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Multi-Objective Chance Constrained Programming of Spare Parts Based on Uncertainty Theory

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ABSTRACT The optimization of spare parts inventory is very important in the modern aerospace engineering system, especially in the environment with low management effectiveness and a wide variety of spare parts. At present, there are many optimization models for spare parts inventory, and the single-objective optimization inventory is mostly used. But the single-objective optimization model has some limitations. First, in the applications of practical engineering, a single-goal decision problem is generally rare, and most of the decisions we have experienced involve many complicated goals. Second, it is difficult to truly present the actual situation when the mathematical programming model is used to discuss the optimization problem in practical engineering application. The solution to solve the model is a hybrid intelligent algorithm by combining the genetic algorithm with the inverse uncertainty distribution function. Finally, an example is given to illustrate the feasibility of the optimization model.

INDEX TERMS Genetic algorithm, multi-objective chance planning, spare parts, uncertainty theory.

I. INTRODUCTION

Under the condition of modern war, support capability of equipment is directly related to the combat readiness and mission sustainability. Support equipment is a key factor in determining the outcome of wars. As one kind of important goods to ensure the maintenance work, the spare parts is the key factor to ensure the good use of the equipment support ability.

Exponential smoothing [1], [2] is a robust method of demand estimation. It is not only a means to predict continuous needs, but also a widely used method for intermittent demand forecasting. Based on the theory of exponential smoothing, Croston gives us a method to forecast demand for intermittent demands, which is called Croston method. The Croston method is suitable for the situation of the lead time demand obeys the normal distribution. When the demand is not normal distribution, the prediction accuracy of this method is low.

Then, the Bootstrap method is given by Gamero *et al.* [3]. Bootstrap method extracts samples from the demand historical data to get the virtual data, so that it can be applied to predict the demand by the data generated in the historical demand. The shortcomings of the Bootstrap method are low accuracy, poor stability, and a lack of accurate estimation of the differences when demand changes in the first phase of the method [4].

On the basis of fault information in the early part of the spare parts, Cheng *et al.* [5] gave a method to evaluate the reliability of spare parts. And a forecasting model of spare parts demand is put forward, which accords with a certain degree of support. Fu and Zhao [6] put forward a mathematical model when the demand for spare parts yields to Poisson distribution. This model is used to predict spare parts demand when spare parts meet the expected guarantee probability [6]. The model is suitable for the prediction of spare parts demand which has maintenance period, wear and tear period, the period of purchase.

On the basis of how to accurately predict the spare parts demand for aeronautical equipment, Duan and Li [7] constructed a model block diagram in a object-oriented simulation way. The model block diagram is built under a relatively steady demand. For the needs of spare parts in combat period caused by operational damage, the key of the current way is to analyze the factors that restrict the demand of spare parts, and build the analytical model of demand prediction and calculation, which is also called direct calculation method [8], [9]. This method is simple and easy to understand, but the values of parameters related to the request is not easy gotten, so as to the fix parameters related especially relevant indicators of battle damage. And this method is also not easy to be applied directly to the estimation of spare parts demand in the operational environment.

The METRIC model built by Sherbrooke [10] takes account of the economic and military benefits, which is a very influential inventory model in the field of spare parts logistics support model. Graves improves the performance of the METRIC model in solving the shortage of the expected value. The Poisson distribution is replaced by negative two distribution to describe the expected shortage value at the base level, and the VARI-METRIC model is given. Meanwhile, the accuracy of the model is explored through simulation analysis. Two main advantages of the METRIC model [12] as follows. The multi-stage storage maintenance structure of spare parts logistics is considered. And this method considers the multilevel storage and maintenance structure of spare parts logistics support, which is integrated and comprehensively takes into account the characteristics of the problem.

Li Ya firstly analyzes the two-level logistics support system corresponding to the repairable spare parts of complicated weapons and equipment. And then provided an algorithm based on the optimization of Pareto. Finally the relationship between spare parts supply and equipment combat readiness is explored. Based on the ordering cost and storage cost of spare parts, and aiming at the benefit of logistics support of equipment system, the model of repairable spare parts is built [13]–[15].

The probability theory needs a large number of historical data, the theory of stochastic spare parts inventory model based on probability theory also needs sufficient statistical data to ensure the accuracy and availability of the premise. Because of the complex structure, systematicness and high test cost of weapon system, it is difficult to get a lot of statistical data of failure and maintenance in practical applications. So, the traditional method of decision making for spare parts support based on probability theory is not applicable [16]. Therefore, many scholars have begun to try to use other methods to study the problem. Under this circumstance, the uncertainty theory was proposed by Liu [17] in 2007 and refined by Liu [18] in 2010 to treat human's belief degree.

The rest of this paper will be organized as follows: Firstly, uncertainty theory will be introduced in Section 2; Then Section 3 will give a description of the multi-objective chance constrained programming model. An genetic algorithm will be introduced in Section 4, and in Section 5, a numerical example will be given to illustrate the availability and effectiveness of these models.

II. UNCERTAINTY THEORY

Let Γ be a nonempty set, and L is a σ-algebra over Γ. Each Λ in L is called an event. An uncertain measure is a set function M from L to [0,1] satisfying the following axioms.

Axiom 1 (Normality Axiom): $M\{\Gamma\} = 1$ for the universal set Γ.

Axiom 2 (Duality Axiom): $M\{\Lambda\} + M\{\Lambda^c\} = 1$ for any event Λ

Axiom 3 (Subadditivity Axiom): For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots$

$$M\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} M\{\Lambda_i\} \tag{1}$$

Axiom 4 (Product Axiom): Let (Γ_k, L_k, M_k) be uncertainty spaces for $k = 1, 2, \ldots$ The product uncertain measure M is an uncertain measure satisfying

$$M\left\{\prod_{i=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} M_k\{\Lambda_k\} \tag{2}$$

Where Λ_k are arbitrarily chosen events from L_K for $k = 1, 2, \ldots$ respectively.

Definition 1: An uncertain variable is a measurable function ξ from an uncertainty space (Γ, L, M) to the set of real numbers, i.e., for any Borel set B of real numbers, we have

$$\{\xi \in B\} = \{\gamma \in T | \xi(\gamma) \in B\} \tag{3}$$

Definition 2: The uncertain variables $\xi_1, \xi_2, \ldots, \xi_n$ are said to be independent, If

$$M\left\{\bigcup_{i=1}^{n} (\xi_i \in B_i)\right\} = \bigwedge_{i=1}^{n} M\{\xi_i \in B\}$$

for any Borel sets $B_1, B_2, \ldots, B_N$ of real numbers.

To describe the uncertain variables, we define the indeterminate distribution.

Definition 3: The uncertainty distribution Φ of an uncertain variable ξ is defined by

$$\Phi(x) = M\{\xi \leq x\} \tag{4}$$

for any real number x.

Example 1: An uncertain variable ξ is called zigzag if it has a zigzag uncertainty distribution

$$\Phi(x) = \begin{cases} 0 & \text{if } x < a \\ (x-a)/2(b-a) & \text{if a} \leq x \leq b \\ (x+c-2b)/2(c-b) & \text{if b} \leq x \leq c \\ 1 & \text{if } x > c \end{cases} \tag{5}$$

denoted by $Z(a, b, c)$ where a,b,c are real numbers with $a < b < c$. An uncertain variable ξ is called normal if it has a normal uncertainty distribution.

Definition 4: An uncertainty distribution $\Phi(x)$ is said to be regular if it is a continuous and strictly increasing function with respect to x at which $0 < \Phi(x) < 1$, and

$$\lim_{x \to -\infty} \Phi(x) = 0, \quad \lim_{x \to +\infty} \Phi(x) = 1$$

Then its inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0, 1)$. In this case, the inverse function $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of ξ.

Theorem 1: Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$ respectively. If f is a strictly increasing function, then $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ is an uncertain variable with inverse uncertainty distribution

$$\Phi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha)) \quad (6)$$

Example 2: Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$ respectively. Then $\xi = \xi_1 + \xi_2 + \cdots + \xi_n$ is an uncertain variable with inverse uncertainty distribution

$$\Phi^{-1}(\alpha) = \Phi_1^{-1}(\alpha) + \Phi_2^{-1}(\alpha) + \cdots + \Phi_n^{-1}(\alpha) \quad (7)$$

Theorem 2: The expected value of an uncertain variable ξ is defined by

$$E[\xi] = \int_0^{\infty} M\{\xi \geq X\}dx - \int_{-\infty}^{0} M\{\xi \leq X\}dx \quad (8)$$

provided that at least one of the two integrals is finite.

Theorem 3: Let ξ be an uncertain variable with an uncertainty distribution Φ. If $E[\xi]$ exists, then

$$E[\xi] = \int_0^{+\infty} (1 - \Phi(x))dx - \int_{-\infty}^{0} \Phi(x)dx \quad (9)$$

Example 3: Let $\xi \sim Z(a, b, c)$ be a zigzag uncertain variable. Then it has an expected value

$$E[\xi] = \frac{a + 2b + c}{4} \quad (10)$$

Theorem 4: Assume that $\xi_1, \xi_2, \ldots, \xi_n$ are independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$ respectively. If $f(x_1, x_2, \ldots, x_n)$ is strictly increasing with respect to $x_1, x_2, \ldots, x_m$, and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \ldots, x_n$, then the uncertain variable $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ has an expected value

$$E[\xi] = \int_0^1 f(\Phi_1^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha) \ldots, \Phi_n^{-1}(1-\alpha))d\alpha \quad (11)$$

III. MODEL

As an part of aeronautical equipment, spare parts inventory plays an important role. The spare parts must be sufficient. The shortage of spare parts in the aviation equipment system will cause immeasurable loss. In terms of availability, the more spare parts, the better. But the increase in spare parts inventory means the increase of the cost, which is not our expectation. Therefore, it is a major target to keep balance between costs for spare parts and readiness capability of equipment system. Combined with the analysis of the characteristics of spare parts inventory of aviation equipment, the expected shortage and total cost (holding cost and shortage cost) are selected as the objective function in the end. It not only guarantees the supply efficiency to achieve the proper operational efficiency of the equipment, but also minimizes the related support costs. Compared with the single objective model, the optimal balance between the operational efficiency and economic efficiency of the equipment can be considered more intuitively. Obviously, the two objective functions are conflicting. And there is no way to meet two goals optimal at the same time. This is a typical multi-objective problem.

In practical engineering applications, a single goal determination problem is generally rare. Many of the problems we experience are decisions involving multiple complex objectives, and these goals are interrelated and constrained. How to balance multiple goals, It is often necessary to think of a multi-objective programming method to select a suitable scheme. And a mathematical programming model on the practical application of the optimization problem is difficult to truly present the actual situation. The solution is using the uncertain variables of uncertain programming to describe the optimization problem, so it is necessary to establish a multi-objective chance constrained programming model. Based on the theory of uncertainty, this paper will optimize the spare parts inventory and establish a multi-objective chance programming model, so as to realize the rational allocation and optimization of spare parts resources.

A. UNCERTAIN PARAMETER

1) MULTI-OBJECTIVE CHANCE CONSTRAINED PROGRAMMING MODEL

The decision system of this article includes multiple targets. We can deal with multiple objective functions by the measure generated by the objective function, in order to avoid the the expectations retained shortcomings of objective function, then, an uncertain multi-objective chance constrained programming model which is not less than a certain reliability is obtained by multiple objective functions. x is decision variable, ξ is uncertainty variable, $f_1(x, \xi), f_2(x, \xi)$, K, $f_m(x, \xi)$, $(i = 1, 2, K, m)$ are objective functions of x. There are many possible values $\bar{f}_m$ to make $M\left\{f_i(x, \xi) \leq \bar{f}_i\right\} \geq \beta_i$, we want to achieve the goal of minimizing the $\bar{f}_i$. An uncertain multi-objective chance constrained programming model is showing as followed:

$$\begin{cases} \min\left[\bar{f}_1, \bar{f}_2, \ldots, \bar{f}_{\mathrm{m}}\right] \\ M\left\{f_i(x, \xi) \leq f_i-\right\} \geq \beta_i \quad i = 1, 2, \ldots, m \\ M\left\{g_j(x, \xi) \leq 0\right\} \geq \alpha_j \quad j = 1, 2, \ldots, p \end{cases} \quad (12)$$

Based on the theory above, this part applies the idea of chance programming to deal with uncertain variables, and establishes corresponding optimization models to achieve the goal of this paper.

2) OPTIMISTIC VALUE AND PESSIMISTIC VALUE OF UNCERTAIN VARIABLES

a: OPTIMISTIC VALUE OF UNCERTAIN VARIABLES

Definition 5: Assuming that ξ is an uncertain variable and satisfies $\alpha \in (0, 1)$, then $\xi_{\sup}(\alpha) = \sup\{\gamma \mid M\{\zeta \geq \gamma\} \geq \alpha\}$ is the α optimistic value of ξ.

Theorem 5: Assuming that ξ is an uncertain variable and an uncertain distribution is Φ, then the α optimism value of ξ is $\xi_{\sup}(\alpha) = \Phi^{-1}(1 - \alpha)$.

b: PESSIMISTIC VALUE OF UNCERTAIN VARIABLES

Definition 6: Assuming that ξ is an uncertain variable and satisfies $\alpha \in (0,1)$, then $\xi_{\inf}(\alpha) = \inf\{\gamma \,|M\{\zeta \le \gamma\} \ge \alpha\}$ is the α pessimistic value of ξ.

Theorem 6: Assuming that ξ is an uncertain variable and an uncertain distribution is Φ, then the α pessimistic value of ξ is $\xi_{\inf}(\alpha) = \Phi^{-1}(\alpha)$.

The establishment of Multi-objective chance constrained programming model

Based on the theory we has been known, we establish the multi-objective chance constrained programming model as follows:

$$\begin{cases} \min\limits_{x} \{f, \quad g\} & (13)\\ \text{s.t.} & \\ M\left\{\sum_{j=1}^{J}\sum_{i=1}^{I}\left(\xi_{ij}-x_{ij}\right) \le f\right\} \ge a & (14)\\ M\left\{\sum_{j=1}^{J}\sum_{i=1}^{I}\left[c_{ij}^{H}\,T\left(x_{ij}-0.5\xi_{ij}\right)+c_{ij}^{O}\,x_{ij}\right] \le g\right\} \ge b & (15)\\ M\left\{\sum_{j=1}^{J}\left\{\prod_{i=1}^{I}\left[1-\left(E\left[\xi_{ij}\right]-x_{ij}\right)/\left(K_jM_{ij}\right)\right]^{M_{ij}}\right\}K_j \right. & \\ \left. /\left(\sum_{j=1}^{J}K_j\right) \ge C^{-}\right\} \ge \eta & (16)\\ M\left\{\sum_{i=1}^{I}\frac{1}{\Lambda_0}\left\{\sum_{j=1}^{J}\left(\xi_{ij}-x_{ij}\right)-x_{i0}\right\} \ge A_0\right\} \le \beta_0 & (17)\\ M\left\{\sum_{i=1}^{I}\frac{1}{\Lambda_j}\left(\xi_{ij}-x_{ij}\right) \ge A_j\right\} \le \beta_j, j=1,2,\cdots,J & (18)\\ M\left\{\sum_{i=1}^{I}\left\{\sum_{j=1}^{J}\left(\xi_{ij}-x_{ij}\right)-x_{i0}\right\} \ge B_0\right\} \le \gamma_0 & (19)\\ M\left\{\sum_{i=1}^{I}\left(\xi_{ij}-x_{ij}\right) \ge B_j\right\} \le \gamma_j, j=1,2,\cdots,J & \\ x_{ij} \in N, i=1,2,\cdots,I, \quad j=0,1,2,\cdots,J & (20) \end{cases}$$

In which f, g are the target values, that is, the minimum value when the confidence level is guaranteed to be at least a, b. The equation (14) and (15) are the guarantee to make the objective function obtain the optimal value under the confidence degree. The equation (16) is a constraint that makes supply availability under the reliability of η for the normal operation of the supply system. The equation (17) and the equation (18) are the constraints that make the reaction time to guarantee the supply efficiency under confidence β_0 and β_j. The equations (19) and (20) are the constraints that make the expected shortage of the base level and the grass-roots level at the level of γ_0, γ_j as minimum as possible.

3) EQUIVALENT MODEL

When solving the uncertain multi-objective constrained programming model, we need to use its deterministic equivalent model. According to the relevant theorems in the operation rules of uncertain variables, and theorem 5-6, we can obtain the following form of the deterministic equivalent model

$$\begin{cases} \min\limits_{x}\left\{\begin{array}{l}\sum_{j=1}^{J}\sum_{i=1}^{I}\left(\Phi_{ij}^{-1}(a)-x_{ij}\right)\\ \sum_{j=1}^{J}\sum_{i=1}^{I}\left[c_{ij}^{H}\,T\left(x_{ij}-0.5\,\Phi_{ij}^{-1}(1-b)\right)+c_{ij}^{O}\,x_{ij}\right]\end{array}\right\} & (21)\\ \text{s.t} & \\ \sum_{j=1}^{J}\left\{\prod_{i=1}^{I}\left[1-\left(\Phi_{ij}^{-1}(\eta)-x_{ij}\right)/\left(K_jM_{ij}\right)\right]^{M_{ij}}\right\}K_j & \\ /\left(\sum_{j=1}^{J}K_j\right) \ge C^{-} & (22)\\ \frac{1}{\Lambda_0}\sum_{i=1}^{I}\left\{\sum_{j=1}^{J}\left[\Phi_{ij}^{-1}(\beta_0)-x_{ij}\right]-x_{i0}\right\} \ge A_0 & (23)\\ \sum_{i=1}^{I}\frac{1}{\Lambda_j}\left[\Phi_{ij}^{-1}(\beta_j)-x_{ij}\right] \ge A_j, j=1,2,\cdots,J & (24)\\ \sum_{i=1}^{I}\left\{\sum_{j=1}^{J}\left[\Phi_{ij}^{-1}(\gamma_0)-x_{ij}\right]-x_{i0}\right\} \ge B_0 & (25)\\ \sum_{i=1}^{I}\left[\Phi_{ij}^{-1}(\gamma_j)-x_{ij}\right] \ge B_j, \quad j=1,2,\cdots,J & \\ x_{ij} \in N, \quad i=1,2,\cdots,I, \quad j=0,1,2,\cdots,J & (26) \end{cases}$$

Equation (21) is a definite form after the equivalent transformation of equations (14) and (15).The equations (22) is the equivalent forms of equations (16).The equations (23) and (24) are the equivalent forms of equations (17) and (18), respectively. The same as (25)-(26) and (19)-(20).

IV. ALGORITHM

Genetic algorithms (GAs) as efficient algorithms for solution of optimization problems have been shown to be effective at exploring a large and complex search space in an adaptive way guided by the equivalent biological evolution mechanisms of reproduction, crossover and mutation. They are random search algorithms which have been derived based on the 'Darwin's theory of survival of the fittest' [19], [20]. In this paper, a hybrid intelligent algorithm is proposed to solve this model by integrating genetic algorithm and inverse uncertain distribution function. Based on the existing genetic algorithms, the two processes of cross and mutation are improved. And a hybrid intelligent algorithm is constructed synthetically with the inverse uncertainty distribution function method. Finally, the rationality of the model will be explained and verified by the numerical example in the following chapter.

1) Gene coding design and confirm: the binary code and the real value coding are usually seen to be selected as the appropriate encoding method.
2) The selection of the initial population and the size of the population: random generation of initial groups is generally used. Group size is one of the control parameters of evolutionary algorithm, and its selection has an effect on the efficiency of evolutionary algorithm. The size of the group is between tens to hundreds. The value of group size is different according to the complexity of the problem. The more difficult the problem is, the higher the dimension is, the larger the scale of the population is, on the contrary, it is small.
3) Determine the equivalent transformation: transform the function into its equivalent form, and then generate a new evaluation function.
4) Operator selection: The best individual retention method was used to copy the highly adaptable individuals directly to the next generation without mating. Then the roulette method is used to randomly select a certain number of individuals from the group.
5) Crossover operator: according to the crossover probability P_c adopts sigle-point crossing or multi-point crossing.
6) Mutation operator: m chromosomes were randomly selected according to the m times of variation determined by the variation rate of P_m. And the mutation operation was performed respectively on chromosomes.
7) Iterative termination principle: the upper limit of iterative is G times or the iteration stops when optimal solution continuous Q times without change.

The following steps are given to solve this kind of model:

Step 1: The binary form or real number are chosen as coding form to determine the population size N, the cross rate P_c, the mutation rate P_m, and the number of termination iterations G.

Step 2: Solving the problem according to the equivalent determination model.

Step 3: Pop_size chromosomes were generated randomly. At the same time, the feasibility of detecting chromosomes is done by the formula of the equivalent model, and then form an initial group of $G(t)$, and t=0.

Step 4: Calculate the fitness value of each chromosome in $G(t)$ and decide whether or not it meets the optimization criteria. If it is conformed, the best individual and the optimal solution or the satisfactory solution of the best individual are output, then procedure stops; otherwise, go to step 5.

Step 5: Using the optimal individual reservation method, the high adaptive individuals in $G(t)$ can be copied directly to the next generation without mating. Then the roulette method was used to select N individuals randomly from the group to form the parent chromosome $G(t)$.

TABLE 1. Other parameters of the system.

Items	Parameters				
	c_{ij}^{H}	c_{ij}^{o}	K_j	M_{ij}	$\bar{B}_j$
1 at Base 1	4.6	0.8	12	6	5
2 at Base 1	4.8	1.2		9	
1 at Base 2	4.6	0.8	10	6	6
2 at Base 2	4.8	1.2		9	

TABLE 2. Inventory optimization level of multi objective opportunity constrained programming model for spare parts.

Optimal solution	Parameters					
	x_{11}	x_{21}	x_{12}	x_{22}	EBO	C
1	59	75	69	66	27	1845.3
2	63	77	73	68	26	1900.0
3	66	79	77	73	25	2020.5

Step 6: The cross operation of the chromosomes in the set $G(t)$ is done by crossing probability $P_c$to obtain $G_1(t)$.

Step 7: Mutation operation of chromosomes in $G_1(t)$ were conducted according to the mutation rate P_m, and $G_2(t)$, $G_3(t)$ were obtained.

Step 8: Select the best N individuals from $G_1(t) \cup G_2(t) \cup G_3(t)$ as the next generation population $G(t+1)$;

Step 9: If it reaches the maximum generation G, and can satisfy the termination conditions, then stop; otherwise makes t=t+1 and go to step 3.

V. NUMERICAL EXAMPLE

Consider a spare part supply system consisting of 1 depot and 2 bases: each base supplies 2 items. In the system, we firstly specify two kinds of spare parts, which are defined as 1 and 2 respectively. Then, we give the relevant parameters and the distribution function of uncertain variables in the spare parts supply system. Parameters of items at all bases and depot in details are given as follows: T=20 (days), $A_0^-=A_j^-=0.5$(day), $\Lambda_0=\Lambda_j=0.02$ (units/day), $B_0^-=5$, f=5, g=800, a=0.8; b=0.8; $\overline{C}=0.9$, $\eta=0.7$; $\beta_0=0.6$; β_j=0.75; $\gamma_0=\gamma_j$=0.8. At the same time, the size of the population is N=50, cross probability is P_c=0.9,variation rate is P_m=0.02. The iterated algebra is G=2000. Other parameters of the system as shown in Table 1:

In this paper, we need to find appropriate uncertain demand distribution function as example for analysis and verification, which requires us to give some uncertain demand distribution functions. Through the analysis of this paper, we assume that the uncertainty variable obeys the Zigzag distribution, and gives the uncertain distribution function of this paper as follows:

$$\xi_{11} \sim Z\,(56,\ 59,\ 65), \quad \xi_{12} \sim Z\,(64,\ 68,\ 76),$$
$$\xi_{21} \sim Z\,(72,\ 74,\ 79), \quad \xi_{22} \sim Z\,(62,\ 64,\ 70).$$

We can get the optimal solution set after optimizing the inventory level of the spare parts at each site. Part of optimal solutions are shown in Table 2.

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