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Sub-Nyquist Spectrum Sensing Based on Modulated Wideband Converter in Cognitive Radio Sensor Networks

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ABSTRACT The large-scale deployment of wireless sensor networks is indispensable to the success of Internet of Things. Considering dynamic spectrum access and the limited spectrum resources in cognitive radio sensor networks, sub-Nyquist spectrum sensing based on the modulated wideband converter is introduced. Since the transmission signals are usually modulated by different carrier frequencies, the interested spectrum can be modeled as the multiband signal. Modulated wideband converter (MWC) is an attractive alternative among several sub-Nyquist sampling systems because it has been implemented in practice and the frequency support reconstruction algorithm is the most important part in MWC. However, most existing reconstruction methods require the sparse information, which is difficult to acquire in practical scenarios. In this paper, we propose a blind multiband signal reconstruction method, referred to as the statistics multiple measurement vectors (MMV) iterative algorithm to bypasses the above problem. By exploiting the jointly sparse property of MMV model, the supports can be obtained by statistical analysis for the reconstruction results. Simulation results show that, without the sparse prior, the statistics MMV iterative algorithm can accurately determine the support of the multiband signal in a wide range of signal-to-noise ratio by using various numbers of sampling channels.

INDEX TERMS Cognitive radio sensor networks, blind multiband signal reconstruction, sub-Nyquist sampling, multiple measurement vectors, modulated wideband converter.

I. INTRODUCTION

As the Internet of Things develops into globalization gradually, the ubiquity of wireless sensor networks (WSN) is becoming imperative. In order to achieve efficient spectrum utilization, the new paradigm of the cognitive radio sensor networks (CRSN) has been drawn more attentions, which integrates cognitive radio (CR) into WSN. This concept aims to fuse the benefits of dynamic spectrum access into WSN. Sensor networks are usually densely deployed, with hundreds of nodes. However, there are few nodes transmitting signals at a certain time, so it is not reasonable to allocate spectrum for each node. Due to the scarcity of spectrum resource, the best way to utilize spectrum resources is to share spectrum, which is based on spectrum sensing technology. Wideband spectrum sensing (WSS) has been widely recognized as an effective means to deal with the increasing demand for broadband access and the scarcity of available spectrum [1]. However, the increasing bandwidth brings a great challenge to the implementation of conventional WSS techniques, thus sub-Nyquist spectrum sensing attracts significant attentions.

In CRSN, the active nodes achieve dynamic spectrum access on spectrum holes. Therefore, the transmission signals in CRSN are radio frequency (RF) signals, modulated by different carrier frequencies randomly. Many RF signals reside within several continuous frequency intervals spreading over a board spectrum range, which is modeled as the multiband signal [2], [3], depicted in Fig.1. The multiband signal model is also an important signal in radar and communication systems [4]–[6].

Landau [7] developed a minimal rate, called the Landau rate for multiband signal to achieve prefect reconstruction with the prior knowledge of bands locations and their bandwidth. The Landau rate is equal to the total bandwidth of

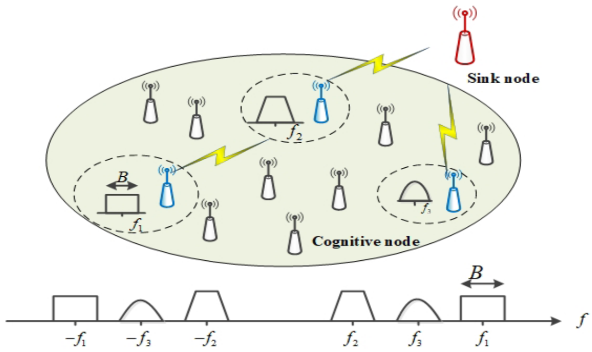

FIGURE 1. Illustration of the multiband signal model in a CRSN.

multiband signal. However, it is just a minimal sampling rate requirement for an arbitrary sampling mode. The multiband feature of the signals in the frequency domain is exploited to achieve the signal reconstruction from finite samples and the uniform sampling rate is set by the bandwidth of single sub-band, which can reduce the sampling rate [8].

Periodically nonuniform sampling was studied in [9]–[12], which allowed the lower average rate approaching the Landau rate for bandpass and multiband signals. In [9], nonuniform periodic sampling was given as a generalization of the classical sampling theorem and coest sampling was proposed as a representative. And periodic nonuniform samples from several components with different phases were used to achieve exact reconstruction, but its sampling pattern, called minimum rate sampling must satisfy the relevant theorem [10]. Minimum rate sampling also can process the multiband signal with arbitrary frequency support [11]. Periodically Nonuniform Sampling of Lth-order can be considered as the prototype of multicoest sampling, which can recover a broader class of bandpass or multiband signals [12]. At this point, based on periodically nonuniform sampling, a theoretical study on sub-Nyquist sampling of multiband signals is carried out and some conclusions are given, such as perfect reconstruction formula, bounds on aliasing error and a necessary condition on the optimal sampling density [13]–[16]. All above mentioned sampling schemes require exact knowledge of the spectrum support to recover the original multiband signal from the samples.

With the emergence of the compressed sensing (CS) theory, it brings an opportunity for blind processing of the multiband signal, in which the exact location of each sub-band is no longer needed. Meanwhile, the sparse multiband structure is also considered as the basis for exploiting the CS theory and the requirement for realizing the sub-Nyquist sampling. There are several common sub-Nyquist sampling systems, such as an analog to information converter (AIC), a multi-rate sampling (MRS), a multicoset sampling (MCS) and a modulated wideband converter (MWC). AIC system has single sampling channel, which contains a pseudorandom demodulator and a low rate sampler [17]. But it can only deal with discrete multitone signal (i.e. sinusoids with sparse frequencies), unable to process the signal with wideband sub-bands. The other three sub-Nyquist systems belong to multichannel structure. MRS includes multi-rate asynchronous sampling [18] and multi-rate synchronous sampling [19]. The asynchronous sampling scheme does not require the knowledge of the time offset between the sampling channels, which makes hardware implementation simple. The sufficient condition for an accurate reconstruction is that the frequency support of the multiband signal must be unaliased in at least one of the sampling channels. However, the synchronous scheme is more complex because of synchronization. But it can reconstruct a signal in many cases, not limited to the above case in the asynchronous scheme. MCS system has a universal sampling pattern, in which there are a bank of ADCs clocked at the same rate to sample signals at different delays [20]. The accurate control of time delay becomes the biggest obstacle for the success reconstruction, and it also makes the system more complex and expensive. Asynchronous coprime sampling was proposed in [6]. Compared to the conventional MCS, it has fewer samplers and does not require synchronous clock phase, but the clock phase needs to be compensated in the reconstruction stage and the start time differences between two samplers should be measured. MWC system also can achieve blind processing for the multiband signal [21]. It has a fixed analog front end to pre-process the input multiband signal in multiple channels simultaneously, in which the input signal is multiplied by a bank of periodic waveforms, filtered by a lowpass filter, and then sampled uniformly by a low-rate ADC. Meanwhile, both ADC and DSP rates are substantially below Nyquist. Above all, MWC hardware has been available and it can be implemented with off-the-shelf ADCs [22]. Though the precision that MWC can achieve is lower than that of MCS at the same total sampling rate, its error range can be accepted in practice applications and MWC has attracted increasingly more attention.

Since most sub-Nyquist sampling systems are based on the CS theory, many CS reconstruction algorithms can be used for reference. A partial list includes orthogonal matching pursuit (OMP) [23], regularized orthogonal matching pursuit (ROMP) [24], stagewise orthogonal matching pursuit (StOMP) [25], subspace pursuit (SP) [26] and CoSaMP [27]. These algorithms are proposed to solve the single measurement vector (SMV) problem, which do not apply to the multiband signal. However, the reconstruction for the multiband signal can be transformed to multiple measurement vectors (MMV) problem [20]. There are several reconstruction algorithms to solve MMV problem for the multiband spectrum processing [28]. In [29], it has developed performance limits for support reconstruction of sparse signals based on MMV and the proposed methodology also has the potential to address other theoretical and practical issues associated to sparse signal reconstruction. In order to solve the problem of jointly sparse reconstruction, five greedy algorithms designed for SMV sparse approximation problem have been extended to the MMV problem [30]. Another novel approach to obtain the solution of a sequence of SMV problems with a joint support has been presented, which could be adaptive to solve it as a sequence of weighted SMV problems rather than collecting the measurement vectors and solving the MMV problem [31]. The multiband signal has the feature of the group sparsity, which can be used to achieve reconstruction, such as block MMV algorithm [32] and group binary compressive sensing method [33]. And block MMV algorithm uses block coherence to recover multiband signals, making computational process easy. Since MWC has drawn more attentions, many other improved algorithms based on MWC have been proposed, including simultaneous smoothed ℓ^0 norm (SL0) [34], iterative support detection (ISD) [35] and sparse Bayesian learning (SBL) [36].

As mentioned above, MWC system is an attractive alternative among several sub-Nyquist sampling systems because it has been implemented in practice and most reconstruction algorithms are proposed to solve the MMV problem based on MWC. In MWC, the interested spectrum is divided into several intervals and each interval has its label. All labels of the occupied spectrum intervals make up a frequency support set. Therefore, to recover the frequency support is to acquire the spectrum status occupancy information, which is the purpose of spectrum sensing. In order to recover the accurate frequency support, several MMV-based reconstruction algorithms need the sparse prior information, namely the number of sub-bands. According to the prior knowledge, the new data frame must be constructed by the sub-Nyquist samples and this data frame is used to recover the frequency support through the reconstruction algorithms. Specially, this operation is necessary for the accuracy of reconstruction results. However, the number of sub-bands is difficult to obtain in practical applications, especially in spectrum sensing scenarios. In CRSN, we consider a distributed MWC-based scheme, which regards one sensor node as one sampling channel and combines MWC technology with a sub-Nyquist spectrum sensing network perfectly [37]. In this paper, we propose a blind multiband signal reconstruction method, referred to as the statistics MMV iterative algorithm to achieve sub-Nyquist spectrum sensing. This method can recover the frequency support without the prior information about the number of sub-bands. Different from other MMV-based reconstruction algorithms, the MMV model is no longer used to simplify the algorithm in the proposed algorithm. Each measurement vector in MMV model is used to recover the information as an individual respectively, and then combining the characteristic of MMV jointly sparse, we analyze all the reconstruction results from a statistical point of view. Finally, the frequency support set of the multiband signal can be obtained. For each measurement vector reconstruction, the optimization iterative method is exploited. By statistical analysis, the proposed algorithm can be used to realize the support information reconstruction without the sparsity prior.

The remainder of this paper is organized as follows. In Section II, the MWC sub-Nyquist sampling system is first briefly reviewed. Then, the statistics MMV iterative algorithm is proposed in Section III. Finally, simulation results are presented in Section IV for demonstrating the performance of the proposed approach.

II. MODULATED WIDEBAND CONVERTER

The multiband signal $x(t)$ is continuous and bandlimited to $F=\left[-1/2T,\ 1/2T\right]$. Formally, the Fourier transform of $x(t)$ is defined by

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft} dt \tag{1}$$

The Nyquist rate of $x(t)$ is denoted by $f_{Nyq} = 1/T$. For the multiband signal $x(t)$, $X(f) = 0$ for every $f \notin F$ and the frequency support of $X(f)$ contains within a union of N disjoint bands in F, as well the bandwidth of each sub-band does not exceed B.

MWC system is considered as an attractive alternative to process the multiband signal and it is also the successful example to extend the CS theory from discrete domain to analog domain. Since the energy of multiband signal does not cover the whole frequency range, it provides the opportunity to sample such a signal at low rate below the Nyquist rate. Meanwhile, the sparse characteristic of the multiband signal is the basis for adopting the CS theory. In MWC system, there are two parts, namely the sub-sampling stage and the signal reconstruction phase.

A. SUB-SAMPLING

MWC system has a fixed analog mixing front-end, which belongs to multichannel parallel structure. There are a mixer, a low pass filter and a sampler in each sampling channel. As shown in Fig.2, MWC system contains m parallel sampling channels.

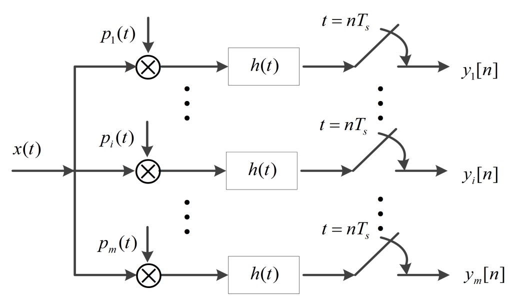

FIGURE 2. The sub-Nyquist sampling stage in MWC.

The purpose of mixer operation is to make the spectrum of the multiband signal extend periodically and the spectrum portion from each sub-band can appear in the baseband. This operation is similar to spread-spectrum technology.

The mixing function $p_i(t)$ is T_p-periodic and piecewise smooth, like a pseudo-random sequence. Here, $p_i(t)$ is chosen as a piecewise constant function that alternates between the levels ± 1 for each of M equal time intervals, depicted in Fig.3. Formally,

$$p_i(t) = \alpha_{ik}, \quad k\frac{T_p}{M} \le t \le (k+1)\frac{T_p}{M},\ 0 \le k \le M-1 \tag{2}$$

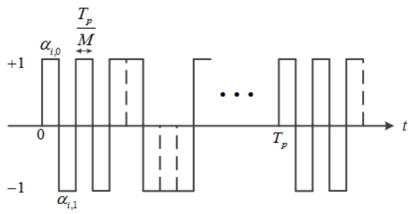

FIGURE 3. The mixing function $p_i(t)$.

with $\alpha_{ik} \in \{+1, -1\}$ and $p_i\left(t + nT_p\right) = p_i(t)$ for every $n \in Z$.

For the ith channel, the mixing function $p_i(t)$ has a Fourier expansion,

$$p_i(t) = \sum_{l=-\infty}^{\infty} c_{il} e^{j\frac{2\pi}{T_p}lt} \tag{3}$$

where,

$$c_{il} = \frac{1}{T_p}\int_0^{T_p} p_i(t)\, e^{-j\frac{2\pi}{T_p}lt} dt \tag{4}$$

Next, the mixing function is applied to the multiband signal $x(t)$, obtaining the modulated signal $\tilde{x}_i(t) = x(t)\, p_i(t)$. Its Fourier transform is evaluated as

$$\begin{aligned}\tilde{X}_i(f) &= \int_{-\infty}^{\infty} \tilde{x}_i(t)\, e^{-j2\pi ft} dt \\ &= \int_{-\infty}^{\infty} x(t) \left(\sum_{l=-\infty}^{\infty} c_{il} e^{j\frac{2\pi}{T_p}lt}\right) e^{-j2\pi ft} dt \\ &= \sum_{l=-\infty}^{\infty} c_{il} X\left(f - lf_p\right)\end{aligned} \tag{5}$$

where $X(f)$ is the Fourier representation of $x(t)$ and $f_p = 1/T_p$.

After mixer, the spectrum of the modulated signal achieves periodic expansion, causing spectrum aliasing. Subsequently, a linear combination of f_p-shift copies of $X(f)$ is acted as the input of low pass filter with cutoff $1/(2T_s)$, as shown in Fig.4. After low-pass filtered, only the spectrum in baseband is retained, containing spectrum information from each sub-band. Finally, the filtered signal is sampled uniformly at low rate $f_s = 1/T_s$. The sampling rate is matched to the cutoff frequency of low pass filter.

The period T_p determines the aliasing of $X(f)$ by setting the shift intervals to $f_p = 1/T_p$ and we choose $f_p \geq B$ so that

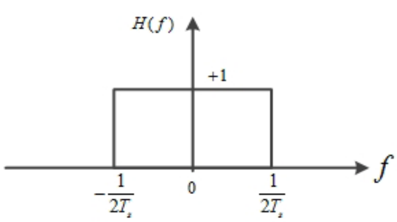

FIGURE 4. Frequency response of low pass filter.

each sub-band itself is no aliasing. In practice f_p is chosen slightly more than B to avoid edge effects. In order to get the whole information of each sub-band, the relationship between f_p and f_s must satisfy $f_s \geq f_p$. The simplest choice $f_s = f_p \simeq B$ allows the lowest sampling rate of each sampling channel. The overall sampling rate of MWC is equal to the product of channel number m and single channel sampling rate f_s, namely $f_{sys} = m \cdot f_s$. Actually, this sampling rate is much lower than the Nyquist rate, so it is called a sub-Nyquist rate.

B. RECONSTRUCTION

The uniform sequence $y_i[n]$ from the ith channel contains only frequency in F and its discrete-time Fourier transform (DTFT) is expressed as

$$\begin{aligned}Y_i\left(e^{j2\pi fT_s}\right) &= \sum_{n=-\infty}^{\infty} y_i[n]\, e^{-j2\pi fnT_s} \\ &= \sum_{l=-L_0}^{+L_0} c_{il} X\left(f - lf_p\right)\end{aligned} \tag{6}$$

$L = 2L_0 + 1$ denotes the amount of periodic spread spectrum in the Nyquist frequency range. Rewrite (6) in matrix form as

$$Y(f) = Az(f) \tag{7}$$

where $Y(f)$ is a vector of length m with ith element $y_i(f) = Y_i\left(e^{j2\pi fT_s}\right)$, the DTFT of $y_i[n]$. The matrix A contains the coefficients c_{il} and $z(f)$ consists of f_p-shifted copies of $X(f)$.

Equation (7) is similar to the typical CS problem and can be solved referring to the CS reconstruction algorithms. In MWC, continuous to finite (CTF) block is used to recover the frequency support, depicted in Fig.5.

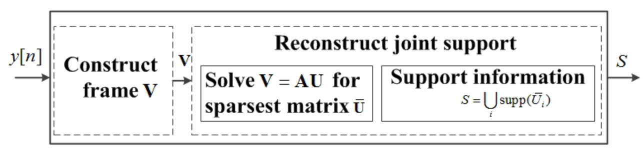

FIGURE 5. Continuous to finite block.

First, a data frame V is constructed as follows,

$$Q = \int_{f \in F_s} y(f) y^H(f) df = \sum_{n=-\infty}^{+\infty} y[n] y^T[n] \tag{8}$$

The matrix V is obtained by $Q = VV^H$. Then, next step is to solve $V = AU$ for the sparsest matrix $\bar{U}$ and get its support $S = \cup supp\left(\bar{U}\right)$. It can be proved that the sparsest matrix $\bar{U}$ has the same frequency support as $X(f)$. According to the support information, the analog multiband signal $x(t)$ can be recovered.

However, in order to recover the accurate frequency support, the frame V construction must meet the requirement, which is $rank(V) \leq 2N$ in terms of the prior knowledge. This operation is to separate the signal space and noise space, so as to reduce the influence of noise on the support reconstruction. Otherwise, it will have a huge impact on the reconstruction results. It means that the sparse prior information is required by both data frame construction and the support reconstruction process. In the support reconstruction stage, the prior is used to terminate the iteration process.

Section III describes another algorithm to solve (7), which can avoid the above problem.

III. ALGORITHM DESCRIPTIONS

In MWC, the frequency support reconstruction is transformed into solving MMV problem. For many cases in CS theory framework, MMV is considered as a potential condition to simplify the CS reconstruction algorithms. Several methods based on MMV model can improve the reconstruction performance and fewer measurements are needed. However, in this paper, it can be used to realize the support information reconstruction without the sparsity prior. In MMV model, nonzero locations in each measurement vector are similar to each other. With no prior knowledge, the position of the nonzero can be found by comprehensive analysis of the result from each measurement vector and then the signal frequency support set is obtained.

A. MMV SIGNAL MODEL

In the MMV setting, the typical problem is described as follow,

$$Y = A \cdot X \tag{9}$$

where Y is a combination of s measurements and vector x_i, $1 \leq i \leq s$ is the column of matrix X.

Specially, each vector x_i is k-sparse and exhibit the same indices for their nonzero locations. This is the so-called jointly sparse. There will be at most k nonzero rows in X, which means the nonzero values occur on a common location set. An example of such a matrix X is depicted in Fig.6. In Fig.6 (a), the sparse vectors ($s = 6$) share a common support. Each square corresponds to a vector entry, in which the filled squares represent nonzero elements and blank squares indicate zeros. The corresponding matrix $X = [x_1\, x_2 \cdots x_s]$ has a small number of nonzero rows in Fig.6 (b).

B. OPTIMIZATION PROBLEM ANALYSIS

Referring to the classical formula of CS theory, equation (7) can be written,

$$\hat{z}(f) = \arg\min \|z(f)\|_0 \quad s.t.\ Y(f) = Az(f) \tag{10}$$

where

$$Y(f) = \begin{bmatrix} y_1(f) \\ y_2(f) \\ \vdots \\ y_m(f) \end{bmatrix}, \quad z(f) = \begin{bmatrix} X(f + L_0 f_p) \\ X(f + (L_0 - 1) f_p) \\ \vdots \\ X(f - L_0 f_p) \end{bmatrix} \tag{11}$$

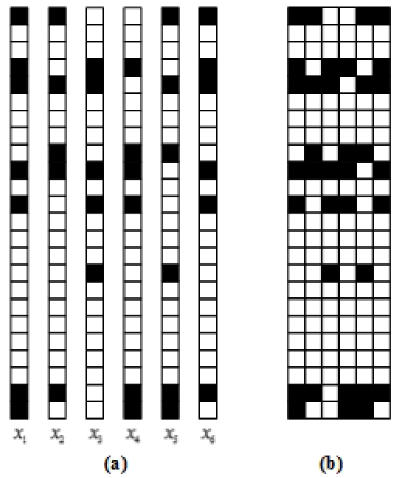

FIGURE 6. MMV signal model.

Since $y_i(f)$, $1 \leq i \leq m$ and $X(f - lf_p)$, $-L_0 \leq l \leq L_0$ are not values but row vectors, rewrite $Y(f)$ and $z(f)$ as follow,

$$Y(f) = \begin{bmatrix} y_1(f) \\ y_2(f) \\ \vdots \\ y_m(f) \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1s} \\ y_{21} & y_{22} & \cdots & y_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1} & y_{m2} & \cdots & y_{ms} \end{bmatrix} = \begin{bmatrix} \tilde{y}_1 & \tilde{y}_2 & \cdots & \tilde{y}_s \end{bmatrix} \tag{12}$$

and

$$z(f) = \begin{bmatrix} X(f + L_0 f_p) \\ X(f + (L_0 - 1) f_p) \\ \vdots \\ X(f - L_0 f_p) \end{bmatrix} = \begin{bmatrix} X_{-L_0,1} & X_{-L_0,2} & \cdots & X_{-L_0,s} \\ X_{-L_0+1,1} & X_{-L_0+1,2} & \cdots & X_{-L_0+1,s} \\ \vdots & \vdots & \ddots & \vdots \\ X_{L_0,1} & X_{L_0,2} & \cdots & X_{L_0,s} \end{bmatrix} = \begin{bmatrix} \tilde{x}_1 & \tilde{x}_2 & \cdots & \tilde{x}_s \end{bmatrix} \tag{13}$$

where $\tilde{y}_j = \left[y_{1j}, y_{2j}, \cdots, y_{mj}\right]^T$, $1 \leq j \leq s$ is a $m \times 1$ column vector and $\tilde{x}_j = \left[X_{-L_0,j}, X_{-L_0+1,j}, \cdots, X_{L_0,j}\right]^T$, $1 \leq j \leq s$ is a $L \times 1$ column vector, $L = 2L_0 + 1$. s denotes the length of multiple measurement vectors.

In order to solve the problem (10), we need to estimate $z(f)$ from $Y(f)$ by minimizing the objective function,

$$J(z) = \|Y(f) - Az(f)\|_2^2 = \sum_{j=1}^{s} \left\|\tilde{y}_j - A\tilde{x}_j\right\|_2^2 \tag{14}$$

The function $J(z)$ can be minimized by minimizing each term $\left\|\tilde{y}_j - A\tilde{x}_j\right\|_2^2$ individually to obtain $\tilde{x}_j$, for $1 \leq j \leq s$. And we need only consider the scalar minimization of the function

$$R(\tilde{x}) = \|\tilde{y} - A\tilde{x}\|_2^2 \tag{15}$$

The majorization-minimization (MM) approach [38] can be used to solve the problem for the minimization of (15), because $R(\tilde{x})$ cannot be easily minimized, but a new function is needed. According to [39], the new function form can be chosen,

$$G_k(\tilde{x}) = \|\tilde{y} - A\tilde{x}\|_2^2 + (\tilde{x} - \tilde{x}_k)^T \left(\alpha I - A^T A\right)(\tilde{x} - \tilde{x}_k) \quad (16)$$

where the scalar parameter α must be equal to or greater than the maximum eigenvalue of $A^T A$.

Following the MM procedure, we need to minimize $G_k(\tilde{x})$ to obtain $\tilde{x}_k$. Expanding $G_k(\tilde{x})$ in (16) gives

$$\begin{aligned} G_k(\tilde{x}) &= (\tilde{y} - A\tilde{x})^T (\tilde{y} - A\tilde{x}) \\ &\quad + (\tilde{x} - \tilde{x}_k)^T \left(\alpha I - A^T A\right)(\tilde{x} - \tilde{x}_k) \\ &= \tilde{y}^T \tilde{y} + \tilde{x}_k^T \left(\alpha I - A^T A\right)\tilde{x}_k \\ &\quad - 2\left(\tilde{y}^T A + \tilde{x}_k^T \left(\alpha I - A^T A\right)\right)\tilde{x} + \alpha \tilde{x}^T \tilde{x} \end{aligned} \quad (17)$$

Then, it is easily to minimize $G_k(\tilde{x})$,

$$\frac{\partial}{\partial \tilde{x}} G_k(\tilde{x}) = -2A^T \tilde{y} - 2\left(\alpha I - A^T A\right)\tilde{x}_k + 2\alpha\tilde{x} \quad (18)$$

Next, let $\frac{\partial}{\partial \tilde{x}} G_k(\tilde{x}) = 0$,

$$\tilde{x} = \tilde{x}_k + \frac{1}{\alpha} A^T (\tilde{y} - A\tilde{x}_k) \quad (19)$$

Finally, the iterative formula to solve the problem for the minimization of (15) is obtained,

$$\tilde{x}_{k+1} = \tilde{x}_k + \frac{1}{\alpha} A^T (\tilde{y} - A\tilde{x}_k) \quad (20)$$

In this approach, an iteration stop condition can be set by using two numeric residuals. However, the results is just to recover each $\tilde{x}_j$, for $1 \le j \le s$ and $\tilde{x}_j$ is badly effected by noise. We can analyze the all $\tilde{x}_j$ from the statistical point of view and then, the frequency support can be finally obtained. In this paper, the statistics MMV iterative algorithm is proposed to exploit the MMV model and statistics analysis method to achieve the reconstruction of the joint sparse vector for adaptive sparsity.

C. STATISTICS MMV ITERATIVE ALGORITHM

The proposed statistics MMV iterative algorithm is first to obtain the sparse vector $\tilde{x}_j$ from each measurement vector $\tilde{y}_j$ exploiting the above iterative method. Then, the frequency support information of each $\tilde{x}_j$ can be acquired to make a statistical analysis. Finally, the frequency support set of the multiband signal is got by combining the characteristic of joint sparse. The implementation of statistics MMV iterative algorithm is given in Fig.7.

In the iteration process, a non-linear operation H_h is added. The specific method is to retain the half elements with the largest magnitude of $\tilde{x}_j$ and make the other remaining elements equal to zero. Since the noise exists, all the elements in $\tilde{x}_j$ have the magnitude. After this operation, it can reduce the amount of computation to speed up the iteration process.

algorithm 1 Statistics MMV Iterative Algorithm

Input: $\mathbf{A}$, $\mathbf{Y}$

Output: Support set $S = \text{supp}(\mathbf{Z})$

Initialize: $k = 0$, $i = 0$, $t = 0$, $\mathbf{A}_{mA \times nA}$,
  $\mathbf{Z}^{(k)}_{nA \times s} = \left\{\tilde{\mathbf{x}}^{(k)}_0, \tilde{\mathbf{x}}^{(k)}_1, \ldots, \tilde{\mathbf{x}}^{(k)}_{s-1}\right\}$,
  $\left(\tilde{\mathbf{x}}^{(k)}_j\right)_{nA \times 1} \leftarrow 0$ for $0 \le j \le s-1$,
  $\mathbf{Y}_{m \times s} = \{\tilde{\mathbf{y}}_0, \tilde{\mathbf{y}}_1, \ldots, \tilde{\mathbf{y}}_{s-1}\}$;
for $i = 0$ to $s - 1$ **do**
  External loop: Decompose MMV model and compute each measurement vector to obtain the independent sparse vector.
  while $R(\tilde{\mathbf{x}}) > \varepsilon$ **do**
    Internal loop: Minimize $R(\tilde{\mathbf{x}})$ *by multiple iterations.*
    $\tilde{\mathbf{x}}^{(k+1)}_i \leftarrow \tilde{\mathbf{x}}^{(k)}_i + \frac{1}{\alpha}\mathbf{A}^T\left(\tilde{\mathbf{y}}_i - \mathbf{A}\tilde{\mathbf{x}}^{(k)}_i\right)$
    $\mathbf{H}_h$: retain half of the largest elements in $\tilde{\mathbf{x}}^{(k+1)}_i$ and the remaining elements are equal to zero;
    Update vector $\tilde{\mathbf{x}}^{(k+1)}_i \leftarrow \mathbf{H}_h \cdot \tilde{\mathbf{x}}^{(k+1)}_i$;
    Update the iteration counter: $k \leftarrow k + 1$;
  end while
end for
Statistics and analysis: Collect all sparse vectors $\mathbf{Z}_{nA \times s} = [\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \cdots, \tilde{\mathbf{x}}_s]$;
for $t = 0$ to $nA - 1$ **do**
  Compute the average Avg_t of tth row in $\mathbf{Z}_{nA \times s}$;
end for
Get matrix $\mathbf{D}$ by comparing the elements in $\mathbf{Z}$ with the corresponding Avg_t and compute the sum row by row in matrix $\mathbf{D}$ to get vector $\mathbf{P}$;
Obtain the support position according to the value of $\mathbf{P}$.

FIGURE 7. **Statistics MMV iterative algorithm.**

Here, the iteration termination condition is no longer the sparse condition, but the mean square error

$$R(\tilde{x}) = \|\tilde{y} - A\tilde{x}\|^2 < \varepsilon \quad (21)$$

where ε denotes the threshold value.

In the statistical analysis phase, all sparse column vectors $\tilde{x}_j$ are considered together to make up a matrix $Z = [\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_s]$ and we use this matrix to make a statistical analysis. The first step is to compute the average of Z row by row and make the elements of each row in Z compare to the corresponding average. Next, if the element is larger than its corresponding average value, it will be set to one. Otherwise, it will be set to zero. We will get another $L \times s$ matrix D with the element 1 or 0 and compute the sum of each row element of matrix D to get a $L \times 1$ judgment vector P.

Since the spectrum of the multiband signal is symmetric in frequency domain, the frequency support is also symmetric. In general, we consider the multiband signal within one sub-band signal to simplify the analysis. Divide the interested spectrum into L intervals of length f_p and use number l, $-L_0 \le l \le L_0$ as labels for each interval. Supposed that the two parts of sub-band signal locate in interval l and l', which are symmetric, the relationship between the labels l and l' is

$$l + l' = L + 1, \quad L = 2L_0 + 1 \tag{22}$$

Then, we fold up the judgment vector P and calculate the sum of the corresponding overlap elements. Specifically, the elements with labels l and l' are added, where l and l' satisfy the relationship (22). The purpose is to enhance the credibility of the results. According to the joint sparsity, the large elements in P is likely to denote the support set with great possibilities. Finally, if the value of element in P is larger than that of the half length of multiple measurement vectors, its corresponding position is determined as the support position.

As we can see, the proposed algorithm does not need to construct the other data frame and the sparse prior knowledge is also not required to run the CS reconstruction algorithm.

IV. SIMULATION RESULTS

In this section, we will verify the above algorithm and compare the performance along with several CS reconstruction algorithms based MMV. Here, we choose the QPSK signal as the type of the sub-band signal,

$$x(t) = \sum_{i=1}^{N/2} \sqrt{\frac{2E_{si}}{T_{si}}} \left\{ \begin{array}{l} \sum_n I[n]\, s(t - nT_s) \cos(2\pi f_i t) \\ + \sum_n Q[n]\, s(t - nT_s) \sin(2\pi f_i t) \end{array} \right\} \tag{23}$$

where N denotes the number of the sub-bands and one sub-signal has two symmetrical sub-bands. The symbol duration $T_{si} = 4 \times 10^{-2} \mu s$ and the symbol energy E_{si} is random selection. There are $n = 150$ symbols generated uniformly at random. The carriers f_i are chosen uniformly at random over a wideband range within $[0, 2GHz]$, which means the Nyquist rate f_{Nyq} is up to $4GHz$.

In the sampling stage, the frequency of the mixing function f_p is set to 25.2 MHz, slightly larger than the bandwidth of sub-band $B = 1/T_{si}$. In order to minimize the sampling rate, the sampling rate f_s in each channel equals to f_p. Moreover, the white Gaussian noise ω is added and define the desired signal-to-noise ratio (SNR) $10 \log \left(\|x\|^2 / \|\omega\|^2\right)$ to scale. In Fig.8, the multiband signal composed of three QPSK signals, $N = 6$ is shown and $SNR = -5dB$. The proposed statistics MMV iterative algorithm is used to recover the signal supports with $m = 30$ sampling channels. The signal processing results shows that the proposed algorithm can accurately locate the sub-band position.

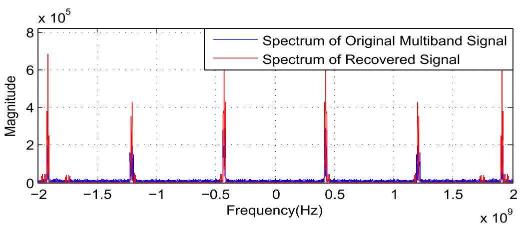

FIGURE 8. Spectrum comparison diagram of signal reconstruction.

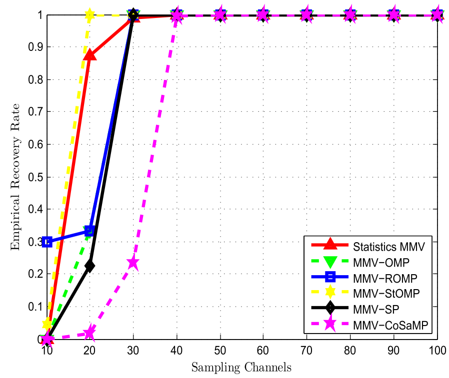

FIGURE 9. Percentage of correct support reconstruction, when SNR = 5dB.

The most important step to reconstruct the multiband signal is to recover the frequency support. Generally speaking, correct support reconstruction is considered when the estimated support set contains all true supports, represented by empirical reconstruction rate. To evaluate the performance of the proposed statistics MMV iterative reconstruction algorithm, we compare it with several reconstruction algorithms based on MMV, such as MMV-OMP, MMV-ROMP, MMV-StOMP, MMV-SP, MMV-CoSaMP. Fig.9 reports the percentage of correct support recoveries for various numbers of sampling channels and Fig.10 shows the reconstruction performance for several SNRs.

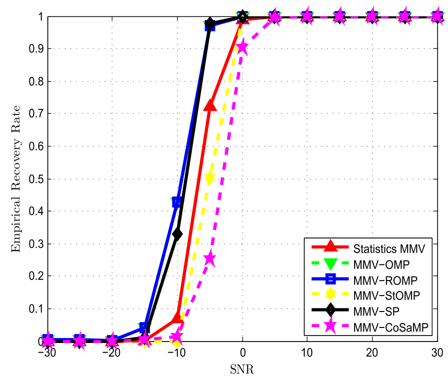

FIGURE 10. Percentage of correct support reconstruction, when sampling channels m = 40.

An obvious trend which appears in all results is that the reconstruction rate is proportional to the number of channels used for reconstruction and inversely proportional to the SNR level. Overall, the proposed algorithm has the good behavior in the condition of less sampling channels and there is no obvious advantage for SNR level. But it is worth noting that the other MMV-based algorithms must undergo the separation process between signal and noise, which needs the sparse prior knowledge. The proposed algorithm skips the space separation process and such prior information is no longer needed.

Fig.11 shows the performance of all above algorithms for different numbers of sub-bands, in other words, for different sparsity. Since the number of sub-bands grows, all

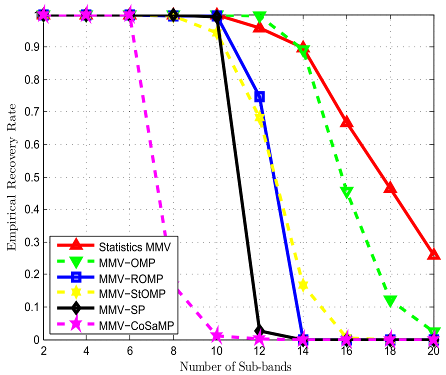

FIGURE 11. Percentage of correct support reconstruction, when SNR = 0dB and m = 50.

correct support reconstruction rates decrease and the proposed algorithm, by contrast, has better performance.

For multiband signal processing, the mentioned empirical reconstruction rate reflects the probability of accurate positioning for the locations of sub-bands and a few additional entries caused by noise are not considered, which uses the probability of false targets to measure. In this paper, the probability of accurate positioning P_{ap} and the probability of false targets P_{ft} are defined as follows,

$$P_{ap} = \frac{\sum_{l=1}^{L} [(d_l = 1) \cap (\hat{d}_l = 1)]}{\sum_{l=1}^{L} (d_l = 1)} \tag{24}$$

$$P_{ft} = \frac{\sum_{l=1}^{L} [(d_l = 0) \cap (\hat{d}_l = 1)]}{\sum_{l=1}^{L} (d_l = 0)} \tag{25}$$

where d_l and $\hat{d}_l$ denote the spectrum state and detection result of the $l th$ spectrum interval respectively,

$$d_l = \begin{cases} 0, & Status : idle \\ 1, & Status : occupied \end{cases} \tag{26}$$

$$\hat{d}_l = \begin{cases} 0, & Result : idle \\ 1, & Result : occupied \end{cases} \tag{27}$$

The proposed algorithm performs statistical analysis on several iterative results from multiple measurements, which is thought to greatly prolong the processing time. But we

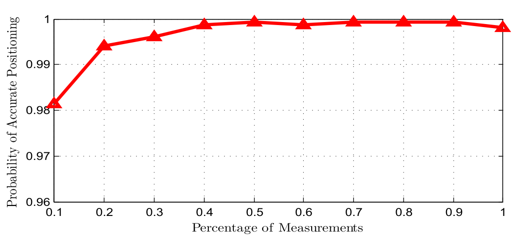

FIGURE 12. Probability of accurate positioning, when SNR = 0dB and m = 50.

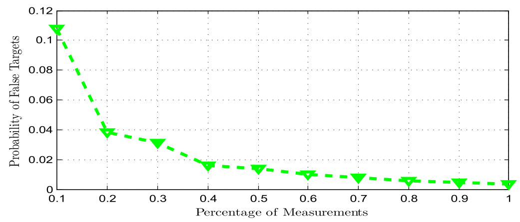

FIGURE 13. Probability of false targets, when SNR = 0dB and m = 50.

can use partial measurements for reconstruction, not all of them. Fig.12 and Fig.13 report the probability of accurate positioning and the probability of false targets versus the percentage of all the measurements. The number of measurements used in the proposed algorithm can be appropriately reduced according to practical application requirements.

V. CONCLUSION

In this paper, a blind multiband signal reconstruction method, referred to as the statistics MMV iterative algorithm, is proposed to achieve sub-Nyquist spectrum sensing in CRSN. The statistics MMV iterative algorithm is an MWC-based sub-Nyquist scheme for support reconstruction. By exploiting the properties of multiple measurement vectors from MWC and statistical analysis, the frequency support set of the multiband signal can be obtained. Simulations results show that, even with no prior knowledge, the statistics MMV iterative algorithm can achieve high empirical reconstruction rate for a wide range of SNR in additive white noise channel and various numbers of sampling channels, as well as robust recovery performance with different numbers of occupied bands.

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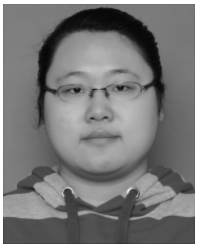

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