Received June 18, 2018, accepted July 19, 2018, date of publication July 24, 2018, date of current version August 15, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2859346

A Novel Index for Assessing the Robustness of Integrated Electrical Network and a Natural Gas Network

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This work was supported in part by the National High Technology Research and Development Program 863 under Grant 2011AA05A119, in part by the National Natural Science Foundation of China under Grant 51207113, in part by the State Grid Corporation of China, Major Projects on Planning and Operation Control of Large Scale Grid under Grant SGCC-MPLG001-029-2012, and in part by the Applied Basic Research (Key Project) of Sichuan Province under Grant 2017JY0095.

ABSTRACT The robustness of an integrated power grid and natural gas network can be transformed into a graph theory problem theoretically. With the concept of tree-coritivity, a novel assessment to analyze the robustness of the interdependent network is proposed in this paper. This method contains a two-stage optimization method and the calculation framework to reduce the computation burden and examine the results, respectively. Compared with existing robustness indices, the proposed tree-coritivity shows the advanced performance in terms of the connected and unconnected network. Furthermore, this index is able to measure the vulnerability of graphs with the same coritivity. Case studies on the standard and modified IEEE New England 39-bus systems integrated with the natural gas network show the effectiveness of the proposed algorithm and index.

INDEX TERMS Integrated electrical network and natural gas network, tree-coritivity, interdependent network, robustness, NP-complete.

I. INTRODUCTION

Due to the severe environmental pollution and global climate change problems, people are making effort to replace the coal with the clean energy in the recent decades. Natural gas is playing a more and more indispensable role in supplying power system loads and balancing the demand and supply because of its rapid response capability [1]–[8]. Moreover, gas-fired generating units have significant advantages such as high efficiency and short installation time. These units contribute to protecting the environment, as they do not emit SO2 and have substantially less NOx than coal when burned [9]. Given the wide range of combined heat and power (CHP) units used, the interactions between electricity and natural gas are tighten. In the United States, the consumption of the natural gas by power generation has increased from 32% in 2007 to 39% in 2009 [10]. The U.S. natural gas pipeline network is a highly integrated with the transmission and distribution grid that can transport natural gas to nearly any location in the 48 states. Figure 1 shows the American pipeline network in 2016 [11]. Assessment of the entire system as an integrity is critical to improve the robustness and safety of the system.

Since the power grid and natural gas network have tight relationship, extensive security analysis on the coordinated operation of the power grid and natural gas network have been conducted in recent years [9], [10], [12]–[26]. The robustness of the whole system is the prior requirement. Security constraints are widely considered in optimal operation, unit commitment and other researches [9], [10], [12]. An integrated model has been proposed to study the impact of the interdependency of electricity and natural gas networks on power system security [14]. Likewise, the impact of the gas network on the power security and economic dispatch was investigated previously [15]–[17]. Reports contained the demand response and wind uncertainty [10] as well as the variability and uncertainty of wind energy in the power

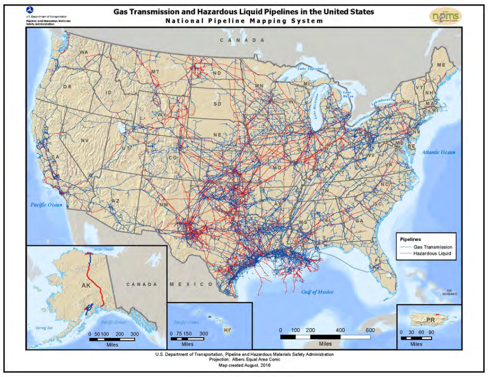

FIGURE 1. American pipeline network in 2016.

grid to analyze the interdependency among natural gas, coal and electricity infrastructures [19]–[21]. A partial differential equation model of the natural gas pipelines was proposed [22] to explore the effect of intermittent wind generation on the pressure fluctuations, which affect the reliability of natural gas deliveries to those same generators and the safety of pipeline operations. A hub-planning model that determines the least cost network of the transmission lines and natural gas pipelines has been proposed to satisfy the reliability criteria [17]. Existing researches merely focus on energy flow in terms of the security of whole system. However, information flow literally plays an important role in operation and robustness of system and few papers have studied robustness in terms of the graph theory. This paper proposes a novel tree-coritivity index based on coritivity to assess the robustness of the integrated system. Tree-coritivity proposed in [27] which is used in the single-layer network is extended to the interdependent network in this passage. The robustness of the integrated power grid and natural gas is defined as the capability of the whole system to sustain services under attack or failure. The contributions of this paper can be summarized as follows:

1) The benefits of tree-coritivity compared to other indices are presented, and an interdependent network model of the integrated power grid and natural gas network is developed.
2) The framework of the interdependent network is implemented to calculate the tree-coritivity of the power grid and natural gas system. A two-stage optimization method can simplify the calculations and reduce complexity.

The remaining content of the paper is organized as follows. Section 2 describes the theory of coritivity, tree-coritivity and exhibits the advantage of tree-coritivity as compared to coritivity, the connectivity theory, and indices used in the interdependent network. Section 3 applies the tree-coritivity to the interdependent network for assessing the robustness and developing a calculation method. A case study is presented in Section 4 to illustrate the effectiveness of the proposed model. Section 5 concludes this paper.

II. THEORY OF CORITIVITY AND TREE-CORITIVITY

A. GRAPH THEORY

There are three quantities, which influence the vulnerability of a graph. The first quantity is the number of elements that are not functioning and the second one is the number of remaining connected subnetworks. The third quantity is the number of cycles and their distribution in a graph. Coritivity involves the first and second quantities but it does not include the third quantity. Therefore, coritivity has its own limitation in measuring the vulnerability of graphs due to the missing of the third quantity. Tree coritivity improving the concept of coritivity is therefore used to assess the vulnerability of a graph [16].

Existing research uses various indices to explain the structure characteristic and robustness of the interdependent network, although these indices cannot describe the network performance in a more comprehensive way. The most widely used index in interdependent network is the giant mutually connected cluster [17]–[20], which can partially reflect the ability of the network to maintain connectivity after removing the vertices or edges. Therefore, it cannot take into account the survivability of islanding after cascading failures, because only the giant mutually connected cluster can live at last. The research of threshold is mainly to solve the stochastic network with small-world and scale-free networks that uses the generating function. So far, no approach can handle and calculate the percolation threshold in the specific network.

B. CORITIVITY AND TREE-CORITIVITY

The feedback vertex set of graph G is a set of vertices S^* whose removal leaves a graph without cycles. Let G be a connected graph, if S is a vertex-cut, $G - S$ is disconnected, and the set of S is denoted by $C(G)$. All set of S^* is denoted by $T(G)$.

Definition 1 [27]: Let G be a connected graph and $n \geq 4$ be the number of nodes. The coritivity of G is defined as

$$h(G) = \max\{\omega(G - s) - |S| : S \in C(G)\} \quad (1)$$

where $\omega(G - S)$ is the number of connected components of G after deleting nodes from cut set S and $|S|$ is the cardinality of S. When the value of $\omega(G - s) - |S|$ reaches its maximum value, $h(G)$ is the coritivity of G and S is the core of G on this occasion.

Theorem 1 [27]: For graphs G_1 and G_2, if $h(G_1) < h(G_2)$, the survivability of G_1 is higher than that of G_2.

Definition 2: Let G be a graph with at least one pair of distinct nonadjacent vertices. The tree-coritivity of G is defined as

$$h_t(G) = \max\{\omega(G - s) - |S| : S \in T(G)\} \quad (2)$$

where $\omega(G - S)$ is the number of connected components of G after deleting nodes from cut set S and $|S|$ is the cardinality of S. When the value of $\omega(G - s) - |S|$ reaches its maximum value, $h_t(G)$ is the tree-coritivity of G and S is the tree-core of G on this occasion.

Theorem 2: For graphs G_1 and G_2, if $h_t(G_1) < h_t(G_2)$, the survivability of G_1 is higher than that of G_2.

Many graphs with same connectedness but different coritivity are created. For example, in (a) and (b) of Fig. 2, the removal of u_1 from G_1 shows two separated regions and two regions still connected such that the two regions form a circle and the network nodes still can work properly. However, after G_2 removes u_2, it is divided into five connected branches, and no loops exist in each of the connected branches. In the actual networks, many areas will be isolated and the directly connection with the outside world would be cut off. G_2 is more fragile than G_1, but connectivity of G_1 is 1, which is identical with G_2, and its cut point is v. Therefore, the vertex connectivity does not distinguish between G_1 and G_2. According to Eq. (1), coritivity of Fig. 2 (a) is 1 and coritivity of Fig. 2 (b) is 4, so the survivability of G_1 is higher than that of G_2 which agrees with the theoretical analysis.

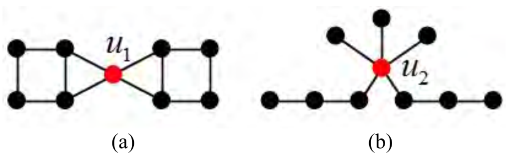

FIGURE 2. Topology of G_1 and G_2. (a) G_1. (b) G_2.

However, coritivity is only defined for connected graphs so that it cannot measure unconnected graphs. The tree-coritivity theory is more effective than the connectivity theory in analyzing the fragility of graphs. It not only take the third quantity into account, but also be used in an unconnected network. What's more, the tree-coritivity, which generalizes the concept of core and coritivity, can be used to further measure the vulnerability of graphs with the same coritivity. For example, in Fig.3, the removal of v_1 from H_1 and the removal of v_2 from H_2 break connectivity of two graphs, so core of H_1 and H_2 are v_1 and v_2 respectively. Coritivity of H_1 is 1, which is identical with H_2. Coritivity cannot distinguish between H_1 and H_2. Tree-core of H_1 is v_1 and tree-coritivity is 1. Tree-core of H_1 is v_2 and w and tree-coritivity is 0. According to Eq. (2), the survivability of H_1 is lower than that of H_2. Therefore, tree-coritivity is proposed to assess the robustness of an interdependent network which is constituted by a power grid and natural gas.

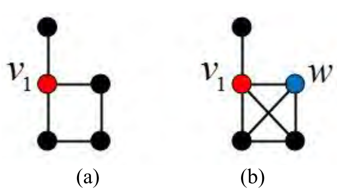

FIGURE 3. Topology of H_1 and H_2. (a) H_1. (b) H_2.

III. TREE-CORITIVITY IN INTERDEPENDENT NETWORK

A. INTERDEPENDENT NETWORK

Modern power systems integrate electricity, natural gas, solar energy, and other energies. In addition, it is a highly integrated physical and information system. Figure 4 depicts the schematic diagram of the integrated energy system [34]. From the aspect of energy flow, LNG (liquefied natural gas) station, CHP/CCHP and P2G (power-to-gas) facility build out two-way transformation of gas and electricity [8]. Therefore, the power system and natural gas network are becoming increasingly coupled and interdependent. Large-scale power units and distributed energy resources (DER) transfer power to customers through transmission and distribution networks. Meanwhile, DER can directly serve for customers, while some controllable load such as storage can transfer power into the grid in turn.

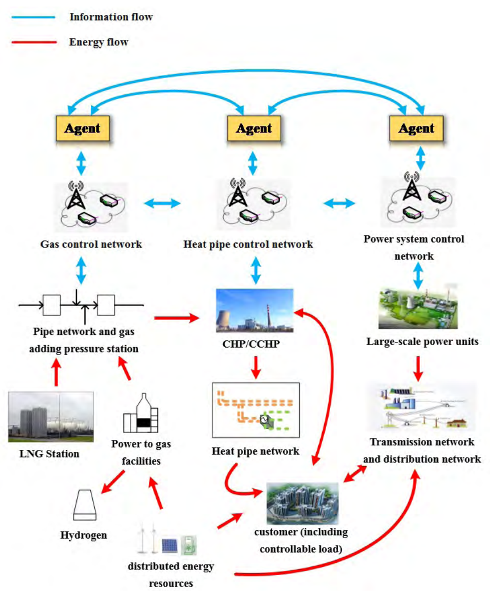

FIGURE 4. Schematic diagram of integrated energy system.

Taking advantage of information flow, each network has a control center that can communicate with each other. Each control center is an agent of information exchange such as the operating status and control sign. The operation and control of the smart grid are highly dependent on a comprehensive information system containing numerous pieces of measurement, monitoring, and management information [35]. Failures in communication system may cause cascading failures of the whole system because of huge losses like the blackout in Italy in 2003 [28]. Some errors and inaccurate information may not only affect own system's operation and security, but also those in other systems caused by information exchange. Similarly, operation and security of physical system are also critical to communication system due to energy supply [36]. The control architecture for a system with three interconnected control areas adopts the distributed control strategy because conflicting objectives of individual hubs are often explicitly considered and to achieve higher robustness. Moreover, this requires shorter computation times as compared to centralized control where a central coordinator manages three interconnected control areas, particularly for larger-scale systems.

From the perspective of the electric market, the natural gas market price would directly affect the commitment, dispatch, and cost of supplying the hourly load in electric utilities. If natural gas prices lose competitiveness compared with those of alternative fossil fuels such as coal or oil, power system could switch to other generating units with lower fuel costs. On the contrary, an electricity power system would commit more gas-fired generating units. The power system network can be represented by a graph composed of vertices (buses in power grids and gas nodes in natural gas networks) and edges (branches in power grids and pipelines in natural gas networks). Any edge can be weighted/unweighted and directed/undirected. Natural gas network mainly consists of natural gas pipelines, pressure regulators, valves, and compressor stations. The electricity network mainly contains generators, transformer substation, loads, and transmission lines. Natural gas pipelines and power grids are very similar and used to transport energy from the supply side to the consumer side. The similarities can be summarized as:

1) Supply side (power station or natural gas field)
2) Delivery (high-voltage power grid or high-pressure pipe network)
3) Distribution (medium/low voltage grid or medium/low pressure pipe network)
4) Users (power users or gas users).

However, some differences do exist between the two networks. Natural gas is always maintained in the form of primary energy directly from the natural gas field, which is a secondary energy source that is converted from primary energy at the power plant. Natural gas is delivered to the user from the natural gas field (supply side) through the pipe network, while power is transmitted through the power line. In addition, the natural gas pipeline network can store some natural gas for peak load use, whereas electricity cannot be effectively stored. The storage of natural gas may cause cascading failures response delay. In this paper, a time delay coefficient ξ is defined, when $\xi < \xi_0$ (ξ_0 is the threshold decided by the system delay tolerance ability), time delay effect can be negligible, and this paper only considers this occasion. As they interconnected with each other, to some extent, the two networks operate as one.

Based on the above theories, the power system is an interdependent network with a fundamental property such that the failure of nodes in one network may lead to the failure of the dependent nodes in other networks. For example, an error happening in the gas network may results in another failure in the power system due to incorrect information transmission or delays. On the contrary, if failure occurs in the power system, pressure regulators in natural gas system do not work due to lacking of energy supply, the communication facilities in natural gas system also need power to support its operation and supervision.

Power grid and natural gas system can be modeled as interdependent networks [28]–[33]. Figure 5 depicts the topology of the coupled energy subsystem and natural gas network. To analyze the interaction of power system and natural gas system, the energy hubs can be used to describe nodes that determine the information and energy exchange between the energy and natural gas subsystems, which are mapped as the connected nodes in Fig. 5. In the coupling relationship of the energy and information flows, there are two dotted lines between the two systems. However, only the mutually connected clusters are potentially functional. A set of nodes, a, is in network A and the corresponding set of nodes, b, is in network B form a mutually connected set. If the following statements can be satisfied:

(1) each pair of nodes in a is connected by a path that consists of a nodes and the links of network A;
(2) each pair of nodes in b is connected by a path that consists of b nodes and the links of network B.

We call a mutually connected set a mutually connected cluster if it cannot be enlarged by adding other nodes while still satisfying the conditions above.

(a), (b) and (c) in Fig.5 show the failure of natural gas system affects power grid, and (d), (e) and (f) show the failure of power grid affects natural gas system. When the node a6 in the natural gas system fails in Fig. 3(a), its corresponding node, b8, in the electricity network fails as well. As a result, due to the insufficient capacity or wrong information, all edges linked to b8 or a6 fail as well in Fig. 3(b). b9, b10, b11, and b12 become an isolated island and they cannot work based on the theory of the mutually connected cluster. However, for cases in the real world, every part of the system is isolated, the islands can still perform part of their functionality. When substation b7 in electricity network fails in Fig.5 (e), a7 fails due to lack of energy supply. Given the failure of b9, a5 cannot work. Following the dynamic interaction process, the whole system will eventually come to a steady state [Fig. 4(c)].

B. OPTIMIZATION METHOD

The first step of calculating tree-coritivity is to find all Minimum feedback vertex sets (MFVS) of the graph. The MFVS problem is an NP-complete problem, which is difficult to solve accurately. Some special types of graph have a polynomial time algorithm. However, calculations for the minimum feedback node set for general graphs still utilize the approximation algorithm. Pardalos P.M. [37] proposed the greedy random adaptive search procedure (GRASP) to solve problems with the feedback node set of directed graphs. To obtain an accurate solution, a simple algorithm can be adopted such where the process is to delete one, two, ..., k nodes in turn, and then judge whether a cycle exists in the graph. However, given that there are kinds of possibilities in each case, these result in low efficiencies. To reduce computation complexities, a two-stage optimization method is introduced in detail.

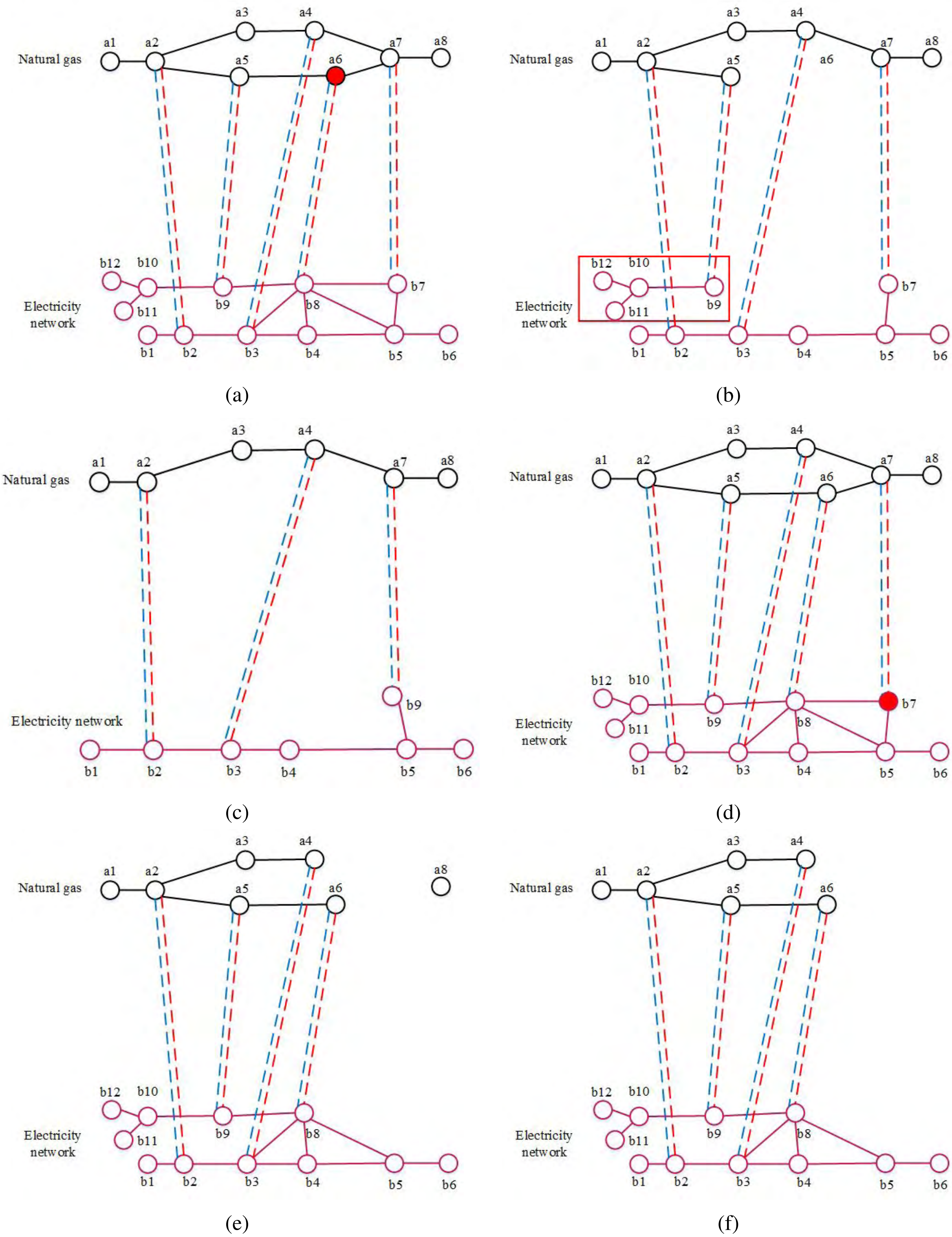

FIGURE 5. Dynamic interaction process of energy system and natural gas network. (a) Initial situation. (b) Dynamic process after a6 fails. (c) Steady state. (d) Initial situation. (e) Dynamic process after b7 fails. (f) Steady state.

Stage 1:

1) For a graph G, if the deletion of k nodes meets the requirements of the case, do not consider the deletion of $k+1$ and the larger case.
2) Nodes with a degree of 1 in the graph do not have to be considered, which narrows the range of n.
3) Nodes with a degree of 2 in the graph do not have to be considered unless there are special circumstances.

Stage 2:

1) If there is a ring under the deletion of k nodes, when deleting $(k+1)$ nodes, only the nodes in the rings under k nodes are considered.
2) Only delete k nodes if there is a ring. If rings exist, count which nodes in the ring. When deleting $(k+1)$ nodes, there is no need to compute the degree of nodes again. However, it is acceptable to update the degrees of the nodes when k nodes are deleted.

After a one-stage optimization, the efficiency of the proposed algorithm is about 1,000 fold higher than the simple algorithm. After two-stage optimizations, the efficiency of the proposed algorithm is about 100,000 times higher than the simple algorithm.

The case of IEEE-39 bus system is tested by using MATLAB 2010b on 2.6 GHz Intel two core processors with 4GB of RAM, running 64-b Windows. The calculation time using simple algorithm is about 2h while using two-stage optimizations, the calculation time is about 2∼3s. Figure 6 shows three kinds of MFVS in this case. Tree-coritivity of this case is 1 when the tree core is {5, 13, 16, 26}. Tree-coritivity of double-layer network is studied in Section 3.3 and Section 4.1.

C. CALCULATION FRAMEWORK OF TREE-CORITIVITY IN INTERDEPENDENT NETWORK

Figure 7 presents the process for calculating the tree-coritivity in the interdependent network. When creating a set of deletable nodes, stage 1 can be optimized. Depending on the presence of cycles in the graph, stage 2 can be optimized as well. Some nodes and processes do not need to be considered to simplify the computation. If the preset max level is m, each subset with $m-1$ vertices of is a tree-tore of $V(K_n)$ such that level $i >$ max *level* cannot appear. If the preset max level is smaller than m such that the tree tore can't be found during the whole process, then the max level should be changed and the process should be repeated. When deleting nodes, the property of the nodes should be mentioned. If the deleted nodes have links with other work, the corresponding nodes in the other work should be deleted as well. Figure 8 depicts a simple recursive process of a single-layer graph when the maximum level is 4. Three main methods are used to determine if the undirected graph has a cycle.

1) N ALGORITHM

If a cycle is in the graph, a subgraph cycle must also be presented. The degrees of all vertices in the cycle are greater or equal to 2.

1) The vertices and associated edges of all degrees less than or equal to 1 are deleted and the degrees of other vertices associated with these edges are reduced by 1.
2) Place the vertex that has a degree of 1 into the queue and remove a vertex from the queue. Repeat step one.

If there are still undeleted vertices in the end, a cycle exits. Otherwise, there is no cycle in the graph.

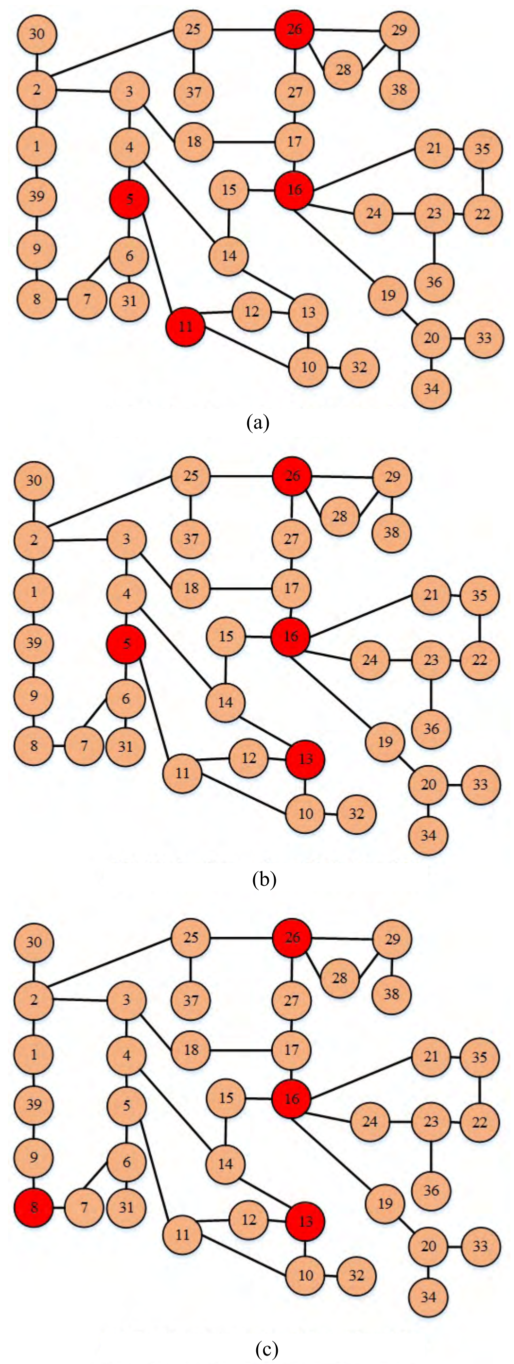

FIGURE 6. **Case of IEEE 39-bus system nodes. (a) Deletion four nodes of 4, 11, 16 and 26. (b) Deletion four nodes of 5, 13, 16 and 26. (c) Deletion four nodes of 8, 13, 16 and 26.**

2) DEPTH-FIRST-SEARCH (DFS) ALGORITHM

The edge of the graph may only be a tree edge or a reverse side. The presence of a reverse edge indicates the existence of a ring.

3) CONNECTED COMPONENT METHOD

Suppose that the number of vertices is M and the number of edges is E.

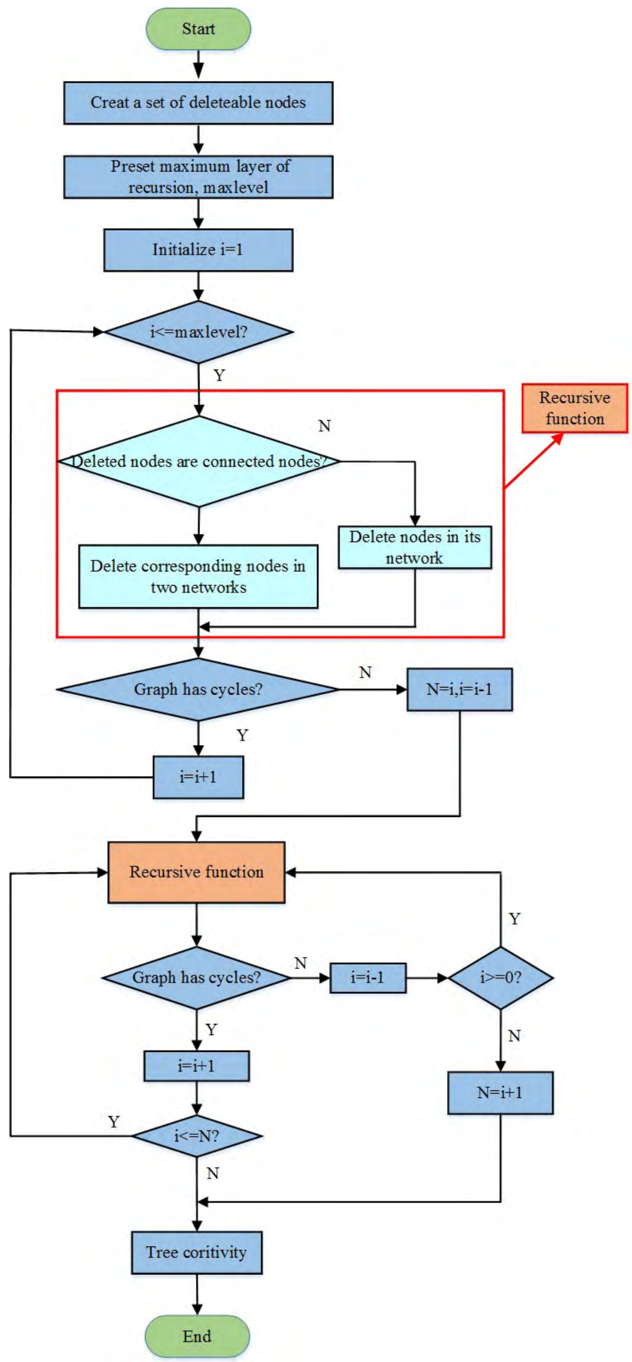

FIGURE 7. Tree-coritivity flow chart in the interdependent network.

1) Traverse the graph and determine the number of parts that graph is divided into (assuming P parts, that is, there are P connected components).
2) For each connected component, the lack of a cycle indicates that it can only be a tree, that is, the number of edges equals the number of nodes minus one.

The original graph has a cycle given that one connected component exists that satisfies the relationship (e.g., the number of edges is greater than the number of nodes minus 1).

TABLE 1. Tree-coritivity of the gas-electricity integrated system.

Case	Tree-coritivity	Electricity network	Gas network
1	8	2,4,6,8,10,16,19,22,23,26	28,25,31,66,41,32
1	8	2,4,6,8,10,16,19,22,26	28,25,31,66,32
1	8	2,4,6,8,10,16,19,23,26	28,25,31,66,41,32
1	8	2,4,6,8,10,16,22,26	28,25,31,66,32
1	8	2,4,6,8,10,16,23,26	28,25,31,66,41,32

TABLE 2. Components of two networks after deleting a tree tore in Table 1.

Case	Electricity network components	Gas network components
1	{1,9,39}, {5}, {7}, {11,12,13,14, 15}, {3,17,18,27}, {21}, {24}, {25,37}, {30}, {31}, {32}, {33}, {35}, {20,34}, {36},{28,29,38}	{1,2,...24,43,44,...,60}, {26,27}, {29,30}, {33,34,...,40}, {42}, {67,68,...,73}, {74,75,76,81,84,85,...,89}, {77,78,79,80,82,83,90,91,92}
2	{1,9,39}, {3,17,18,27}, {5}, {7}, {11,12,...,15}, {20,34}, {21}, {23,24,26}, {25,37}, {28,29,38}, {30}, {31}, {32}, {33}, {35}	{1,2,...,24,43,44,...,65}, {26,27}, {29,30}, {33,34,...,42}, {67,68,...,73}, {74,75,76,81,84,85,...,89}, {77,78,79,80,82,83,90,91,92}
3	{1,9,39}, {3,17,18,27}, {5}, {7}, {11,12,...,15}, {20,34}, {21,22,35}, {24}, {25,37}, {28,29,38}, {30}, {31}, {32}, {33}, {36}	{1,2,...,24,43,44,...,65}, {26,27}, {29,30}, {33,34,...,40}, {42}, {67,68,...,73}, {74,75,76,81,84,...,89}, {77,78,79,80,82,83,90,91,92}
4	{1,9,39}, {3,17,18,27}, {5}, {7}, {11,12,...,15}, {19,20,33,34}, {21}, {23,24,36}, {25,37}, {28,29,38}, {30}, {31}, {32}, {35}	{1,2,...,24,43,44,...,65}, {26,27}, {29,30}, {33,34,...,42}, {67,68,..., 73}, {74,75,76,81,84,...,89}, {77,78,79,80,82,83,90,91,92}
5	{1,9,39}, {3,17,18,27}, {5}, {7}, {11,12,...,15}, {19,20,33,34}, {21,22,35}, {24}, {25,37}, {28,29,38}, {30}, {31}, {32}, {36}	{1,2,...,24,43,44,...,65}, {26,27}, {29,30}, {33,34,...,40}, {42}, {67,68,...,73}, {74,75,76,81,84,...,89}, {77,78,79,80,82,83,90,91,92}

By adding the inequalities of the P connected components, we get:

$$\begin{aligned} P_1 &: E_1 = M_1 - 1 \\ P_2 &: E_2 = M_2 - 1 \\ &\cdots \\ P_N &: E_N = M_N - 1 \end{aligned} \tag{3}$$

The number of all edges (E) is greater than the number of all nodes (M) minus the number of connected components (P). As long as the results satisfy the inequality, there is a cycle in the original graph. Otherwise, there is no cycle.

These are three common methods to judge if an undirected graph has a cycle. Three methods are parallel but have different algorithm complexity. When N Algorithm is used in a graph consisting of V nodes and E edges, the algorithm complexity is $O(EV)$. The complexity of Depth-First-Search (DFS) algorithm is O(V). Connected component method is more suitable for unconnected graphs but not suitable for direct graphs. not suitable for direct graph. Different methods

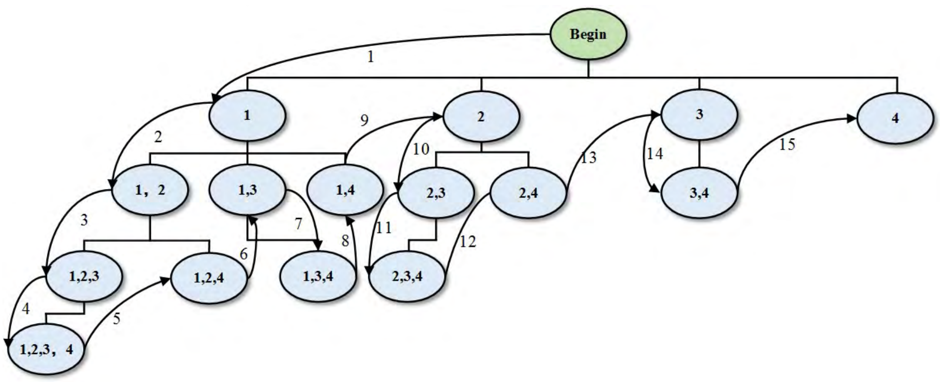

FIGURE 8. Flow diagram of recursive process.

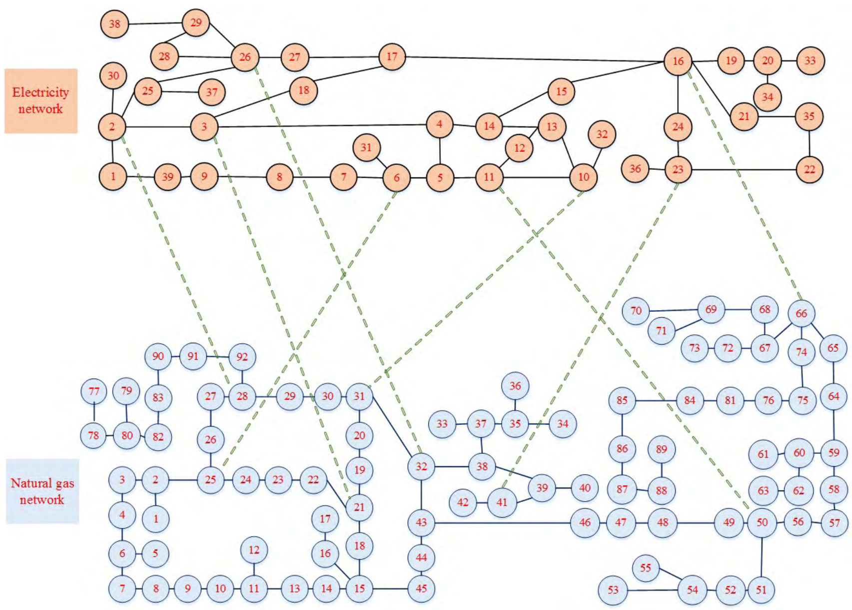

FIGURE 9. Topological structure of integrated IEEE 39-bus system and natural gas system.

can be used based on different systems. After finding all the feedback vertex sets (tree tore) of the graph, the tree-coritivity can be calculated using Eq. (2).

IV. CASE STUDY

A case study for the integrated IEEE 39-bus system and a natural gas system [38] is presented in Fig. 9, where the orange network represents the electricity network and blue network represents the natural gas network. Each node in the electricity network represents a substation and each node in the natural gas network represents pressure regulators, valves, and compressor stations and so on. The edges stand for the transmission lines and pipelines, respectively.

This graph is used to calculate 108 kinds of tree core cases. When the tree cores follow the cases presented in Table 1, $\omega(G - S^*) - |S^*|$ gets its maximum value, which is the tree-coritivity. It means when these parts in power system and natural gas system fail, impacts on system operation are the most serious. In each case of Table 2, the integrated system breaks into some of the components below. When tree cores are minimum feedback vertex sets, the values of $\omega(G - S^*) - |S^*|$ and corresponding feedback vertexes in two networks are presented in Table 3. For example, when $\{3, 5, 13, 22, 26\}$ in electricity network and $\{21, 32\}$ in gas network are deleted, the value of $\omega(G - S^*) - |S^*|$ is -1. However, though S^* is the MFVS, the value of $\omega(G - S^*) - |S^*|$ is not the tree-coritivity when compared with Table 1, the value of $\omega(G - S^*) - |S^*|$ is 8.

According to Tables 1, 3, 5, and 7, when cut set S^* is a minimum feedback cut set such that the value of $\omega(G-S^*) - |S^*|$ is not the largest. Because when calculating the tree-coritivity, the number of nodes in the feedback cut set and the number of connected components should be considered

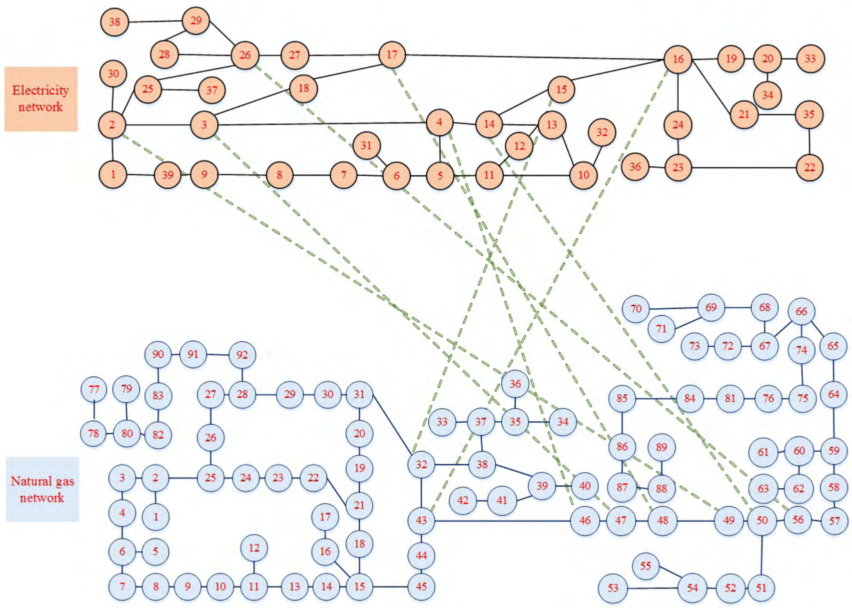

FIGURE 10. Topological structure of modified integrated IEEE 39-bus system and natural gas system.

TABLE 3. MFVS of the gas-electricity integrated system.

$\omega(G-S')-\|S'\|$	Electricity network	Gas network
-1	3,5,13,22,26	21,32
-1	3,8,13,22,26	21,32
-1	4,6,13,22,26	25,32

TABLE 4. The five nodes of the highest betweenness of the electrical network and natural gas network, respectively.

Electrical network		Gas network	
Nodes	Betweenness	Nodes	Betweenness
16	0.8511	43	1.048
14	0.5272	50	0.9814
4	0.5071	46	0.9546
3	0.4552	47	0.9460
17	0.4476	48	0.9365
2	0.4451	49	0.9259
15	0.3936	32	0.849
26	0.3374	56	0.8256

at the same time. In fact, if the power system integrating with natural gas system is fragile, in each system only few facilities need to be attacked and two systems will be broken into many pieces, thus lose the ability to work properly. The degree of fragmentation higher is, the degree of damage higher is. When assessing the robustness of a system, the number of critical infrastructure and effect of failure are equally important.

Table 2 and Table 6 present the connected components in the graph following node deletion. Some components may still function because of the islanding operation. For example, the island, which meets the requirements of the power supply and demand balance and other operation conditions, can still work properly. In the other case, the natural gas system retains

TABLE 5. Tree-coritivity of the gas-electricity integrated system.

Case	Tree-coritivity	Electricity network	Gas network
6	10	2,4,6,8,10,16,19,22,23,26	2,6,11,15,21,49,46,43,56
7	10	2,4,6,8,10,16,19,22,23,26	2,6,11,15,28,49,46,43,56
8	10	2,4,6,8,10,16,19,22,23,26	2,6,11,15,31,49,46,43,56

TABLE 6. Components of two networks after deleting tree tore in Table 5.

Case	Electricity network components	Gas network components
6	{1,9,39}, {3,17,18,27}, {5}, {7}, {11,12,13,14,15}, {20,34}, {21}, {24}, {25,37}, {28,29,38}, {30}, {31}, {32}, {33}, {35}, {36}	{1}, {3,4}, {5}, {7,8,9,10}, {12}, {13,14}, {16,17}, {18}, {19,20,22,23,...,42,77,78, 79,80,82,83,90,91,92}, {44,45}, {47,48}, {50,51,...,55}, {57,58,...,76,81,84,85,...,89}
7	{1,9,39}, {3,17,18,27}, {5}, {7}, {11,12,13,14,15}, {20,34}, {21}, {24}, {25,37}, {28,29,38}, {30}, {31}, {32}, {33}, {35}, {36}	{1}, {3,4}, {5}, {7,8,9,10}, {12}, {13,14}, {16,17}, {18,19,...,27,29,30,...,42}, {44,45}, {47,48}, {50,51,...,55}, {57,58,...,73}, {77,78,79,80,82,83,90,91,92}
8	{1,9,39}, {3,17,18,27}, {5}, {7}, {11,12,13,14,15}, {20,34}, {21}, {24}, {25,37}, {28,29,38}, {30}, {31}, {32}, {33}, {35}, {36}	{1}, {3,4}, {5}, {7,8,9,10}, {12}, {13,14}, {16,17}, {18,19,...,27}, {29,30}, {32,33,...,42}, {44,45}, {47,48}, {50,51,...,55}, {57,58,...,73}, {77,78,79,80,82,83,90,91,92}

main function module and realizes natural gas supply in the island. However, based on the theory of mutually connected clusters, some small clusters have no possibility of working.

TABLE 7. MFVS of the gas-electricity integrated system.

$\omega(G-S')-\lvert S'\rvert$	Electricity network	Gas network
0	5,11,16,26	25,43,56
0	5,13,16,26	21,43,56
0	5,13,16,26	25,43,56
0	8,13,16,26	21,43,56
0	8,13,16,26	25,43,56

Therefore, tree-coritivity is more practical than mutually connected clusters.

The interdependent system in Fig. 9 is more robust than the system in Fig. 10. The tree-coritivity in Fig. 9 is smaller than that of Fig. 10 which can be concluded from Table 1 and Table 5. Meanwhile, Fig. 9 has fewer possibilities to present a feedback vertex-cut, which can be concluded by comparing Table 3 with Table 7. From the planning point of view, the structure of Fig.8 is more suitable for the whole system. High betweenness nodes means these nodes take up a key position in the graph. From the aspect of an engineer, important power substations in power grid interact with important compressor stations in the natural gas system. When high betweenness facilities fail in one system, it strongly affect not only own system's ability but also relevant systemars, because the high betweenness facility in another system also fails due to interconnection. This kind of interconnection has double damage effect.

V. CONCLUSION

The tree-coritivity is applied to the interdependent network to analyze the robustness of the integrated electrical and natural gas networks. In order to implement computational simplicity, a two-stage optimization method is proposed to reduce the computational complexity of the NP-complete problem. The results of case study demonstrate that the proposed methodology is able to assess the robustness of the integrated system considering interdependence of networks. The value of the tree-coritivity can reflect the robustness of the topology of the different systems. Considering the number of cycles and their distributions in a graph, this index can be used in both connected graphs and unconnected graphs. Moreover, this index is highly applicable for islanding after cascading failures.

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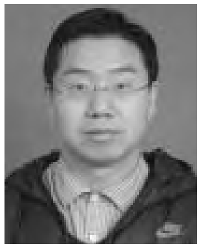

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