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A Robust Finite-Time Output Feedback Control Scheme for Marine Surface Vehicles Formation

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ABSTRACT In view of the marine surface vehicles are subject to unknown time-varying external disturbance and its velocities are not available for feedback, a robust finite-time output feedback control scheme is presented for multiple marine surface vehicles to achieve precise formation control in this paper. Without neglecting the first time derivative of disturbance, a novel finite-time extended state observer (FTESO) is proposed to precisely estimate the external disturbance and velocity information in finite time. Based on the outputs of FTESO, the distributed formation controller is designed to guarantee that a group of marine surface vehicles can track the time-varying virtual leader in finite time with precise tracking performance. The errors of the proposed FTESO and formation controller can be guaranteed to converge to zero in finite time by using the homogeneous method and the Lyapunov theory. Numerical simulations are presented to illustrate the effectiveness of the proposed finite-time output feedback formation control scheme.

INDEX TERMS Marine surface vehicles, finite-time extended state observer, formation control, output feedback, disturbances.

I. INTRODUCTION

Formation control of marine surface vehicles (MSVs) has attracted significant attention recently, because of its widespread applications in sea investigation, exploration, maritime rescue, and surveillance. Furthermore, the vehicles teamwork obtains more robustness, redundancy and efficiency than a single vehicle. Therefore, many strategies were proposed to achieve the desired formation in the literature, such as: guided leader-follower control approach [1], passivity-based scheme [2], sliding mode control method [3], dynamic surface control technique [4], and guided leaderless control algorithm [5].

However, it is well known that the dynamics of MSV in three degree-of-freedom (DOF) (surge, sway, and yaw) are strongly coupled, and the motion of MSV inevitably suffers from environmental disturbance induced by waves, winds and ocean currents [6]. For estimating external disturbance, a widely used technique is neural network. The neural network-based controllers were designed for multiple MSVs to accurately identify the uncertainties and time-varying ocean disturbances in [7]–[9]. Although this method is an effective way to deal with disturbance, the asymptotic stability of the closed-loop tracking system can not be realized. This is because the neural network has an approximate error. Therefore, the disturbance can not be estimated and compensated accurately. To precisely estimate the disturbance acting on MSVs, an alternative solution is to design nonlinear observer [10]. In [11], a disturbance observer was employed to compensate disturbance uncertainties for trajectory tracking control of ship. Asymptotic estimation of disturbance was achieved. In [12], a finite-time disturbance observer was proposed for fully actuated surface vehicles. In [13], a terminal sliding mode observer-based estimation approach was developed to provide the formation controller design for vessels. The actual external disturbance can be precisely estimated with zero estimation error after finite time.

In addition, it should be stressed that most of the existing formation controller require velocity measurements of all MSVs, which are not easy to be measured because of the noise contamination. Moreover, velocity sensors will increase the weight and cost of MSVs. But, the position information can be easily obtained by using Global Positioning System [14] . Hence, in [15], a high-gain observer-based cooperative path following controller was developed without measuring the velocity of each vehicle, where the neural network adaptive technique was used to handle the dynamical uncertainty and ocean disturbances. In [16], a nonlinear saturated observer was introduced to estimate velocities of the followers, and the multi-layer neural network was developed to against uncertain nonlinearities and environmental disturbances. In the literature, to estimate both the velocity information and the external disturbance, another solution is to design external state observer (ESO). The ESO was first proposed in [17], which has the capability of state observation and can provide real-time estimation of system uncertainties and disturbances, does not dependent on a accurate system model. Because of its effectiveness, a variety of applications for the ESO-based control schemes were emerged in [18]–[22], but few investigations are available for MSV. In [23], the Generalized Extended State Observer (GESO) was presented to estimate the effects of the disturbances along with the system state vector for steering autopilot of MSVs.

Although the preceding output feedback formation control schemes are able to achieve the desired formation for MSVs, almost of them are asymptotically convergent algorithms, which means that the optimal convergence rate is exponential with infinite settling time. In other words, the motion control with high-accuracy cannot be achieved in finite time. Recently, given that the finite-time control has the advantages of the fast convergence rate, high accuracy, and disturbance rejection properties, many studies were arisen in [24] and [25]. Yan *et al.* [26] discussed the problem of finite-time trajectory tracking control for underactuated unmanned underwater vehicles (UUVs) with model parameter perturbation and constant unknown current in the horizontal plane. In [27], a nonlinear disturbance observer-based backstepping finite-time sliding mode control scheme for trajectory tracking of underwater vehicles subject to unknown system uncertainties and time-varying external disturbances was proposed. With angle constraints and system fault tolerant, a finite-time leader-follower formation control scheme was proposed in [28].

Suppose that the velocity measurements and the external disturbance cannot be estimated for MSVs within finite time. Then, the corresponding observer-based controller would not compensate for the external disturbance and velocity measurements in finite time, and this will result in deterioration of control performance. Therefore, to solve this problem, a novel robust finite-time output feedback formation control scheme for MSVs is proposed in this paper. The main contributions of this work are expressed as follows: (1) A novel finite-time extended state observer (FTESO) is proposed. In comparison with the existing observer design for MSVs such as [12], [13], [15], and [16], the velocity measurements of the MSVs and the external disturbance acting on the MSVs can be estimated simultaneously by the FTESO. (2) The convergence of ESO was discussed in [29] and [30]. However, the convergence cannot be achieved in finite time. In contrast, the estimation errors of the proposed FTESO can be governed to converge to zero in finite time. A fast and precise estimation of external disturbance and velocity measurements can thus be achieved. (3) Based on the outputs of FTESO, the finite-time formation control laws are provided for MSVs to achieve the desired formation with precise control performance. Compared with many previous works, all of the errors in the closed-loop system can converge to zero in finite time by using homogeneous method and Lyaunov theory.

The rest of the paper is organized as follows. The preliminaries and problem formulation are presented in the next section. Section 3 presents the main result of the robust finite-time output feedback control scheme, including the finite-time stability analysis. In Section 4, comparison simulation results are provided to illustrate the effectiveness of the proposed formation control scheme. Conclusions are given in Section 5.

Throughout this paper, the notation $\|\cdot\|$ denotes the Euclidean norm of a vector. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ refers to the maximum and minimum eigenvalues of a matrix. I_n represents the $n \times n$ identity matrix. The Kronecker product is denoted by $\otimes$. Given a vector $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}$, define $x^\alpha = [{x_1}^\alpha, {x_2}^\alpha, \ldots, {x_n}^\alpha]^T \in \mathbb{R}^n$, $\mathrm{sig}^\alpha(x) = \left[|x_1|^\alpha \mathrm{sgn}(x_1), |x_2|^\alpha \mathrm{sgn}(x_2), |x_3|^\alpha \mathrm{sgn}(x_3)\right]^T$, $\mathrm{sgn}(\cdot)$ is the signum function.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. DEFINITIONS AND LEMMAS

Definition 1 [31]: Consider the following system :

$$\dot{x} = f(x, t), \quad f(0, t) = 0, \ x \in U \subset \mathbb{R}^n \tag{1}$$

where $f : U \times \mathbb{R}^+ \to \mathbb{R}^n$ is continuous on an open neighborhood U of the origin $x = 0$. The zero solution of (1) is (locally) finite-time stable if it is Lyapunov stable and finite-time convergent in a neighborhood $U_0 \subseteq U$ of the origin. The finite-time convergence means: for any initial condition $x(t_0) = x_0 \in U_0$ at any given initial time t_0, if there is a settling time $T > t_0$, such that every solution $x(t; t_0, x_0)$ of system (1) satisfies $x(t; t_0, x_0) \in U_0 \backslash \{0\}$ for $t \in [t_0, T)$, and $\lim_{t \to T} x(t; t_0, x_0) = 0$, $x(t; t_0, x_0) = 0$, $\forall t > T$. When $U = U_0 = \mathbb{R}^n$, then the zero solution is said to be globally finite-time stable.

Lemma 1 [32]: Suppose that there is a positive definite continuous Lyapunov function $V(x, t)$ defined on $U_1 \times \mathbb{R}^+$, where $U_1 \subseteq U \in \mathbb{R}^n$ is a neighborhood of the origin, and

$$\dot{V}(x, t) \leq -cV^\alpha(x, t), \quad \forall x \in U_1 \backslash \{0\} \tag{2}$$

where $c > 0$ and $0 < \alpha < 1$. Then the origin of system (1) is locally finite-time stable. The settling time satisfies $T \leq \frac{V^{1-\alpha}(x(t_0), t_0)}{c(1-\alpha)}$ for a given initial condition $x(t_0) \in U_1$.

Lemma 2 [33]: Suppose that there is a positive definite continuous Lyapunov function $V(x, t)$ defined on $U_1 \in \mathbb{R}^n$ of the origin, and

$$\dot{V}(x, t) \leq -c_1 V^\alpha(x, t) + c_2 V(x, t), \quad \forall x \in U_1 \backslash \{0\} \tag{3}$$

where $c_1 > 0, c_2 > 0$ and $0 < \alpha < 1$. Then the origin of system (1) is locally finite-time stable. The set $U_2 = \{x | V^{1-\alpha}(x, t) \leq \frac{c_1}{c_2}\}$ is contained in the domain of attraction of the origin. The settling time satisfies $T \leq \ln(1-(c_2/c_1)V^{1-\alpha}(x_0,t_0))/(c_2\alpha - c_2)$ for a given initial condition $x(t_0) \in \{U_1 \cap U_2\}$.

Lemma 3 [34]: For $\forall x, y \in \mathbb{R}$, if $c > 0$, $d > 0$, and $\gamma > 0$, then

$$|x|^c|y|^d \leq c\gamma|x|^{c+d}/(c+d) + d|y|^{c+d}/[\gamma^{c/d}(c+d)]$$

Lemma 4 [35]: For $\forall x_i \in \mathbb{R}$, $i = 1, \ldots, n$, and $0 < p \leq 1$, then

$$\left(\sum_{i=1}^{n}|x_i|\right)^p \leq \sum_{i=1}^{n}|x_i|^p \leq n^{1-p}\left(\sum_{i=1}^{n}|x_i|\right)^p$$

Lemma 5 [36]: For any $x_i \in \mathbb{R}$, $i = 1, 2, \ldots, n$, and a real number $p > 1$,

$$\sum_{i=1}^{n}|x_i|^p \leq \left(\sum_{i=1}^{n}|x_i|\right)^p \leq n^{p-1}\sum_{i=1}^{n}|x_i|^p$$

B. GRAPH THEORY

An undirected connected graph $\mathcal{G} = \mathcal{G}(\nu, \varepsilon)$ is used to describe the communication topology between the n MSVs, where $\nu = \{1, 2, \ldots, n\}$ represents the set of vehicles, and $\varepsilon \subseteq \nu \times \nu$ denotes the edge set. The edge $(i,j) \in \varepsilon$ denotes that there is an interaction link between ith vehicle and jth vehicle. Since the graph is undirected, $(i,j) \in \varepsilon \Leftrightarrow (j,i) \in \varepsilon$. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ associated with $\mathcal{G}$ is defined as $a_{ij} > 0$ if $(i,j) \in \varepsilon$, and $a_{ij} = 0$ otherwise. That is, the adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ is a symmetric matrix. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ of the graph $\mathcal{G} = \mathcal{G}(\nu, \varepsilon)$ is defined as $l_{ij} = \begin{cases} -a_{ij}, i \neq j \\ \sum\limits_{j=1, j\neq i}^{n} a_{ij}, i = j \end{cases}$, which is a symmetric matrix. The interaction among the virtual leader and the n followers is described by $\bar{\mathcal{G}}$. Let $B = \text{diag}\{b_1, b_2, \ldots, b_n\}$ be the adjacency matrix associated with $\bar{\mathcal{G}}$, where $b_i > 0$ if the ith vehicle has access to the leader, otherwise $b_i = 0$. For the graph $\bar{\mathcal{G}}$, if there exists a path from the leader to every vehicle, then the graph $\bar{\mathcal{G}}$ is connected.

Lemma 6 [37]: If $\bar{\mathcal{G}}$ is connected, then the matrix $L + B$ associated with $\bar{\mathcal{G}}$ is symmetric and positive definite.

C. PROBLEM FORMULATION

Let $\eta_i = [x_i, y_i, \psi_i]^T$, $[x_i, y_i]^T$ be the position of the ith $(i = 1, \ldots n)$ MSV, and ψ_i be the heading angle of the ith MSV in the Earth-fixed inertial frame, and $\nu_i = [u_i, v_i, r_i]^T$ be the corresponding velocity vector of the ith MSV in the Body-fixed frame, where the Earth-fixed inertial frame and the Body-fixed frame are shown in Fig.1. Then, the dynamical model of vehicle i in the three degrees of freedom (DOF) is presented as follows [38]:

$$\dot{\eta}_i = R_i(\psi_i)\nu_i \tag{4}$$

$$M_i\dot{\nu}_i + C_i(\nu_i)\nu_i + D_i(\nu_i)\nu_i = \tau_i + M_i{R_i}^T(\psi_i)\omega_i \tag{5}$$

where $R_i(\psi_i) = \begin{bmatrix} \cos\psi_i & -\sin\psi_i & 0 \\ \sin\psi_i & \cos\psi_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the rotation matrix from the Body-fixed frame to the Earth-fixed inertial frame,

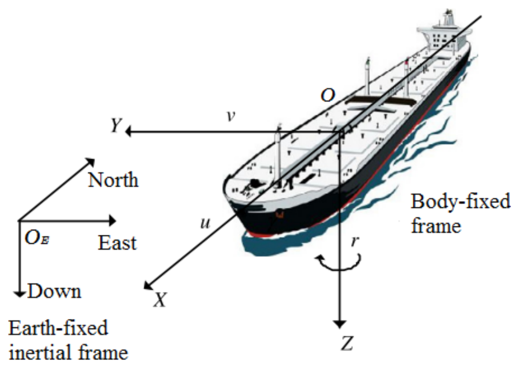

FIGURE 1. The Earth-fixed inertial frame and Body-fixed frame.

${R_i}^T R_i = I_3$. $M_i = {M_i}^T \in \mathbb{R}^{3\times 3}$ is the inertia matrix, $C_i(\nu_i) \in \mathbb{R}^{3\times 3}$ is a skew-symmetric matrix of coriolis term, $D_i(\nu_i) \in \mathbb{R}^{3\times 3}$ is a hydrodynamic damping matrix. $\tau_i = [\tau_{ui}, \tau_{vi}, \tau_{ri}]^T$ denotes the control input. $\omega_i = [\omega_{ui}, \omega_{vi}, \omega_{ri}]^T$ denotes the time-varying external disturbances caused by winds, waves, and ocean currents acting on the ith MSV.

By defining $\mu_i = \dot{\eta}_i$, we transform the dynamical model of vehicle i as follows:

$$\dot{\eta}_i = \mu_i \tag{6}$$

$$\dot{\mu}_i = R_i M_i^{-1}\tau_i + f(\eta_i, \mu_i) + \omega_i \tag{7}$$

where$f(\eta_i, \mu_i) = S(r_i)\mu_i - R_i M_i^{-1}(C(\nu_i) + D(\nu_i))R_i^T\mu_i$, $S(r_i) = \begin{bmatrix} 0 & -r_i & 0 \\ r_i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

The finite-time formation control problem for MSVs is to maintain a desired formation and track the virtual leader $\eta_d(t)$ within finite time. In this paper, the signal $\eta_d(t)$ is differentiable, and η_d, $\dot{\eta}_d$, $\ddot{\eta}_d$ are bounded. Then, the control objective of this paper is to design a robust finite-time output feedback formation control law τ_i for each vehicle to satisfy the following formulas (1) $\eta_i + \delta_i \rightarrow \eta_j + \delta_j \rightarrow \eta_d$, (2)$\dot{\eta}_i + \dot{\delta}_i \rightarrow \dot{\eta}_j + \dot{\delta}_j \rightarrow \dot{\eta}_d$ within finite time, where $\delta_i = R(\psi_i - \psi_{0i})l_i$ denote the relative deviations in Earth-fixed inertial frame, $l_i = [x_{0i}, y_{0i}, \psi_{0i}]^T \in \mathbb{R}^3 (-\pi \leq \psi_{0i} \leq \pi)$ are the configuration vectors in Body-fixed frame that determine the configuration of each MSV in the formation.

Assumption 1: The rate of external disturbance $\dot{\omega}_i = \Delta_i$ is unknown but bounded, which satisfies the following inequality: $\|\Delta_i\| \leq \bar{\Delta}_i$, where $\bar{\Delta}_i$, $i = 1, \ldots n$ denote the upper bound of $\dot{\omega}_i$.

Assumption 2: The signals of vehicle i are assumed to be bounded, and there exist a positive constant ε and a compact set Ω_1, such that $\Omega_1 = \{(\eta_i, \mu_i, \dot{\eta}_i, \dot{\mu}_i) | \, \|\eta_i\| \leq \varepsilon, \|\mu_i\| \leq \varepsilon, \|\dot{\eta}_i\| \leq \varepsilon, \|\dot{\mu}_i\| \leq \varepsilon\}$.

Remark 1: The external disturbance ω_i acting on the ith MSV is caused by winds, waves, and ocean currents. Their rates are bounded in practical engineering. Therefore, Assumption 1 is reasonable. In addition, the motion of MSV is a rigid body movement, so the signals of the MSV are considered to be bounded.

III. MAIN RESULTS

A. FINITE-TIME EXTENDED STATE OBSERVER

To estimate both the velocity measurements and the time-varying disturbance in finite time, the FTESO for MSV described by (6)(7) is designed as follows:

$$\begin{cases} \dot{\hat{\eta}}_i = \hat{\mu}_i + k_{1i}\mathrm{sig}^{\alpha_1}(z_{1i}) + \lambda_{1i}\mathrm{sgn}(z_{1i}) \\ \dot{\hat{\mu}}_i = R_i M_i^{-1}\tau_i + \hat{\omega}_i + f(\eta_i, \hat{\mu}_i) + k_{2i}\mathrm{sig}^{\alpha_2}(z_{1i}) \\ \quad +\lambda_{2i}\mathrm{sgn}(z_{1i}) \\ \dot{\hat{\omega}}_i = k_{3i}\mathrm{sig}^{\alpha_3}(z_{1i}) + \lambda_{3i}\mathrm{sgn}(z_{1i}) \end{cases} \tag{8}$$

where $z_{1i} = \eta_i - \hat{\eta}_i$, $z_{2i} = \dot{\eta}_i - \hat{\mu}_i$, and $z_{3i} = \omega_i - \hat{\omega}_i$. $\hat{\eta}_i$, $\hat{\mu}_i$, and $\hat{\omega}_i$ denote the estimates of η_i, μ_i, and ω_i, respectively. In addition, $\frac{2}{3} < \alpha_1 < 1$, $\alpha_2 = 2\alpha_1 - 1$, $\alpha_3 = 3\alpha_1 - 2$. $k_{mi} > 0$, and $\lambda_{mi} > 0$, $(m = 1, 2, 3)$ are control parameters.

Together with (6), (7) and (8), the estimation error system of FTESO can be expressed as

$$\begin{cases} \dot{z}_{1i} = z_{2i} - k_{1i}\mathrm{sig}^{\alpha_1}(z_{1i}) - \lambda_{1i}\mathrm{sgn}(z_{1i}) \\ \dot{z}_{2i} = z_{3i} + f(\eta_i, \mu_i) - f(\eta_i, \hat{\mu}_i) - k_{2i}\mathrm{sig}^{\alpha_2}(z_{1i}) \\ \quad -\lambda_{2i}\mathrm{sgn}(z_{1i}) \\ \dot{z}_{3i} = \Delta_i - k_{3i}\mathrm{sig}^{\alpha_3}(z_{1i}) - \lambda_{3i}\mathrm{sgn}(z_{1i}) \end{cases} \tag{9}$$

Theorem 1: Consider the MSV described in (4) and (5) with external disturbances, and suppose that the Assumptions 1-2 are satisfied. The FTESO is designed in (8) for MSV to estimate the velocity measurements and external disturbances simultaneously. Then, the estimated errors of FTESO can converge to the following residual set

$$\Omega_2 = \left\{ (z_{1i}, z_{2i}, z_{3i}) \,\middle|\, \|(z_{1i}, z_{2i}, z_{3i})\| \le \frac{3^{1-\frac{\sigma}{2}}}{\sqrt{\lambda_{\min}(P_i)}^{\sigma}} \left(\frac{c_{2i}+c_{3i}+c_{4i}}{c_{1i}-c_{5i}}\right)^{\frac{1}{\alpha_1}} + \frac{3^{1-\frac{\sigma\alpha_1}{2}}}{\sqrt{\lambda_{\min}(P_i)}^{\sigma\alpha_1}}\left(\frac{c_{2i}+c_{3i}+c_{4i}}{c_{1i}-c_{5i}}\right) + \frac{3^{1-\frac{\sigma\alpha_2}{2}}}{\sqrt{\lambda_{\min}(P_i)}^{\sigma\alpha_2}}\left(\frac{c_{2i}+c_{3i}+c_{4i}}{c_{1i}-c_{5i}}\right)^{\frac{\alpha_2}{\alpha_1}} \right\}$$

in finite time $T_{1i} \le t_{1i}+t_{2i}$, $i = 1, \ldots n$. Here the convergence times t_{1i} and t_{2i} are given as

$$t_{1i} \le \ln(1 - (c_{6i}/c_{1i})V_{1i}(0)^{1-\gamma_1})/(c_{6i}\gamma_1 - c_{6i})$$
$$t_{2i} \le 2V_{1i}(t_{1i})^{(2/\sigma)}/(H_i(c_{1i} - c_{5i})\sigma)$$

where the control parameters are chosen as

$$c_{1i} = -\max_{\{x_i:V_\alpha(x_i)=1\}} L_{f_\alpha}V_\alpha(x_i),$$
$$c_{2i} = \frac{2\times 3^{(1+\sigma)/2}\lambda_{1i}\lambda_{\max}(P_i)}{\sigma\lambda_{\min}(P_i)^{1-2/\sigma}},$$
$$c_{3i} = \frac{6\sqrt{3}\lambda_{\max}(P_i)\xi_i}{\sigma\alpha_1\lambda_{\min}(P_i)},$$
$$c_{4i} = \frac{2\times 3^{(1+\sigma\alpha_1)/2}\lambda_{2i}\lambda_{\max}(P_i)}{\sigma\alpha_1\lambda_{\min}(P_i)^{1-\sigma\alpha_1/2}},$$
$$c_{5i} = \frac{2\times 3^{(1+\sigma\alpha_2)/2}\lambda_{\max}(P_i)(\bar{\Delta}_i+\lambda_{3i})}{\sigma\alpha_2\lambda_{\min}(P_i)^{1-\sigma\alpha_2/2}},$$
$$c_{6i} = c_{2i} + c_{3i} + c_{4i} + c_{5i}.$$

Proof: if we omit the terms $-\lambda_{1i}\mathrm{sgn}(z_{1i})$, $f(\eta_i, \mu_i) - f(\eta_i, \hat{\mu}_i) - \lambda_{2i}\mathrm{sgn}(z_{1i})$ and $\Delta_i - \lambda_{3i}\mathrm{sgn}(z_{1i})$ simultaneously, then the estimation error system (9) can be rewritten as

$$\begin{cases} \dot{z}_{1i} = z_{2i} - k_{1i}\mathrm{sig}^{\alpha_1}(z_{1i}) \\ \dot{z}_{2i} = z_{3i} - k_{2i}\mathrm{sig}^{\alpha_2}(z_{1i}) \\ \dot{z}_{3i} = -k_{3i}\mathrm{sig}^{\alpha_3}(z_{1i}) \end{cases} \tag{10}$$

We can get that the system (10) is homogeneous of degree $\alpha_1 - 1$ with respect to the weights $(1, \alpha_1, 2\alpha_1 - 1)$. From (10), it is noted that the matrix $A_i \in \mathbb{R}^{9\times 9}$ given by $A_i = \begin{bmatrix} -k_{1i}I_3 & I_3 & 0 \\ -k_{2i}I_3 & 0 & I_3 \\ -k_{3i}I_3 & 0 & 0 \end{bmatrix}$ is Hurwitz. Then, consider the following differentiable positive-definite function $V_\alpha(z_{1i}, z_{2i}, z_{3i}) = \tilde{Z}_i^T P_i \tilde{Z}_i$, $\tilde{Z}_i = \left[\tilde{Z}_{1i}^T, \tilde{Z}_{2i}^T, \tilde{Z}_{3i}^T\right]^T = \left[\left[\mathrm{sig}^{\frac{1}{\sigma}}(z_{1i})\right]^T, \left[\mathrm{sig}^{\frac{1}{\sigma\alpha_1}}(z_{2i})\right]^T, \left[\mathrm{sig}^{\frac{1}{\sigma\alpha_2}}(z_{3i})\right]^T\right]^T$, where $\sigma{=}\alpha_1\alpha_2\alpha_3$, P_i is the solution of the following Lyapunov equation $A_i^T P_i + P_i A_i = -I_9$. From [39], it can be deduced that the function $V_\alpha(z_{1i}, z_{2i}, z_{3i})$ is a Lyapunov function for the system (10). Let f_α be the vector field of system (10), and $L_{f_\alpha}V_\alpha$ be the Lie derivative of $V_\alpha(z_{1i}, z_{2i}, z_{3i})$ along the vector field f_α. It can be verified that $V_\alpha(z_{1i}, z_{2i}, z_{3i})$ and $L_{f_\alpha}V_\alpha(z_{1i}, z_{2i}, z_{3i})$ are homogeneous of degree $2/\sigma$ and $2/\sigma + \alpha_1 - 1$ with respect to the weights $(1, \alpha_1, 2\alpha_1 - 1)$. Then, from [40, Lemma 4.2], the following inequality $L_{f_\alpha}V_\alpha(z_{1i}, z_{2i}, z_{3i}) \le -c_{1i}V_\alpha(z_{1i}, z_{2i}, z_{3i})^{\gamma_1}$ holds, where $c_{1i} = -\max_{\{x_i:V_\alpha(x_i)=1\}} L_{f_\alpha}V_\alpha(x_i)$, $\gamma_1 = 1 + \frac{\alpha_1\sigma}{2} - \frac{\sigma}{2} < 1$.

Therefore, the following Lyapunov function is designed for the estimation error system of FTESO.

$$V_{1i}(z_{1i}, z_{2i}, z_{3i}) = \tilde{Z}_i^T P_i \tilde{Z}_i \tag{11}$$

According to the analysis of (10), then, taking the first time derivative of the Lyapunov function $V_{1i}(z_{1i}, z_{2i}, z_{3i})$ along (9) yields

$$\dot{V}_{1i} = L_{f_\alpha}V_\alpha(z_{1i}, z_{2i}, z_{3i}) + 2\tilde{Z}_i^T P_i \times \begin{bmatrix} \dfrac{-\mathrm{diag}(|z_{1i}|^{\frac{1}{\sigma}-1})\lambda_{1i}\mathrm{sgn}(z_{1i})}{\sigma} \\ \dfrac{\mathrm{diag}(|z_{2i}|^{\frac{1}{\sigma\alpha_1}-1})(f(\eta_i, \mu_i) - f(\eta_i, \hat{\mu}_i) + \lambda_{2i}\mathrm{sgn}(z_{1i}))}{\sigma\alpha_1} \\ \dfrac{\mathrm{diag}(|z_{3i}|^{\frac{1}{\sigma\alpha_2}-1})(\Delta_i - \lambda_{3i}\mathrm{sgn}(z_{1i}))}{\sigma\alpha_2} \end{bmatrix} \tag{12}$$

From Assumption 2, we can get that there exist positive constants ξ_i such that $\|f(\eta_i, \mu_i) - f(\eta_i, \hat{\mu}_i)\| \le \xi_i z_{2i}$.

Then, (12) becomes

$$\begin{aligned}\dot{V}_{1i} \leq & -c_{1i}V_{1i}^{\gamma_1} + 2\lambda_{1i}\lambda_{\max}(P_i)\left\|\tilde{Z}_i\right\| \\ & \times \frac{\sum_{m=1}^{3}\left|z_{1i,m}\right|^{\frac{1}{\sigma}-1}}{\sigma} + 2\lambda_{\max}(P_i)\left\|\tilde{Z}_i\right\|\xi_i \\ & \times \frac{(\sum_{m=1}^{3}\left|z_{2i,m}\right|)(\sum_{m=1}^{3}\left|z_{2i,m}\right|^{\frac{1}{\sigma\alpha_1}-1})}{\sigma\alpha_1} \\ & + \frac{2\lambda_{2i}\lambda_{\max}(P_i)\left\|\tilde{Z}_i\right\|(\sum_{m=1}^{3}\left|z_{2i,m}\right|^{\frac{1}{\sigma\alpha_1}-1})}{\sigma\alpha_1} \\ & + \frac{2\lambda_{\max}(P_i)(\bar{\Delta}_i+\lambda_{3i})\left\|\tilde{Z}_i\right\|(\sum_{m=1}^{3}\left|z_{3i,m}\right|^{\frac{1}{\sigma\alpha_2}-1})}{\sigma\alpha_2} \end{aligned} \tag{13}$$

In addition, by using Lemma 4 and the inequality $(a+b+c)^2 \leq 3(a^2+b^2+c^2)$, we obtain the following inequalities

$$\begin{aligned}(\sum_{m=1}^{3}\left|z_{1i,m}\right|^{\frac{1}{\sigma}-1}) & \leq 3^{\sigma}(\sum_{m=1}^{3}\left|z_{1i,m}\right|^{\frac{1}{\sigma}})^{1-\sigma} \\ & \leq 3^{(1+\sigma)/2}\left\|\tilde{Z}_i\right\|^{1-\sigma}\end{aligned} \tag{14}$$

$$\begin{aligned}(\sum_{m=1}^{3}\left|z_{2i,m}\right|^{(\frac{1}{\sigma\alpha_1}-1)})(\sum_{m=1}^{3}\left|z_{2i,m}\right|) & \\ & \leq 3\sum_{m=1}^{3}\left|z_{2i,m}\right|^{\frac{1}{\sigma\alpha_1}-1} \\ & \leq 3\sqrt{3}\left\|\tilde{Z}_i\right\|\end{aligned} \tag{15}$$

$$\begin{aligned}(\sum_{m=1}^{3}\left|z_{2i,m}\right|^{\frac{1}{\sigma\alpha_1}-1}) & \leq 3^{\sigma\alpha_1}(\sum_{m=1}^{3}\left|z_{2i,m}\right|^{\frac{1}{\sigma\alpha_1}})^{1-\sigma\alpha_1} \\ & \leq 3^{(1+\sigma\alpha_1)/2}\left\|\tilde{Z}_i\right\|^{1-\sigma\alpha_1}\end{aligned} \tag{16}$$

$$\begin{aligned}(\sum_{m=1}^{3}\left|z_{3i,m}\right|^{\frac{1}{\sigma\alpha_2}-1}) & \leq 3^{\sigma\alpha_2}(\sum_{m=1}^{3}\left|z_{3i,m}\right|^{\frac{1}{\sigma\alpha_2}})^{1-\sigma\alpha_2} \\ & \leq 3^{(1+\sigma\alpha_2)/2}\left\|\tilde{Z}_i\right\|^{1-\sigma\alpha_2}\end{aligned} \tag{17}$$

Substituting (14)-(17) into (13) yields

$$\begin{aligned}\dot{V}_{1i} \leq & -c_{1i}V_{1i}^{\gamma_1} + \frac{6\sqrt{3}\lambda_{\max}(P_i)\xi_i\left\|\tilde{Z}_i\right\|^2}{\sigma\alpha_1} \\ & + \frac{2\times 3^{(1+\sigma)/2}\lambda_{1i}\lambda_{\max}(P_i)\left\|\tilde{Z}_i\right\|^{2-\sigma}}{\sigma} \\ & + \frac{2\times 3^{(1+\sigma\alpha_1)/2}\lambda_{2i}\lambda_{\max}(P_i)\left\|\tilde{Z}_i\right\|^{2-\sigma\alpha_1}}{\sigma\alpha_1} \\ & + \frac{2\times 3^{(1+\sigma\alpha_2)/2}\lambda_{\max}(P_i)(\bar{\Delta}_i+\lambda_{3i})\left\|\tilde{Z}_i\right\|^{2-\sigma\alpha_2}}{\sigma\alpha_2} \\ \leq & -c_{1i}V_{1i}^{\gamma_1} + c_{2i}V_{1i}^{1-\frac{\sigma}{2}} + c_{3i}V_{1i} \\ & + c_{4i}V_{1i}^{1-\frac{\sigma\alpha_1}{2}} + c_{5i}V_{1i}^{1-\frac{\sigma\alpha_2}{2}}\end{aligned} \tag{18}$$

where

$$\begin{aligned}c_{2i} &= \frac{2\times 3^{(1+\sigma)/2}\lambda_{1i}\lambda_{\max}(P_i)}{\sigma\lambda_{\min}(P_i)^{1-2/\sigma}}, \\ c_{3i} &= \frac{6\sqrt{3}\lambda_{\max}(P_i)\xi_i}{\sigma\alpha_1\lambda_{\min}(P_i)}, \\ c_{4i} &= \frac{2\times 3^{(1+\sigma\alpha_1)/2}\lambda_{2i}\lambda_{\max}(P_i)}{\sigma\alpha_1\lambda_{\min}(P_i)^{1-\sigma\alpha_1/2}}, \\ c_{5i} &= 2\times 3^{(1+\sigma\alpha_2)/2}\lambda_{\max}(P_i)\times\frac{(\bar{\Delta}_i+\lambda_{3i})}{\sigma\alpha_2\lambda_{\min}(P_i)^{1-\sigma\alpha_2/2}}.\end{aligned}$$

Since $0 < 1-\frac{\sigma}{2} < 1-\frac{\sigma\alpha_1}{2} < 1-\frac{\sigma\alpha_2}{2} < 1$, we consider the following two cases for further analysis.

Case 1: If $V_{1i} \geq 1$, the following inequality can be obtained

$$\dot{V}_{1i} \leq -c_{1i}V_{1i}^{\gamma_1} + c_{6i}V_{1i} \tag{19}$$

where $c_{6i} = c_{2i} + c_{3i} + c_{4i} + c_{5i}$. Thus, according to Lemma 2, we can conservatively obtain the time t_{1i} that V_{1i}, $i = 1, 2, \ldots n$ converge to $V_{1i} = 1$.

$$t_{1i} \leq \ln(1-(c_{6i}/c_{1i})V_{1i}(0)^{1-\gamma_1})/(c_{6i}\gamma_1 - c_{6i}), \quad i=1,2,\ldots n$$

Case 2: If $V_{1i} < 1$, the inequality (18) can be simplified as

$$\begin{aligned}\dot{V}_{1i} & \leq -(c_{1i}-c_{5i})V_{1i}^{\gamma_1} + (c_{2i}+c_{3i}+c_{4i})V_{1i}^{1-\frac{\sigma}{2}} \\ & = -(c_{1i}-c_{5i})H_iV_{1i}^{1-\frac{\sigma}{2}}\end{aligned} \tag{20}$$

where $H_i = V_{1i}^{\gamma_1-1+\frac{\sigma}{2}} - \frac{(c_{2i}+c_{3i}+c_{4i})}{(c_{1i}-c_{5i})}$. Choose the appropriate parameters k_m ($m = 1, 2, 3$) satisfying that $c_{1i} > c_{5i}$. If the inequality

$$V_{1i} < (\frac{(c_{2i}+c_{3i}+c_{4i})}{(c_{1i}-c_{5i})})^{\frac{2}{2\gamma_1-2+\sigma}} \tag{21}$$

holds, then according to Lemma 1, we get

$$t_{2i} \leq 2V_{1i}(t_1)^{(2/\sigma)}/(H_i(c_{1i}-c_{5i})\sigma), \quad i=1,2,\ldots n$$

Thus, we can obtain that the Lyapunov function V_{1i} converge to the domain in (21) within the time $T_{1i} \leq t_{1i} + t_{2i} < \infty$. Since $2\gamma_1 - 2 + \sigma = \sigma\alpha_1$, then, form (21), the estimation errors $\tilde{Z}_i$ can be derived as follows:

$$\left\|\tilde{Z}_i\right\| < \frac{1}{\sqrt{\lambda_{\min}(P_i)}}\left(\frac{c_{2i}+c_{3i}+c_{4i}}{c_{1i}-c_{5i}}\right)^{\frac{1}{\sigma\alpha_1}} \tag{22}$$

Noticing that

$$\begin{aligned}\|(z_{1i},z_{2i},z_{3i})\| \leq & \sum_{m=1}^{3}(\left|z_{1i,m}\right|^{\frac{1}{\sigma}})^{\sigma} \\ & + \sum_{m=1}^{3}(\left|z_{2i,m}\right|^{\frac{1}{\sigma\alpha_1}})^{\sigma\alpha_1} \\ & + \sum_{m=1}^{3}(\left|z_{3i,m}\right|^{\frac{1}{\sigma\alpha_2}})^{\sigma\alpha_2} \\ \leq & 3^{1-\sigma}\left(\sum_{m=1}^{3}\left|z_{1i,m}\right|^{\frac{1}{\sigma}}\right)^{\sigma} \\ & + 3^{1-\sigma\alpha_1}\left(\sum_{m=1}^{3}\left|z_{2i,m}\right|^{\frac{1}{\sigma\alpha_1}}\right)^{\sigma\alpha_1} \\ & + 3^{1-\sigma\alpha_2}\left(\sum_{m=1}^{3}\left|z_{3i,m}\right|^{\frac{1}{\sigma\alpha_2}}\right)^{\sigma\alpha_2} \\ \leq & 3^{1-\frac{\sigma}{2}}\left\|\tilde{Z}_{1i}\right\|^{\sigma} + 3^{1-\frac{\sigma\alpha_1}{2}}\left\|\tilde{Z}_{2i}\right\|^{\sigma\alpha_1} \\ & + 3^{1-\frac{\sigma\alpha_2}{2}}\left\|\tilde{Z}_{3i}\right\|^{\sigma\alpha_2}\end{aligned}$$

Therefore, the residual set of the estimation errors can be expressed as

$$\Omega_2=\left\{\begin{array}{l}(z_{1i},z_{2i},z_{3i})|\ \|(z_{1i},z_{2i},z_{3i})\|\le \dfrac{3^{1-\frac{\sigma}{2}}}{\sqrt{\lambda_{\min}(P_i)}^{\sigma}}\\ \left(\dfrac{c_{2i}+c_{3i}+c_{4i}}{c_{1i}-c_{5i}}\right)^{\frac{1}{\alpha_1}}+\dfrac{3^{1-\frac{\sigma\alpha_1}{2}}}{\sqrt{\lambda_{\min}(P_i)}^{\sigma\alpha_1}}\left(\dfrac{c_{2i}+c_{3i}+c_{4i}}{c_{1i}-c_{5i}}\right)^{\frac{\alpha_2}{\alpha_1}}\\ +\dfrac{3^{1-\frac{\sigma\alpha_2}{2}}}{\sqrt{\lambda_{\min}(P_i)}^{\sigma\alpha_2}}\left(\dfrac{c_{2i}+c_{3i}+c_{4i}}{c_{1i}-c_{5i}}\right)^{\frac{\alpha_2}{\alpha_1}}\end{array}\right\}$$

Hence, the proof of Theorem 1 is completed.

B. GAIN DETERMINATION

The above analysis shows that the estimation errors z_{1i}, z_{2i}, z_{3i} can converge to a bounded residual set around origin in finite time. In the next, we will analyze the case that the estimation errors converge to zero in finite time with appropriate parameters $\lambda_{mi}(m = 1, 2, 3)$.

Define the Lyapunov function $V_{01i}(z_{1i}) = \frac{1}{2}{z_{1i}}^T z_{1i}$. By taking the first time derivative of V_{01i} along (9), we have

$$\begin{aligned}\dot V_{01i} &= {z_{1i}}^T\dot z_{1i} = {z_{1i}}^T(z_{2i}-k_{1i}\mathrm{sig}^{\alpha_1}(z_{1i})-\lambda_{1i}\mathrm{sgn}(z_{1i}))\\ &\le \|z_{1i}\|\,\|z_{2i}\| - k_{1i}\sum_{m=1}^{3}\left|z_{1i,m}\right|^{\alpha_1+1}-\lambda_{1i}\sum_{m=1}^{3}\left|z_{1i,m}\right|\\ &\le -(\lambda_{1i}-\|z_{2i}\|)\,\|z_{1i}\|-\frac{k_{1i}}{3^{\alpha_1}}\|z_{1i}\|^{\alpha_1+1} \qquad (23)\end{aligned}$$

where Lemma 5 is used.

Since $\|z_{2i}\| \le 3^{1-\frac{\sigma\alpha_1}{2}}\frac{1}{\sqrt{\lambda_{\min}(P_i)}^{\sigma\alpha_1}}\left(\frac{c_{2i}+c_{3i}+c_{4i}}{c_{1i}-c_{5i}}\right)$, choose $\lambda_{1i} > \|z_{2i}\|$. By recalling c_{2i}, then we get $\lambda_{1i} > \frac{\sigma\lambda_{\min}(P_i)^{1-\frac{2}{\sigma}}3^{\frac{2-\sigma\alpha_1}{2}}(c_{3i}+c_{4i})}{\sigma\lambda_{\min}(P_i)^{1-\frac{2}{\sigma}+\frac{\sigma\alpha_1}{2}}(c_{1i}-c_{5i})-2\times 3^{\frac{(3+\sigma-\sigma\alpha_1)}{2}}\lambda_{\max}(P_i)}$. It can be shown that the parameters λ_{1i} are independent of system variables. Then, (23) becomes $\dot V_{01i} \le -c_{01i}V_{01i}^{\frac{\alpha_1+1}{2}}$, where $c_{01i} = \frac{2^{\frac{\alpha_1+1}{2}}k_{1i}}{3^{\alpha_1}}$. Therefore, z_{1i} converge to zero within finite time $t_{3i} < \frac{2V_{01i}^{\frac{1-\alpha_1}{2}}(z_{1i}(T_{1i}))}{c_{01i}(1-\alpha_1)}$.

After the time $T_{2i} = T_{1i} + t_{3i}$, the second formula of (9) becomes

$$\dot z_{2i} = z_{3i} + f(\eta_i,\mu_i) - f(\eta_i,\hat\mu_i) - \lambda_{2i}\mathrm{sgn}(z_{2i}) \qquad (24)$$

Similarity, consider the following Lyapunov function for z_{2i}, $V_{02i}(z_{2i}) = \frac{1}{2}{z_{2i}}^T z_{2i}$. Whose first time derivative along (24) is given by

$$\begin{aligned}\dot V_{02i} &= {z_{2i}}^T\dot z_{2i}\\ &= {z_{2i}}^T(z_{3i}+f(\eta_i,\mu_i)-f(\eta_i,\hat\mu_i)-\lambda_{2i}\mathrm{sgn}(z_{2i}))\\ &\le -(\lambda_{2i}-\|z_{3i}\|-\xi_i)\,\|z_{2i}\| \qquad (25)\end{aligned}$$

Let the inequality $\lambda_{2i} > \|z_{3i}\| + \xi_i + \varepsilon_{1i}$ holds, then we obtain that z_{2i} converge to zero within finite time $t_{4i} < \frac{\sqrt{2}}{\varepsilon_{1i}}V_{02i}(z_{2i})^{\frac{1}{2}}_{T_{2i}}$. After $T_{3i} = T_{2i} + t_{4i}$, consider the Lyapunov function for z_{3i}, $V_{03i}(z_{3i}) = \frac{1}{2}{z_{3i}}^T z_{3i}$.

$$\begin{aligned}\dot V_{03i} &= {z_{3i}}^T\dot z_{3i} = {z_{3i}}^T(\Delta_i-\lambda_{3i}\mathrm{sgn}(z_{3i}))\\ &\le -(\lambda_{3i}-\|\Delta_i\|)\,\|z_{3i}\| \qquad (26)\end{aligned}$$

By choosing $\lambda_{3i} > \bar\Delta_i + \varepsilon_{2i}$. then we can get that z_{3i} converge to zero within finite time $t_{5i} < \frac{\sqrt{2}}{\varepsilon_{2i}}V_{03i}(z_{3i})^{\frac{1}{2}}_{T_{3i}}$. Therefore, we can conclude that the estimated errors z_{1i}, z_{2i}, and z_{3i} converge to zero in finite time $T_{4i} = T_{3i} + t_{5i}$.

Remark 2: If the external disturbance acting on the MSVs is constant, then, the rate of external disturbance is $\Delta_i = 0$. The estimation error system of FTESO becomes

$$\begin{cases}\dot z_{1i} = z_{2i} - k_{1i}\mathrm{sig}^{\alpha_1}(z_{1i}) - \lambda_{1i}\mathrm{sgn}(z_{1i})\\ \dot z_{2i} = z_{3i} + f(\eta_i,\mu_i) - f(\eta_i,\hat\mu_i) - k_{2i}\mathrm{sig}^{\alpha_2}(z_{1i})\\ \quad - \lambda_{2i}\mathrm{sgn}(z_{1i})\\ \dot z_{3i} = -k_{3i}\mathrm{sig}^{\alpha_3}(z_{1i}) - \lambda_{3i}\mathrm{sgn}(z_{1i}).\end{cases}$$

It is obvious that the finite-time stability of the FTESO can be obtained by following the analysis of Theorem 1. As a matter of fact, the ocean environment is changeable, so it is more reasonable to consider the time-varying disturbances for the MSVs formation.

C. FINITE-TIME FORMATION CONTROLLER

Based on the outputs of FTESO, the finite-time formation control law is designed for MSVs to achieve the desired formation with precise control performance. Thus, to track the time-varying virtual leader, the position tracking error e_{1i} and velocity tracking error e_{2i} for each vehicle are given as follows:

$$e_{1i} = \eta_i + \delta_i - \eta_d \qquad (27)$$
$$e_{2i} = \dot\eta_i + \dot\delta_i - \dot\eta_d \qquad (28)$$

and to keep the consistency between adjacent MSVs, the position synchronization error $\eta_i + \delta_i - \eta_j - \delta_j$ and velocity synchronization error $\dot\eta_i + \dot\delta_i - \dot\eta_j - \dot\delta_j$ are considered. Therefore, the formation errors for each vehicle can be expressed as

$$E_{1i} = \sum\nolimits_{j=1}^{n} a_{ij}(e_{1i}-e_{1j}) + b_i(e_{1i}) \qquad (29)$$
$$E_{2i} = \sum\nolimits_{j=1}^{n} a_{ij}(e_{2i}-e_{2j}) + b_i(e_{2i}) \qquad (30)$$

Because of the unknown velocity information, the errors E_{2i} $(i = 1, 2, \ldots, n)$ cannot be used directly in the design of formation control law. Therefore, an auxiliary variable $\hat e_{2i} = \hat\mu_i + \dot\delta_i - \dot\eta_d$ is introduced based on the outputs of FTESO. Thus, the formation errors for MSVs become

$$E_{1i} = \sum\nolimits_{j=1}^{n} a_{ij}(e_{1i}-e_{1j}) + b_i(e_{1i}) \qquad (31)$$
$$\hat E_{2i} = \sum\nolimits_{j=1}^{n} a_{ij}(\hat e_{2i}-\hat e_{2j}) + b_i(\hat e_{2i}) \qquad (32)$$

Therefore, to achieve the desired formation in finite time, the distributed finite-time formation control law τ_i for the ith vehicle is chosen as follows:

$$\begin{aligned}\tau_i = M_iR^T(\psi_i)[&-f(\eta_i,\hat\mu_i) - k_{2i}\mathrm{sig}^{\alpha_2}(z_{1i}) - \lambda_{2i}\mathrm{sgn}(z_{1i})\\ &-\hat\omega_i - \ddot\delta_i + \ddot\eta_d - \rho_1\mathrm{sig}^{\alpha_2}(E_{1i}) - \rho_2\mathrm{sig}^{2-(1/\alpha_1)}(\hat E_{2i})]\end{aligned} \qquad (33)$$

where ρ_1 and ρ_2 are positive constants.

By using the graph theory, the error system for MSVs can be expressed as $\dot E_1 = \Theta e_2$, $\dot{\hat E}_2 = \Theta\dot{\hat e}_2$, whose time derivative along (8) and (33) are given

$$\dot{E}_1 = \Theta e_2 = \hat{E}_2 + \Theta z_2 \quad (34)$$
$$\dot{\hat{E}}_2 = \Theta\dot{\hat{e}}_2 = \Theta[-\rho_1 \mathrm{sig}^{\alpha_2}(E_1) - \rho_2 \mathrm{sig}^{2-(1/\alpha_1)}(\hat{E}_2)] \quad (35)$$

where $E_1 = [E_{11}^T, \ldots, E_{1i}^T, E_{1n}^T]^T$, $\hat{E}_2 = [\hat{E}_{21}^T, \ldots, \hat{E}_{2i}^T, \hat{E}_{2n}^T]^T$, $\Theta = (L+B) \otimes I_3$, $e_1 = [e_{11}^T, \ldots, e_{1i}^T, e_{1n}^T]^T$, $\hat{e}_2 = [\hat{e}_{21}^T, \ldots, \hat{e}_{2i}^T, \hat{e}_{2n}^T]^T$, $z_2 = [z_{21}^T, \ldots, z_{2i}^T, z_{2n}^T]^T$.

Let $N = \begin{bmatrix} 0 & I_{3n} \\ -\rho_1\Theta & -\rho_2\Theta \end{bmatrix}$. Since the matrix N is Hurwitz, there exists $Q = Q^T > 0$ such that $N^T Q + QN = -I_{6n}$. Then, consider the following Lyapunov function:

$$V_2 = \tilde{E}^T Q \tilde{E} \quad (36)$$

where $\tilde{E} = [E_1{}^T, \mathrm{sig}^{1/\alpha_1}(\hat{E}_2^T)]$.

If we omit the term Θz_2, then (34) and (35) can be rewritten as

$$\dot{E}_1 = \Theta e_2 = \hat{E}_2 \quad (37)$$
$$\dot{\hat{E}}_2 = \Theta\dot{\hat{e}}_2 = \Theta[-\rho_1 \mathrm{sig}^{\alpha_2}(E_1) - \rho_2 \mathrm{sig}^{2-(1/\alpha_1)}(\hat{E}_2)] \quad (38)$$

Similarly, the error system described in (37) and (38) is homogeneous of degree $\alpha_1 - 1$ with respect to the weights $(1, \alpha_1)$. Then, $V_\beta(\tilde{E})$ and $L_{f_\beta} V_\beta(\tilde{E})$ are homogeneous of degree 2 and $\alpha_1 + 1$. By using [40, Lemma 4.2] again, the following inequality is obtained. $L_{f_\beta} V_\beta(\tilde{E}) \leq -c_7 V_\beta(\tilde{E})^{\gamma_2}$, where $c_7 = -\max_{\{x:V_\beta(x)=1\}} L_{f_\beta} V_\beta(x)$, $\gamma_2 = \frac{\alpha_1+1}{2} < 1$.

In combination with the above analysis of (37) and (38), then, we take the first time derivative of V_2 along (34) and (35)

$$\begin{aligned}\dot{V}_2 &\leq -c_7 V_2(\tilde{E})^{\gamma_2} + 2\tilde{E}^T Q \Theta z_2 \\ &\leq -c_7 V_2(\tilde{E})^{\gamma_2} + 2\lambda_{\max}(Q)\lambda_{\max}(\Theta)\left\|\tilde{E}\right\|\|z_2\| \\ &\leq -c_7 V_2(\tilde{E})^{\gamma_2} + \zeta_1\left(\sum_{j=1}^{2}\sum_{i=1}^{n}\sum_{m=1}^{3}\left|\tilde{E}_{ji,m}\right|\right) \\ &\quad\times\left(\sum_{i=1}^{n}\sum_{m=1}^{3}\left|\tilde{Z}_{2i,m}\right|^{\sigma\alpha_1}\right) \end{aligned} \quad (39)$$

where $\zeta_1 = 2\lambda_{\max}(Q)\lambda_{\max}(\Theta)$.

According to Lemma 3 and Lemma 4, we have

$$\begin{aligned}\dot{V}_2 &\leq -c_7 V_2(\tilde{E})^{\gamma_2} + \frac{6n\sigma\alpha_1\theta}{1+\sigma\alpha_1}\left(\sum_{i=1}^{n}\sum_{m=1}^{3}\left|\tilde{z}_{2i,m}\right|^{1+\sigma\alpha_1}\right) \\ &\quad + \frac{1}{(6n)^{\frac{1-\sigma\alpha_1}{2}}}\left(\sum_{j=1}^{2}\sum_{i=1}^{n}\sum_{m=1}^{3}\left|\tilde{E}_{ji,m}\right|^{1+\sigma\alpha_1}\right) \\ &\leq -c_7 V_2(\tilde{E})^{\gamma_2} + \frac{6n\sigma\alpha_1\theta}{1+\sigma\alpha_1}\left(\sum_{i=1}^{n}\sum_{m=1}^{3}\left(\left|\tilde{z}_{2i,m}\right|^2\right)^{\frac{1+\sigma\alpha_1}{2}}\right) \\ &\quad + \frac{1}{(6n)^{\frac{1-\sigma\alpha_1}{2}}}\left(\sum_{j=1}^{2}\sum_{i=1}^{n}\sum_{m=1}^{3}\left(\left|\tilde{E}_{ji,m}\right|^2\right)^{\frac{1+\sigma\alpha_1}{2}}\right) \\ &\leq -c_7 V_2(\tilde{E})^{\gamma_2} + \frac{1}{\lambda_{\min}^{\frac{1+\sigma\alpha_1}{2}}(Q)} V_2(\tilde{E})^{\frac{1+\sigma\alpha_1}{2}} \\ &\quad + \frac{6n\sigma\alpha_1\theta \times 3^{\frac{1-\sigma\alpha_1}{2}}}{(1+\sigma\alpha_1)\lambda_{\min}^{\frac{1+\sigma\alpha_1}{2}}(Q)}\sum_{i=1}^{n} V_1(\tilde{Z}_i)^{\frac{1+\sigma\alpha_1}{2}} \\ &\leq -(c_7 - c_8)V_2(\tilde{E})^{\frac{1+\sigma\alpha_1}{2}} + c_9\sum_{i=1}^{n} V_1(\tilde{Z}_i)^{\frac{1+\sigma\alpha_1}{2}} \end{aligned} \quad (40)$$

where $c_8 = \frac{1}{\lambda_{\min}^{\frac{1+\sigma\alpha_1}{2}}(Q)}$, $c_9 = \frac{6n\sigma\alpha_1\zeta_2 \times 3^{\frac{1-\sigma\alpha_1}{2}}}{(1+\sigma\alpha_1)\lambda_{\min}^{\frac{1+\sigma\alpha_1}{2}}(Q)}$, $\zeta_2 = \zeta_1\left[(6n)^{\frac{1-\sigma\alpha_1}{2}} 3n\zeta_1/1+\sigma\alpha_1\right]^{1/\sigma\alpha_1}$.

Theorem 2: Consider the MSVs described in (4) and (5) with unavailable velocity measurements and external disturbances. Suppose that Assumptions 1-2 are satisfied and the topology graph among the MSVs is undirected and connected. The FTESO is designed as (8), and the distributed formation control laws are given by (33). If initial conditions of the system satisfy the compact set $\Omega_3 = \left\{(\tilde{Z}_i, E_1, \hat{E}_2) | \sum_{i=1}^{n} V_{1i}(\tilde{Z}_i) + V_2(E_1, \hat{E}_2) < V_M\right\}$, where $V_M > 0$ denotes any given constant, $V_{1i}(\tilde{Z}_i)$ and $V_2(E_1, \hat{E}_2)$ are defined by (11) and (36), then there exist appropriate parameters k_{mi}, λ_{mi} $(m = 1, 2, 3)$ such that the finite-time stability of the overall closed-loop system can be obtained.

Proof: The following Lyapunov function is constructed for the closed-loop system.

$$V_3 = \sum_{i=1}^{n} V_{1i}(\tilde{Z}_i) + V_2(E_1, \hat{E}_2) \quad (41)$$

whose time derivative along with (20) and (40) is given by

$$\dot{V}_3 \leq -\sum_{i=1}^{n}(c_{1i} - c_{5i})H_i V_{1i}{}^{1-\frac{\sigma}{2}} - (c_7 - c_8)V_2(\tilde{E})^{\frac{1+\sigma\alpha_1}{2}} + \varpi \quad (42)$$

where $\varpi = c_9 \sum_{i=1}^{n} V_{1i}{}^{\frac{1+\sigma\alpha_1}{2}}$. It can be easily verified that ϖ is bounded in the compact set Ω_3, i.e., $\|\varpi\| \leq \varpi_M$, where ϖ_M is a positive constant. Then, (42) becomes

$$\begin{aligned}\dot{V}_3 \leq &-\sum_{i=1}^{n}(c_{1i} - c_{5i})H_i V_{1i}{}^{1-\frac{\sigma}{2}} \\ &- (c_7 - c_8)V_2(\tilde{E})^{\frac{1+\sigma\alpha_1}{2}} + \varpi_M \end{aligned} \quad (43)$$

Therefore, if the following inequalities $\sum_{i=1}^{n}(c_{1i} - c_{5i}) H_i V_{1i}{}^{1-\frac{\sigma}{2}} > \varpi_M$, $(c_7 - c_8)V_2(\tilde{E})^{\frac{1+\sigma\alpha_1}{2}} > \varpi_M$ holds, then it follows that $\dot{V}_3 < 0$, which in turn implies that $\tilde{Z}_i, E_1, \hat{E}_2$ are uniformly ultimately bounded. Hence, we obtain that $\|\eta_i\| \leq \varepsilon$, $\|\mu_i\| \leq \varepsilon$, $\|\dot{\eta}_i\| \leq \varepsilon$, $\|\dot{\mu}_i\| \leq \varepsilon$. From Theorem 1, there exists T_{4i}, $i = 1, 2, ..n$ such that the estimation errors $\tilde{Z}_i$ of the proposed FTESO are converge to zero. Then, we get

$e_2 = \hat{e}_2$, (34) and (35) become

$$\dot{E}_1 = \Theta e_2 = E_2 \quad (44)$$
$$\dot{E}_2 = \Theta \dot{e}_2 = \Theta[-\rho_1 \mathrm{sig}^{\alpha_2}(E_1) - \rho_2 \mathrm{sig}^{2-(1/\alpha_1)}(E_2)] \quad (45)$$

It can be verified that there exists the time $T_5 > \max\{T_{4i}\}$ such that E_1 and E_2 converge to zero. According to (29) and (30), the objective that the control errors e_{1i}, $e_{1i} - e_{1j}$, e_{2i}, and $e_{2i} - e_{2j}$ converge to zero in finite time can be achieved. Thereby, the proof is completed here.

Remark 3: In [41], the finite-time extended state observer-based distributed formation control problem for marine surface vehicles has been solved. However, the control errors of the closed-loop system can only be proved bounded in finite time. To further improve the accuracy of control errors in the steady state, a formation control scheme that converges the control errors to zero in finite time is proposed for MSVs in this paper.

IV. SIMULATION RESULTS

To demonstrate the effectiveness of the proposed robust finite-time output feedback control scheme for MSVs formation, the following simulation studies are considered. A scenario where there are five vehicles and one virtual leader is considered. Accordingly, the communication topology graph between the leader and vehicles is shown in Fig.2, where the virtual leader is labeled as 0. In addition, the ship CyberShip II is used in the simulation, and the main model parameters can be found in [42].

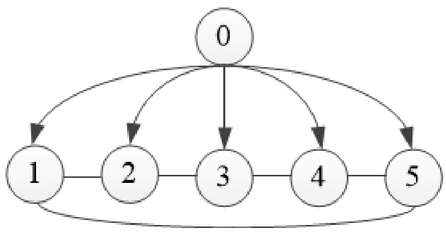

FIGURE 2. The communication topology graph.

The desired trajectory for the MSVs formation is given as $\begin{cases} y(t) = 0.2t \ \ (m) \\ x(t) = 20\sin(0.01t) \ \ (m). \end{cases}$ The complex environment disturbances including winds, waves, and ocean currents can be simulated by $\omega_i = \begin{bmatrix} 0.1v_i^3 + 0.06u_i + 0.01\sin(t) \\ u_i r_i + 0.1u_i + 0.01\sin(t) \\ 0.4u_i r_i + v_i^2 + 0.01\sin(t) \end{bmatrix}^T$. The initial positions and orientations of the MSVs are given as: $\eta_1=[1, -4, \pi/3]^T$, $\eta_2=[0, -2, \pi/3]^T$, $\eta_3=[0, 1, \pi/3]^T$, $\eta_4=[-2, 0, \pi/3]^T$, and $\eta_5=[-4, 0, \pi/3]^T$, respectively. The initial velocities are $u_i(0) = v_i(0) = 0$(m/s), $r_i(0) = 0$(rad/s). The configuration vectors for MSVs are given as $l_1=[3, 4, 0]^T$, $l_2=[3/2, 2, 0]^T$, $l_3=[0, 0, 0]^T$, $l_4=[3/2, -2, 0]^T$, and $l_5=[3, -4, 0]^T$, respectively. The control parameters α_m, β_m, k_{mi}, and λ_{mi} ($m = 1, 2, 3$, $i = 1, 2, 3, 4, 5$) can be tuned to adjust the final tracking accuracy and the convergence rate of the observation errors and the tracking errors. However, in practice, the control forces and torques of MSVs are bounded. Therefore, the control parameters are selected as $\alpha_1 = 0.8$, $\alpha_2 = 0.6$, $\alpha_3 = 0.4$, $\beta_1 = 1.25$, $\beta_2 = 1.05$, $\beta_3 = 0.85$, $k_{1i} = 5$, $k_{2i} = 1$, $k_{3i} = 0.1$, $\lambda_{1i} = 0.05$, $\lambda_{2i} = 0.01$, $\lambda_{3i} = 0.05$, $\rho_1 = 1$, $\rho_2 = 1$.

The formation trajectories of the five vehicles are shown in Fig.3, it can be observed that each vehicle, from different initial positions and orientations, can maintain the desired formation while tracking the virtual leader. The estimation errors of FTESO are plotted in Fig.4. From which we can see that the estimated errors of the position converge to $|z_{1i}| < 1 \times 10^{-3}$ within 6 sec, the estimated errors of the velocity measurements drop to $|z_{2i}| < 1 \times 10^{-3}$ at approximately 24 sec, and the estimated errors of the external disturbance drop to $|z_{3i}| < 1 \times 10^{-3}$ at approximately 25 sec. Therefore, it can be concluded that the estimated values of FTESO, i.e., velocity measurements and time-varying disturbance, converge to the real values with fast transient response and high steady-state accuracy. The position errors and the velocity errors of the finite-time formation controller are plotted in Fig.5 and Fig.6. It is clear that the followers can not only track the leader's position and velocity with fast and

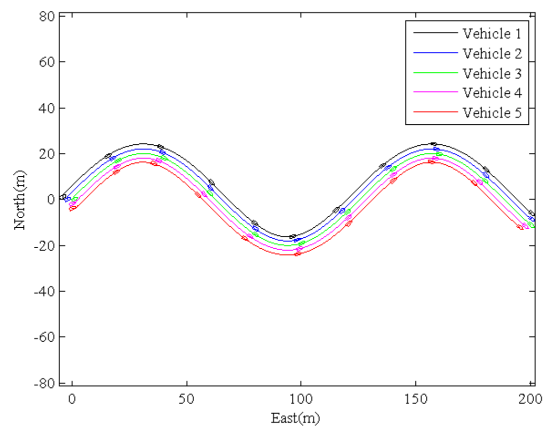

FIGURE 3. The formation trajectories of the five vehicles.

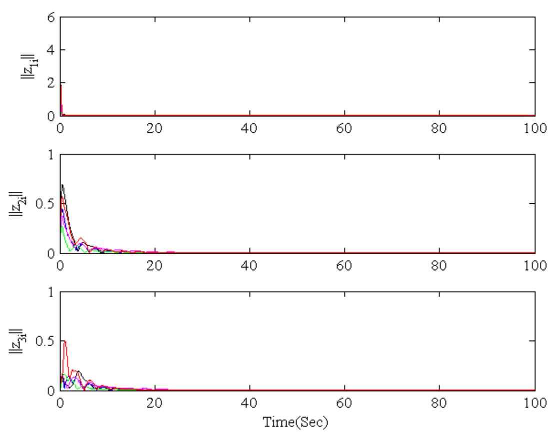

FIGURE 4. The estimation errors of FTESO (8).

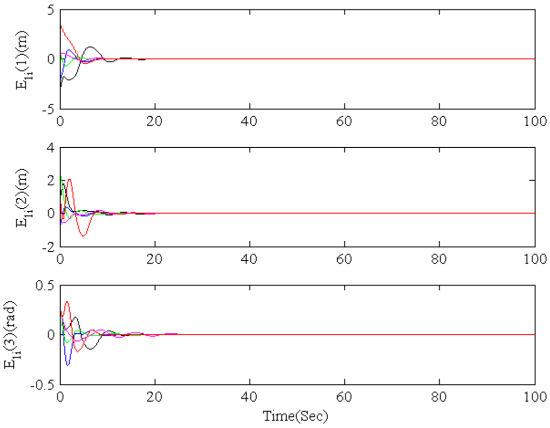

FIGURE 5. The position errors E_{1i} of the finite-time formation controller (33).

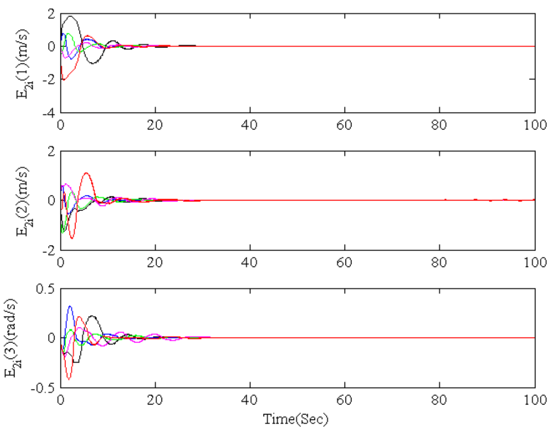

FIGURE 6. The velocity errors E_{2i} of the finite-time formation controller (33).

precise tracking performance, but also keep the consistency of each other. Fig.7 shows the control inputs in surge, sway and yaw directions. Because of the signum function, there is a slight chattering in the control forces. These results hence

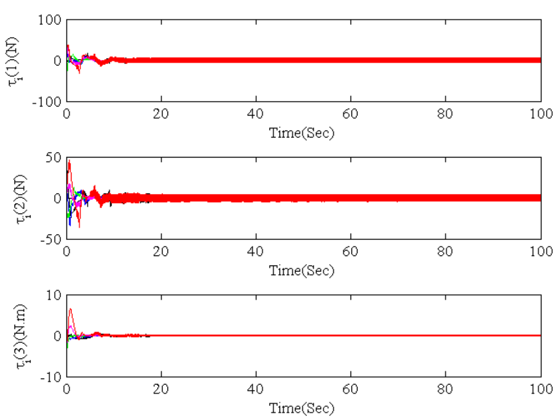

FIGURE 7. The control inputs in surge, sway and yaw directions of the finite-time formation controller (33).

verify the conclusions in Theorem 1 and 2 that the proposed FTESO and the formation control law (33) are able to achieve finite-time control with the errors converging to zero in finite time.

To further show the superior control performance of the proposed robust finite-time output feedback formation control scheme, comparison is constructed with the following linear extend state observer (LESO) [43] and asymptotic formation controller.

$$\begin{cases} \dot{\hat{\eta}}_i = \hat{\mu}_i + k_1 z_{1i} \\ \dot{\hat{\mu}}_i = R_i M_i^{-1} \tau_i + \hat{\omega}_i + f(\eta_i, \hat{\mu}_i) + k_2 z_{1i} \\ \dot{\hat{\omega}}_i = k_3 z_{1i} \end{cases} \tag{46}$$

$$\tau_i = M_i R^T(\psi_i)[-\hat{\omega}_i - f(\eta_i, \hat{\mu}_i) - k_2 z_{1i} - \ddot{\delta}_i + \ddot{\eta}_d - \rho_1 E_{1i} - \rho_2 \hat{E}_{2i}] \tag{47}$$

where the design parameters of LESO and asymptotic formation controller are selected as $k_1 = 10$, $k_2 = 3$, $k_3 = 1$, $\rho_1 = 1$, $\rho_2 = 1$ in the simulation.

Fig.8 shows the estimation errors of LESO. It is seen that a longer time is required to accomplish the estimation compare with the FTESO. These verify that the FTESO (8) guarantees

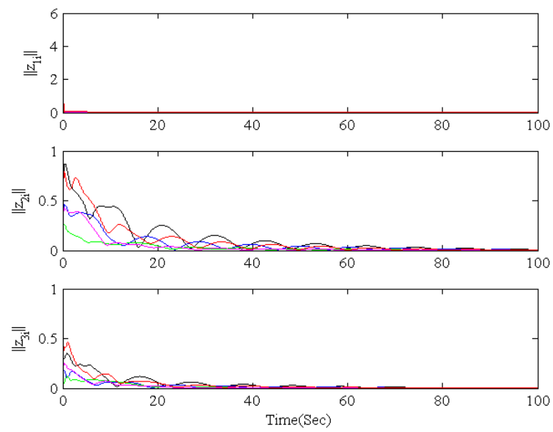

FIGURE 8. The estimation errors of LESO (46).

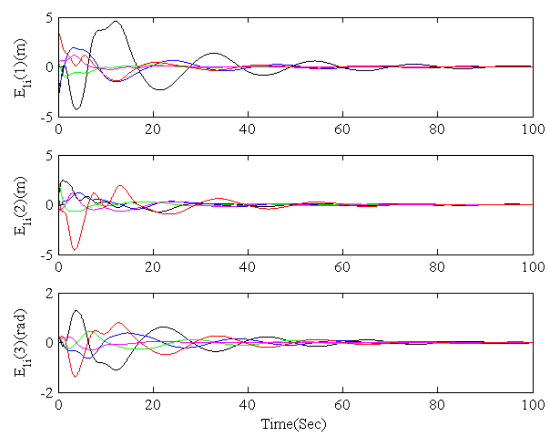

FIGURE 9. The position errors E_{1i} of the asymptotic formation controller (47).

a superior estimation performance than the LESO (46). The position errors and the velocity errors of the asymptotic formation controller are plotted in Fig.9 and Fig.10. Compare with Figs.5-6, the transient response time is longer, and the tracking precision of the errors is poor. Fig.11 plots the control inputs of the asymptotic formation controller in surge, sway and yaw directions. From Figs.3-11, we concluded that the proposed finite-time output feedback formation control scheme (8)(33) for MSVs with unavailable velocity measurements and external disturbances can guarantee the errors of the closed-loop system converge to zero in finite time, and can provide faster convergence speed and better disturbance rejection ability with higher accuracy than the asymptotic one (46)(47).

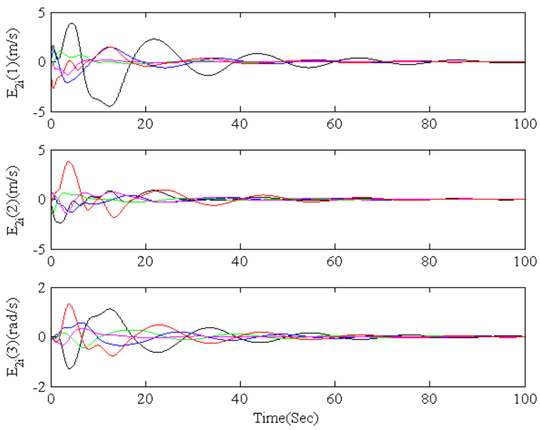

FIGURE 10. The velocity errors E_{2i} of the asymptotic formation controller (47).

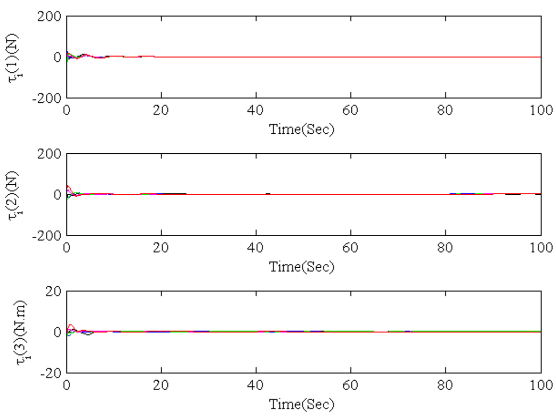

FIGURE 11. The control inputs in surge, sway and yaw directions of the asymptotic formation controller (47).

V. CONCLUSIONS

In this paper, a novel finite-time output feedback formation control scheme for MSVs with external disturbances was proposed. The velocity measurements of the MSVs and the external disturbance acting on the MSVs were simultaneously estimated by the proposed FTESO. The finite-time formation control laws were designed for the MSVs to precisely track the relative configuration with respect to the leader based on the outputs of FTESO. Both the estimation errors and the tracking errors were proofed to converge to zero in finite time by using homogeneous method and Lyapunov theory. Further, to show the superiority of the proposed finite-time output feedback formation control scheme, comparison simulations were carried out with some existing designs appearing in the literature. It is shown that the proposed finite-time output feedback formation control scheme can provide faster convergence speed and better disturbance rejection ability with higher accuracy than the asymptotic ones. Future researches are devoted to reducing chattering and considering the actuator saturation and uncertain parameters in this work. Moreover, it is also a meaningful investigation to extend the proposed formation control scheme to under-actuated MSVs.

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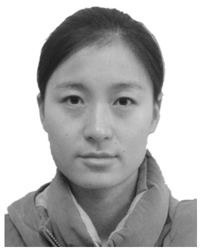

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