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Advanced Air Path Control in Diesel Engines Accounting for Variable Operational Conditions

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ABSTRACT In this paper, we develop an extended linear parameter-varying (LPV) model to design an LPV controller for air path system control in diesel engines. The objective is to use a widely used and accepted nonlinear diesel engine air path model, minimize simplifying assumptions on the model, and then design a model-based gain scheduling controller to work in a wide range of engine operating points. To that end, we transform the nonlinear model to linear parameter-varying form by defining state-dependent inputs and scheduling parameters. Some of the defined state-dependent scheduling parameters are synthetic in the sense that they are not obvious from the model and are created through algebraic operations. The control law we design is parameter-dependent and allows a large range of operating points to be considered. The robust performance of the controller (with respect to parametric uncertainties in the control design model) under variable operating points (depending on engine speed, fuel flow rate, and intake-exhaust manifold temperatures) is tested on simulations by tracking reference exhaust manifold pressure and compressor air mass flow signals. Finally, the performance of the designed extended LPV controller is compared to an H_{∞} controller and to an LPV controller from the existing literature to see its superior performance under variable operating points.

INDEX TERMS Diesel engines, air path control, gain-scheduling, LPV modeling, LPV control.

I. INTRODUCTION

Today a challenge for modern turbocharged diesel engines is the satisfaction of emission legislations which are becoming stricter with time. A turbocharged diesel engine with exhaust gas recirculation (EGR) is shown schematically in Figure 1. The environmental problem associated with diesel engines is their high rate of particulate matter and NO_x emissions. Advanced control strategies are key solutions for reduction of engine exhaust emissions. Among the control strategies model-based control strategies are preferred thanks to their potential of being optimal compared to other control approaches. A number of model-based control development studies appeared in the literature regarding air path regulation in turbocharged diesel engines with EGR, which directly affects emissions. In [1], Stefanopoulou et al. considered a direct injected diesel engine with EGR and variable geometry turbine (VGT). A controller architecture consisting of a combination of nonlinear feed-forward and gain-scheduled multi-variable linear feedback controllers was used to coordinate EGR and VGT actuators to achieve the maximum utilization. Jankovic and Kolmanovsky [2] applied a control Lyapunov function (CLF)-based nonlinear approach to



FIGURE 1. Diesel engine flow diagram. The subscripts "a", "c", "i","e" and "x" are used to represent ambient, compressor, intake manifold, engine cylinder and exhaust manifold locations, respectively.

control the diesel engine air path where CLF was constructed using input-output linearization of a reduced-order diesel engine model. Plianos *et al.* [3] used a nonlinear control strategy based on property of flatness and dynamic

feedback linearization to control the air path system in a turbocharged engine with EGR. In [4], Rajamani considered a turbocharged diesel engine with EGR and VGT. The applied control method is an observer-based nonlinear feedback control approach. Air fuel ratio and burned gas fraction in the intake manifold were key states to be controlled and an observer based on flow and pressure sensor measurements was developed for their real time estimations and was used with the designed nonlinear controller. Jung and Glover [5] present air path control in diesel engines using a linear parameter-varying (LPV) approach. A third-order, mean-value, nonlinear diesel engine model was used. The model was transformed into a quasi-LPV form and the scheduling parameter was taken to be intake manifold pressure. The work of Wei and del Re [6] used LPV identification methods to identify a quasi-LPV model and used it for air path regulation. In the recent work of Zhao et al. [7] an explicit model predictive approach was used for air path control in the diesel engines, and in Yin et al. [8] a receding horizon sliding control approach was used to solve the tracking problem of the air path. Other closely related model predictive control-based studies are by Liao-McPherson et al. [9] and by Esteban and Dahl [10] where a model predictive control strategy was used to maximize fuel economy while maintaining drivability and reducing emissions, and by Hung et al. [11] where a nonlinear model predictive control approach under different constraint handling strategies is used for assessment of the computation time and constraint violation.

Finally, as examples of other control methods in [12] Zentner *et al.* presented a novel model-based approach for airpath control based on cascaded control which is is applicable to various types of air path configurations; in [13] Chauvin and Jorde used a motion planning technique combined with an observer and two inner loop controllers for regulation of air path in diesel engines; in [14] Mohamed *et al.* used a higher order adaptive sliding mode controller for engine air path regulation. In the proposed controller, the super twisting control algorithm which is well known for its ability to reduce the so-called chattering phenomenon was used to minimize actuator oscillations; in [15] and [16], the effects of dual EGR and/or VGT loops and their control on engine emission are studied.

In all the studies mentioned above, although great success was obtained, still the designed control algorithms are not general enough to be used in a wide range of engine operating points. This is mainly due to the simplifying assumptions made on the engine model during the control design phase. First of all, in all studies, the manifold temperatures were assumed to be constant, their values being the optimized model tuning parameter values. Manifold temperatures may be considered as slowly time-varying parameters in some situations but there exist situations where they vary considerably [17]. For example, exhaust gas temperature varies considerably during engine operation when it goes through different operating points, and intake manifold temperature may not be stable for engines without an EGR cooler.

In addition, in [1], linear controllers designed around several operating points were scheduled. It is well known in the literature that a scheduled set of controllers generally lacks guarantee of stability [18]. In [3], the engine model parameters, which also include constant manifold temperatures as tuning parameters, were determined at a constant engine speed of 1600 rpm and at a fueling rate of 7.2 kg/h, and hence the model is accurate only around this operating point. Furthermore, in [5] exhaust manifold pressure was assumed to be 2.5 kPa greater than the intake manifold pressure to reduce the number of scheduling parameters. Such an assumption is very restrictive and it may hold only at some steady-state points. In [6], although modeling of air path system was done successfully by LPV identification for the considered operating points, an identified model may not be accurate enough when a wide range of operating points is considered. The explicit model predictive control approach used in [7] is a computationally demanding approach, and the model predictive control approach used in [8] and [9] have to assume constant scheduling parameters over the prediction horizon, which can give suboptimal results. Moreover, in general, the success of any model predictive approach requires a very accurate prediction model over the prediction horizon, which is not easy to obtain for diesel engine air path dynamics when a wide range of engine operating points is considered.

In this paper, our contributions can be listed as follows: (1) we use a widely accepted nonlinear mean-value diesel engine air path model which was used in many studies (in [2], [3], [5], [7], [19], and [20]) and minimize simplifying assumptions on the model. I.e., we use this nonlinear model as a control design model. This is in contrast to other studies ([2], [3], [5], [7], [19]) where the control models are significantly simplified versions of this nonlinear model; (2) we obtain an extended LPV model from the nonlinear air path model and then using this model we design an LPV controller (to be called accordingly "extended LPV controller") for diesel engines to be used in variable operating conditions. Other existing controllers developed in the literature work for narrow engine operating points and their extension to a wide range of operating points requires tuning of the model and controller (or re-design of controller). This tuning or redesign task is avoided in the presented control design; (3) robustness of the developed extended LPV controller against modeling uncertainties is tested, and the superior performance of the designed controller is compared to performances of an H_{∞} controller and an existing LPV controller in the literature; (4) a simple guideline is presented on how to use this widely used nonlinear air path model for the design of an extended LPV controller for a given tubocharged engine with EGR.

To design an extended LPV controller for variable operating conditions, first we take engine manifold temperatures into account as time-varying parameters (scheduling parameters). This results in an improved engine control design model and leads to the possibility of a better controller design. To the best of our knowledge, manifold temperatures were assumed to be constant in all diesel engine air path control system designs in the literature. This is partly due to the fact that taking manifold temperatures as time-varying parameters complicates the model and the subsequent controller design. Second, no simplification on exhaust manifold pressure is assumed, which is a nonlinear state in the used model. Finally, engine speed is also taken as a scheduling parameter to schedule the controller. The designed controller uses a modified LPV control design framework where some state-dependent synthetic scheduling parameters are created to have feasibility of the linear matrix inequality-based optimization problem during controller design phase of the air path controller to work in wide range of operating conditions.

The paper is organized as follows. In Section II, we present the developed extended LPV model (extended to represent a wide range of operating points) of the air path dynamics for the considered diesel engine. Section III gives a short introduction of the LPV-based control design method used in this paper. In Section IV, we start with some comments on the LPV modeling of diesel engines by pointing out important points which are crucial during control design phase and can help in other LPV applications. Next, in Section IV the developed LPV model is used for designing an extended LPV controller for air path regulation of turbocharged diesel engines with EGR. The robust performance of the extended LPV controller is shown through simulations. In Section V, the performance of the extended LPV controller is compared to an H_{∞} controller and to the simplified LPV controller from [5]. A simple guideline for the design of the developed extended LPV controller for a given tubocharged diesel engine with EGR is presented in Section VI. Lastly, in Section VII, we conclude with the main findings of this study and present some future work on this subject.

II. LPV MODELING OF DIESEL ENGINES

Consider the schematic flow diagram of a typical turbocharged diesel engine with EGR as shown in Figure 1 where temperatures, pressures, air mass flows and powers are denoted by T, p, W and P, respectively. N and N_t are engine and turbocharger speeds, x_v and x_r are VGT vane and EGR valve positions, varying between 0 and 1. Flow variables are shown with a double subscript which shows the path from the source to the sink location. All related system variables and used abbreviations are described in the nomenclature part in Appendix A. The dynamics describing the air path system [2], [3], [5], [7], [19] is

$$\dot{p}_i = \frac{RT_i}{V_i} (W_{ci} + W_{xi} - W_{ie}) + \frac{\dot{T}_i}{T_i} p_i,$$
 (1a)

$$\dot{p}_x = \frac{RT_x}{V_x} (W_{ie} + W_f - W_{xi} - W_{xt}) + \frac{\dot{T}_x}{T_x} p_x,$$
 (1b)

$$\dot{P}_c = \frac{1}{\tau} \left(-P_c + P_t \right). \tag{1c}$$

Unless manifold temperatures change very rapidly, the contribution of the terms $\frac{\dot{T}_i}{T_i}p_i$ and $\frac{\dot{T}_x}{T_x}p_x$ to p_i and p_x dynamics, respectively, are very small compared to the contributions coming from other terms. As a result, these terms are neglected as done in [2], [3], [5], [7], and [19]. However, note that manifold temperatures are taken variable in the expressions $\frac{RT_i}{V_i}$ and $\frac{RT_x}{V_x}$ to take into account manifold temperature variations. The expressions for the air mass flows and power variables are

$$\begin{split} W_{ci} &= \frac{\eta_c}{c_p T_a} \frac{P_c}{\left(\frac{p_i}{p_a}\right)^{\mu} - 1}, \quad W_{ie} = \frac{\eta_v V_d N p_i}{120 R T_i}, \\ W_{xi} &= \frac{A_r(x_r) p_x}{\sqrt{R T_x}} \sqrt{2 \frac{p_i}{p_x} \left(1 - \frac{p_i}{p_x}\right)}, \\ W_{xt} &= (a(1 - x_v) + b) \left(c \left(\frac{p_x}{p_a} - 1\right) + d\right) \frac{p_x}{p_{ref}} \sqrt{\frac{T_{ref}}{T_x}} \\ &\times \sqrt{2 \frac{p_a}{p_x} \left(1 - \frac{p_a}{p_x}\right)}, \\ P_t &= W_{xt} c_p T_x \eta_t \left(1 - \left(\frac{p_a}{p_x}\right)^{\mu}\right). \end{split}$$

The term A_r (effective EGR area) in the definition of W_{xi} is a quadratic function of x_r and given by

$$A_r(x_r) = -1.370135 \times 10^{-4} x_r^2 + 3.156976 \times 10^{-4} x_r.$$

Since $A_r(x_r)$ is a monotone increasing function for $x_r \in [0, 1]$, it is invertible. This allows us to determine x_r uniquely once A_r is determined. The constants a, b, c, d in W_{xt} , the polynomial coefficients in A_r , the values of volumetric, compressor and turbine efficiency terms η_v , η_c , η_t and other engine physical parameters were taken from [5] for the considered specific engine and for low-to-medium load-speed signal range of New European Drive Cycle, which are given in Appendix B.

To make controller design easier, we take $\tilde{u}_1 := W_{xt}$ and $\tilde{u}_2 := W_{xi}$ as modified inputs as done in [2], [3], and [19]. This is possible since the mappings $x_v \to W_{xt}$ and $x_r \to W_{xi}$ are invertible. After inserting W_{ci} , W_{ie} and P_t expressions into (1), we get

$$\dot{p}_i = -k_3 N p_i + k_1 T_i \frac{P_c}{\left(\frac{p_i}{p_a}\right)^{\mu} - 1} + k_2 T_i \tilde{u}_2,$$
 (2a)

$$\dot{p}_x = l_1 N \frac{T_x}{T_i} p_i + l_2 T_x W_f - l_2 T_x \tilde{u}_1 - l_2 T_x \tilde{u}_2,$$
 (2b)

$$\dot{P}_c = -\frac{1}{\tau} P_c + m_1 \tilde{u}_1 T_x \left(1 - \left(\frac{p_a}{p_x}\right)^{\mu} \right), \tag{2c}$$

where

$$k_{1} := \frac{R\eta_{c}}{V_{i}c_{p}T_{a}}, \quad k_{2} := \frac{R}{V_{i}}, \quad k_{3} := \frac{\eta_{v}V_{d}}{120V_{i}}, \\ l_{1} := \frac{\eta_{v}V_{d}}{120V_{x}}, \quad l_{2} := \frac{R}{V_{x}}, \quad m_{1} := \frac{c_{p}\eta_{t}}{\tau}.$$

Next, we define the time-varying parameters (scheduling parameters)

$$\rho_1 := \frac{1}{(p_i/p_a)^{\mu} - 1}, \quad \rho_2 := N, \ \rho_3 := 1 - (p_a/p_x)^{\mu}$$
$$\rho_4 := \frac{P_c}{p_x}, \quad \rho_5 := \frac{Np_i}{p_x}, \ \rho_6 := T_i, \ \rho_7 := T_x,$$

so that with $m_2 := -\frac{1}{\tau}$, we have

$$-\frac{1}{\tau}P_{c} = -\frac{1}{\tau}\frac{P_{c}}{p_{x}}p_{x} = m_{2}\rho_{4}p_{x},$$
$$l_{1}N\frac{T_{x}}{T_{i}}p_{i} = l_{1}\frac{T_{x}}{T_{i}}N\frac{p_{i}}{p_{x}}p_{x} = l_{1}\frac{\rho_{7}}{\rho_{6}}\rho_{5}p_{x}$$

Then, (2) becomes

$$\dot{p}_i = -k_3 \rho_2 p_i + k_1 \rho_1 \rho_6 P_c + k_2 \rho_6 \tilde{u}_2, \qquad (3a)$$

$$\dot{p}_x = l_1 \frac{\rho_5 \rho_7}{\rho_6} p_x + l_2 \rho_7 W_f - l_2 \rho_7 \tilde{u}_1 - l_2 \rho_7 \tilde{u}_2,$$
 (3b)

$$\dot{P}_c = m_2 \rho_4 p_x + m_1 \rho_3 \rho_7 \tilde{u}_1.$$
(3c)

System (3) can be written as

$$\begin{pmatrix} \dot{p}_{i} \\ \dot{p}_{x} \\ \dot{P}_{c} \end{pmatrix} = \begin{bmatrix} -k_{3}\rho_{2} & 0 & k_{1}\rho_{1}\rho_{6} \\ 0 & \frac{l_{1}\rho_{5}\rho_{7}}{\rho_{6}} & 0 \\ 0 & m_{2}\rho_{4} & 0 \end{bmatrix} \begin{pmatrix} p_{i} \\ p_{x} \\ P_{c} \end{pmatrix} + \begin{bmatrix} 0 & k_{2}\rho_{6} \\ -l_{2}\rho_{7} & -l_{2}\rho_{7} \\ m_{1}\rho_{3}\rho_{7} & 0 \end{bmatrix} \begin{pmatrix} \tilde{u}_{1} \\ \tilde{u}_{2} \end{pmatrix} + \begin{pmatrix} 0 \\ l_{2}\rho_{7} \\ 0 \end{pmatrix} W_{f}.$$
(4)

With the definitions $x := [p_i p_x P_c]^T$, $w := W_f$, $u := [\tilde{u}_1 \tilde{u}_2]^T$, equation (4) can be expressed as

$$\dot{x} = \mathcal{A}(\rho)x + \mathcal{B}_{w}(\rho)w + \mathcal{B}_{u}(\rho)u_{z}$$

where $\rho := [\rho_1 \cdots \rho_7]^T$ denotes the vector of time-varying parameters in the system. Such dynamical systems are known as *linear parameter-varying* systems. When the measured (y) and controlled outputs (z) are considered, which will be introduced later on, the system can be put in the form

$$\dot{x} = \mathcal{A}(\rho)x + \mathcal{B}_{w}(\rho)w + \mathcal{B}_{u}(\rho)u, \qquad (5a)$$

$$z = C_z(\rho)x + \mathcal{D}_{zw}(\rho)w + \mathcal{D}_{zu}(\rho)u, \tag{5b}$$

$$y = C_{y}(\rho)x + \mathcal{D}_{yw}(\rho)w + \mathcal{D}_{yu}(\rho)u.$$
 (5c)

There exists a large literature on control of such systems [21]–[25]. The approaches presented in [22]–[25] require gridding of the scheduling parameter space, which becomes computational very hard if the number of parameters is large (for example, if more than three parameters). In this study, the number of parameters is relatively large (seven), and hence we will use the method of [21] which is for LPV systems in rational form and which does not require gridding. In the next section, the underlying LPV control theory of [21] will be explained very shortly. The interested reader can find more detailed information than presented here in [21] and the references therein.

III. RECAP OF LPV CONTROL THEORY

A. THE LPV CONTROL PROBLEM

Consider the system (5) where the dependence on ρ is assumed to be fractional in each term. We assume $\mathcal{D}_{yu}(\rho) = 0$. The scheduling parameter vector ρ can be "pulled out" from the system such that the system of equations in (5) becomes

$$\dot{x} = Ax + B_p p + B_w w + B_u u, \tag{6a}$$

$$q = C_q x + D_{qp} p + D_{qw} w + D_{qu} u \text{ and } p = \Delta q, \quad (6b)$$

$$z = C_z x + D_{zp} p + D_{zw} w + D_{zu} u, ag{6c}$$

$$y = C_{y}x + D_{yp}p + D_{yw}w + D_{yu}u,$$
 (6d)

where $\Delta := \operatorname{diag} \left(\rho_1 I_{n_1}, \rho_2 I_{n_2}, \cdots, \rho_{n_\rho} I_{n_\rho} \right)$ for some positive integers n_1, \cdots, n_ρ, q is the input signal to the Δ block, and p is the output signal from the block. The Δ block can be considered as a perturbation mapping for the plant. The transformation of the system (5) to the system (6) is said to be an LFR (linear fractional representation) form. The control design problem is to find a controller K with state-space equations

$$\dot{x}_c = A_c x_c + B_{p_c} p_c + B_c y, \tag{7a}$$

$$q_c = C_{q_c} x_c + D_{q_c p_c} p_c + D_{q_c y} \text{ and } p_c = \Delta_c q_c, \quad (7b)$$

$$u = C_u x_c + D_{up_c} p_c + D_{uy} y, ag{7c}$$

where $\Delta_c = \Delta$ is the controller scheduling function. The plant with controller is shown in Figure 2.

For all admissible parameter trajectories $\rho(t) \in \mathbb{R}^{n_{\rho}}$, the closed-loop system is required to be stable and to guarantee the possible minimum \mathcal{L}_2 -gain from the disturbance channel *w* to the controlled output channel *z* (necessary for minimization of the effect of disturbance *w* on the controlled output *z*):

$$\sup_{e \in \mathcal{L}_{2}, w \neq 0} \frac{||z||_{2}}{||w||_{2}} < \gamma.$$
(8)

Here, the numerical value of γ changes from system to system and from disturbance channel to controlled output channel. Rather than its numerical value, its minimization is important. Note that the entries in the parameter vector ρ can

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FIGURE 2. Scheduled LTI-plant with scheduled LTI-controller.

always be normalized to $\bar{\rho}_i$ s such that $|\bar{\rho}_i(t)| \le 1$ for all $t \ge 0$. For the rest of the paper, we replace ρ with $\bar{\rho}$ and assume that each parameter is bounded in magnitude by 1. With such a parameter vector, we associate the "multiplier" sets

$$\mathcal{P} = \left\{ \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} : Q \prec 0, S = -S^T, R = -Q \right\}$$

and

$$\widetilde{\mathcal{P}} = \left\{ \begin{pmatrix} \widetilde{Q} & \widetilde{S} \\ \widetilde{S}^T & \widetilde{R} \end{pmatrix} : \widetilde{R} \succ 0, \widetilde{S} = -\widetilde{S}^T, \widetilde{Q} = -\widetilde{R} \right\}$$

In general, multiplier sets are sets of variables used to expresses system stability and performance as LMI conditions. The multiplier matrices in the multiplier sets \mathcal{P} and $\widetilde{\mathcal{P}}$ will constitute some of the linear matrix inequality (LMI) variables in the LMI optimization-based formulation of the LPV control design problem.

B. EXISTENCE CONDITIONS FOR THE REQUIRED CONTROLLER

We now give sufficient conditions for the existence of a stabilizing controller of the form (7) that guarantee the minimum bound on the \mathcal{L}_2 -gain from the disturbance to the controlled output. In the statement of the theorem, for a given matrix M, M_{\perp} denotes a basis for the orthogonal complement of the image of M and the symbol \star stands for B in $\begin{pmatrix} A & \star \\ B^T & C \end{pmatrix}$ and $\star^T MB$.

For $P \in \mathcal{P}, \widetilde{P} \in \widetilde{\mathcal{P}}, X = X^T \in \mathbb{R}^{n_x \times n_x}$ and $Y = Y^T \in \mathbb{R}^{n_x \times n_x}$, let

$$F_{1} = \begin{pmatrix} 0 & | & 0 & | & X & | & 0 \\ - & | & - & | & - & | & - & | & - & | \\ 0 & | & P & | & 0 & | & 0 \\ - & - & | & - & - & | & - & - & | \\ X & | & 0 & | & 0 & | & 0 \\ - & - & | & - & - & | & - & - & | \\ 0 & | & 0 & | & Y & | & 0 \\ - & - & - & - & - & | & - & - & - \\ 0 & | & P & | & 0 & | & 0 \\ - & - & - & - & - & - & - & - \\ Y & | & 0 & | & 0 & | & \gamma I \end{pmatrix},$$

$$F_{2} = \begin{pmatrix} 0 & | & 0 & | & Y & | & 0 \\ - & - & - & - & - & - & - \\ Y & | & 0 & | & 0 & | & 0 \\ - & - & - & - & - & - & - \\ 0 & | & 0 & | & 0 & | & \gamma I \end{pmatrix},$$

$$G_{1} = \begin{pmatrix} A & B_{p} & B_{w} \\ - & - & - & - & - \\ Q & D_{qp} & D_{qw} \\ 0 & I & 0 \\ - & - & - & - & - \\ I & 0 & 0 \\ - & - & - & - & - \\ 0 & 0 & I \end{pmatrix}, \quad G_{2} = - \begin{pmatrix} -I & 0 & 0 \\ -I & 0 & 0 \\ B_{p}^{T} & D_{qp}^{T} & D_{zp}^{T} \\ -I & -I & - & - \\ A^{T} & C_{q}^{T} & C_{z}^{T} \\ -I & -I & - & - \\ 0 & 0 & -I \end{pmatrix}$$

Theorem 1 [21]: There exists a controller of the form (7) such that the closed-loop system is stable for all possible

parameter trajectories and the \mathcal{L}_2 -gain from w to z is less than γ if there exist $P \in \mathcal{P}$, $\tilde{P} \in \tilde{\mathcal{P}}$, $X = X^T \in \mathbb{R}^{n_x \times n_x}$ and $Y = Y^T \in \mathbb{R}^{n_x \times n_x}$ such that

$$\begin{pmatrix} \left(\star\right)_{\perp}^{T}\left(\star\right)^{T}F_{1}G_{1}\begin{pmatrix}C_{y}^{T}\\D_{yp}^{T}\\D_{yy}^{T}\end{pmatrix}_{\perp} \\ \left(C_{z} \quad D_{zp} \quad D_{zw}\right)\begin{pmatrix}C_{y}^{T}\\D_{yp}^{T}\\D_{yw}^{T}\end{pmatrix}_{\perp} -\gamma I \\ \left(\star\right)_{\perp}^{T}\left(\star\right)^{T}F_{2}G_{2}\begin{pmatrix}B_{u}\\D_{qu}\\D_{zu}\end{pmatrix}_{\perp} \\ \left(B_{w}^{T} \quad D_{qw}^{T} \quad D_{zw}^{T}\right)\begin{pmatrix}B_{u}\\D_{qu}\\D_{zu}\end{pmatrix}_{\perp} \gamma I \\ \left(I \quad X\right) \succ 0 \qquad (9c)$$

Proof: The lengthy and detailed steps of the proof can be found in [21].

The controller is constructed from X, Y, the multipliers P, \tilde{P} and from the system matrices. The controller construction steps are also very lengthy and technical, and hence are skipped here. The details can be found in [21].

IV. GAIN SCHEDULED AIR PATH CONTROL

A. COMMENTS ON LPV MODELING OF AIR PATH SYSTEM IN DIESEL ENGINES

In LPV modeling of the diesel engine air path model given by (2), the first thing to notice is that an LPV model could be obtained with a smaller number of scheduling parameters. For example, we could obtain an LPV model by just considering the five scheduling parameters N, $\frac{1}{(p_i/p_a)^{\mu}-1}$, 1 – $(p_a/p_x)^{\mu}$, T_i and T_x . Therefore, the immediate question that will arise is that why P_c/p_x and Np_i/p_x were considered as additional parameters so that the number of parameters increased from five to seven. Before answering this question, it is helpful to know that in design and real-time application of LPV controllers it is better to have a small number of scheduling parameters as much as possible. As the number of scheduling parameters increases, from design perspective the LPV control design LMIs "may" become infeasible since the controller has to stabilize the system and give the desired performance for all independent scheduling parameter trajectories. From application perspective, the underlying controller may require more sensors for the measurement of the additional scheduling parameters or the requirements of more observers if they should be estimated. Therefore, in general, the objective is to have an LPV-model with a small number of scheduling parameters.

In this study, for the above suggested LPV model with a smaller number of five scheduling parameters the controller existence conditions in (9) did not produce any feasible solu-

tion. In fact, not only this selection of parameters but also many other possible parameter selections either did not give a feasible solution, or gave a feasible solution under very small ranges for the considered scheduling parameters, and hence the associated controller failed to work in variable operational conditions. In the LPV model given by (3), note that the selected extra scheduling parameters P_c/p_x and Np_i/p_x are not obvious at all from (2). I.e., they were created "synthetically". Luckily, this set of scheduling parameters resulted in a feasible solution for very wide ranges of the considered parameters (see Section IV-C) and subsequently a controller working in a wide range of operating points and having satisfactory results. Therefore, optimal scheduling parameter selection brings an additional freedom in LPV-based control systems [26]. In general, unfortunately, there is no method to tell how to model a system in LPV form with a minimum number of scheduling parameters so that satisfactory results can be obtained from the designed LPV controller. The only way is trial of different parameterizations of the system (meaning different LPV models) until satisfactory results are obtained. The fact that in LPV control design beforehand it is impossible to know the best LPV model form is the weakness of LPV control design paradigm (irrespective of which LPV control design approach is used!). It is impossible to know the best LPV model form because we solve a semi-definite optimization problem (an LMI optimization problem) and the LPV controller is constructed from the determined optimization variables. As a result, without solving the optimization problem, it is impossible to guess the best LPV model form.

Although we do not know how the LPV model form and the conservatism of the corresponding designed LPV controller are related, there is an implicit relation because for some LPV models of the same system the corresponding LPV control design LMIs are not feasible.

To summarize, although our created LPV model has much more scheduling parameters, the resulting extended LPV controller works in a wide range of operating points, which is what we target for.

B. SELECTION OF CONTROL PERFORMANCE VARIABLES

In diesel engines, the most directly emission-related variables are air fuel ratio (AFR) and EGR fraction in the intake manifold (EGR_f), and they must be controlled by regulating them to set points (shown below as variables with bars) which are determined from the static engine data:

$$\overline{AFR} = \overline{AFR}(N, W_f), \quad \overline{EGR}_f = \overline{EGR}_f(N, W_f).$$

Since it is not easy to measure AFR and EGR_f, we will transform the set points of these quantities into set points of measurable variables, namely, among many possibilities, to set points of compressor air mass flow W_{ci} and exhaust manifold pressure p_x . As a result, the control problem will boil down to the regulation of W_{ci} and p_x . Using the same

lines as in [27] and the system equations (1), we obtain

$$\begin{split} \overline{W}_{ci} &= \frac{W_f}{2} \bigg[\overline{K}_{AFR} \overline{K}_{\text{EGR}f} - \alpha \overline{\text{EGR}}_f \\ &+ \sqrt{\left(\overline{K}_{AFR} \overline{K}_{\text{EGR}f} - \alpha \overline{\text{EGR}}_f \right)^2 + 4 \overline{AFR} \overline{K}_{\text{EGR}f}} \ \bigg], \\ \overline{W}_{xi} &= \frac{\overline{\text{EGR}}_f}{\overline{K}_{\text{EGR}f}} \overline{W}_{ci}, \quad \overline{p}_x = p_i + \frac{RT_x \overline{W}_{xi}^2}{2A_r p_i}, \end{split}$$

where $\overline{K}_{AFR} = \overline{AFR} - 1$, $\overline{K}_{EGR_f} = 1 - \overline{EGR}_f$ and α is the stoichiometric ratio. The derivation steps were skipped due to space limitation.

The current study does not involve controlling turbocharger over-speeding because the model does not involve the turbocharger speed as a state. The turbocharger overspeeding either must be controlled through another control loop using a different model, or as a partial remedy we can find a relation (based on static engine data) between p_x , W_{ci} and turbo speed to modify the reference set points for p_x , W_{ci} so that turbo speed will be taken into account indirectly and tracking of these modified references will indirectly prevent turbocharger over-speeding.

C. TESTING THE ROBUSTNESS OF THE EXTENDED LPV CONTROLLER UNDER MODELING ERRORS AND VARIABLE OPERATING POINTS

The LPV diesel engine model with variable manifold temperatures is

$$\dot{p}_i = -k_3 \rho_2 p_i + k_1 \rho_1 \rho_6 P_c + k_2 \rho_6 \tilde{u}_2, \tag{10a}$$

$$\dot{p}_x = l_1 \frac{\rho_5 \rho_7}{\rho_6} p_x + l_2 \rho_7 W_f - l_2 \rho_7 \tilde{u}_1 - l_2 \rho_7 \tilde{u}_2, \quad (10b)$$

$$\dot{P}_c = m_2 \rho_4 p_x + m_1 \rho_3 \rho_7 \tilde{u}_1.$$
 (10c)

In this paper, we assume the following ranges for the engine variables: $p_i \in [103, 160]$ kPa, $p_x \in [105, 170]$ kPa, $P_c \in [150, 2000]$ W, $N \in [1000, 2500]$ rpm, $T_i \in [300, 350]$ K and $T_x \in [350, 700]$ K. As we see, the considered ranges represent a wide range of operating points which can be encountered during engine running.

The model in (10) is put in LFR framework for controller design phase and we obtain $\Delta = \text{diag}(\bar{\rho}_1, \bar{\rho}_2, \bar{\rho}_3, \bar{\rho}_4, \bar{\rho}_5, \bar{\rho}_6 I_2, \bar{\rho}_7 I_2)$ where $\bar{\rho}_i$, i = 1: 7 are parameters normalized about the mean of each ρ_i . W_f is taken as a disturbance signal. As air path variables to be controlled, we choose to track exhaust manifold pressure (p_x) and compressor air mass flow (W_{ci}) . As we discussed before, the operating points giving minimal engine emissions can be represented by reference set points for these variables. The tracking configuration is shown in Figure 3.

Note that the designed LPV controller is applied in an anti-windup (AW) form, which is used to prevent degradation of control performance when control inputs $(x_v \text{ and } x_r)$ are saturated. The AW method used is the one developed in [28]. Using manifold pressures and exhaust temperature,



FIGURE 3. Closed-loop system with inversion mappings, AW and weights.

determined modified control inputs are inverted to obtain VGT vane and EGR valve positions, which are then passed through the saturation block to guarantee achievement of physically meaningful control inputs (which are called real process inputs). The possible negative effect of saturation block is compensated by first inverting the the real process inputs to obtain the corresponding modified real process inputs and then feeding the difference between modified control inputs and modified real process inputs to the AW-LPV controller. The AW-LPV controller is also scheduled by $\bar{\rho} = [\bar{\rho}_1 \cdots \bar{\rho}_7]^T$.

Weight selection, which involves trial and error most of the time, is an important task and strongly affects the performance of the controller. Here three kinds of weights are used: weights for disturbances, weights for tracking errors and weights for control inputs. The disturbances are $p_{x_{ref}}$, $W_{ci_{ref}}$ and W_f . Since the references $p_{x_{ref}}$, $W_{ci_{ref}}$ are steps or slowly varying signals, typically a low pass filter is preferred. Hence, we chose the block low pass filter in Matlab-Simulink and set its band frequency to 2π . The weight for W_{W_f} was set to 1. The effect of other choices is negligible since W_f has a small effect on the dynamics of the system. As to the wights for tracking errors, this is the classical choice of an integrator to minimize the integral of the tracking error but since $\frac{1}{s}$ can make the system unstable a small number 0.1 was added to the denominator. The coefficients of numerators were increased until the tracking error was acceptable. Input weights were chosen to be unity. If the frequency of control inputs were very high, we would choose high pass filters. However, that is not the case in this study. As a result, we have the following transfer functions for the chosen weights:

$$W_{p_x} = W_{W_{ci}} = \frac{6.283}{s + 6.283}, \quad W_{W_f} = 1,$$

$$W_{e_1} = \frac{300}{s + 0.1}, \quad W_{e_2} = \frac{600}{s + 0.1}, \quad W_{\tilde{u}_1} = W_{\tilde{u}_2} = 1.$$

The robustness aspect of the extended LPV controller against modeling uncertainties will be tested against uncertainties in the parameters η_c , η_t , η_v , and τ . To that end, we will assume that η_c , η_t , η_v vary dynamically by 10% around their nominal values (η_c^0 , η_t^0 , η_v^0) given in Appendix B as follows:

$$\eta_c(t) = \eta_c^0 + \delta_{\eta_c}(t)\eta_c^0$$

$$\eta_t(t) = \eta_t 0 + \delta_{\eta_t}(t)\eta_t^0$$

$$\eta_v(t) = \eta_v^0 + \delta_{\eta_v}(t)\eta_v^0$$

and 10% static uncertainty in τ (with respect to its nominal value τ^0 in Appendix B) will be assumed:

$$\tau = \tau^0 + \delta_\tau \tau^0.$$

The specific values of perturbation functions $\delta_{\eta_c}(t)$, $\delta_{\eta_t}(t)$, $\delta_{\eta_v}(t)$, $\delta_{\tau_v}(t)$, δ_{τ} will be given for each considered case below.

As to testing performance of extended LPV controller under variable operating points, N, T_i , T_x will be taken as variable over large intervals. In the provided simulations below, both aspect will be tested simultaneously on a model which has the specified parametric uncertainties and variable operating points.

The order of designed extended LPV controller is seven (three due to system states and four due to dynamic weights). The achieved \mathcal{L}_2 -gain (γ^*) is 217.5. Next, we consider two case studies for the illustration of the robust performance of the designed extended LPV controller. For both case studies, the initial conditions are $p_i(0) = 103$ kPa, $p_x(0) =$ 109 kPa and $P_c(0) = 150$ W. Since we do not have a testbed to test the designed controller experimentally, we show the controlled system results through detailed simulations. However, the nonlinear engine model we used in this study is a widely accepted and common model used in many of the mentioned references (for example, in [2], [3], [5], [7], [19], and [20]). Hence, testing the robustness of the designed controller together with this fact should give confidence on the validity of the results presented in this paper.

1) CASE I

In this case study, we consider the following uncertainties: $\delta_{\eta_c}(t) = 0.1 \sin(t)$, $\delta_{\eta_t}(t) = 0.1 \cos(t)$, $\delta_{\eta_v}(t) = 0.05 \sin(t) + 0.05 \cos(t)$, and $\delta_{\tau} = 0.1$. For disturbance input, scheduling parameters and set points, we assume that fuel flow rate consists of one step, engine speed is a sinusoid, manifold temperatures and $p_x - W_{ci}$ references consist of three steps. These plots together with the corresponding control simulation results are shown through Figure 4.

2) CASE II

In the second case study, we consider the following uncertainties: $\delta_{\eta_c}(t) = 0.1 \cos(t)$, $\delta_{\eta_t}(t) = 0.1 \sin(t)$, $\delta_{\eta_v}(t) = 0.05 \cos(t) - 0.05 \sin(t)$, and $\delta_{\tau} = -0.1$. For disturbance



FIGURE 4. First robust performance test case results: disturbance, scheduling parameters, references, controlled outputs and control inputs. (a) Fuel flow rate and engine speed. (b) Intake and exhaust manifold temperatures. (c) Reference signals and the corresponding controlled system response. (d) VGT vane and EGR valve positions.

input, scheduling parameters and set points, we assume that fuel flow rate and manifold temperatures are sinusoids, engine speed and $p_x - W_{ci}$ references are multi-step signals.



FIGURE 5. Second robust performance test case results: disturbance, scheduling parameters, references, controlled outputs and control inputs. (a) Fuel flow rate and engine speed. (b) Intake and exhaust manifold temperatures. (c) Reference signals and the corresponding controlled system response. (d) VGT vane and EGR valve positions.

These plots together with the corresponding control simulation results are shown through Figure 5.

From the simulation results we see that the performance of the LPV controller is satisfactory. The considered scheduling parameter ranges are wide enough to cover practical applications with variable operating points. The settling time of the system under the AW-LPV controller is a little bit high. This is due to the high range of temperature variables. In the first case study, Figure 4(d) shows that AW is effective during t = 8s-10.1s (x_r saturates), and in the second case study Figure 5(d) shows that AW is effective during t = 5.4s-5.6s (x_r saturates).

Note that in principle, T_i, T_x, N and other scheduling parameters can vary freely in their specified ranges. However, from physical considerations, this may not be possible. In the diesel engine considered, the scheduling parameters T_i, T_x, N (which are states for the real physical system but for the constructed LPV model they are scheduling parameters) have to be consistent with other engine states, inputs, disturbances, *etc.* From control side, whether they are chosen freely or mapped to other data is not important unless they are out of the specified bounds. This is the main paradigm and advantage of LPV-based control strategies.

D. REMARKS

The following remarks are important.

- Although we assumed variable manifold temperatures during LPV modeling and the subsequent extended LPV controller design, for diesel engines with an EGR cooler, intake manifold temperature is relatively stable since the charge air cooler can cool down the compressed air after the compressor and EGR cooler can cool the EGR gas. In such cases the intake manifold temperature can be taken as a constant model tuning parameter instead of a scheduling parameter. However, for diesel engines without EGR cooler intake manifold temperature varies due to the effect of EGR and for such situations intake manifold temperature should be taken as a scheduling parameter.
- Since currently measuring exhaust temperature accurately is not practical in diesel engines, for the application of the proposed method exhaust temperature should be estimated through an observer.

V. COMPARISON OF EXTENDED LPV CONTROLLER TO OTHER CONTROLLERS

In this section, we will compare the superior performance of the designed extended LPV controller to two types of controllers: first to an H_{∞} controller to see the failure of a linear controller under variable operating points, then to an LPV controller developed in [5] where manifold temperatures were assumed constant model tuning parameters, $p_x = p_i + 2.5$ kPa assumption was done, and engine speed was taken as a disturbance in the developed LPV model.

A. COMPARISON TO H_{∞} CONTROLLER

In this subsection, to appreciate the importance of air path system control using an extended LPV control approach, we will compare performances of the extended LPV controller under variable manifold temperatures and an H_{∞} controller based on a linearized model of the engine around an equilibrium point of the nonlinear engine model. The ranges for the non-state scheduling parameters to the system are $N \in [1000, 2500]$ rpm, $T_i \in [300, 350]$ K and $T_x \in [350, 700]$ K. To find an equilibrium point of the system we first choose nominal values of these non-state scheduling parameters. Here, we take $N_{\text{nom}} = 1750$ rpm, $T_{i_{\text{nom}}} = 325$ K and $T_{x_{\text{nom}}} = 525$ K. The equilibrium values for the engine inputs (modified inputs) and fuel flow rate are taken as $W_{xt_{\text{eq}}} = 25$ g, $W_{xi_{\text{eq}}} = 10$ g and $W_{f_{\text{eq}}} = 1$ g. The corresponding equilibrium states are determined as $p_{i_{eq}} = 125$ kPa, $p_{x_{eq}} = 133.5$ kPa and $P_{c_{eq}} = 823$ W. The linearized model around the equilibrium points is

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u},$$
$$\tilde{y} = C\tilde{x} + D\tilde{u},$$

where

$$\begin{split} A &= \begin{pmatrix} -17.58 & 0 & 453.22 \\ 40.99 & 0 & 0 \\ 0 & 0.18 & -9.09 \end{pmatrix}, \\ B &= \begin{pmatrix} 0 & 0 & 15.54 \\ 150.67 & -150.67 & -150.67 \\ 0 & 0.29 & 0 \end{pmatrix}, \\ C &= \begin{pmatrix} 0 & 1 & 0 \\ -3.09 & 0 & 104.95 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ \tilde{x} &= \begin{pmatrix} p_i - p_{i_{eq}} \\ p_x - p_{x_{eq}} \\ P_c - P_{c_{eq}} \end{pmatrix}, \quad \tilde{u} = \begin{pmatrix} W_f - W_{f_{eq}} \\ W_{xt} - W_{xt_{eq}} \\ W_{xi} - W_{xi_{eq}} \end{pmatrix}, \\ \tilde{y} &= \begin{pmatrix} p_x - p_{x_{eq}} \\ W_{ci} - \frac{\eta_c}{c_p T_a} \frac{P_{c_{eq}}}{\left(\frac{p_{i_{eq}}}{p_a}\right)^{\mu} - 1} \end{pmatrix}. \end{split}$$

For the H_{∞} controller, the weight selection is the same as in Section IV-C. The obtained \mathcal{L}_2 -gain, γ , is 1.0664. The controller implementation is the same as before.

Next, we will consider a comparison case test. We will assume that N, T_i and T_x vary far from their nominal values and track set points for p_x and W_{ci} which are also far from their equilibrium values. We take fuel flow rate and engine speed as sinusoids, manifold temperatures as steps, and p_x and W_{ci} references consist of two steps. These plots together with the corresponding control simulation results are shown through Figure 6.

The comparison results in Figure 6 clearly illustrates that H_{∞} controller does not work when N, T_i and T_x vary far from their nominal values and/or when the reference set points for the outputs are far from their equilibrium values. As a conclusion, when a wide range of engine operating



(d)

FIGURE 6. Comparison of extended LPV controller to an H_{∞} controller: disturbance, scheduling parameters, references, controlled outputs and control inputs. (a) Fuel flow rate and engine speed. (b) Intake and exhaust manifold temperatures. (c) Reference signals and the corresponding system responses from the two controllers. (d) VGT vane and EGR valve positions.

points is considered, it is very possible that a linear controller will not work. This implies the necessity of gain-scheduled controllers for engine air path control.



FIGURE 7. Performance comparison of designed extended LPV controller in this study (LPV_e) with the simplified LPV controller from [5] (LPV_s).

B. COMPARISON TO ANOTHER LPV CONTROLLER

Now we compare the performance of the designed extended LPV controller to that of the LPV controller designed in [5] where the following set of assumptions was done during LPV controller design: (i) manifold temperatures were assumed constant optimal model tuning parameters with the values $T_i = 305$ K and $T_x = 509$ K; (ii) engine speed was taken as a disturbance; and (iii) $p_x = p_i + 2.5$ kPa assumption was done to reduce the number of scheduling variables to one: p_i .

We compare the two controllers in a scenario of variable operating points where the fuel flow rate and scheduling parameters were chosen as $W_f = 1 + 0.8 \cos(t)$ (g), $N = 2000 + 300 \sin(t)$ (rpm), $T_i = 320 + 20 \sin(10t)$ (K) and $T_x = 450 + 50 \sin(10t)$ (K). The performances of the two controllers are shown in Figure 7 where LPV_e means the "extended" LPV controller designed in this study and LPV_s means the "simplified" LPV controller designed in [5].

From the comparison results in Figure 7 we see that LPV_s is not robust to variable operating points due to manifold temperature variations, engine speed variations or $p_x = p_i + 2.5$ kPa assumption. In contrast, the performance of LPV_e is satisfactory. The main finding from the comparison done here is that an LPV controller with one scheduling variable (p_i) may not work properly in variable operating conditions. Moreover, taking engine speed as a disturbance degrades the performance of the LPV controller, and hence it should also be considered as a scheduling parameter as we did. Finally, the assumption of $p_x = p_i + 2.5$ kPa may also degrade the LPV controller performance.

VI. A GUIDELINE FOR USE OF THE PROPOSED METHOD

Given an arbitrary turbocharged diesel engine with EGR, next we give a set of guideline steps for designing the proposed extended LPV controller discussed in this study.

Step1: Run the engine through multiple operating points so that variable engine operating situations are covered. Collect static turbine mass flow rate data and from these data calculate a, b, c, d for W_{xt} . This approach was also utilized in [5].

Step 2: Get the p_i , p_x and P_c data either through direct dynamic measurements of these variables when engine runs through variable points or indirectly estimate them from other measured variables. Then, optimize the model parameters η_c , η_t , η_v and τ to fit the air path model to the above data. It is better to calculate η_c , η_t , η_v and τ through dynamic measurements since these parameters have important effects on the dynamic response of the air path model. If desired, the volumes V_i , V_x and V_d could be taken as additional tuning variables with their initial values equal to the values of the considered engine. Since in this case the total number of tuning parameters increases, a sensitivity analysis can be done before determining the set of important tuning parameters in parameter optimization.

Step 3: Construct the extended LPV model and then design the corresponding extended LPV controller developed in this study.

VII. CONCLUSION AND FUTURE WORK

In this paper, we considered the problem of controlling the air path system in diesel engines using an extended linear parameter-varying control approach by considering a general nonlinear, mean-value engine model used extensively in the literature. In the control design phase, typical simplifications on the engine model and selection of a specific engine operating point were avoided. Moreover, manifold temperatures were considered as time-varying parameters in addition to the engine speed. Based on the developed high-fidelity LPV model, an LPV controller was designed and its robust performance was tested through detailed simulations. Finally, a comparison of the performance of the developed extended LPV controller was done with two other controllers: an H_{∞} controller and an LPV controller from the literature. From these comparisons, we clearly observe that the developed controller is much more general to work better in variable operating conditions.

In the control part, we considered volumetric, turbine and compressor efficiencies as some optimal constant values. Their modeling may be hard and even when this is possible, they should be considered as additional scheduling parameters which may complicate the LPV controller design. A better way will be interpreting them as uncertain parameters and then applying a control approach for partly-measured parameters in LPV systems [29], which is an LPV control method which includes robustness property in its design framework.

APPENDIX A NOMENCLATURE

Symbol	Description	
AFR	Air fuel ratio	
AW	Anti-windup	
EGR	Exhaust gas recirculation	
LFR	Linear fractional representation	
LMI	Linear matrix inequality	
LPV	Linear parameter-varying	
VGT	Varibale geometry turbine	
Ν	Engine speed	
N_t	Turbocharger speed	
p_i	Intake manifold pressure	
p_x	Exhaust manifold pressure	
P_c	Compressor power	
P_t	Turbine power	
T_i	Intake manifold temperature	
T_x	Exhaust manifold temperature	
x_v	Tubine vane position	
x_r	EGR valve position	
W_{ci}	Compressor mass flow rate	
W_f	Engine fuel flow rate	
W_{ie}	Mass flow rate to the intake manifold	
W_{xe}	Mass flow rate from the exhaust manifold	
W_{xi}	EGR mass flow rate	
W_{xt}	Turbine mass flow rate	

APPENDIX B DIESEL ENGINE MODEL PARAMETERS

The engine model parameters are shown in the Table 1.

TABLE 1. Engine model parameters.

Sym.	Val.	Unit	Descr.
V_i	6×10^{-3}	m ³	intake manifold vol.
V_x	1×10^{-3}	m ³	exhaust manifold vol.
V_d	2×10^{-3}	m ³	displacement vol.
c_p	1014.4	J/kg/K	specific heat at const. pres.
c_v	727.4	J/kg/K	specific heat at const. vol.
R	287	J/kg/K	gas constant ($R = c_p - c_v$)
p_a	99.2	kPa	ambient pres.
T_a	302	K	ambient temp.
p_{ref}	101.3	kPa	reference pres.
T_{ref}	298	K	reference temp.
γ	1.3936	-	specific heat rat.
a	-490.4/3600	-	-
b	633.7/3600	-	-
c, d	0.4, 0.6	-	-
η_c, η_t	0.61, 0.76	-	compressor, turbine eff.
η_v	0.87	-	volumetric eff.

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