IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received May 31, 2018, accepted July 17, 2018, date of publication July 23, 2018, date of current version August 15, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2858246

Adaptive Lag Synchronization of Memristive Neural Networks With Mixed Delays

CHUAN CHEN[1], LIXIANG LI[1], HAIPENG PENG[1], AND YIXIAN YANG[1,2]
[1]Information Security Center, State Key Laboratory of Networking and Switching Technology,
Beijing University of Posts and Telecommunications, Beijing 100876, China
[2]State Key Laboratory of Public Big Data, Guizhou 550025, China

Corresponding author: Lixiang Li (li_lixiang2006@163.com)

This work was supported in part by the National Key R&D Program of China under Grant 2016YFB0800602, in part by the National Natural Science Foundation of China under Grant 61573067, Grant 61771071, and Grant 61472045, and in part by the BUPT Excellent Ph.D. Students Foundation under Grant CX2017315.

ABSTRACT This paper investigates the lag synchronization of memristive neural networks (MNNs) with mixed delays via adaptive control. Based on the switching jump properties of memristors and the assumption that the activation functions are bounded, three lemmas are derived first to deal with the theoretical analysis difficulties caused by the existences of time delays and time lag. By designing a series of suitable adaptive controllers, we prove that the considered MNNs can achieve asymptotic lag synchronization, exponential lag synchronization, and finite-time lag synchronization, respectively. Adaptive control can avoid the large control gains very well, and adaptive control can be used even when the system parameters are unknown. Moreover, extra calculations are not required to determine the appropriate control gains. Numerical simulations are presented to verify the effectiveness of the obtained theoretical results.

INDEX TERMS Memristive neural networks, adaptive controllers, lag synchronization, mixed delays.

I. INTRODUCTION

In 1971, Chua [1] predicted that besides inductor, capacitor and resistor, there should exist another basic circuit element—memristor. In 2008, HP Laboratory manufactured the prototype of memristor [2], since then, memristor has attracted great attention from many research areas. Memristor reflects the relationship between flux and charge, and its memristance varies with the amount of the passed charge [3], so memristor has the memory function.

An important application of memristor is to construct memristive neural network (MNN) [4]. In the circuit implementment of neural network, scholars usually used resistors to imitate synapses. However, we know that synapses play an important part in memory formation, while the common resistors don't have the memory function. Relatively speaking, memristor behaves more like the real synapse. If the above-mentioned resistors are replaced by memristors, the usual artificial neural network [5] will become a memristive neural network, which can simulate the function of human brain better [6].

Recently, the dynamic analysis of MNNs has become a hot topic [7]–[12]. Especially, considerable publications on the synchronization of MNNs have been reported [13]–[26]. The first correct result about the synchronization of MNNs has been published in [23], where detailed analysis was given why the classical state feedback controller cannot synchronize MNNs. Then, the techniques have been extended to impulsive control in [24]. Moreover, new analytical techniques were provided to further explain the control method in [25] and [26], and the finite-time synchronization of delayed MNNs has also been considered in [25].

To the best of our knowledge, most works about the synchronization of MNNs were related to complete synchronization [27]. However, it is believed that complete synchronization may be unrealistic in some application fields. For example, in the large-scale network, because of the finite signal transmission speed, the existence of time delay is unavoidable, so the signal that the receiver receives at time $t+\delta$ may be the one that was sent at time t. As an important synchronization type, lag synchronization means that two systems can achieve synchronization with a constant time lag $\delta > 0$ [28]. Obviously, lag synchronization is very applicable to the aforementioned case. Up to now, the lag synchronization of MNNs has been investigated in [29]–[37].

In [30], by designing feedback controllers, the exponential lag synchronization of coupled MNNs with discrete delays was investigated via ω-Measure. By adopting hybrid switching control, the exponential lag synchronization of MNNs with constant delays was addressed in [31].

Because of the limitation of theoretical derivations, the control gains of the controllers in [30], [31], and [33]–[37] may be too large in some cases. However, adaptive controllers can solve this problem very well, and adaptive controllers can be applied even when the system parameters are unknown. In [29], [32], and [38]–[43], adaptive control was used to discuss the synchronization of MNNs with discrete delays. In [39], the adaptive finite-time synchronization of MNNs with discrete delays was considered, but the designed adaptive controller was very complex. As far as we know, there still has been no publication about the adaptive lag synchronization of MNNs with distributed delays nowadays. What is more, the analysis techniques used in [29]–[43] cannot be utilized to deal with the adaptive lag synchronization of MNNs with distributed delays. However, since there exist parallel pathways with various axon lengths and sizes, we should also consider distributed delay in the studies of neural networks. Fortunately, the results in [44] and [45] have shed some light on studying the adaptive asymptotic/exponential lag synchronization of MNNs with distributed delays.

In view of the above analysis, this paper investigates the adaptive lag synchronization of MNNs with mixed delays. By designing a series of adaptive controllers, we will prove the considered MNNs can achieve asymptotic lag synchronization, exponential lag synchronization and finite-time lag synchronization, respectively. The contributions of our paper include: (1) In engineering applications, finite-time synchronization [46]–[52] is more valuable than asymptotic/exponential synchronization. Based on novel finite-time synchronization analysis techniques, the finite-time lag synchronization of MNNs is discussed in this paper by designing a simple adaptive controller. (2) In recent years, the scholars have done some researches on the lag/adaptive synchronization of MNNs. However, the time delays that they considered were mainly discrete delays. In this paper, MNNs with mixed delays are considered. (3) In the existing works about the lag synchronization of MNNs with time-varying discrete delays, when t was replaced by $t-\delta, f_k(x_k(t-\tau_k(t)))$ was replaced by $f_k(x_k(t-\delta-\tau_k(t)))$. Of course that is not right. In this paper, $f_k(x_k(t-\tau_k(t)))$ is replaced by $f_k(x_k(t-\delta-\tau_k(t-\delta)))$ when t is changed into $t-\delta$. (4) The existences of time delays [53] and time lag usually make the theoretical analysis of lag synchronization difficult. In this paper, we first derive Lemmas 2,3 and 4, which can make the remaining theoretical derivations very concise.

We organize the remainder of this paper as follows. Some necessary preliminaries are introduced in Section 2. In Section 3, the main results of this paper are presented. Numerical simulations are given to validate the effectiveness of our theoretical results in Section 4. The conclusion of this paper is provided in Section 5.

II. NETWORK MODEL AND PRELIMINARIES

Consider such a MNN model with mixed delays:

$$\begin{aligned}\dot{x}_j(t) = &-d_j(x_j(t))x_j(t) + \sum_{k=1}^{n} a_{jk}(x_j(t))f_k(x_k(t))\\ &+\sum_{k=1}^{n} b_{jk}(x_j(t))f_k(x_k(t-\tau_k(t)))\\ &+\sum_{k=1}^{n} c_{jk}(x_j(t))\int_{t-\rho_k(t)}^{t} f_k(x_k(s))ds + I_j, \quad t\geq 0,\end{aligned} \tag{1}$$

$j=1,2,\ldots,n$, where $x_j(t)$ denotes the voltage of capacitor C_j; $d_j(\cdot) > 0$ represents the rate of neuron self-inhibition; $f_k(\cdot)$ stands for the activation function; $\tau_k(t)$ denotes the discrete delay, which satisfies $0 \leq \tau_k(t) \leq \tau_k$; $\rho_k(t)$ denotes the distributed delay, which satisfies $0 \leq \rho_k(t) \leq \rho_k$; I_j is the external input; $d_j(x_j(t))$, $a_{jk}(x_j(t))$, $b_{jk}(x_j(t))$ and $c_{jk}(x_j(t))$ denote the memristive connection weights, and

$$\begin{aligned}&d_j(x_j(t)) = \frac{1}{C_j}\left[\sum_{k=1}^{n}(M_{jk}+\widetilde{M}_{jk}+\widehat{M}_{jk})\times sign_{jk}+\frac{1}{R_j}\right],\\ &a_{jk}(x_j(t)) = \frac{M_{jk}}{C_j}\times sign_{jk}, \quad b_{jk}(x_j(t)) = \frac{\widetilde{M}_{jk}}{C_j}\times sign_{jk},\\ &c_{jk}(x_j(t)) = \frac{\widehat{M}_{jk}}{C_j}\times sign_{jk}, \quad sign_{jk} = \begin{cases}1, & j\neq k,\\ -1, & j=k,\end{cases}\end{aligned} \tag{2}$$

here $M_{jk}, \widetilde{M}_{jk}, \widehat{M}_{jk}$ express the memductances of memristors $R_{jk}, \widetilde{R}_{jk}, \widehat{R}_{jk}$, respectively. What is more, R_{jk} represents the memristor between $f_k(x_k(t))$ and $x_j(t)$, $\widetilde{R}_{jk}$ represents the memristor between $f_k(x_k(t-\tau_k(t)))$ and $x_j(t)$, $\widehat{R}_{jk}$ represents the memristor between $\int_{t-\rho_k(t)}^{t} f_k(x_k(s))ds$ and $x_j(t)$, and R_j is the parallel-resistor. The circuit implementation of MNN (1) is illustrated in Fig.1. Based on the property of memristor, we set

$$\begin{aligned}&d_j(x_j(t)) = \begin{cases}\acute{d}_j, & |x_j(t)|\leq T_j,\\ \grave{d}_j, & |x_j(t)| > T_j,\end{cases}\\ &a_{jk}(x_j(t)) = \begin{cases}\acute{a}_{jk}, & |x_j(t)|\leq T_j,\\ \grave{a}_{jk}, & |x_j(t)| > T_j,\end{cases}\\ &b_{jk}(x_j(t)) = \begin{cases}\acute{b}_{jk}, & |x_j(t)|\leq T_j,\\ \grave{b}_{jk}, & |x_j(t)| > T_j,\end{cases}\\ &c_{jk}(x_j(t)) = \begin{cases}\acute{c}_{jk}, & |x_j(t)|\leq T_j,\\ \grave{c}_{jk}, & |x_j(t)| > T_j,\end{cases}\end{aligned} \tag{3}$$

for $j,k = 1,2,\ldots,n$, where $\acute{d}_j, \grave{d}_j, \acute{a}_{jk}, \grave{a}_{jk}, \acute{b}_{jk}, \grave{b}_{jk}, \acute{c}_{jk}, \grave{c}_{jk}$ are known constants. The initial condition of MNN (1) is $x_j(s) = \varphi_j(s) \in C([-\tau,0],R)$, $j = 1,2,\ldots,n$, where $\tau = \max_{1\leq k\leq n}\{\tau_k, \rho_k\}$.

Throughout this paper, set $\underline{d}_j = \min\{\acute{d}_j, \grave{d}_j\}$, $a^v_{jk} = \max\{|\acute{a}_{jk}|, |\grave{a}_{jk}|\}$, $b^v_{jk} = \max\{|\acute{b}_{jk}|, |\grave{b}_{jk}|\}$, $c^v_{jk} = \max\{|\acute{c}_{jk}|, |\grave{c}_{jk}|\}$, for $j,k = 1,2,\ldots,n$.

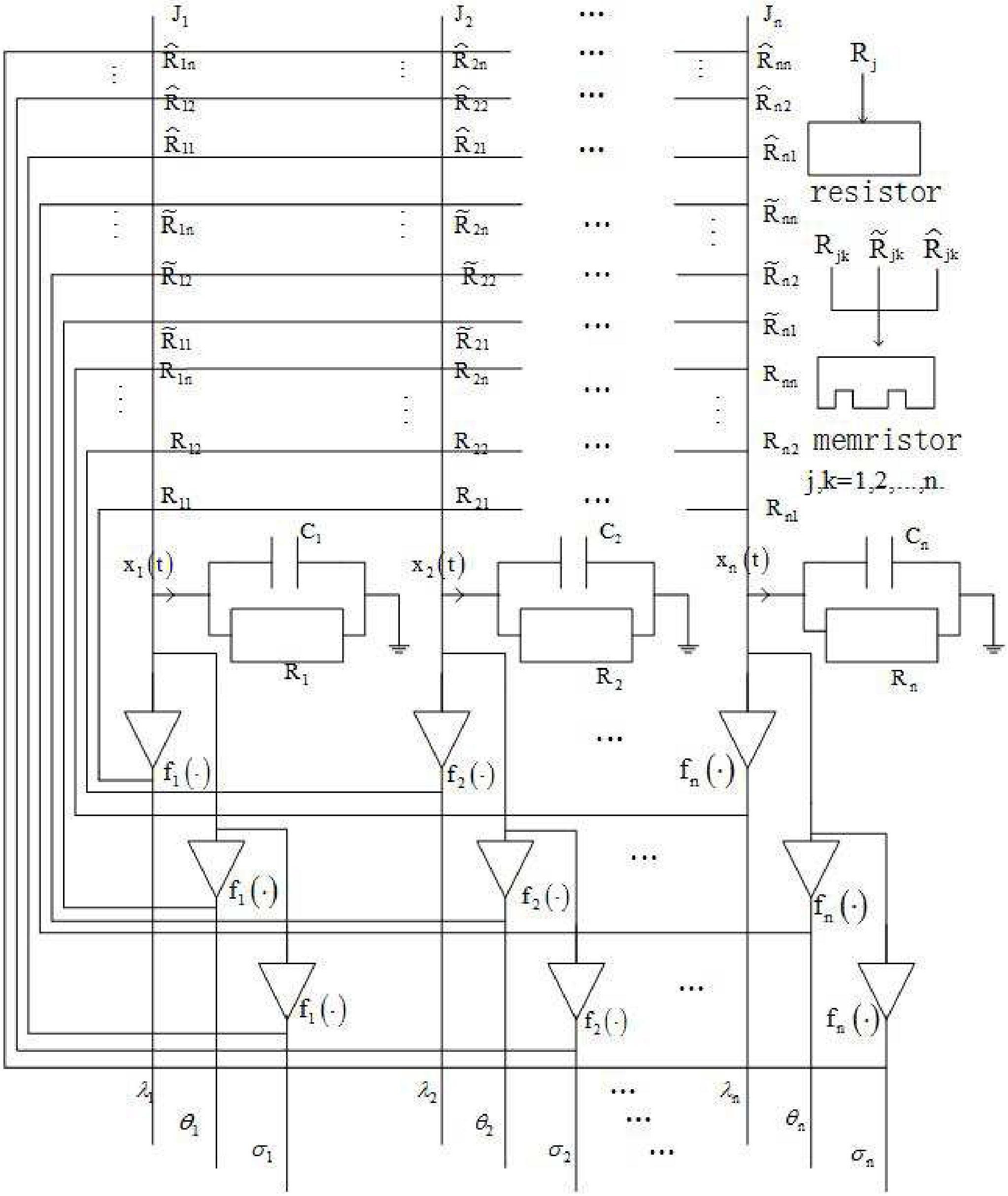

FIGURE 1. The circuit implementation of MNN (1), where $J_j = I_j C_j$ is the external inputs, λ_j, θ_j and σ_j are the outputs, $j = 1, 2, \ldots, n$.

MNN (1) is referred to as the drive system, the corresponding response system can be written as

$$\begin{aligned}\dot{y}_j(t) &= -d_j(y_j(t))y_j(t) + \sum_{k=1}^{n} a_{jk}(y_j(t))f_k(y_k(t)) \\ &\quad + \sum_{k=1}^{n} b_{jk}(y_j(t))f_k(y_k(t-\tau_k(t))) \\ &\quad + \sum_{k=1}^{n} c_{jk}(y_j(t)) \int_{t-\rho_k(t)}^{t} f_k(y_k(s))ds + I_j + u_j(t), \quad (4)\end{aligned}$$

$t \geq \delta$, $j = 1, 2, \ldots, n$, here $u_j(t)$ denotes the appropriate controller. The initial condition of MNN (4) is $y_j(s) = \psi_j(s+\delta) \in C([-\tau, 0], R)$, $j = 1, 2, \ldots, n$. $d_j(y_j(t))$, $a_{jk}(y_j(t))$, $b_{jk}(y_j(t))$ and $c_{jk}(y_j(t))$ are given by

$$\begin{aligned} d_j(y_j(t)) &= \begin{cases} \acute{d}_j, & |y_j(t)| \leq T_j, \\ \grave{d}_j, & |y_j(t)| > T_j, \end{cases} \\ a_{jk}(y_j(t)) &= \begin{cases} \acute{a}_{jk}, & |y_j(t)| \leq T_j, \\ \grave{a}_{jk}, & |y_j(t)| > T_j, \end{cases} \\ b_{jk}(y_j(t)) &= \begin{cases} \acute{b}_{jk}, & |y_j(t)| \leq T_j, \\ \grave{b}_{jk}, & |y_j(t)| > T_j, \end{cases} \\ c_{jk}(y_j(t)) &= \begin{cases} \acute{c}_{jk}, & |y_j(t)| \leq T_j, \\ \grave{c}_{jk}, & |y_j(t)| > T_j, \end{cases} \quad (5) \end{aligned}$$

for $j, k = 1, 2, \ldots, n$.

Let the synchronization errors be $e_j(t) = y_j(t) - x_j(t - \delta)$. From MNNs (1) and (4), it follows that

$$\begin{aligned}\dot{e}_j(t) = &-d_j(y_j(t))e_j(t) + R_j(t)\\ &- (d_j(y_j(t)) - d_j(x_j(t-\delta)))x_j(t-\delta) + u_j(t), \quad (6)\end{aligned}$$

$t \geq \delta,\ j = 1, 2, \ldots, n$, where

$$\begin{aligned}R_j(t) = &\sum_{k=1}^{n} a_{jk}(y_j(t))f_k(y_k(t))\\ &- \sum_{k=1}^{n} a_{jk}(x_j(t-\delta))f_k(x_k(t-\delta))\\ &+ \sum_{k=1}^{n} b_{jk}(y_j(t))f_k(y_k(t-\tau_k(t)))\\ &- \sum_{k=1}^{n} b_{jk}(x_j(t-\delta))f_k(x_k(t-\delta-\tau_k(t-\delta)))\\ &+ \sum_{k=1}^{n} c_{jk}(y_j(t)) \int_{t-\rho_k(t)}^{t} f_k(y_k(s))ds\\ &- \sum_{k=1}^{n} c_{jk}(x_j(t-\delta)) \int_{t-\delta-\rho_k(t-\delta)}^{t-\delta} f_k(x_k(s))ds. \quad (7)\end{aligned}$$

Let $e(t) = (e_1(t), e_2(t), \ldots, e_n(t))^T$. The initial condition of error system (6) is $e_j(s) = \psi_j(s+\delta) - \varphi_j(s), j = 1, 2, \ldots, n$.

Assumption 1: $|f_k(\cdot)| \leq M_k$ for some constants $M_k > 0$, $k = 1, 2, \ldots, n$.

Lemma 1 [54]: Suppose $V(x) : R^n \to R$ is a C−regular function. If $x(t)$ is absolutely continuous on each compact subinterval of $[0, +\infty)$, then $x(t)$ and $V(x(t)) : [0, +\infty) \to R$ are derivable for $a.a.t \in [0, +\infty)$, what is more,

$$\frac{d}{dt}V(x(t)) = v(t)\dot{x}(t), \quad \forall v(t) \in \partial V(x(t)),$$

here $\partial V(x(t))$ denotes the generalized gradient of V.

Lemma 2: $sign(e_j(t))(-d_j(y_j(t))y_j(t) + d_j(x_j(t-\delta))x_j(t-\delta)) \leq -\underline{d}_j\left|e_j(t)\right| + T_j\left|\acute{d}_j - \grave{d}_j\right| \cdot \left|sign(e_j(t))\right|$, $j = 1, 2, \ldots, n$.

Proof: Four cases will be discussed respectively.

(1) When $\left|x_j(t-\delta)\right| < T_j$ and $\left|y_j(t)\right| < T_j$,

$$\begin{aligned}&sign(e_j(t))(-d_j(y_j(t))y_j(t) + d_j(x_j(t-\delta))x_j(t-\delta))\\ &\quad = -sign(e_j(t))(\acute{d}_j y_j(t) - \acute{d}_j x_j(t-\delta))\\ &\quad = -\acute{d}_j\left|e_j(t)\right| \leq -\underline{d}_j\left|e_j(t)\right|. \quad (8)\end{aligned}$$

(2) When $\left|x_j(t-\delta)\right| > T_j$ and $\left|y_j(t)\right| > T_j$,

$$\begin{aligned}&sign(e_j(t))(-d_j(y_j(t))y_j(t) + d_j(x_j(t-\delta))x_j(t-\delta))\\ &\quad = -\grave{d}_j\left|e_j(t)\right| \leq -\underline{d}_j\left|e_j(t)\right|. \quad (9)\end{aligned}$$

(3) When $\left|x_j(t-\delta)\right| \leq T_j$ and $\left|y_j(t)\right| \geq T_j$,

$$\begin{aligned}&sign(e_j(t))(-d_j(y_j(t))y_j(t) + d_j(x_j(t-\delta))x_j(t-\delta))\\ &\quad = -sign(e_j(t))[d_j(y_j(t))e_j(t)\\ &\qquad + (d_j(y_j(t)) - d_j(x_j(t-\delta)))x_j(t-\delta)]\\ &\quad \leq -\underline{d}_j\left|e_j(t)\right| + T_j\left|\acute{d}_j - \grave{d}_j\right| \cdot \left|sign(e_j(t))\right|. \quad (10)\end{aligned}$$

(4) When $\left|x_j(t-\delta)\right| \geq T_j$ and $\left|y_j(t)\right| \leq T_j$,

$$\begin{aligned}&sign(e_j(t))(-d_j(y_j(t))y_j(t) + d_j(x_j(t-\delta))x_j(t-\delta))\\ &\quad = -sign(e_j(t))[(d_j(y_j(t)) - d_j(x_j(t-\delta)))y_j(t)\\ &\qquad + d_j(x_j(t-\delta))e_j(t)]\\ &\quad \leq -\underline{d}_j\left|e_j(t)\right| + T_j\left|\acute{d}_j - \grave{d}_j\right| \cdot \left|sign(e_j(t))\right|. \quad (11)\end{aligned}$$

The proof is completed.

Lemma 3: $sign(e_j(t))(-d_j(y_j(t))y_j(t) + d_j(x_j(t-\delta))x_j(t-\delta)) \leq -\underline{d}_j\left|e_j(t)\right| + T_j\left|\acute{d}_j - \grave{d}_j\right|, j = 1, 2, \ldots, n$.

Proof: The proof of Lemma 3 is similar to that of Lemma 2.

Lemma 4: $\left|R_j(t)\right| \leq \Lambda_j,\ j = 1, 2, \ldots, n$, where $\Lambda_j = \sum_{k=1}^{n} 2M_k(a_{jk}^v + b_{jk}^v + \rho_k c_{jk}^v)$.

Proof: By means of Assumption 1 and the assumption $0 \leq \rho_k(t) \leq \rho_k$, this lemma can be easily proved.

Definition 1: If there exist constants $\gamma > 0$ and $M^* > 0$ satisfying

$$\|e(t)\| \leq M^* e^{-\gamma(t-\delta)}, \quad t \geq \delta,$$

we say MNNs (1) and (4) realize exponential lag synchronization.

Definition 2: If there exists a constant $t^*(e(0)) > 0$ such that $\lim_{t \to t^*(e(0))} \|e(t)\| = 0$ and $\|e(t)\| \equiv 0$ for $t > t^*(e(0))$, we say MNNs (1) and (4) realize finite-time lag synchronization, and we call $t^*(e(0))$ the settling time.

III. MAIN RESULTS

We will design different kinds of adaptive controllers in this part, which can guarantee MNNs (1) and (4) reach asymptotic lag synchronization, exponential lag synchronization and finite-time lag synchronization, respectively.

A. ADAPTIVE ASYMPTOTIC LAG SYNCHRONIZATION

Theorem 1: If Assumption 1 holds, MNNs (1) and (4) can realize asymptotic lag synchronization under such an adaptive controller:

$$u_j(t) = -\xi_j(t)e_j(t) - \eta_j(t)sign(e_j(t)), \quad j = 1, 2, \ldots, n, \quad (12)$$

with

$$\begin{cases}\dot{\xi}_j(t) = \alpha_j\left|e_j(t)\right|^2,\\ \dot{\eta}_j(t) = \beta_j\left|e_j(t)\right|,\end{cases} \quad (13)$$

where $\alpha_j > 0$ and $\beta_j > 0$ are constants.

Proof: Lyapunov function can be designed as:

$$\begin{aligned}V(t) = \frac{1}{2}\sum_{j=1}^{n}\left|e_j(t)\right|^2 + \sum_{j=1}^{n}\Bigg[&\frac{1}{2\alpha_j}(\xi_j(t) - \xi_j)^2\\ &+ \frac{1}{2\beta_j}(\eta_j(t) - \eta_j)^2\Bigg],\end{aligned}$$

where ξ_j and η_j are constants that will be determined later.

Calculating the derivative of $V(t)$, we can get that

$$\begin{aligned}
\dot{V}(t) &= \sum_{j=1}^{n} e_j(t)\dot{e}_j(t) + \sum_{j=1}^{n} \frac{\xi_j(t)-\xi_j}{\alpha_j}\cdot \alpha_j \left|e_j(t)\right|^2 \\
&\quad + \sum_{j=1}^{n} \frac{\eta_j(t)-\eta_j}{\beta_j}\cdot \beta_j \left|e_j(t)\right| \\
&= \sum_{j=1}^{n} \left|e_j(t)\right| sign(e_j(t)) \big[-d_j(y_j(t))y_j(t) \\
&\quad + d_j(x_j(t-\delta))x_j(t-\delta) + R_j(t) + u_j(t)\big] \\
&\quad + \sum_{j=1}^{n}(\xi_j(t)-\xi_j)\left|e_j(t)\right|^2 + \sum_{j=1}^{n}(\eta_j(t)-\eta_j)\left|e_j(t)\right| \\
&\le \sum_{j=1}^{n} \left|e_j(t)\right| [-\underline{d}_j\left|e_j(t)\right| + T_j\left|\acute{d}_j - \grave{d}_j\right| + \Lambda_j \\
&\quad - \xi_j(t)\left|e_j(t)\right| - \eta_j(t)] + \sum_{j=1}^{n}(\xi_j(t)-\xi_j)\left|e_j(t)\right|^2 \\
&\quad + \sum_{j=1}^{n}(\eta_j(t)-\eta_j)\left|e_j(t)\right| \\
&= \sum_{j=1}^{n}(-\underline{d}_j - \xi_j)\left|e_j(t)\right|^2 \\
&\quad + \sum_{j=1}^{n}(T_j\left|\acute{d}_j - \grave{d}_j\right| + \Lambda_j - \eta_j)\left|e_j(t)\right|,
\end{aligned}$$

where Lemma 3 and Lemma 4 have been used.

Choose $\xi_j > -\underline{d}_j$, $\eta_j \ge T_j\left|\acute{d}_j - \grave{d}_j\right| + \Lambda_j, j = 1, 2, \ldots, n$, then $\dot{V}(t) < 0$. It follows that MNNs (1) and (4) can realize asymptotic lag synchronization.

B. ADAPTIVE EXPONENTIAL LAG SYNCHRONIZATION

Theorem 2: If Assumption 1 holds and $0 < \mu \le 2\min_{1\le j\le n} \underline{d}_j$, MNNs (1) and (4) can realize exponential lag synchronization under such an adaptive controller:

$$u_j(t) = -\xi_j(t)sign(e_j(t)) - \eta_j(t)x_j(t-\delta)sign(e_j(t)x_j(t-\delta)), \tag{14}$$

$j = 1, 2, \ldots, n$, with

$$\begin{cases} \dot{\xi}_j(t) = \alpha_j\left|e_j(t)\right|e^{\mu(t-\delta)}, \\ \dot{\eta}_j(t) = \beta_j\left|e_j(t)x_j(t-\delta)\right|e^{\mu(t-\delta)}, \end{cases} \tag{15}$$

where $\alpha_j > 0$ and $\beta_j > 0$ are constants.

Proof: Lyapunov function can be designed as:

$$\begin{aligned}
V(t) = e^{\mu(t-\delta)}\sum_{j=1}^{n} e_j^2(t) + \sum_{j=1}^{n}\bigg[\frac{1}{\alpha_j}(\xi_j(t)-\xi_j)^2 \\
+ \frac{1}{\beta_j}(\eta_j(t)-\eta_j)^2\bigg],
\end{aligned}$$

where ξ_j and η_j are constants that will be determined later.

Calculating the derivative of $V(t)$, we can get that

$$\begin{aligned}
\dot{V}(t) &= \mu e^{\mu(t-\delta)}\sum_{j=1}^{n} e_j^2(t) + e^{\mu(t-\delta)}\sum_{j=1}^{n} 2e_j(t)\dot{e}_j(t) \\
&\quad + \sum_{j=1}^{n}\frac{\xi_j(t)-\xi_j}{\alpha_j}\cdot\alpha_j\left|e_j(t)\right|e^{\mu(t-\delta)} \\
&\quad + \sum_{j=1}^{n}\frac{\eta_j(t)-\eta_j}{\beta_j}\cdot\beta_j\left|e_j(t)x_j(t-\delta)\right|e^{\mu(t-\delta)}.
\end{aligned}$$

By means of Lemma 4, it can be obtained that

$$\begin{aligned}
&2e_j(t)\dot{e}_j(t) \\
&= 2e_j(t)(-d_j(y_j(t))e_j(t) + R_j(t) + u_j(t) \\
&\quad - (d_j(y_j(t)) - d_j(x_j(t-\delta)))x_j(t-\delta)) \\
&= -2d_j(y_j(t))e_j^2(t) + 2e_j(t)R_j(t) - 2\xi_j(t)\left|e_j(t)\right| \\
&\quad - 2\eta_j(t)\left|e_j(t)x_j(t-\delta)\right| \\
&\quad - 2(d_j(y_j(t)) - d_j(x_j(t-\delta)))e_j(t)x_j(t-\delta) \\
&\le -2\underline{d}_je_j^2(t) + 2\Lambda_j\left|e_j(t)\right| - 2\xi_j(t)\left|e_j(t)\right| \\
&\quad - 2\eta_j(t)\left|e_j(t)x_j(t-\delta)\right| + 2\left|\acute{d}_j - \grave{d}_j\right|\cdot\left|e_j(t)x_j(t-\delta)\right|.
\end{aligned}$$

Then

$$\begin{aligned}
&\dot{V}(t) \\
&\le e^{\mu(t-\delta)}\sum_{j=1}^{n}\Big[\mu e_j^2(t) - 2\underline{d}_je_j^2(t) + 2\Lambda_j\left|e_j(t)\right| \\
&\quad - 2\xi_j(t)\left|e_j(t)\right| - 2\eta_j(t)\left|e_j(t)x_j(t-\delta)\right| \\
&\quad + 2\left|\acute{d}_j - \grave{d}_j\right|\cdot\left|e_j(t)x_j(t-\delta)\right| + 2(\xi_j(t)-\xi_j)\left|e_j(t)\right| \\
&\quad + 2(\eta_j(t)-\eta_j)\left|e_j(t)x_j(t-\delta)\right|\Big] \\
&\le e^{\mu(t-\delta)}\sum_{j=1}^{n}\Big[(-2\min_{1\le j\le n}\underline{d}_j + \mu)e_j^2(t) \\
&\quad + 2(\left|\acute{d}_j - \grave{d}_j\right| - \eta_j)\left|e_j(t)x_j(t-\delta)\right| + 2(\Lambda_j - \xi_j)\left|e_j(t)\right|\Big].
\end{aligned}$$

Choose $\xi_j \ge \Lambda_j$, $\eta_j \ge \left|\acute{d}_j - \grave{d}_j\right|, j = 1, 2, \ldots, n$. Since $0 < \mu \le 2\min_{1\le j\le n}\underline{d}_j$, then $\dot{V}(t) \le 0$, that means

$$\begin{aligned}
V(t) \le V(\delta) = \sum_{j=1}^{n} e_j^2(\delta) \\
+ \sum_{j=1}^{n}\left[\frac{1}{\alpha_j}(\xi_j(\delta)-\xi_j)^2 + \frac{1}{\beta_j}(\eta_j(\delta)-\eta_j)^2\right], \quad t \ge \delta.
\end{aligned}$$

In view of

$$V(t) \ge e^{\mu(t-\delta)}\|e(t)\|^2,$$

we have

$$\|e(t)\|^2 \le V(t)e^{-\mu(t-\delta)} \le V(\delta)e^{-\mu(t-\delta)}, \quad t \ge \delta.$$

Therefore,

$$\|e(t)\| \le \sqrt{V(\delta)}e^{-\frac{\mu}{2}(t-\delta)}, \quad t \ge \delta.$$

C. ADAPTIVE FINITE-TIME LAG SYNCHRONIZATION

Theorem 3: If Assumption 1 holds, MNNs (1) and (4) will realize finite-time lag synchronization under such an adaptive controller:

$$u_j(t)=-\xi_j(t)e_j(t)-\eta_j(t)sign(e_j(t)),\quad j=1,2,\ldots,n, \tag{16}$$

with

$$\begin{cases} \dot{\xi}_j(t)=\alpha_j\left|e_j(t)\right|, \\ \dot{\eta}_j(t)=\beta_j\left|sign(e_j(t))\right|, \end{cases} \tag{17}$$

where $\alpha_j>0$ and $\beta_j>0$ are constants.

Proof: Lyapunov function can be designed as:

$$V(t)=\sum_{j=1}^{n}\left|e_j(t)\right|+\sum_{j=1}^{n}\left[\frac{1}{2\alpha_j}(\xi_j(t)-\xi_j)^2+\frac{1}{2\beta_j}(\eta_j(t)-\eta_j)^2\right],$$

where ξ_j and η_j are constants that will be determined later.

By means of Lemma 1, we calculate the derivative of $V(t)$:

$$\begin{aligned}\dot{V}(t)&=\sum_{j=1}^{n}sign(e_j(t))\dot{e}_j(t)\\&\quad+\sum_{j=1}^{n}\frac{1}{2\alpha_j}\cdot 2(\xi_j(t)-\xi_j)\cdot\alpha_j\left|e_j(t)\right|\\&\quad+\sum_{j=1}^{n}\frac{1}{2\beta_j}\cdot 2(\eta_j(t)-\eta_j)\cdot\beta_j\left|sign(e_j(t))\right|\\&=\sum_{j=1}^{n}sign(e_j(t))\left[-d_j(y_j(t))y_j(t)\right.\\&\quad\left.+d_j(x_j(t-\delta))x_j(t-\delta)+R_j(t)+u_j(t)\right]\\&\quad+\sum_{j=1}^{n}\left[(\xi_j(t)-\xi_j)\left|e_j(t)\right|+(\eta_j(t)-\eta_j)\left|sign(e_j(t))\right|\right].\end{aligned}$$

By Lemma 2 and Lemma 4, it can be obtained that

$$\begin{aligned}\dot{V}(t)&\leq\sum_{j=1}^{n}\left[-\underline{d}_j\left|e_j(t)\right|+T_j\left|\acute{d}_j-\grave{d}_j\right|\cdot\left|sign(e_j(t))\right|\right]\\&\quad+\sum_{j=1}^{n}\Lambda_j\left|sign(e_j(t))\right|-\sum_{j=1}^{n}\xi_j(t)\left|e_j(t)\right|\\&\quad-\sum_{j=1}^{n}\eta_j(t)\left|sign(e_j(t))\right|+\sum_{j=1}^{n}(\xi_j(t)-\xi_j)\left|e_j(t)\right|\\&\quad+\sum_{j=1}^{n}(\eta_j(t)-\eta_j)\left|sign(e_j(t))\right|\\&=\sum_{j=1}^{n}(-\underline{d}_j-\xi_j)\left|e_j(t)\right|\\&\quad+\sum_{j=1}^{n}(T_j\left|\acute{d}_j-\grave{d}_j\right|+\Lambda_j-\eta_j)\left|sign(e_j(t))\right|.\end{aligned}$$

Choose $\xi_j\geq-\underline{d}_j$, $\eta_j=T_j\left|\acute{d}_j-\grave{d}_j\right|+\Lambda_j+\omega_j$, $j=1,2,\ldots,n$, where $\omega_j>0$, we can get that

$$\dot{V}(t)\leq-\sum_{j=1}^{n}\omega_j\left|sign(e_j(t))\right|.$$

Let $\omega=\min_{1\leq j\leq n}\omega_j$, then we have

$$\dot{V}(t)\leq-\omega\sum_{j=1}^{n}\left|sign(e_j(t))\right|.$$

Our aim is to prove there exist $t^*>0$ which satisfies

$$\lim_{t\to t^*}\|e(t)\|=0\quad and\ \|e(t)\|\equiv 0,\quad\forall t\geq t^*.$$

Obviously, if $\|e(t)\|\neq 0$, then

$$\dot{V}(t)\leq-\omega\sum_{j=1}^{n}\left|sign(e_j(t))\right|\leq-\omega.$$

So there always exists a constant $t_0=\frac{V(0)}{\omega}$ such that

$$\lim_{t\to t_0}\|e(t)\|=0\quad and\ \|e(t)\|\equiv 0,\quad t\geq t_0.$$

Let $t^*=\inf\{t\in(0,t_0]:\|e(s)\|\equiv 0,\ s\geq t\}$, then we have

$$\lim_{t\to t^*}\|e(t)\|=0\quad and\ \|e(t)\|\equiv 0,\quad t\geq t^*.$$

According to Definition 2, MNNs (1) and (4) can realize finite-time lag synchronization.

Remark 1: As far as we know, the lag synchronization of delayed MNNs has been considered in [29], [30], and [32]–[34]. However, the authors believed that $f_k(x_k(t-\tau_k(t)))$ should be replaced by $f_k(x_k(t-\delta-\tau_k(t)))$ when t was changed into $t-\delta$. Obviously, it is not right. In this paper, $f_k(x_k(t-\tau_k(t)))$ is replaced by $f_k(x_k(t-\delta-\tau_k(t-\delta)))$ when t is changed into $t-\delta$.

Remark 2: Because of the conservativeness of strict derivations, the control gains of the controllers in [30], [31], and [33]–[37] may be very large sometimes. In this paper, the controllers that we adopt are adaptive controllers, which can avoid the large control gains very well.

Remark 3: Among the results about the lag synchronization of MNNs [29]–[37], most were associated with asymptotic synchronization or exponential synchronization. In this paper, not only asymptotic lag synchronization and exponential lag synchronization but also finite-time lag synchronization is considered.

Remark 4: Due to traffic jams and the finite speeds of signals transmission, discrete delays [55]–[57] inevitably exist in neural networks. On the other hand, because there exist parallel pathways with various axon lengths and sizes, we should also consider distributed delay in the studies of neural networks. However, most results about the dynamic analysis of MNNs only considered MNNs with discrete delays. The mixed delays of this paper include discrete delays and distributed delays, so the MNN model considered in this paper is less conservative.

Remark 5: In this paper, it is proved that MNNs (1) and (4) can realize synchronization with a lag time δ. If the lag time $\delta = 0$, the results of this paper imply that MNNs (1) and (4) can achieve complete synchronization.

Remark 6: As far as we know, it is usually difficult to study the lag synchronization of MNNs with distributed delays. In this paper, by adopting the analysis techniques used in [21] and [45], we can effectively solve the difficulty caused by the existence of time delays, including distributed delays. Undeniably, the finite-time synchronization analysis techniques in this paper are similar to those used in [12], [47]–[50], and [52]. However, the controllers in this paper are adaptive controllers, while the controllers in [12], [47]–[50], and [52] were feedback controllers.

IV. NUMERICAL EXAMPLES

Next we provide some numerical simulations to verify the theoretical results in Section 3.

Consider a delayed MNN model:

$$\begin{aligned}\dot{x}_j(t) = &-d_j(x_j(t))x_j(t) + \sum_{k=1}^{2} a_{jk}(x_j(t))f_k(x_k(t)) \\ &+ \sum_{k=1}^{2} b_{jk}(x_j(t))f_k(x_k(t-\tau_k(t))) + I_j \\ &+ \sum_{k=1}^{2} c_{jk}(x_j(t)) \int_{t-\rho_k(t)}^{t} f_k(x_k(s))ds, \quad j=1,2, \quad (18)\end{aligned}$$

where

$$d_1(x_1(t)) = \begin{cases} 0.9, & |x_1(t)| \le 1.2, \\ 1.1, & |x_1(t)| > 1.2, \end{cases}$$

$$d_2(x_2(t)) = \begin{cases} 1.1, & |x_1(t)| \le 1.2, \\ 0.9, & |x_1(t)| > 1.2, \end{cases}$$

$$a_{11}(x_1(t)) = \begin{cases} -0.5, & |x_1(t)| \le 1.2, \\ -0.6, & |x_1(t)| > 1.2, \end{cases}$$

$$a_{12}(x_1(t)) = \begin{cases} 3, & |x_1(t)| \le 1.2, \\ 3.2, & |x_1(t)| > 1.2, \end{cases}$$

$$a_{21}(x_2(t)) = \begin{cases} 5, & |x_2(t)| \le 1.2, \\ 5.1, & |x_2(t)| > 1.2, \end{cases}$$

$$a_{22}(x_2(t)) = \begin{cases} -0.5, & |x_2(t)| \le 1.2, \\ -0.4, & |x_2(t)| > 1.2, \end{cases}$$

$$b_{11}(x_1(t)) = \begin{cases} -0.1, & |x_1(t)| \le 1.2, \\ -0.2, & |x_1(t)| > 1.2, \end{cases}$$

$$b_{12}(x_1(t)) = \begin{cases} 5, & |x_1(t)| \le 1.2, \\ 5.2, & |x_1(t)| > 1.2, \end{cases}$$

$$b_{21}(x_2(t)) = \begin{cases} 3, & |x_2(t)| \le 1.2, \\ 3.1, & |x_2(t)| > 1.2, \end{cases}$$

$$b_{22}(x_2(t)) = \begin{cases} -0.1, & |x_2(t)| \le 1.2, \\ -0.2, & |x_2(t)| > 1.2, \end{cases}$$

$$c_{11}(x_1(t)) = \begin{cases} -0.4, & |x_1(t)| \le 1.2, \\ -0.2, & |x_1(t)| > 1.2, \end{cases}$$

$$c_{12}(x_1(t)) = \begin{cases} 0.5, & |x_1(t)| \le 1.2, \\ 0.5, & |x_1(t)| > 1.2, \end{cases}$$

$$c_{21}(x_2(t)) = \begin{cases} 0.7, & |x_2(t)| \le 1.2, \\ 0.2, & |x_2(t)| > 1.2, \end{cases}$$

$$c_{22}(x_2(t)) = \begin{cases} -0.4, & |x_2(t)| \le 1.2, \\ -0.5, & |x_2(t)| > 1.2. \end{cases}$$

Let $f_1(v) = f_2(v) = \frac{|v+1|-|v-1|}{2}$, $I_1 = sint$, $I_2 = cost$, $\tau_1(t) = \tau_2(t) = \frac{e^t}{1+e^t}$ and $\rho_1(t) = \rho_2(t) = 1 + sint$, we can get that $M_1 = M_2 = 1$, $\tau_1 = \tau_2 = 1$, $\rho_1 = \rho_2 = 2$, $\tau = 2$. The initial condition of MNN (18) is $x_1(s) = -0.5$, $x_2(s) = 1.2$, $s \in [-2, 0]$.

This is the corresponding response system:

$$\begin{aligned}\dot{y}_j(t) = &-d_j(y_j(t))y_j(t) + \sum_{k=1}^{2} a_{jk}(y_j(t))f_k(y_k(t)) \\ &+ \sum_{k=1}^{2} b_{jk}(y_j(t))f_k(y_k(t-\tau_k(t))) + I_j \\ &+ \sum_{k=1}^{2} c_{jk}(y_j(t)) \int_{t-\rho_k(t)}^{t} f_k(y_k(s))ds + u_j(t), \quad j=1,2. \end{aligned} \quad (19)$$

Let the initial condition of MNN (19) be $y_1(s) = 0.7$, $y_2(s) = 0.4$, $s \in [-2, 0]$. The synchronization errors are defined as $e_j(t) = y_j(t) - x_j(t-\delta)$, $j = 1, 2$, here $\delta = 1$. Fig.2 shows the evolutions of the synchronization errors $e_1(t)$ and $e_2(t)$ when $u_1(t) = u_2(t) = 0$.

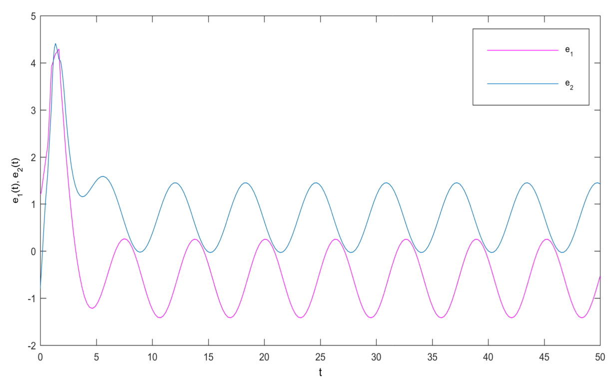

FIGURE 2. **Evolutions of the synchronization errors when $u_1(t) = u_2(t) = 0$.**

According to Theorem 1, we choose $\alpha_j = 1$, $\beta_j = 1$, $\xi_j(0) = 0$, $\eta_j(0) = 0$, $j = 1, 2$, then MNNs (18) and (19) can realize asymptotic lag synchronization under the controller (12). Fig.3 illustrates the corresponding

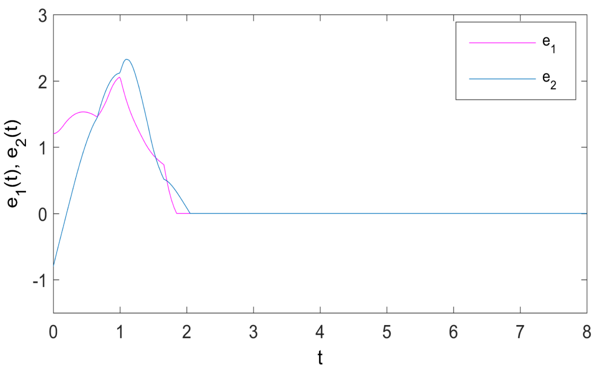

FIGURE 3. Evolutions of the synchronization errors under controller (12).

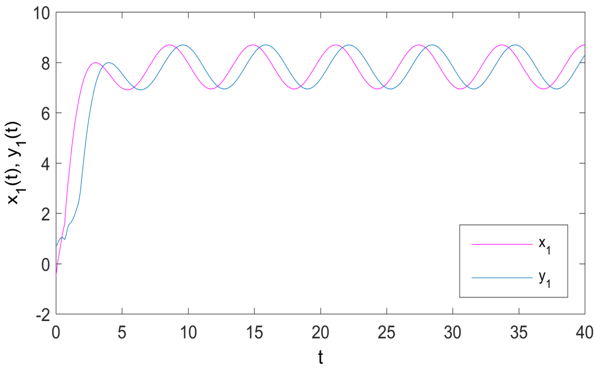

FIGURE 4. Evolutions of x_1 and y_1 under controller (12).

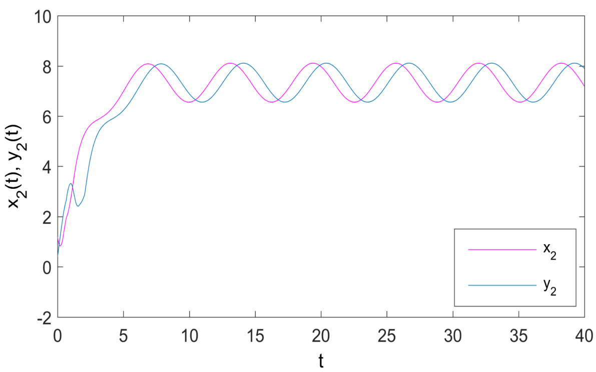

FIGURE 5. Evolutions of x_2 and y_2 under controller (12).

synchronization errors, and Fig.4 and Fig.5 present the evolutions of x_1 and y_1, x_2 and y_2. The evolutions of the control gains of controller (12) are given in Fig.6 and Fig.7 respectively.

Remark 7: In Fig.2, the synchronization errors when $u_j(t) = 0$, $j = 1, 2$ are not equal to zero obviously, that means MNNs (18) and (19) have not achieved lag synchronization. In Fig.3, the synchronization errors between MNNs (18) and (19) under controller (12) tend to zero within $t = 3$, that means MNNs (18) and (19) have achieved lag synchronization. Fig.4 and Fig.5 show that MNNs (18) and (19) have

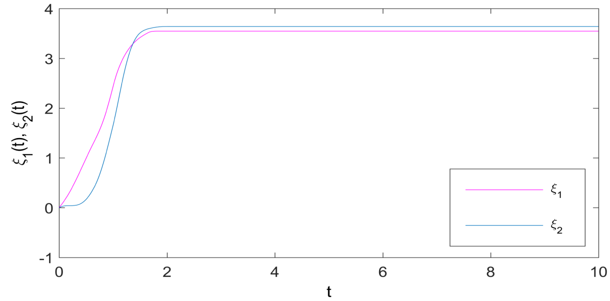

FIGURE 6. Evolutions of $\xi_1(t)$ and $\xi_2(t)$ in controller (12).

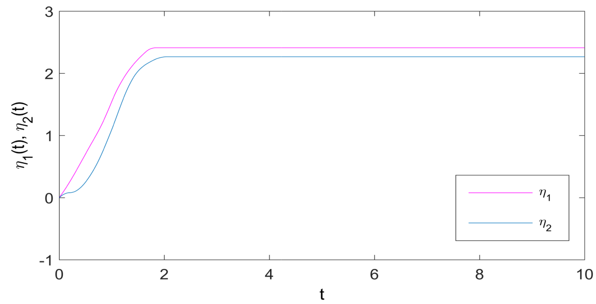

FIGURE 7. Evolutions of $\eta_1(t)$ and $\eta_2(t)$ in controller (12).

realized lag synchronization with a lag time $\delta = 1$. Fig.6 and Fig.7 illustrate that $\xi_j(t)$ and $\eta_j(t)$ tend to some small constants when $t > 2$.

Remark 8: Although some system parameters in MNNs (18) and (19) are quite large, one can see from Fig.6 and Fig.7 that the control gains are always very small, which fully illustrates the advantage of the proposed adaptive controllers. In [29], [32], adaptive control has also been utilized to deal with the lag synchronization of delayed MNNs. However, the evolutions of the control gains were not provided in the Numerical Simulations of [29], [32].

V. CONCLUSION

This paper discusses the adaptive lag synchronization control of MNNs with mixed delays. By adopting a series of suitable adaptive feedback controllers, we prove that the considered MNNs can achieve asymptotic lag synchronization, exponential lag synchronization and finite-time lag synchronization, respectively. Adaptive control can avoid the large control gains perfectly, and adaptive control can be applied even when the system parameters are unknown. What is more, no excessive calculation is needed to determine the desirable control gains. We also provide some numerical simulations to illustrate the validity of our theoretical results. In the future, we will consider the applications of MNNs in the fields of image encryption and optimal computation.

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CHUAN CHEN received the B.S. degree in mathematics from Henan University, Kaifeng, China, in 2005, and the M.S. degree in mathematics from Capital Normal University, Beijing, China, in 2008. He is currently pursuing the Ph.D. degree with the Beijing University of Posts and Telecommunications, Beijing, China. His current research interests include synchronization theory, control theory, complex networks, and memristive neural networks.

LIXIANG LI received the M.S. degree in circuit and system from Yanshan University, Qinhuangdao, China, in 2003, and the Ph.D. degree in signal and information processing from the Beijing University of Posts and Telecommunications, Beijing, China, in 2006. She was a Visiting Scholar with the Potsdam Institute for Climate Impact Research, Germany, in 2011. She was involved in the research of swarm intelligence and network security. She is currently a Professor with the School of Cyberspace Security, Beijing University of Posts and Telecommunications. She has authored or co-authored over 100 papers and a monograph in recent five years. She was a recipient of the National Excellent Doctoral Theses, the New Century Excellent Talents in University, the Henry Fok Education Foundation, the Hong Kong Scholar Award, and the Beijing Higher Education Program for Young Talents.

HAIPENG PENG received the M.S. degree in system engineering from the Shenyang University of Technology, Shenyang, China, in 2006, and the Ph.D. degree in signal and information processing from the Beijing University of Posts and Telecommunications, Beijing, China, in 2010. He is currently a Professor with the School of Cyberspace Security, Beijing University of Posts and Telecommunications. He has co-authored 50 scientific papers. His research interests include information security, network security, complex networks, and control of dynamical systems.

YIXIAN YANG received the M.S. degree in applied mathematics and the Ph.D. degree in electronics and communication systems from the Beijing University of Posts and Telecommunications, Beijing, China, in 1986 and 1988, respectively. He is currently the Managing Director of the Information Security Center, Beijing University of Posts and Telecommunications. He has authored over 40 national and provincial key scientific research project, and published over 300 high-level papers and 20 monographs. His major research interests include coding and cryptography, information and network security, and signal and information processing. He was the Yangtze River Scholar Program Professor, National Outstanding Youth Fund Winner, and the National Teaching Master.