j(a_t) - \sum_{j=1}^{N} sell_j(a_t)$	e_{t+T_p}
money	m_t	$-(a_t - e_t) \cdot C_t$	$-\sum_{j=1}^{N} buy_j(a_t) \cdot C_B + \sum_{j=1}^{N} sell_j(a_t) \cdot C_S$	m_{t+T_p}

$$m_{t+T_p} = m_t - (a_t - e_t) \cdot C_t - \sum_{j=1}^{N} buy_j(a_t) \cdot C_B + \sum_{j=1}^{N} sell_j(a_t) \cdot C_S, \quad (3)$$

where C_t is the trading price of energy among retailers at time t, C_B is the purchasing price of energy from the macrogrid, and C_S is the selling price of energy to the macrogrid. We then need to aggregate two assets to determine the reward of the retailer. One reasonable way is to convert two assets into money values based on the trading prices of energy at time t among retailers, which results in the following reward function R_t:

$$\begin{aligned} &R_t(s_t, a_t) \\ &= E_t\left[\left(e_{t+T_p} - e_t\right) C_t + \left(m_{t+T_p} - m_t\right)\right] \\ &= E_t\left[\sum_{j=1}^{N} buy_j(a_t) \cdot (C_t - C_B) + \sum_{j=1}^{N} sell_j(a_t) \cdot (C_S - C_t)\right] \end{aligned}$$

where the expectation with subscript t means the conditional expectation with given trading price C_t of energy at time t. Recall that $buy_j(a_t)$ and $sell_j(a_t)$ are buy_j and $sell_j$ from Algorithm 1 with action a_t.

When the trading price C_t is in the interval $[C_S, C_B]$, the reward function R_t cannot exceed 0. This may be confusing, but it is natural because it is better to trade energy among retailers, not from/to the macrogrid and hence, if there occurs an energy trading from/to the macrogrid, it is reasonable to have some negative reward. In this case, this negative reward is interpreted as a dependency on the macrogrid. One more remarkable fact is that the rewards are determined only by s_t and a_t not by e_t, the amount of energy of the retailer at time t (just before trading among retailers).

B. DETERMINING OPTIMAL ACTIONS FOR RETAILERS

In this subsection, we will find optimal actions for each retailer. First, for a policy $\pi = (\pi^0, \pi^{T_p}, \cdots, \pi^{kT_p})$, we define the expected total reward as follows:

$$v_s^{\pi} := E^{\pi}\left\{\sum_{t \in \{0, T_p, \cdots, kT_p\}} R_t(X_t, \pi^t(X_t)) | X_0 = s\right\}.$$

Our goal is to find a policy that maximizes the expected total reward. In our model, the state space and the action set we define are at most countable sets. Therefore, in finite horizon ($T < \infty$) cases, we could obtain the optimal policy by using the backward induction algorithm introduced in [31]. This algorithm offers us the optimal action at each trading epoch that maximizes the expected total reward during the finite expiration time. Theoretically, the backward induction algorithm is valid, but it costs too much to run the algorithm. Fortunately, from the observation in the previous subsection, we can develop Algorithm 2, called the ***One Period Consideration Algorithm (OPCA)*** to find optimal actions.

Algorithm 2 One Period Consideration Algorithm (OPCA)

Step 1 (Initialization) :
At trading epoch t, an initial state s_t is given.
Step 2 (Determination):
Calculate $R_t(s_t, a)$
for $a = 0, T_p\epsilon/N, 2T_p\epsilon/N, \cdots, E_{max}$.
Determine a^* that maximizes R_t.
Step 3 (Finalization)
Change the retailer's energy level to a^* by purchasing/selling $a^* - e_t$ amount of energy from/to other retailers. (i.e. the retailer take an optimal action a^*.)

Since the state transition probabilities are independent of the choice of action as explained in the previous section, unlike general MDP models, no matter what action you take at the beginning of a period, the state transition behaviors during one period are always identical. Thus, finding the optimal action that maximizes the reward for each period leads to maximizing the expected total reward. From this observation, Algorithm 2 considers only one period to determine the optimal policy. Moreover, even if we want to find an optimal policy for infinite horizon cases ($T = \infty$), our observation and Algorithm 2 still works.

IV. EQUILIBRIUM TRADING PRICE DETERMINATION

In this section, we provide how to determine ***Equilibrium Trading Price*** of energy with which the energy trading among retailers actually occurs in our model. In a microgrid, if the trading price is suggested from the distributor, each retailer determines its optimal action based on Algorithm 2 given in the previous section. If the suggested trading price is appropriate, the total energy demand and supply in the microgrid at the trading epoch coincide, and all retailers can purchase or sell energy with the suggested price and the corresponding optimal actions. Otherwise, the distributor suggests a new price which is better than the previous price. So, it is important to find an efficient way to determine the equilibrium trading price.

A. BEHAVIORS OF OPTIMAL ACTIONS

In this subsection, we investigate several important behaviors of optimal actions as the price changes, which helps finding an equilibrium trading price.

Recall that C_B is the purchasing price of energy from the macrogrid and C_S is the selling price of energy to the macrogrid. Note that the trading price in the microgrid is always between C_S and C_B because, if the trading price is lower than C_S, selling energy to the macrogrid is better than trading energy in the microgrid and there is no energy supply from the microgrid. Similarly, if the trading price is higher than C_B, purchasing energy from the macrogrid is better than trading energy in the microgrid and there is no energy demand from the microgrid.

Proposition 4 describes the optimal actions for the boundary trading prices C_S and C_B in the microgrid. The proof is given in Appendix.

Proposition 1 (Behavior on Boundary Price): At trading epoch t, let C be the trading price. When $C = C_S$, $a = E_{max}$ is an optimal action, and optimal actions may not be unique and exist continuously. When $C = C_B$, $a = 0$ is an optimal action, and optimal actions may not be unique and exist continuously.

This proposition tells us that, when $C = C_S$, all retailers in a microgrid will try to purchase energy as much as they can, and when $C = C_B$, all retailers in a microgrid will try to sell energy as much as they can. Also, the optimal actions on the boundary prices may not be unique and exist continuously.

Our next proposition describes an important property of the optimal actions. The proof is also given in Appendix.

Proposition 2 (Monotonicity in Action): At trading epoch t, for trading price C and energy level e_t, let a^* be an optimal action. Then, for $C' > (<)C$, there is an optimal action equal to or less (greater) than a^*.

From Proposition 4 and Proposition 5, we see that there exists a decreasing function of optimal actions as the trading price increases from C_S to C_B. We use this decreasing function to determine the equilibrium trading price. The detailed procedure is provided in the next subsection.

B. EQUILIBRIUM TRADING PRICE

From the previous subsection, optimal actions may not be unique and exist continuously. So, for retailer i, we can define the two functions $a^*_{i,max}(C)$ and $a^*_{i,min}(C)$ by the maximum and minimum optimal actions of retailer i with trading price C. These two functions are the upper bound and the lower bound of optimal actions of retailer i.

To calculate the microgrid's trading energy demand/supply, it is enough to know amount of each retailers' energy at the trading epoch. Let e_i be the amount of energy of retailer i before trading among retailers. We next define two functions by

$$b^*_{i,max}(C) := a^*_{i,max}(C) - e_i$$
$$b^*_{i,min}(C) := a^*_{i,min}(C) - e_i.$$

We next add up these functions of all retailers as follows:

$$Eq_{max}(C) := \sum_{i\in\mathbf{M}} b^*_{i,max}(C),$$
$$Eq_{min}(C) := \sum_{i\in\mathbf{M}} b^*_{i,min}(C).$$

We now focus on the condition $Eq_{min}(C) \le 0 \le Eq_{max}(C)$, equivalently

$$\sum_{i\in\mathbf{M}} a^*_{i,min}(C) \le \sum_{i\in\mathbf{M}} e_i \le \sum_{i\in\mathbf{M}} a^*_{i,max}(C). \tag{4}$$

When $\sum_{i\in\mathbf{M}} a^*_{i,min}(C) \le \sum_{i\in\mathbf{M}} e_i$, there exists some retailers who can sell energy in the microgrid. In addition, when $\sum_{i\in\mathbf{M}} e_i \le \sum_{i\in\mathbf{M}} a^*_{i,max}(C)$, there exists some retailers who can purchase energy in the microgrid. Note that, for retailer i, if $b^*_{i,min}(C) := a^*_{i,min}(C) - e_i \le 0$, retailer i can sell energy to other retailers up to $e_i - a^*_{i,min}(C)$ without violating the optimal policy. On the other hand, if $b^*_{i,max}(C) := a^*_{i,max}(C) - e_i \ge 0$, retailer i can purchase up to $a^*_{i,max}(C) - e_i$ without violating the optimal policy. So, if the condition (4) is satisfied, there exists an optimal solution $a^*_i(C) \in [a^*_{i,min}(C), a^*_{i,max}(C)]$, $1 \le i \le \mathbf{M}$ such that

$$\sum_{i\in\mathbf{M}} a^*_i(C) = \sum_{i\in\mathbf{M}} e_i,$$

which means the energy demand and supply are balanced. From this observation, if the condition (4) is satisfied for the price C, we call C an equilibrium trading price. If the condition (4) is not satisfied for price C, then by the decreasing property of the optimal action in Proposition 5 we increase or decrease the trading price C to find an equilibrium trading price. Note that the existence of an equilibrium trading price is guaranteed through a similar proposition to Proposition 4 in the continuous model.

Even though the existence of an equilibrium trading price is guaranteed, it may not exist in our ***quantized*** model because the optimal actions of retailers are multiples of ϵ. So there occurs jumps in the total energy demand or the total energy supply when we change the trading price C. To solve this issue, in our quantized model, we define a δ-equilibrium trading price, a wider sense of equilibrium, instead of the equilibrium trading price.

Definition 1 (δ-equilibrium trading price): If a trading price C_δ satisfies all the following conditions, we call it a δ-equilibrium trading price.

- *When the trading price is $C_\delta - \delta$, there exist optimal actions of retailers for which the total energy demand is greater than or equal to the total energy supply and*
- *when the trading price is $C_\delta + \delta$, there exist optimal actions of retailers for which the total energy supply is greater than or equal to the total energy demand.*

We next show that the δ-equilibrium trading price always exists in our quantized model.

Proposition 3 (Existence of δ-equilibrium Trading Price): In a microgrid with our model, the δ-equilibrium trading price always exists in (C_S, C_B).

The proof is given in Appendix. Based on the proof of Proposition 6 we provide Algorithm 3 given below to get an δ-equilibrium trading price.

Theoretically, the equilibrium trading price is equal to the boundary price C_S and C_B, but in practice, this cannot happen. If the equilibrium trading price is either C_S or C_B, it is obvious that there is no benefit to trade in the microgrid. To avoid this issue, we design Algorithm 3 to have the property that the δ-equilibrium trading price occurs in $[C_S+\gamma, C_B-\gamma]$ for small enough $\gamma > 0$.

The value obtained from Algorithm 3 is either an exact equilibrium trading price or an δ-equilibrium trading price. If it is an exact equilibrium trading price, we have no problem. However, if it is an δ-equilibrium trading price, the total energy demand and supply are slightly different. To fix this problem, we assume the distributor has an ESS to store the energy excess and to make up the energy shortage of the microgrid when the energy trading in the microgrid results in a little energy excess or shortage. That is, the energy in the ESS of the distributor has to be used for balance.

Algorithm 3 Equilibrium Trading Price Determination Algorithm

Step 1 (Boundary Check) :
1) For small $\gamma > 0$, if $Eq_{max}(C_S+\gamma) < 0$, there is no equilibrium. **finish**.
2) For small $\gamma > 0$, if $Eq_{min}(C_B-\gamma) > 0$, there is no equilibrium. **finish**.
3) Let $I := [C_S, C_B]$.

Step 2 (Iteration):
1) Suggest C, a midpoint of interval I.
2) if the length of the interval I is less than 2δ, go to step 4.
3) Check $Eq_{max}(C)$ and $Eq_{min}(C)$.
4) If $Eq_{max}(C) = 0$ or $Eq_{min}(C) = 0$, go to step 3.
5) Else if $Eq_{max}(C) < 0$,
   replace I by the upper half interval of I and repeat step 2.
6) Else if $Eq_{min}(C) > 0$,
   replace I by the lower half interval of I and repeat step 2.
7) Else, go to step 4.

If n iteration is finished, go to step 4.

Step 3 (Equilibrium Trading Price)
C is an equilibrium trading price.
Find optimal actions of retailers that equilibrium can be obtained. Then **finish**.

Step 4 (δ-Equilibrium Trading Price)
C is a δ-equilibrium trading price.
Find optimal actions of retailers near the equilibrium.
Let $Eq(C) := \sum_{i\in\mathbf{M}} a_i^*(C) - e_i$.
If $Eq(C)$ is positive, keep the remaining energy in the ESS of the distributor. Then **finish**.
If $Eq(C)$ is negative, take out the lacking energy from the ESS of the distributor. Then **finish**.

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section we provide numerical results that validate our analysis and investigate some useful properties of our energy trading system in practice.

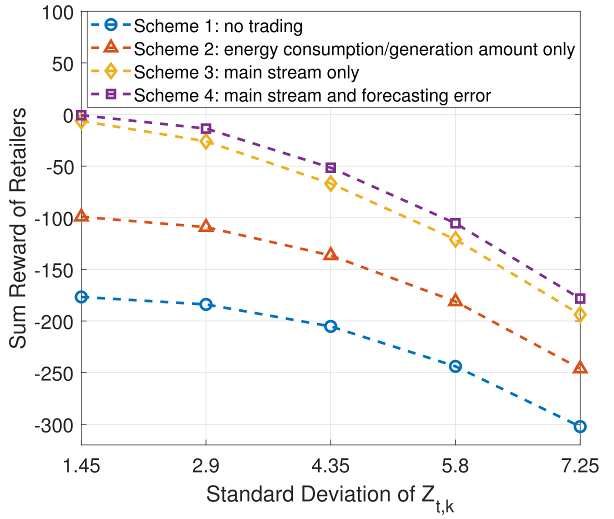

FIGURE 5. Sum reward of retailers with unbiased symmetric forecasting. Z_1. $N = 4$, $\epsilon = 1$, $T_p = 1$, $ESS = 10$, $\alpha = 0.5$, $C_S = 2$, $C_B = 9$, $s_i = 0$ for all retailers. Retailer 1 to retailer 10 have main stream $\hat{h}_1$, retailer 11 to retailer 20 have main stream $\hat{h}_2$, and retailer 21 to retailer 30 have main stream $\hat{h}_3$ in Table 2. $Z_{t,k} \sim l \cdot Z_1$ i.i.d. in Table 3.

TABLE 2. Environments used in simulation. Let $U(-c, c)$ be a discrete uniform distribution that has only integer values in $[-c, c]$.

Environment	N	Forecasted energy change rate in durations
$\hat{h}_1$	4	$[+20, +10, -10, -10] + U(-5, 5) * [1, 1, 1, 1]$
$\hat{h}_2$	4	$[-20, -10, +10, +10] + U(-5, 5) * [1, 1, 1, 1]$
$\hat{h}_3$	4	$U(-5, 5) * [1, 1, 1, 1]$
$\hat{h}_4$	4	$[+20, +10, -20, -15] + U(-5, 5) * [1, 1, 1, 1]$
$\hat{h}_5$	4	$[-20, -10, +20, +15] + U(-5, 5) * [1, 1, 1, 1]$
$\hat{h}_6$	4	$U(-5, 5) * [1, 1, 1, 1]$

A. PERFORMANCE COMPARISON

To evaluate the performance of the energy trading method proposed in this paper, we compare the following four schemes:

- **Scheme 1** is the traditional energy system with no trading among retailers. So retailers can purchase/sell energy only from/to the macrogrid.
- **Scheme 2** is an existing energy trading scheme considering only the total energy consumption/generation amount of each period. So this scheme cannot capture the dynamics of the energy consumption/generation change rate over time. In this scheme, the forecasting error is also ignored.
- **Scheme 3** is the proposed energy trading model considering the forecasted energy change rate, but ignoring the forecasting error. So retailers determine their own optimal actions based only on $\hat{h}_{forecast}$ and do not consider the spread random variable $Z_{t,k}$.
- **Scheme 4** is the proposed energy trading model considering both $\hat{h}_{forecast}$ and $Z_{t,k}$, the forecasted energy change rate and the forecasting error.

TABLE 3. Random Variables used in simulation. We use different types of random variables with quantization level=1.

Random Variable	Value	Probability	Note
Z_1	-3	0.05	symmetric, zero mean, $\sigma = 1.45$
	-2	0.1	
	-1	0.2	
	0	0.3	
	$+1$	0.2	
	$+2$	0.1	
	$+3$	0.05	
Z_2	-2	0.2	not symmetric, zero mean, $\sigma = 1.55$
	-1	0.2	
	0	0.275	
	$+1$	0.15	
	$+2$	0.1	
	$+3$	0.05	
	$+4$	0.025	
Z_3	-2	0.05	not symmetric, positive mean, $\sigma = 1.31$
	-1	0.2	
	0	0.425	
	$+1$	0.15	
	$+2$	0.1	
	$+3$	0.05	
	$+4$	0.025	

To compare the performance of each scheme, as a performance metric we use the **sum reward** R_{sum} defined by, for scheme $k(= 1, 2, 3, 4)$,

$$R_{sum}(k) := \sum_{i \in \mathbf{M}} R_i \left(s_i, a_i^{(k)} | C_{eq}^{(k)} \right)$$

where $R_i \left(s_i, a_i^{(k)} | C_{eq}^{(k)} \right)$ is the reward function of retailer i with selected action $a_i^{(k)}$ according to scheme k and equilibrium trading price $C_{eq}^{(k)}$ according to scheme k. Note that $R_{sum}(k)$ is the summation of the rewards of all retailers for scheme k when an equilibrium occurs.

We do not compare individual retailer's rewards for the schemes because the proposed model and algorithms are to achieve a trading equilibrium in the microgrid. Therefore, we assume in our model that each retailer is not selfish but cooperates to achieve a trading equilibrium. Note that $R_{sum}(k)$ measures how much benefit retailers can gain when scheme k is used. In Figure 5, 6, 7, and 8, the performances of four schemes. Note that the performance of scheme 1, the 'no trading' line (scheme1, circle marked line), is the baseline for the comparison. So the performance of each scheme is considered better if it is much higher than the baseline of scheme 1. In the figures, the main stream (the forecasted energy change rate) denotes $\hat{h}_{forecast}$.

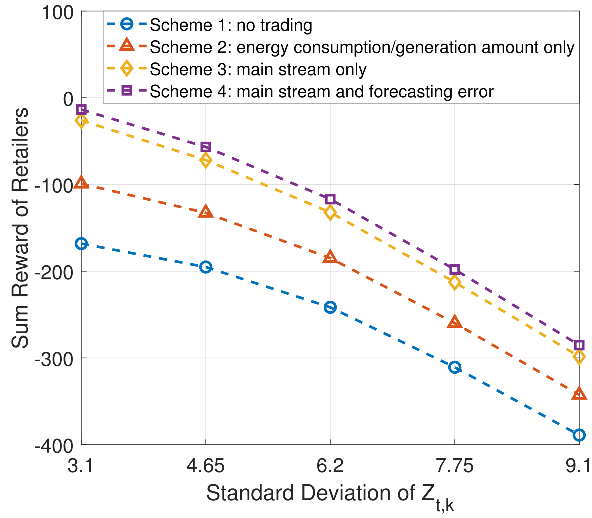

FIGURE 6. Sum reward of retailers with not symmetric but unbiased forecasting. $N = 4$, $\epsilon = 1$, $T_p = 1$, $ESS = 10$, $\alpha = 0.5$, $C_S = 2$, $C_B = 9$, $s_i = 0$ for all retailers. Retailer 1 to retailer 10 have main stream $\hat{h}_1$, retailer 11 to retailer 20 have main stream $\hat{h}_2$, and retailer 21 to retailer 30 have main stream $\hat{h}_3$ in Table 2. $Z_{t,k} \sim I \cdot Z_2$ i.i.d. in Table 3.

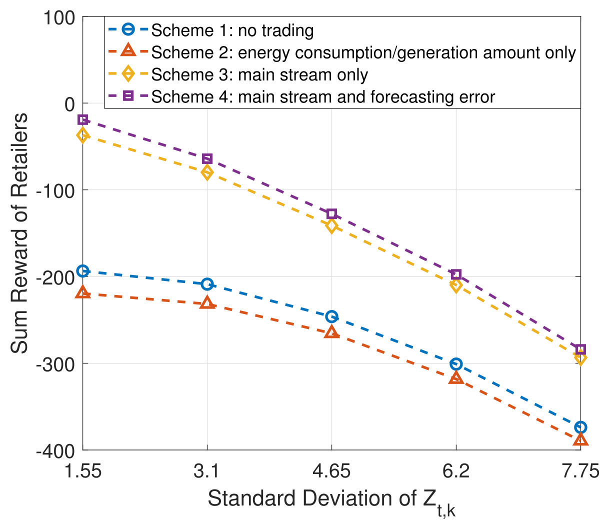

FIGURE 7. Sum reward of retailers with not symmetric but unbiased forecasting. $N = 4$, $\epsilon = 1$, $T_p = 1$, $ESS = 10$, $\alpha = 0.5$, $C_S = 2$, $C_B = 9$, $s_i = 0$ for all retailers. Retailer 1 to retailer 10 have main stream $\hat{h}_4$, retailer 11 to retailer 20 have main stream $\hat{h}_5$, and retailer 21 to retailer 30 have main stream $\hat{h}_6$ in Table 2. $Z_{t,k} \sim I \cdot Z_2$ i.i.d. in Table 3.

B. EFFECT OF FORECAST AND FORECASTING ERROR CONSIDERATION

To investigate the impact of the energy change rate forecast and the forecasting error consideration, we compare scheme 1 (circle marked line), scheme 2 (triangle marked line), scheme 3 (diamond marked line), and scheme 4 (square marked line) in the figures. In Figure 5, 6, 7 and 8, we plot $R_{sum}(k)$, $k = 1, 2, 3, 4$ for different forecasting environments and the x-axis denotes the standard deviation of the spread random variable $Z_{t,k}$.

In Fig. 5, ***real*** energy change rate occurs symmetrically with respect to the forecasted energy change rate $\hat{h}_{forecast}$.

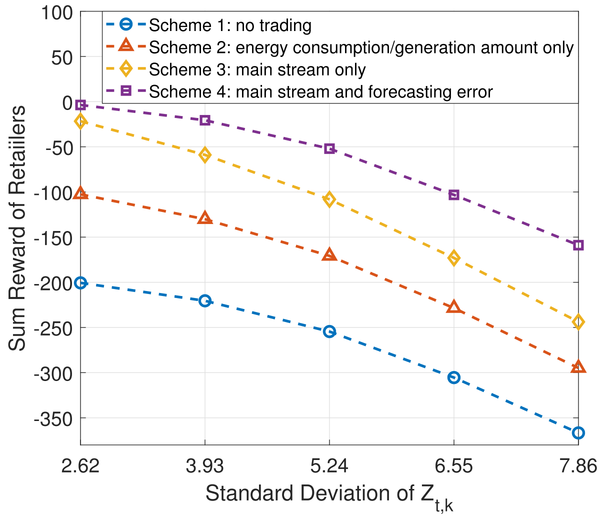

FIGURE 8. **Sum reward of retailers with not symmetric and biased forecasting. $N = 4$, $\epsilon = 1$, $T_p = 1$, $ESS = 10$, $\alpha = 0.5$, $C_S = 2$, $C_B = 9$, $s_i = 0$ for all retailers. Retailer 1 to retailer 10 have main stream $\hat{h}_1$, retailer 11 to retailer 20 have main stream $\hat{h}_2$, and retailer 21 to retailer 30 have main stream $\hat{h}_3$ in Table 2. $Z_{t,k} \sim l \cdot Z_3$ i.i.d. in Table 3.**

To simulate this environment we use the spread random variables $Z_{t,k}$ which are independent and identically distributed (i.i.d.) zero mean symmetric random variables Z_1.

In Fig. 6 and Fig. 7, the ***real*** energy change rate is not symmetric with respect to $\hat{h}_{forecast}$, but it is unbiased in the sense that the ratio that the real energy change rate is higher than $\hat{h}_{forecast}$ and the ratio that the real energy change rate is lower than $\hat{h}_{forecast}$ are equal. To simulate this case we use the spread random variables $Z_{t,k}$ which are i.i.d. zero mean but asymmetric random variables Z_2. The difference between Fig. 6 and Fig. 7 is that they use different $\hat{h}_{forecast}$ in simulation.

In Fig. 8, the ***real*** energy change rate is not symmetric with respect to $\hat{h}_{forecast}$ and it is biased. To simulate this case, we use the spread random variables $Z_{t,k}$ which are i.i.d. ***nonzero*** mean asymmetric random variables Z_3.

In all figures, scheme 4 outperforms all the other schemes. This shows that both the forecasted energy change rate and the forecasting error have significant impacts on the performance of a microgrid and we conclude that both should be well captured in the design of a microgrid. In addition, scheme 3 outperforms scheme 1 and scheme 2, and scheme 2 may have worse performance than scheme 1 (no trading) as seen in Fig. 7.

To see the impact of the forecasting error, we compare $R_{sum}(3)$ and $R_{sum}(4)$ for scheme 3 and scheme 4 in all figures. From Fig. 5, Fig. 6, and Fig. 7 where the real energy change rate is ***unbiased***, it is interesting that the impact of the forecasting error is less significant because the differences between two schemes are small. However, in Fig. 8 where the real energy change rate is ***biased***, the impact of the forecasting error is significant because the differences between two schemes are large.

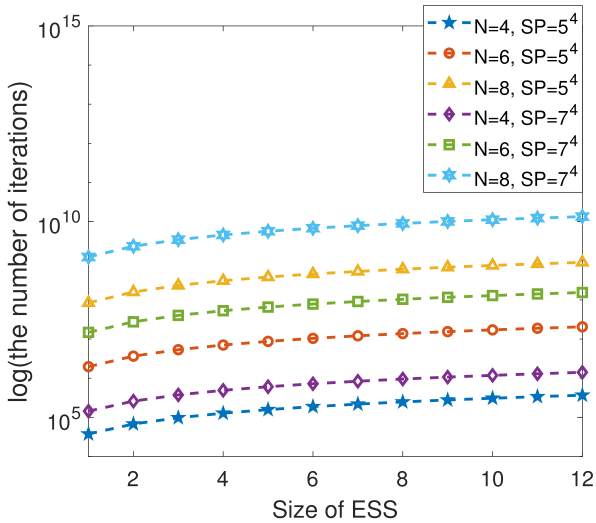

FIGURE 9. **The number of iterations vs. the size of ESS with $\epsilon = 1$ and $T_p = 1$.**

C. COMPUTATIONAL COST

When a trading price among retailers is suggested from the distributor, each retailer should determine its optimal action by running Algorithm 2 with its own hardware. So, it is important to discuss the computational cost. In this subsection, we first discuss the computation cost of Algorithm 2 when applied to our energy trading system in practice. Let SP denote the number of paths of transitions that can be generated in a period. Then, the amount of computation for each retailer is $O\left(\frac{N^2 \cdot SP \cdot E_{max}}{T_p \cdot \epsilon}\right)$ when each retailer finds its optimal action with a given trading price.

Note that finding an optimal action for a given trading price is repeated until an equilibrium trading price is achieved. Hence, we can estimate the required time for each retailer to compute an optimal action. For example, suppose that energy trading among retailers occurs once in a day, and the process of determining an equilibrium trading price and an optimal action of each retailer should be completed within one hour. If you want to restrict the error in the equilibrium trading price to be $\delta = 1/1000$ of the price interval, the feedback between the distributor and the retailers is allowed to perform up to 10 times. Therefore, each calculation by Algorithm 2 of each retailer should be completed within 6 minutes. Like this example, when designing the energy trading system, it is necessary to set the system parameters according to the hardware performance and the computation cost obtained in Fig. 9 and Fig. 10. In addition, by combining the result in the previous subsection, if the real energy change rate ensure symmetric or unbiased, it is possible to reduce the computation cost to $O\left(\frac{N^2 \cdot E_{max}}{T_p \cdot \epsilon}\right)$ by taking scheme 3 and allowing a slight performance degradation compared with scheme 4.

D. DISCUSSION

This system can be used as a basic form of various models. In this paper, retailers achieved a cooperative

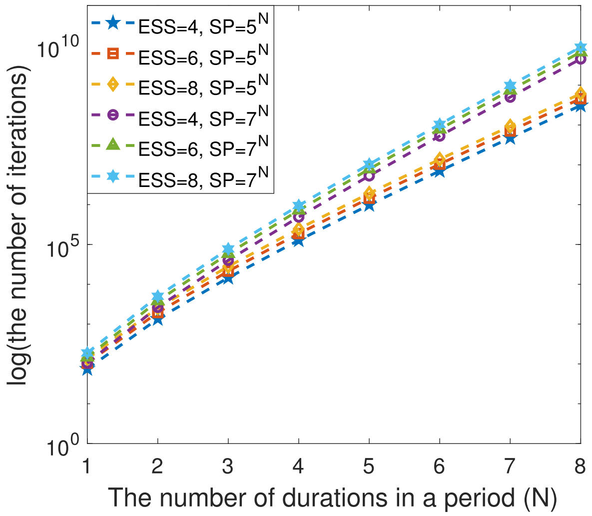

FIGURE 10. **The number of iterations vs. the number of durations in a period with $\epsilon = 1$ and $T_p = 1$.**

equilibrium with maximizing their own rewards, but the reward can be set differently depending on various perspectives and selfishness of retailers. Even in such situations, we can obtain the results by applying a similar argument used in the paper.

Additionally, if the energy is overflowing or lacking in the microgrid, the equilibrium does not occur in our system. However, other trading price determination methods based on game theory seem to be able to deal with this part and we leave this problem as a future study.

VI. CONCLUSION

We proposed a future forecasting based energy trading system among retailers. We analyzed the model, proposed how to find optimal actions for retailers, and suggested an algorithm to find an equilibrium trading price among retailers to balance between energy demand and supply in a microgrid. Some numerical examples and discussions were provided to validate our analysis and to investigate whether the proposed energy trading system works effectively.

APPENDIX
PROOFS OF PROPOSITIONS

Proposition 4 (Behavior on Boundary Price): At trading epoch t, let C be the trading price. When $C = C_S$, $a = E_{max}$ is an optimal action, and optimal actions may not be unique and exist continuously. When $C = C_B$, $a = 0$ is an optimal action, and optimal actions may not be unique and exist continuously.

Proof: For given s_t, let $a^* = E_{max}$ and let $buy_j(a)$ and $sell_j(a)$ be buy_j and $sell_j$ from Algorithm 1 with action a. Then, for any action $a \leq a^*$,

$$buy_j(a^*) \leq buy_j(a),$$

for each path of transition by Lemma 1. Then,

$$\begin{aligned} R_t(s_t, a^*) &= E_t\left[\sum_{j=1}^{N} buy_j(a^*) \cdot (C - C_B)\right] \\ &\geq E_t\left[\sum_{j=1}^{N} buy_j(a) \cdot (C - C_B)\right] \\ &= R_t(s_t, a). \end{aligned}$$

Therefore, $a^* = E_{max}$ is an optimal action.

If there is $a' \leq a^*$ such that $R_t(s_t, a^*) = R_t(s_t, a')$, then, $buy_j(a^*) = buy_j(a')$ for all paths of transitions and all j. So, $buy_j(a) = buy_j(a^*)$ for all $a' < a < a^*$, and all these actions a are optimal.

A similar proof can be done when $C = C_B$. ■

Proposition 5 (Monotonicity in Action): At trading epoch t, for trading price C and energy level e_t, let a^* be an optimal action. Then, for $C' > (<) C$, there is an optimal action equal to or less (greater) than a^*.

Proof: For given s_t, let $R_t(s_t, a|C)$ be the reward function with trading price C and let a^* be an optimal action and trading price C. Then,

$$R_t(s_t, a^*|C) \geq R_t(s_t, a|C)$$

for any action a.

Let $buy_j(a)$ and $sell_j(a)$ be buy_j and $sell_j$ in Algorithm 1 with action a. Then,

$$\begin{aligned} buy_j(a) &\leq buy_j(a^*) \\ sell_j(a) &\geq sell_j(a^*) \end{aligned}$$

for $a > a^*$ by Lemma 1.

If $C' > C$, we have

$$\begin{aligned} &R_t(s_t, a|C) - R_t(s_t, a|C') \\ &\quad = E_t\left[\left(\sum_{j=1}^{N} buy_j(a) - \sum_{j=1}^{N} sell_j(a)\right) \cdot (C - C')\right] \\ &\quad \geq E_t\left[\left(\sum_{j=1}^{N} buy_j(a^*) - \sum_{j=1}^{N} sell_j(a^*)\right) \cdot (C - C')\right] \\ &\quad = R_t(s_t, a^*|C_0) - R_t(s_t, a^*|C') \end{aligned}$$

for $a > a^*$. Therefore,

$$R_t(s_t, a^*|C') \geq R_t(s_t, a|C')$$

for $a > a^*$.

Then, any optimal action with trading price C' cannot be greater than a^*.

A similar result can be obtained when $C' < C$. ■

Proposition 6 (Existence of δ-equilibrium Trading Price): In a microgrid in our model, the δ-equilibrium trading price always exists in (C_S, C_B).

Proof: First, for each retailer, choose a decreasing function of optimal action as the trading price increases from C_S to C_B that starts from $E_{max} - e$ at C_S and ends to $-e$ at C_B.

Let $Supply(C)$ and $Demand(C)$ be the total energy supply and the total energy demand in the microgrid with trading price C, respectively. Then,

$$Supply(C_S) < Demand(C_S)$$
$$Supply(C_B) > Demand(C_B).$$

After that, we check the supply and demand at the midpoint $C' = (C_S + C_B)/2$. If $Supply(C') = Demand(C')$, this price is the equilibrium trading price. When $Supply(C') < Demand(C')$, we could reduce the interval to $[C', C_B]$. Similarly, when $Supply(C') > Demand(C')$ we could reduce the interval to $[C_S, C']$. Then, for the reduced interval, repeat the previous process. If this process is repeated k times, we can obtain an interval of length $(C_B - C_S)/2^k$. So, for large enough k, $(C_B - C_S)/2^k < 2\delta$. Then, the midpoint of this interval is a δ-equilibrium trading price. ∎

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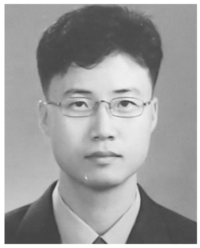

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