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A Self-Adapted Across Neighborhood Search Algorithm With Variable Reduction Strategy for Solving Non-Convex Static and Dynamic Economic Dispatch Problems

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ABSTRACT The economic dispatch problem is a kind of challenging non-convex problem, which minimizes the total operating cost while being subject to a collection of complex equality and inequality constraints. This paper presents a novel meta-heuristic named across neighborhood search (ANS) algorithm to solve both dynamic and static economic dispatch problems. The ANS algorithm is augmented by a solution-difference disturbance mechanism and a parameter self-adaptation strategy. It is generally hard for meta-heuristics to handle complex nonlinear equality constraints, because a meta-heuristic's search behavior is essentially stochastic while the equality constraints require the algorithm to exactly locate feasible solutions at the constraint bound. Therefore, a variable reduction strategy (VRS) is employed to deal with the equality constraint when solving the economic dispatch problem. VRS eliminates the equality constraint and reduces the dimensionality of the problem simultaneously, such that significantly improves the optimization efficiency. Extensive experiments and comparisons suggest that the proposed algorithm could generate the state-of-the-art results for both static and dynamic economic dispatch problems.

INDEX TERMS Economic dispatch problem, across neighborhood search, variable reduction, evolutionary optimization, swarm intelligence.

I. INTRODUCTION

Economic dispatch problems aim at arranging the generation allocation among the committed generating units with minimum costs while subject to various constraints. Economic dispatch problems can be roughly categorized into static economic dispatch (SED) problems and dynamic economic dispatch (DED) problems. The SED problem aims to generate a dispatch solution in a specified time and ignores the system relations between the different operating periods. In contrast, the DED problem considers the connections of different operating periods by taking into account ramp-rate constraints.

Various studies on the economic dispatch problem have been undertaken to date as better solutions would result in significant saving in operating cost [1], [2]. Conventional algorithms for economic dispatch problems use Lagrangian multipliers, which generally require monotonic cost functions [3]. This may lead to an inappropriate dispatch solution as the input–output curve is inherently nonlinear, non-smooth and non-convex due to the effect of multiple steam admission valves (known as the valve-point effect) [4], [5].

Deterministic optimization algorithms such as the interior point method and the dynamic programming method do not work effectively for the non-smooth and non-convex economic dispatch problem. By contrast, meta-heuristics have demonstrated competitive performance in solving both SED and DED problems. Popular meta-heuristics include genetic algorithms (GA) [6], [7], evolutionary programming [3] differential evolution [8]–[10], particle swarm optimization [11]–[17], crisscross optimization [18], [19], immune algorithm [20], artificial bee colony algorithm [21], [22], grey wolf optimization [23], social spider algorithm [24], harmony search [25], Floating search space [26], and teaching–learning-based optimization [27]. Recently, an interesting constrained globalized Nelder-Mead algorithm was proposed to solve SED and DED problems [28].

Across neighborhood search algorithm (ANS) is a recently proposed population-based meta-heuristics [29]. Like other population-based algorithms (e.g., PSO and ACO), in ANS, a group of individuals search solution space with the aim to find the optimal solution of an optimization problem. A memory collection is used in ANS to record a certain number of superior solutions found so far by the whole population. At every generation, each individual updates its position by searching across the neighborhoods of multiple superior solutions biased by a Gaussian distribution. ANS is very easy and convenient for implementation and application. It has only three parameters requiring adjustments to cater for different optimization problems. ANS has shown highly competitive performance in dealing with various benchmark functions, including unimodal, multimodal and rotated functions [29].

In this study, we improve ANS by incorporating a solution-difference perturbation mechanism and a parameter self-adaptation strategy into it, thus obtaining a novel ANS variant named SaANS-SDP. The solution-difference perturbation mechanism is critical in the mutation operator of differential evolution (DE), which greatly contributes to the success of DE. This motivates us to introduce the solution-difference perturbation mechanism to ANS. Therefore, SaANS-SDP can be recognized as a hybridization of ANS and DE.

The required parameter values of a meta-heuristic are generally distinct when solving different optimization problems [30]. Moreover, the most appropriate parameter values may vary at different optimization stages [31]. In order to further enhance the capability of ANS, a parameter self-adaptation strategy is proposed to dynamically control the parameters of ANS along with the optimization process.

Note that ANS is initially proposed for unconstrained numerical optimization like many other evolutionary and swarm intelligence algorithms. However, the economic dispatch problem considered in this study presents complex nonlinear equality and inequality constraints. Effective constraint handling techniques are necessary to apply ANS to the economic dispatch problem. It is known that equality constraints are much harder to satisfy than inequality constraints when meta-heuristics are utilized to deal with constrained optimization problems. We apply the recently proposed variable reduction strategy (VRS) [32], [33] to effectively handle equality constraints in SED and DED problems, which eliminates the equality constraint and reduces the number of variables simultaneously. As a result, VRS shrinks the solution space and speeds up the optimization process. In addition, an epsilon constraint method is adopted to deal with the inequality constraints. SaANS-SDP plus the constraint handling techniques is extensively tested on many benchmarked SED and DED instances and shows highly competitive performance compared with several state-of-the-art methods.

The contributions of this paper are summarized as below.

- For the first time, we propose to apply ANS to static and dynamic economic dispatch problems and generate state-of-the-art results.
- We combine ANS with a solution-difference perturbation mechanism and a self-adapted parameter control strategy, which noticeably enhance the performance of canonical ANS.
- We apply VRS to effectively handle the equality constraint of the economic dispatch problem. This strategy reduces the number of the variables, such that it shrinks the solution space. In addition, it eliminates the equality constraint of the economic dispatch problem, which facilitates the algorithm to find high-quality feasible solutions more quickly and thus significantly speeds up the optimization process.
- Experiments on many well-benchmarked SED and DED instances show that the proposed algorithm could generate state-of-the-art results for both SED and EDE problems. The obtained solutions are highly feasible and much closer to the optimal solution.

The rest of the paper is structured as follows. Section II briefly describes the SED and DED problems. Section III presents SaANS-SDP algorithm. Section IV introduces the used constraint handling techniques including VRS. Extensive experiments and algorithm comparisons are conducted in Section V. Finally, section VI concludes this paper.

II. ECONOMIC POWER DISPATCH PROBLEM

Since DED and SED problems share similarities, we introduce them simultaneously here. The DED problem follows the characteristics of the hourly SED problem, except that the power-demand varies with each hour and the power generation schedule for 24 hours is to be determined. We can view that the number of variables of a DED problem is 24 times that of a corresponding SED problem. Here, we give the formal description the SED problem [7], [34].

The SED problem is about minimizing the fuel cost of generating units for a specific period of operation, usually one hour of operation, so as to accomplish optimal generation dispatch among operating units while satisfying the system load demand, generator operation constraints with ramp rate limits and prohibited operating zones.

The objective function corresponding to the production cost is represented as:

$$\text{Minimize: } F_T = \sum_{i=1}^{N_G} F_i(P_i) \tag{1}$$

where,

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i + |e_i \cdot \sin(f_i \cdot (P_i^{\min} - P_i))|,\quad i = 1, 2, 3, \ldots, N_G \tag{2}$$

is the expression for cost function corresponding to the i^{th} generating unit and a_i, b_i and c_i are its coefficients.

P_i is the real power output (in MW) of the i^{th} generator.

$P_i^{\min}$ is the lower bound of P_i.

N_G is the number of online generating units to be dispatched.

e_i and f_i are the cost coefficients corresponding to valve point loading effect.

The consideration of valve point effect provides a more reasonable representation in relation to the fuel cost of the generation unit. The sinusoidal term in the production cost function reflects the effect of valve points. The economic dispatch problem becomes nonconvex and nondifferentiable because of valve point effects.

Several constraints need to be satisfied, which are described as below.

Power Balance Constraint: This constraint is based on the principle of equilibrium between total system generation and total system loads (P_D) and losses (P_L). That is,

$$\sum_{i=1}^{N_G} P_i = P_D + P_L \tag{3}$$

where P_L is obtained using B-coefficients, given by

$$P_L = \sum_{i=1}^{N_G}\sum_{j=1}^{N_G} P_i B_{ij} P_j + \sum_{i=1}^{N_G} B_{0i} P_i + B_{00} \tag{4}$$

Capacity Constraint: The output power of each generating unit should be among a lower and upper bounds. This constraint is represented by a pair of inequality constraints as below:

$$P_i^{\min} \le P_i \le P_i^{\max} \tag{5}$$

where $P_i^{\min}$ and $P_i^{\max}$ are lower and upper bounds for power output of the i^{th} generating unit.

Ramp Rate Limit: Increasing or decreasing the output generation of each unit should not exceed an amount of power over a time interval due to the physical restrictions of each unit. The generator ramp rate limit constraint is expressed as below.

$$P_i(t-1) - P_i(t) \le DR_i \quad \text{and } P_i(t) - P_i(t-1) \le UR_i \tag{6}$$

where $P_i(t-1)$ is the previous power output of generator i. UR_i and DR_i are the up-ramp and down-ramp limits of generator i, respectively.

Prohibited Operating Zone (POZ): Modern generators with valve point loading have many prohibited operating zones. Therefore, in practical operation, when adjusting the generation output P_i of unit i, the operation of the unit in the prohibited zones must be avoided. The feasible operating zones of the unit i can be described as follows.

$$\begin{aligned} &P_i^{\min} \le P_i \le P_{i,1}^{LB} \\ &P_{i,j-1}^{UB} \le P_i \le P_{i,j}^{LB}, \quad j = 2, 3, \ldots, NP_i \\ &P_{i,NP_i}^{UB} \le P_i \le P_i^{\max} \end{aligned} \tag{7}$$

where NP_i is the number of prohibited zones of unit i. $P_{i,j}^{LB}$ and $P_{i,j}^{UB}$ are, respectively, the lower and upper bounds of the j^{th} prohibited operating zone of unit i.

III. ACROSS NEIGHBORHOOD SEARCH ALGORITHM WITH SOLUTION-DIFFERENCE PERTURBATION AND PARAMETER SELF-ADAPTATION STRATEGY

It is noticeable that SED and DED problems introduced in the former section are highly nonlinear and multi-modal. ANS has demonstrated competitive performance in solving many complex benchmark optimization problems [29]. In this study, we further improve ANS with two advanced strategies with the aim to deal with SED and DED problems effectively. The canonical ANS and the proposed two strategies are introduced in the following subsections.

A. CANONICAL ACROSS NEIGHBORHOOD SEARCH ALGORITHM

It is assumed that there are m individuals search in a solution space cooperatively. Let pos_i denote the current position of individual i; R is a set recording desired superior solutions; r_i denotes a superior solution; The cardinality of collection R is c. ANS is composed of following three parts.

1) *Maintain a Collection of Superior Solutions:* In the canonical ANS, the best solution found by each individual i up to now is recorded in R as a superior solution r_i. Therefore, the superior solution collection R consists of m individuals (i.e. $c = m$).
2) *Search the Neighborhood of a Superior Solution:* The currently best solution r_i (i.e. position) of each individual i is a superior solution, individual i will naturally search the neighborhood of r_i. At every generation, each individual i searches an approximate hyper-box determined by its current position pos_i and the superior position r_i. This hyper-box represents the neighborhood of r_i with respect to pos_i. r_i is the center of the hyper-box. The search range on the dth dimension of individual i is illustrated in Fig. 1. We can observe that r_i^d is the center and $|r_i^d - pos_i^d|$ is the approximate semi-length of the search range. To search this range in a random manner, a Gaussian distribution function is employed. This means that individual i has a higher probability to search the area closer to r_i^d, which is

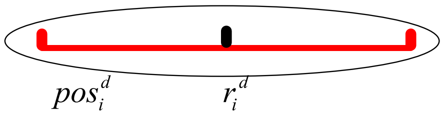

FIGURE 1. **Illustration of the search range on the dth dimension.**

thought to be more promising. The position update strategy of individual i for searching the neighborhood of r_i is formally described in (8), where $G(0, \sigma_i^2)$ is a value produced by a Gaussian distribution with mean value of zero and standard deviation of σ_i.

$$pos_i^d = r_i^d + G(0, \sigma_i^2) * |r_i^d - pos_i^d| \tag{8}$$

3) *Search the Neighborhoods of Multiple Superior Solutions Simultaneously:* Suppose that D denotes the dimensionality of an optimization problem. Individual i is required to search across the neighborhoods of multiple superior solutions at the same time. First, n_i dimensions ($0 \le n_i \le D$) of pos_i are selected randomly (n_i is a parameter called across-search degree). Let collection N_i include the n_i selected dimensions. For each randomly selected dimension d in N_i, a superior solution $r_{g(d)}$ ($g(d) \ne i$) is randomly selected to replace r_i when searching on dimension d. Variables corresponding to the dimensions in N_i are updated as

$$pos_i^d = r_{g(d)}^d + G(0, \sigma_i^2) * |r_{g(d)}^d - pos_i^d|, \quad \text{for } d \in N_i. \tag{9}$$

Considering (8) and (9), the rule for updating the position of individual i is formulated as below.

$$pos_i^d = \begin{cases} r_i^d + G(0, \sigma_i^2) * |r_i^d - pos_i^d| & \text{if } d \notin N_i \\ r_{g(d)}^d + G(0, \sigma_i^2) * |r_{g(d)}^d - pos_i^d|, g(d) \ne i & \\ & \text{if } d \in N_i. \end{cases} \tag{10}$$

As a result, each individual is capable of searching across the neighborhoods of multiple superior solutions. This is why the algorithm is called across neighborhood search algorithm.

B. SOLUTION-DIFFERENCE PERTURBATION MECHANISM

Differential evolution (DE) is a well-known and highly efficient evolutionary algorithm, which combines a solution-difference perturbation mechanism into the mutant operator. The effectiveness of the solution-difference perturbation mechanism probably lies on the fact that it exploits the run-time diversity information of DE population to guide the successive search behaviors and achieves a well balance between exploitation and exploration [35].

In this study, we augment ANS with a solution-difference perturbation mechanism. As a result, the new position update formula is presented as below.

$$pos_i^d = \begin{cases} r_i^d + G(0, \sigma_i^2) * |r_i^d - pos_i^d + r_{k_1}^d - r_{k_2}^d| \\ \qquad \text{if } d \notin N_i \\ r_{g(d)}^d + G(0, \sigma_i^2) * |r_{g(d)}^d - pos_i^d + r_{k_1}^d - r_{k_2}^d|, \\ g(d) \ne i \quad \text{if } d \in N_i \end{cases} \tag{11}$$

where, k_1 and k_2 are two exclusive random integers between 1 and m and they are different from i and $g(d)$.

C. PARAMETER SELF-ADAPTATION STRATEGY

The best parameter values of ANS are generally problem-dependent. Choosing an appropriate parameter configuration by trial-and-error methods is computationally expensive [36]. In addition, different parameter values may be required at the different stages of the optimization process. Several parameter adaptation approaches have been proposed for population-based algorithms in the literature. For example, in [37] and [38], the population distribution information was used to guide the dynamic parameter adjustment. In [39], parameters were coded into the individual vectors and evolved with the population. In [40], the individual-level parameter adaptation and diversity maintenance were adopted and showed impressive effects.

In this study, we use two memory lists to store the recent values of the two parameters that led the algorithm to successfully produce promising solutions in the previous generations. Let memory list $List_i^\sigma$ store the promising values of σ_i with respect to solution i, and $List_i^n$ record the promising values of n_i with respect to solution i.

At each generation if solution i generates a better solution with certain values of σ_i and n_i, the values will be appended to lists $List_i^\sigma$ and $List_i^n$, respectively. If the lists are full, values earliest added to the list will be removed firstly before appending new values. Then the values of σ_i and n_i for the next generation are set to the median values of the two lists, i.e. $\sigma_i = median(List_i^\sigma)$ and $n_i = median(List_i^n)$.

In contrast, if individual i fails to produce a better solution at a generation, the values of σ_i and n_i will not be used to update the lists. In this case, σ_i and n_i for the next generation will be assigned with random values with the probabilities ρ_σ and ρ_n, respectively. The concrete rules are as below.

$$\sigma_i = \begin{cases} Gassian(0.5, 0.15) & \text{if rand()} < \rho_\sigma \\ median(List_i^\sigma) & \text{otherwise,} \end{cases} \tag{12}$$

$$n_i = \begin{cases} \text{random integer value among [1,D],} & \text{if rand()} < \rho_n \\ median(List_i^n) & \text{otherwise.} \end{cases} \tag{13}$$

According to the parameter update rules in (12) and (13), σ_i and n_i have certain probabilities to be assigned with random values if they previously failed to generate promising solutions. On the other hand, the better parameter values have higher probabilities to survive for the successive generations. The usage of the lists potentially make the parameter

Algorithm 1 The Framework of SaANS-SDP

Initialize the population size m, across-search degree n_i, standard deviation σ_i, and parameters ρ_σ and ρ_n;
Randomly initialize the position pos_i of each individual i;
Set the superior solution r_i to pos_i and the best solution g;
Initialize the allowed maximum generations $MaxG$ and set the current generation $k = 1$;
While $k < MaxG$
  $k = k + 1$;
  For $i = 1 \rightarrow m$
   Let set N_i record the randomly selected $n_i (1 \leq n_i \leq D)$ dimensions for individual i;
   For $d = 1 \rightarrow D$
    Generate two random integer values k_1 and k_2;
    If $d \notin N_i$
     $pos_i^d = r_i^d + G(0, \sigma_i^2) * |r_i^d - pos_i^d + r_{k_1}^d - r_{k_2}^d|$;
    Else if $d \in N$
     Randomly select a superior solution $r_{g(d)}$ from R $(g(d) \neq i)$;
     $pos_i^d = r_{g(d)}^d + G(0, \sigma_i^2) * |r_{g(d)}^d - pos_i^d + r_{k_1}^d - r_{k_2}^d|$;
    End if
   End for
   If pos_i is better than r_i
    Update r_i with pos_i;
    Update $List_i^\sigma$ and $List_i^n$
    Calculate $\sigma_i = median(List_i^\sigma)$ and $n_i = median(List_i^n)$
   Else

$$\sigma_i = \begin{cases} Gassian(0.5, 0.15) & \text{if rand()} < \rho_\sigma \\ median(List_i^\sigma) & \text{otherwise} \end{cases}$$

$$n_i = \begin{cases} \text{random integer value among [1,D],} & \text{if rand()} < \rho_n \\ median(List_i^n) & \text{otherwise} \end{cases}$$

   End if
   If pos_i is better than g
    Update g with pos_i;
   End if
  End for
End while

values be changed gradually to cater to faced landscapes, thus enhances the search capability of ANS.

IV. CONSTRAINT HANDLING TECHNIQUE FOR DED AND SED PROBLEMS

A. THE VARIABLE REDUCTION STRATEGY

Since the equality constraint in both DED and SED problems is the same, here we only present the equality constraint handling approach for the SED problem. Wu *et al.* [32] recently proposed a variable reduction strategy (VRS) to efficiently deal with equality constraints. We first give a brief introduction of VRS.

In general a constraint optimization problem (COP) can be formulated as:

$$\text{Minimize: } f(X) \tag{14}$$

$$\text{Subject to: } g_i(X) \leq 0, \quad i = 1, \ldots, p \tag{15}$$

$$h_j(X) = 0, \quad j = 1, \ldots, m \tag{16}$$

$$l_k \leq x_k \leq u_k, \quad k = 1, \ldots, n \tag{17}$$

where $X = (x_1, x_2, \ldots, x_n)$ denotes a solution of n variables; p is the number of inequality constraints; m is the number of equality constraints; l_k and u_k are the lower and upper bounds of x_k, respectively. An equality constraint are usually transformed into an inequality constraint as [27]: $|h_j(X)| - \varepsilon \leq 0$ when an evolutionary algorithm is used to solve a COP. Herein ε is a small threshold value. Various constraint handling techniques were proposed during the last decades [41], [42], including penalty function [43]–[45], feasibility rules [46], [47], stochastic ranking [48], [49], ε-constrained method [50], [51], and multi-objective concepts [52], [53].

It is widely agreed that it is harder to satisfy equality constraints as they usually make the feasible solution space be very small. Statistical analyses of the 24 benchmark COPs presented in CEC 2006 [54] reveal that the feasible region ratio becomes very low if a COP possesses equality constraints, indicating that equality constraints will make it difficult for EAs to search for feasible solutions. Research results have shown that the hardest thing for EAs solving COPs might be how to handle equality constraints efficiently [55].

An equality constraint is formulated as an equation, which actually provides a relationship among some variables in the considered COP. With the relationship involved in the equality constraint, if one variable can be explicitly represented by some other variables, then this variable can be reduced. This is because in the solution search process, the value of the reduced variable can be computed through the relationship and the values of other variables. As a result, the associated equality constraint can always be satisfied by all solutions. On this occasion, the equality constraints are no longer barriers for EAs to search for optimal solutions, but knowledge sources (i.e. variable relationships) being beneficial to the reduction of the complexity of COPs.

Assume that Ω denotes a set including variables in the COP, $\Omega = \{x_k | k = 1, 2, \ldots, n\}$; Ω_j is a collection containing variables involved in equality constraint $h_j(X) = 0$ $(1 \leq j \leq m)$. If we can obtain the following relationship from equation $h_j(X) = 0$

$$x_k = R_{k,j}(\{x_l | l \in \Omega_j, l \neq k\}), \tag{18}$$

then during optimization, x_k can be calculated from the relationship $R_{k,j}$ and the values of variables in set $\{x_l | l \in \Omega_j, l \neq k\}$. Hence, variable x_k can be reduced. Moreover, the equality constraint $h_j(X) = 0$ is eliminated at the same time, because the constraint $h_j(X) = 0$ is completely satisfied by all solutions after calculating the value of x_k. The bound constraint (17) related to the reduced variable x_k is converted to

$$l_k \leq R_{k,j}(\{x_l | l \in \Omega_j, l \neq k\}) \leq u_k. \quad (19)$$

Some key concepts related to the variable reduction strategy are introduced below.

Core Variable: those are the variable(s) used to represent other variables in terms of the derivative equations.

Reduced Variable: the reduced variable(s) is represented by using core variables.

Optimization Variable Core: the collection of all core variables present in the optimization problem.

Eliminated Equality Constraint: The equality constraint eliminated along with the reduction of variables due to full satisfaction by all solutions.

According to the concepts introduced above, we can conclude that the essential task of VRS is to obtain an optimal optimization variable core with minimum cardinality, such that more variables could be reduced and more equality constraints could be eliminated.

B. VARIABLE REDUCTION STRATEGY FOR HANDLING THE EQUALITY CONSTRAINT

VRS is employed to handle the equality constraint of the economic dispatch problem in this study. VRS not only reduces the number of variables but also eliminates the equality constraint from which variable relationships are derived. Through this manner, it speeds up the optimization process and improves the feasibility of the obtained solutions.

The equality constraint (3) is quadratic and thereby we can obtain the following variable relationship

$$P_1 = (-b \pm \sqrt{b^2 - 4ac})/2a, \quad (20)$$

where,

$$a = B_{11}, \quad (21)$$

$$b = \sum_{i=2}^{N_G} B_{1,i} \cdot P_i + \sum_{j=2}^{N_G} P_j \cdot B_{j,1} + B_{01} - 1, \quad (22)$$

$$c = \sum_{i=1}^{N_G} \sum_{j=2}^{N_G} P_i B_{ij} P_j + \sum_{i=2}^{N_G} P_i B_{oi} + B_{00} + P_D - \sum_{i=2}^{N_G} P_i. \quad (23)$$

Therefore, the reduced variable P_1 and core variables $P_i (i = 2, \ldots, N_G)$ are generated. The equality constraint is eliminated accordingly. Some occasions need to be considered to determine a proper value of P_1. If it is impossible to obtain a real value from (20) (i.e. $b^2 - 4ac < 0$), the value of P_1 will not be changed. If there are two values derived from (20) while none of them satisfies the inequality constraints (5), (6) and (7), the value of P_1 will not be changed. If there are two values derived from (20) and only one value satisfy the inequality constraints, P_1 will be set to the feasible value. If there are two values derived from (20) and both values satisfy the inequality constraints, the value producing better objective function will be assigned to P_1.

It is noted that only variable P_1 is reduced, as in the mathematical model there exists just one equality constraint. However, the most important for the variable reduction strategy in this study is its ability to handle the nonlinear equality constraint, i.e., eliminate the equality constraint successfully, thus it enables the search algorithm to find high-quality and feasible solutions more efficiently. This is because nonlinear equality constraints will cause the algorithm to waste too many computational resources to find feasible solutions, and thereby deteriorate the performance of the algorithm in searching for optimal solutions. In addition, although the reduction of one variable seems negligible if the number of total variables is large, it still exerts positive effects on the optimization process.

C. EPSILON CONSTRAINT METHOD FOR HANDLING INEQUALITY CONSTRAINTS

Besides equality constraint, there are also bound constraints (capacity constraints and prohibited operating zone constraints) and linear inequality constraints (ramp rate limit constraints) in the economic dispatch problem. To tackle the bound constraints, we pull it back to a symmetrical value of its current value with respect to the variable bounds if the current value is out of the bounds. Suppose that x_i is a variable in the economic dispatch problem and l_i and u_i are its lower and upper bounds. If the value of x_i is smaller than l_i, update x_i as

$$x_i = l_i + (l_i - x_i). \quad (24)$$

If x_i is greater than u_i, update x_i as

$$x_i = u_i - (x_i - u_i). \quad (25)$$

A core problem in constraint handling when using metaheuristics to solve COPs is solution ranking, which bias the search direction of the algorithm. As during the optimization process, solutions can be feasible or infeasible with different constraint violation degrees, the solution ranking is not as straightforward as that in unconstrained optimization. That is to say, the solution ranking in constrained optimization need to consider the objective function values and constraint violation values simultaneously. The equality constraint is handled by VRS. With regard to the inequality constraints of the economic dispatch problem, we apply the efficient ε-constraint method proposed by Takahama and Sakai [50], [56], which was combined into a DE algorithm and won the CEC 2010 competition on constrained real-parameter optimization [50]. The value of ε, satisfying $\varepsilon > 0$, determines the so-called ε-level comparisons between a pair of solutions x_1 and x_2 with objective function values $f(x_1)$ and $f(x_2)$ and sums of constraint violation $\varphi(x_1)$ and $\varphi(x_2)$ [41].

$$(f(x_1), \varphi(x_1)) \leq_\varepsilon (f(x_2), \varphi(x_2)) \Leftrightarrow \begin{cases} f(x_1) \leq f(x_2), & \text{if } \varphi(x_1), \varphi(x_2) \leq \varepsilon \\ f(x_1) \leq f(x_2), & \text{if } \varphi(x_1) = \varphi(x_2) \\ \varphi(x_1) < \varphi(x_2), & \text{otherwise} \end{cases} \quad (26)$$

Formula (26) indicates that for any two solutions x_1 and x_2, if their constraint violation values are equal, they are ranked according to their objective function values. If both of their constraint violation values are below a threshold value ε, the two solutions are ranked in terms of the objective function values. If the two constraint violation values are not equal and not both of them are smaller than ε, the two solutions will be ranked with respect to the constraint violation values.

V. EXPERIMENTAL STUDY

A. EXPERIMENTAL SETTING

To test the performance of the proposed method, extensive experiments were conducted. Instances used in the experimental studies are described below.

Case 1: DED problem with 5 generators considering power loss. The same case is also widely adopted in other literature [57]–[59]. There are 120 independent variables in this case. Related parameter data can be found in [60].

Case 2: DED problem with 10 generators considering power loss. There are 240 variables included in this case. The parameter data of this case can be found in [8] and [61].

Case 3: SED problem with 6 units considering power loss. There are 6 variables in this case. The parameter data of this case can be found in [62].

Case 4: SED problem with 13 units without considering power loss. Thirteen variables are included in this case. The parameter data of this case can be found in [3].

Case 5: SED problem with 15 units considering power loss. Fifteen variables are included in this case. The parameter data of this case can be found in [62].

Case 6: SED problem with 40 units without considering power loss. Forty variables are included in this case. The parameter data of this case can be found in [62].

The initial values of each parameter n_i and σ_i of SaANS-SDP are set to 2 and 0.5, respectively. The population size is set to 30. The probability values of both ρ_σ and ρ_n are set to 0.1. In addition, the control epsilon level for handling inequality constraints follows the approach provide in [50].

B. COMPARISON WITH OTHER STATE-OF-THE-ART METHODS

In this section, we compare SaANS-SDP combined with VRS presented in this study with some other popular and efficient approaches in recent publications. Here we report the comparison results of Cases 1-6 in Tables 1-6, respectively.

TABLE 1. Comparison of optimization results for case 1.

Method	Fuel Cost ($/hr)			Violation
	Minimum	Average	Maximum	
SaANS-SDP with VRS	**4.3399e+04**	**4.4016e+04**	**4.4126e+04**	**6.8056e-05**
ANS with VRS	4.4012e+04	4.4087e+04	4.4232e+04	6.5412e-05
AIS [57]	4.4385e+04	4.4759e+04	4.5554e+04	NA
GA[59]	4.4862e+04	4.4922e+04	4.5894e+04	NA
ABC[59]	4.4046e+04	4.4065e+04	4.4219e+04	NA
APSO [58]	4.4678e+04	NA	NA	NA
MSL[65]	4.9216e+04	NA	NA	NA

TABLE 2. Comparison of optimization results for case 2.

Method	Fuel Cost ($/hr)			Violation
	Minimum	Average	Maximum	
SaANS-SDP with VRS	**1.0411e+06**	**1.0443e+06**	**1.0474e+06**	**2.0463e-09**
ANS with VRS	1.0450e+06	1.0468e+06	1.0481e+06	2.1548e-09
EP-SQP [66]	1.0527e+06	1.0538e+06	NA	NA
MHEP-SQP[66]	1.0501e+06	1.052,3e+06	NA	NA
IPSO[67]	1.0463e+06	1.0481e+06	NA	NA
AIS [57]	1.0457e+06	1,0471e+06	1.0484e+06	NA
ABC[59]	1.0433e+06	1.0450e+06	1.0468e+06	NA

TABLE 3. Comparison of optimization results for case 3.

Method	Fuel Cost ($/hr)			Violation
	Minimum	Average	Maximum	
Sa**ANS-SDP with VRS**	**1.5444e+04**	**1.5444e+04**	**1.5444e+04**	**3.9571e-05**
ANS with VRS	1.5444e+04	1.5444e+04	1.5444e+04	3.6254e-05
SOH-PSO [12]	1.5446e+04	1.5497e+04	1.5610 e+04	NA
NPSO-LRS[68]	1.5450e+04	NA	NA	NA
GA[68]	1.5451e+04	NA	NA	NA
GAAPI[69]	1.5450 e+04	15450 e+04	1.5450 e+04	NA
SA-PSO[70]	1.5447 e+04	1.5447 e+04	1.5455 e+04	NA

TABLE 4. Comparison of optimization results for case 4.

Method	Fuel Cost ($/hr)			Violation
	Minimum	Average	Maximum	
SaANS-SDP with VRS	**1.7964e+04**	1.7874e+04	1.8024e+04	9.0949e-09
ANS with VRS	1.7976e+04	1.7992e+04	1.8158e+04	9.4586e-09
CEP [3]	1.8048 e+04	1.8190 e+04	1.8404 e+04	NA
MFEP[3]	1.8028 e+04	1.8192 e+04	1.8417 e+04	NA
FEP[3]	1.8018 e+04	1.8201 e+04	1.8454 e+04	NA
IFEP[3]	1.7994 e+04	1.8127 e+04	1.8267 e+04	NA
PSO-SQP[71]	1.7970 e+04	1.8030 e+04	NA	NA
DEC-SQP[72]	1.7964 e+04	**1.7973 e+04**	**1.7985 e+04**	NA
EP-SQP[66]	1.7991 e+04	1.8107 e+04	NA	NA

TABLE 5. Comparison of optimization results for case 5.

Method	Fuel Cost ($/hr)			Violation
	Minimum	Average	Maximum	
SaANS-SDP with VRS	**3.2693e+04**	**3.2693e+04**	**3.2693e+04**	5.6284e-06
ANS with VRS	3.2696e+04	2.3697e+04	3.2699e+04	5.2375e-06
SOH-PSO[12]	3.2751 e+04	3.2878 e+04	3.2945 e+04	NA
GAAPI[69]	3.2733 e+04	32735 e+04	32756 e+04	NA
SA-PSO[70]	32708 e+04	32747 e+04	32807 e+04	NA
FA[63]	32705 e+04	32856 e+04	33175 e+04	NA

The significance test results are not provided in the tables, though they are thought to be important in comparing the results obtained by different stochastic algorithms. This is because the source codes of the comparative algorithms are not available. Therefore, we only use the data reported in the literature, which cannot support the significant test.

In Tables 1-6, we give the results of both objective function values and constraint violation values. It is found that in the previous literature, researchers generally only provided the

TABLE 6. Comparison of optimization results for case 6.

Method	Fuel Cost ($/hr)			Violation
	Minimum	Average	Maximum	
SaANS-SDP with VRS	**1.2141e+05**	**1.2142e+05**	**1.2145e+05**	0.00
ANS with VRS	1.2143e+05	1.2146e+05	1.2178e+05	0.00
ICA-PSO[11]	1.2142 e+05	NA	NA	NA
SOH-PSO[12]	1.2501 e+05	1.2185 e+05	1.2245 e+05	NA
GAAPI[69]	1.3987 e+05	NA	NA	NA
DEC-SQP[72]	1.2174 e+05	1.2229 e+05	1.2284 e+05	NA
PSO-SQP[71]	1.2209 e+05	1.2225 e+05	NA	NA
NPSO-LRS[68]	1.2166 e+05	1.2221 e+05	1.2298 e+05	NA
SA-PSO[70]	1.2143 e+05	1.2257 e+05	1.2371 e+05	NA
FA[63]	1.2142 e+05	1.2142 e+05	1.2142 e+05	NA

TABLE 7. Generator output for the case 3 (six-unit system).

Power output (MW)	SPSO[12]	PC_PSO[12]	SOH_PSO[12]	SaANS-SDP
P1	473.66	473.79	438.21	256.29
P2	140.00	195.98	172.58	173.1
P3	240.06	256.72	257.42	262.81
P4	149.97	149.36	141.09	143.51
P5	173.78	166.20	179.37	163.95
P6	97.91	69.26	86.88	85.369
Total power output	1275.83	1275.31	1275.55	1085.03
Total loss	12.38	12.32	12.55	12.422
Total generation cost	15466.63	15453.09	15446.02	15444.18

final objective function values while related constraint violation values were missing. However, the constraint violation degree definitely exerts significant influences on the final results. For example, Yang *et al.* [63] presented a firefly algorithm for solving the non-convex economic dispatch problem and obtained competitive results. However, according to the data (corresponding to Case 6) reported in [63], Table 9, the power balance constraint is slightly violated with an amount of 1.0. Besides, in [64] the authors proposed a random drift particle swarm optimization algorithm for solving economic dispatch problem. In [64], the reported solution for Case 6 also violates the power balance constraint with violation value of 3.9. Methods may not be fairly compared solely based on objective function values if the obtained results are with different constraint violations.

From the data displayed in Tables 1-6, some observations can be obtained. First, for case 1, Case 2, Case 3, Case 5 and Case 6, compared with other methods, SaANS-SDP with VRS obtain the best results robustly. Second, SaANS-SDP with VRS is slightly worse than DEC-SQP for Case 4 with the fact that although SaANS-SDP produces as good minimum results as DEC-SQP, its average and maximum results are inferior to DEC-SQP. Third, all the results obtained by SaANS-SDP are with very small constraint violation values which demonstrates that, with the aid of VRS, SaANS-SDP could generate high-quality feasible solutions for both SED and DED problems. Fourth, SaANS-SDP with VRS could robustly generate better results than ANS with VRS for all

TABLE 8. Generator output for case 5 (the fifteen-unit system).

Power output (MW)	SPSO[12]	PC_PSO[12]	SOH_PSO[12]	SaANS-SDP
P1	455.00	455.00	455.00	352.65
P2	380.00	380.00	380.00	380.00
P3	130.00	130.00	130.00	130.00
P4	129.28	127.15	130.00	130.00
P5	164.77	169.91	170.00	170.00
P6	460.00	460.00	459.96	460.00
P7	424.52	430.00	430.00	430.00
P8	60.00	108.38	117.53	75.53
P9	25.00	77.41	77.90	54.20
P10	160.00	97.76	119.54	159.89
P11	80.00	67.61	54.50	80.00
P12	72.62	73.26	80.00	80.00
P13	25.00	25.57	25.00	25.00
P14	44.83	19.57	17.86	15.00
P15	49.42	38.93	15.00	15.00
Total power output	2660.44	2660.55	2662.29	2557.27
Total loss	30.49	30.54	32.28	29.62
Total generation cost	32798.69	32775.36	32751.39	32692.51

the experimental instances, which shows that the adopted parameter self-adaptation strategy and the solution-difference perturbation mechanism indeed have strengthen the capability of canonical ANS.

According to the comparison analyses above, it is safely to conclude that SaANS-SDP is a competitive alternative for solving SED and DED problems. The reasons can be explained as below.

First, a solution-difference perturbation mechanism is combined into ANS, such that ANS could dynamically make use of the real-time population diversity information to bias the search behavior. ANS with solution-difference perturbation mechanism could achieve a good balance between exploitation and exploration. Second, the solution landscape of an economic dispatch problem is complicated, as it includes complex nonlinear objective function and several constraints. Therefore, different parameter values are required when confronting different landscapes during the optimization process. The parameter self-adaptation strategy alleviates the parameter configuration process and more importantly it make the parameter values evolve with the solution search process to fit to the current landscape, which significantly improves the search capability of ANS. Third, constraint handling techniques play key roles for meta-heuristics generating high-quality feasible solutions for COPs. Particularly, it is hard for meta-heuristic to generate solutions satisfying the complex nonlinear equality constraint. Therefore, without effective equality constraint handling techniques, a meta-heuristic will spend too many computational resources in finding feasible solutions and this is detrimental to the optimization. In contrast, in this study, VRS guarantees that any generated solution always meets the equality constraint. Furthermore, VRS reduces a variable of the economic dispatch problem, which thereby decreases the

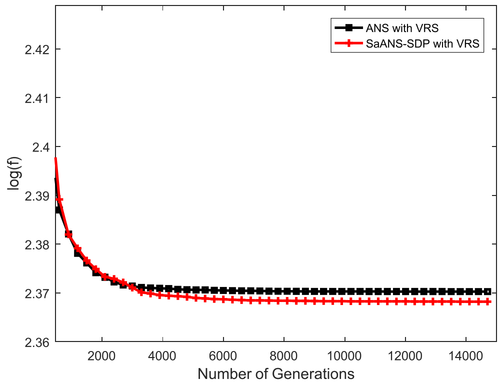

FIGURE 2. The convergence processes of SaANS-SDP with VRS and ANS with VRS for solving Case 1.

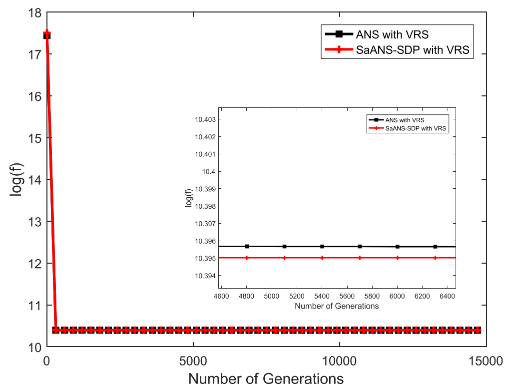

FIGURE 3. The convergence processes of SaANS-SDP with VRS and ANS with VRS for solving Case 5.

dimensionality of the solution space and improves the search efficiency of ANS.

The detailed results on the power output of each unit, total power outputs, total losses and total generation costs for Case 3 and Case 5 are provided as examples in Table 7 and Table 8, respectively.

The convergence processes of SaANS-SDP with VRS and ANS with VRS for solving Case 1 and Case 5 are plotted in Fig. 2 and Fig. 3 as examples. It can be observed that both the algorithms have satisfactory convergence ability. SaANS-SDP with VRS, by contrast, exhibits better performance than ANS with VRS. This is because, on one hand, the parameter self-adaptation strategy makes SaANS-SDP have more appropriate parameter values when searching the solution space, thus speeding up the optimization process. On the other hand, the solution-difference perturbation mechanism has the ability to balance the exploration and exploitation, which efficiently prevents the algorithm from premature convergence.

VI. CONCLUSIONS

In this study, we propose a novel across neighborhood search (ANS) algorithm to efficiently solve both static and dynamic economic dispatch problems. In order to strengthen the optimization capability of ANS, two advanced strategies are incorporated. One is a solution-difference perturbation mechanism, which makes use of the real-time solution distance information to achieve a good balance between exploitation and exploration. Another is a parameter self-adaptation strategy, which controls the parameters of ANS dynamically during the optimization process to fit to different landscapes. In addition, a variable reduction strategy (VRS) is used to handle the nonlinear equality constraint in DED and SED problems. VRS eliminates the equality constraint and reduces the number of variables simultaneously. It makes it easier to generate high-quality feasible solutions and thus speeds up the optimization process noticeably. Extensive experiments show that the proposed method could produce state-of-the-art solutions for DED and SED problems.

In our future research, we plan to further improve the capability of ANS via some other advanced techniques, such as hybridization with other meta-heuristics and mathematical programming approaches, in order to solve the economic dispatch problem more efficiently. In addition, it is noticeable that when we using variable reduction strategy in this study, there are many candidate variables that could be reduced. Then which variable is the best choice for reduction? It is therefore meaningful to investigate this problem theoretically and practically. Lastly, we believe that the proposed method can be applied to more real-world optimization problems especially in power systems, which are often associated with equality constraints expressing the power balance requirements.

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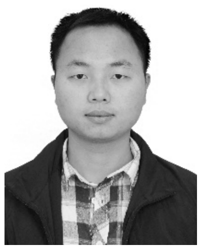

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